Elastic Scattering

1 Classical Rutherford scattering at relativistic energies

Accelerators completely devoted to the study of nuclear collisions at relativistic energies can also be an important tool for investigating nuclear structure in distant collisions without nuclear contact.

In the following we shall calculate the Rutherford scattering of a relativistic projectile with velocity v_∞, impact parameter b, and mass and charge number A_1 and Z_1 on a target nucleus with mass and charge number A_2 and Z_2, respectively. The basic assumption of an elastic scattering is that the charge distribution of the two nuclei do not overlap at any time during the collision. Such collisions take place at extreme forward scattering angles.

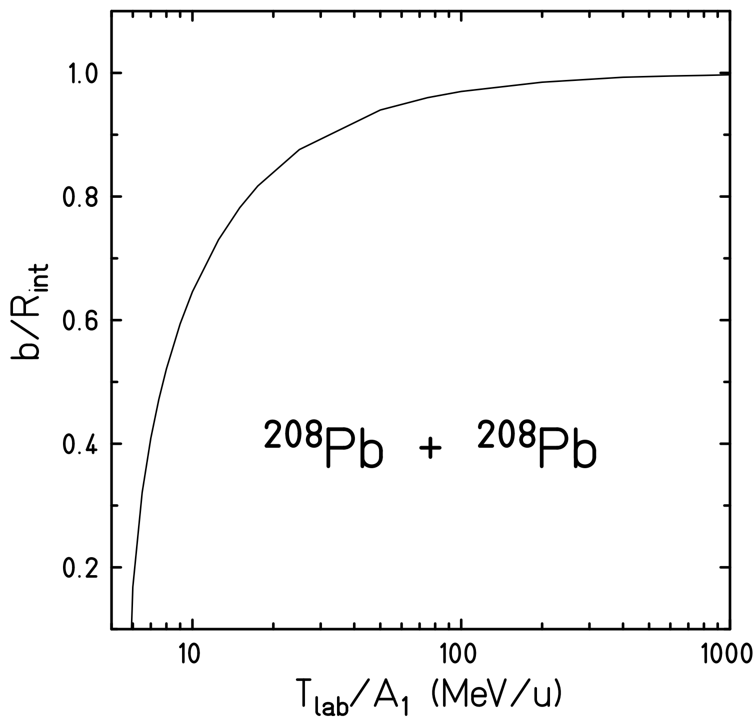

Figure 1: Ratio of the impact parameter b and the nuclear interaction radius R_{int} versus the bombarding energy T_{lab}/A_1 for the system $^{208}Pb +^{208} Pb$.

In the relativistic domain **straight-line trajectories** may be employed in calculations of electromagnetic excitation processes of nuclei. For large bombarding energies ($E_{cm} \gg Z_1 Z_2 e^2/R_{int}$) the impact parameter b becomes identical with the distance of closest approach D, i.e.

$$b = \sqrt{D^2 - \frac{Z_1 Z_2 e^2}{E_{cm}} D} \quad \rightarrow \quad D \tag{1}$$

Fig. 1 illustrates the ratio b/R_{int} (Eq. 1) for a distance of closest approach D given by the nuclear interaction radius R_{int} as a function of the laboratory bombarding energy T/A_1. For the system ${}^{208}Pb + {}^{208}Pb$ electromagnetic excitation at bombarding energies greater than 100 MeV/u are characterized by straight-line trajectories with impact parameters b larger than the sum of the radii of the two colliding ions. For the present case we have calculate a nuclear interaction radius of $R_{int} = 16.0\ fm$.

Since during such collisions the target nucleus only achieves a small recoil velocity, we shall consider the target nucleus as fixed and place the origin of our coordinate system in its centre-of-mass (similar to the non-relativistic case with $A_2 \gg A_1$). In this way we analyse the effects of the electromagnetic field generated by the projectile on the target. In the non-relativistic limit, the half distance of closest approach in a head-on collision is given by

$$a = \frac{Z_1 Z_2 e^2}{p \cdot v_\infty} \tag{2}$$

With the relativistic expression for the momentum

$$p = m_0 \gamma \beta c \tag{3}$$

where m_0 is the rest mass of the projectile, we obtain

$$a = \frac{Z_1 Z_2 e^2}{m_0 c^2 \gamma \beta^2} = \frac{1.44 \cdot Z_1 Z_2 [931.5 + (T_{lab}/A_1)]}{A_1[(T_{lab}/A_1)^2 + 1863 \cdot (T_{lab}/A_1)]} \qquad [fm] \tag{4}$$

The beam velocity v_∞ in units of the velocity of light is given by

$$\beta = \frac{v_\infty}{c} = \frac{\sqrt{T_{lab}^2 + 1863 \cdot A_1 T_{lab}}}{931.5 \cdot A_1 + T_{lab}} \tag{5}$$

and the Lorentz contraction factor is defined by

$$\gamma = (1 - \beta^2)^{-1/2} = \frac{931.5 \cdot A_1 + T_{lab}}{931.5 \cdot A_1} \tag{6}$$

thus the product $\gamma\beta^2$ can be written as

$$\gamma\beta^2 = \frac{(T_{lab}/A_1)^2 + 1863 \cdot (T_{lab}/A_1)}{931.5 \cdot [931.5 + (T_{lab}/A_1)]} \tag{7}$$

where T_{lab} is the laboratory beam energy in MeV. Fig. 2 shows the Lorentz contraction factor γ and the beam velocity β in units of the velocity of light as a function of the bombarding energy T_{lab}/A_1.

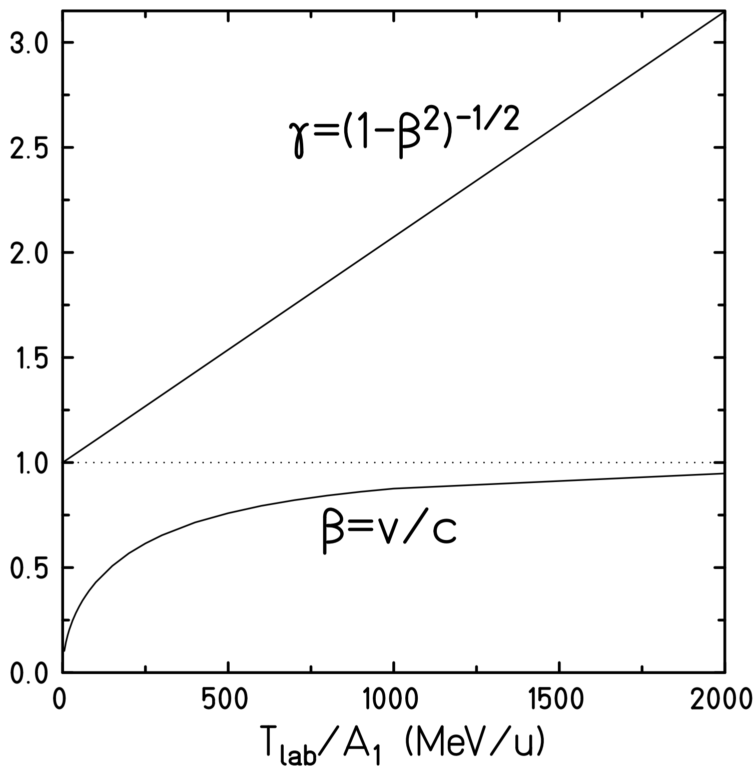

Figure 2: Lorentz contraction factor γ and beam velocity β in units of the velocity of light as a function of the bombarding energy T_{lab}/A_1.

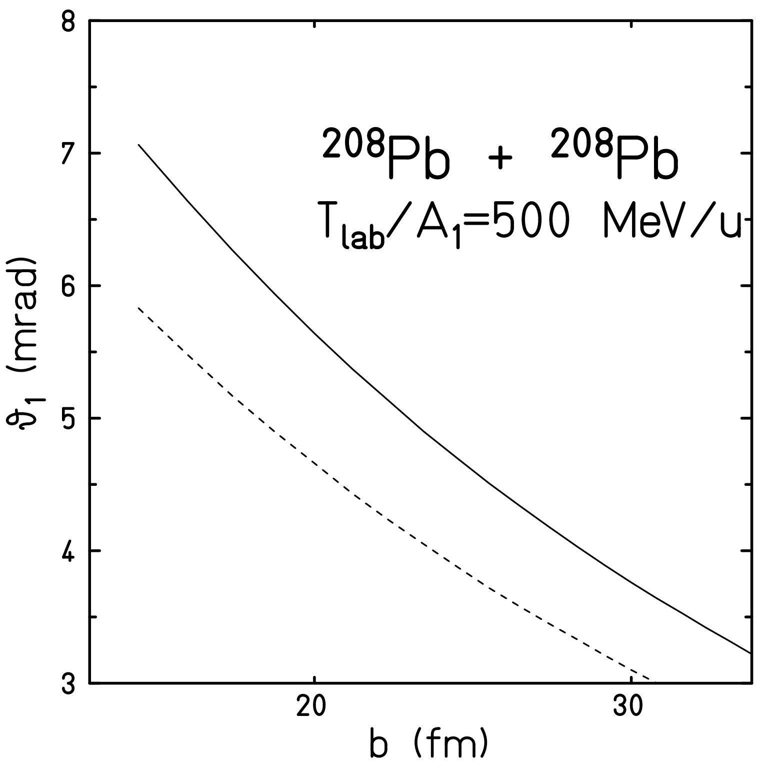

Figure 3: Relativistic (solid line) and non-relativistic (dashed line) deflection function $\vartheta_1(b)$ for the system $^{208}Pb +^{208}Pb$ at a bombarding energy of 500 MeV/u.

In the non-relativistic limit we find for the deflection function $\theta_{cm}(b)$

$$2 \cdot \tan \frac{\theta_{cm}}{2} = \frac{2 \cdot Z_1 Z_2 e^2}{p v_\infty b} \tag{8}$$

Since the target nucleus is almost at rest, the scattering angle θ_{cm} in the centre-of-mass is equal to the one ϑ_1 in the laboratory system. For small angles we find $\tan\vartheta_1 \simeq \vartheta_1$ and obtain for the deflection function

$$\vartheta_1 = \frac{2 \cdot Z_1 Z_2 e^2}{p v_\infty b} \tag{9}$$

With the relativistic expression for the momentum (Eq. 3) we obtain

$$\vartheta_1 = \frac{2 \cdot Z_1 Z_2 e^2}{m_0 c^2 \gamma \beta^2 b} = \frac{2.88 \cdot Z_1 Z_2 [931.5 + (T_{lab}/A_1)]}{A_1[(T_{lab}/A_1)^2 + 1863 \cdot (T_{lab}/A_1)]} \frac{1}{b} \qquad [rad] \tag{10}$$

where T_{lab} is the laboratory beam energy in MeV and the impact parameter b(fm). For the system ^{208}Pb+^{208}Pb at a bombarding energy of 500 MeV/u (a=0.0564 fm) the relativistic deflection function $\vartheta_1(b)$ is compared with the non-relativistic result in Fig. 3.

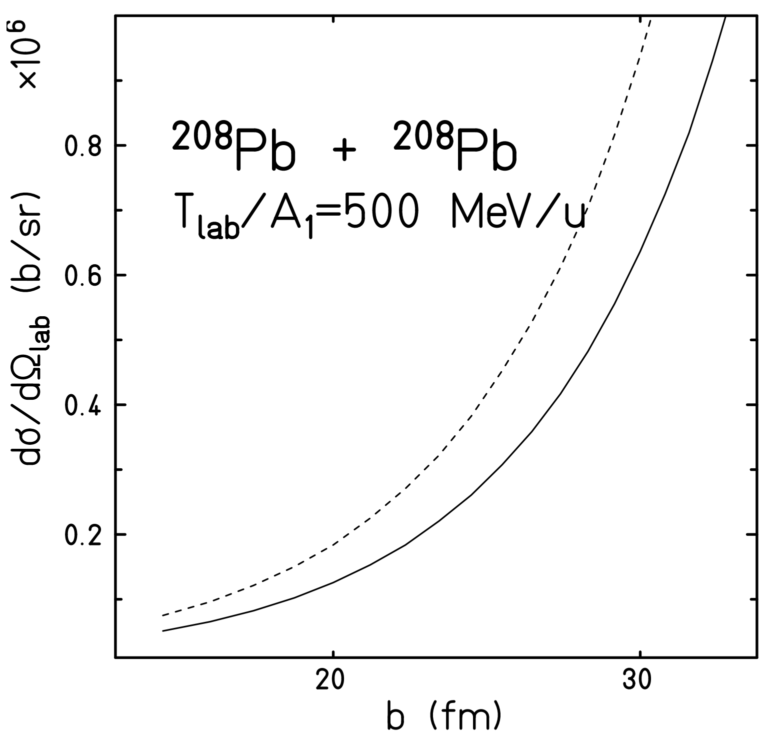

Figure 4: Relativistic (solid line) and non-relativistic (dashed line) cross section as a function of the impact parameter for the system $^{208}Pb +^{208} Pb$ at a bombarding energy of 500 MeV/u.

The Rutherford cross section in relativistic nuclear collisions is given by

$$\frac{d\sigma}{d\Omega_{lab}} = \frac{a^2}{4} \sin^{-4} \frac{\vartheta_1}{2} \tag{11}$$

where the quantity a is defined by eq. 4. For the system $^{208}Pb + ^{208}Pb$ at a bombarding energy of 500 MeV/u (a=0.0564 fm) the relativistic differential cross section is compared with the non-relativistic result in Fig. 4.

For an impact parameter of b=15.01 fm one finds a scattering angle of $\vartheta = 0.431^0 (0.426^0)$ and a differential cross section of $d\sigma/d\Omega_{lab} = 3.99 \cdot 10^4 (4.09 \cdot 10^4)$ $[b/sr]$. The values in brackets are the exact results [Mat87].

References

[Mat87] R. Matzdorf, G. Soff and G. Mehler: Z.Phys.**D6** (1987), 5