Reduced Transition Probabilities

The reduced transition probability $B(E_M^λ)$ is related to the nuclear matrix element by the formula

$$B(E_M^λ, I_i \rightarrow I_f) = \sum_{\mu M_f} |< I_f M_f | M(E_M^λ, \mu) | I_i M_i >|^2$$ \hspace{1cm} (1)

According to the Wigner-Eckart theorem a matrix element of an operator $M(\lambda, \mu)$ can be factorized

$$< I_f M_f | M(E_M^λ, \mu) | I_i M_i > = (I_i \lambda M_i \mu | I_f M_f) < I_f || M(E_M^λ) || I_i >$$ \hspace{1cm} (2)

where $(I_i \lambda M_i \mu | I_f M_f)$ is a Clebsch-Gordan coefficient. Then, with use of the orthonormality of the Clebsch-Gordan coefficients, we have another expression for $B(E_M^λ)$:

$$B(E_M^λ, I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |< I_f || M(E_M^λ) || I_i >|^2$$ \hspace{1cm} (3)

This expression assures us that the lifetime of a state does not depend on its orientation (rotational invariance). The reduced matrix elements $< I_f || M(E_M^λ) || I_i >$ contain the information about the the nuclear wave functions. The relation of the reduced transition probability between the excitation $B(E_M^λ)^\uparrow$ and the decay $B(E_M^λ)^\downarrow$ of the nuclear state is given by

$$B(E_M^λ, I_f \rightarrow I_i) = \frac{2I_i + 1}{2I_f + 1} B(E_M^λ, I_i \rightarrow I_f)$$ \hspace{1cm} (4)

since the absolute value of the reduced matrix element is invariant under the interchange of $I_i$ and $I_f$.

In case of an E2 transition between ground state $0^+_{gs}$ and the first excited state $2^+_1$, we obtain

$$B(E2, 0^+_{gs} \rightarrow 2^+_1) = 5 * B(E2, 2^+_1 \rightarrow 0^+_{gs})$$ \hspace{1cm} (5)