Experimental Errors in Science

Complete certainty is never the mark of a scientific fact, although it is the business of scientific endeavor to reduce the uncertainty as much as possible. The fundamental truth or fact in an experimental science is always an observation, a measurement. Prediction, or a generalized description of the behavior of nature, is an important goal of the science, but the degree of reliability of the prediction is no better than the measurements upon which it is based. Careful analysis of the reliability of measurements therefore is necessarily an early step in achieving scientific maturity.

The observed errors in a number of trial measurements probably include both random and systematic errors. To justify the application of statistical formulas, as we often attempt to do, we assume randomness in the observed variability, an assumption whose validity may be jeopardized by the presence of systematic errors.

Mean (arithmetic average) The experimental mean **m** is defined for **n** trial measurements, $y_i \pm \sigma_i$ with i=1,...,n, by the relation

$$m = \frac{\sum_{i=1}^{n} (y_i/\sigma_i^2)}{\sum_{i=1}^{n} (1/\sigma_i^2)} \tag{1}$$

Experimental standard deviation The experimental standard deviation σ_A is defined as

$$\sigma_A = [\frac{\sum_{i=1}^{n} (y_i - m)^2/\sigma_i^2}{(n-1)\sum_{i=1}^{n} (1/\sigma_i^2)}]^{1/2} \tag{2}$$

The basic problem in experimental science is to deduce from a **limited** number of trial measurements as much information as one can. Eq. 2 for the error σ_A of the weighted mean can yield unphysical values for very small samples. Fig. 1 shows such an example on the left hand side, where the two data points are almost identical, but have large error bars. The resulting error bar σ_A for the weighted mean is too small. Therefore, σ_B is introduced which sets a limit for σ based on the individual errors.

$$\sigma_B = [\sum_{i=1}^{n} (1/\sigma_i^2)]^{-1/2} \tag{3}$$

However, Eq. 3 may also fail for two data points being very different and having small error bars, as seen on the right hand side of Fig. 1. The standard deviation σ of the weighted average **m** may be calculated for a limited number of measurements using the following equation:

$$\sigma = max(\sigma_A, \sigma_B) \tag{4}$$

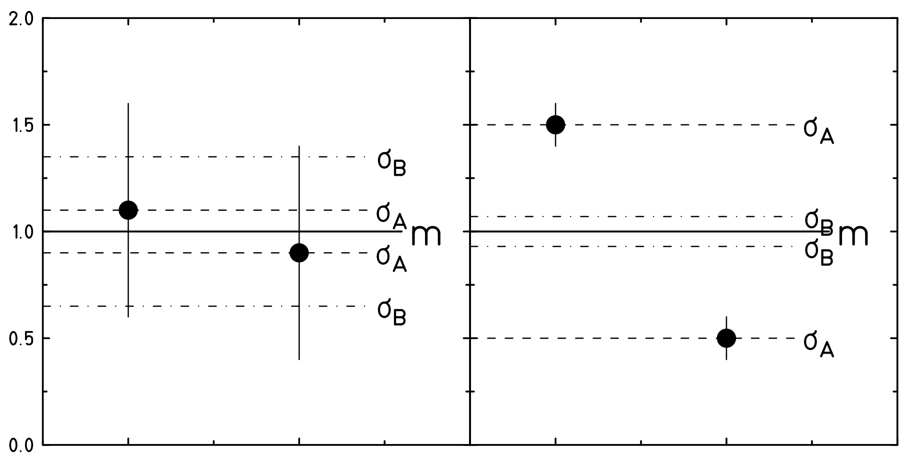

Figure 1: Weighted average **m** and standard deviation σ_A(Eq. 2), σ_B(Eq. 3) calculated for two different sets experimental data points.