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STRUCTURE AND ELECTROMAGNETIC PROPERTIES OF THE DOUBLY EVEN Te ISOTOPES

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Abstract: The doubly even Te isotopes ($122 \leqq A \leqq 134$) have been studied in the two-proton core coupling model. The obtained energy levels, wave functions and electromagnetic properties as well as spectroscopic factors for a (^{3}He, d) stripping reaction are compared with experimental data. A comparison between the results of the afore-mentioned model and the variable moment of inertia model is made.

1. Introduction

Recently, great interest has been shown in the determination of the properties of the energy levels of the doubly even Te ($Z = 52$) isotopes. The spectra of these nuclei have been investigated by β- and γ-spectroscopic measurements [1]) and by neutron capture and inelastic scattering experiments [2,3]). The rapid development of in-beam spectroscopy following (α, xn) reactions made it possible to investigate the levels of even Te nuclei with mass numbers between $A = 114$ and $A = 126$. This technique was used by Luukko *et al.* [4]), Warner and Draper [5]) and Kerek [6]). All the reported positive-parity levels belong to the so-called quasi-ground band†. Kerek publishes also results about low-lying negative-parity states. Measurements of electric quadrupole moments of the first excited 2^+ states have been performed using the Coulomb excitation methods [8-10]). Several reduced E2 transition probabilities are known [9-14]). An isomeric 6^+ state has been observed with half-life ranging from 0.42 ns in ^{128}Te up to 162 ns in ^{134}Te [ref. [1])]. Reactions of the type (p, t) and (^{3}He, d) permitted the assignment of unique I^π values for several states, and tentative values for many others in ^{122,124}Te [ref. [15])]. Analysis of angular distributions yielded strengths for the various two-proton components of the excited state wave functions.

In this paper, we shall investigate the level schemes of the doubly even Te isotopes in a two-proton core coupling model. All but two (the two protons outside the magic

† A definition of the term "quasi-ground band" has been given by Sakai [7]).

$Z = 50$ core) of the nucleons are lumped together to form a core that is described in terms of collective coordinates. A coupling is introduced between the quadrupole vibrations of the core and the shell-model states of the extra nucleons. To reduce the size of the configuration space we shall only take into account one-, two- and three-phonon states of the core, together with the $3s_{\frac{1}{2}}$, $2d_{\frac{3}{2}}$, $2d_{\frac{5}{2}}$, $1g_{\frac{7}{2}}$ and $1h_{\frac{11}{2}}$ single-particle states. Besides the coupling to the core, a residual interaction [approximated by a surface delta interaction (SDI)] between the two extra protons has been considered.

Until now, no detailed calculation for all doubly even Te nuclei has been performed. Only Lopac [16]) has shown the validity of the proposed model by means of a semi-microscopic description of the ^{124}Te nucleus. The residual interaction between the two protons was approximated by a pairing force, which affected only the states with two particles coupled to zero angular momentum. As discussed in ref. [1]) Lopac fails to predict the observed close energy spacing of the 6^+ and 4^+ levels for $A > 126$. Since more experimental information (e.g. the half-lives of the isomeric 6^+ states, the results of the one- and two-particle transfer reaction) is now available, it is straightforward to perform calculations in that energy region in order to improve the agreement with the experimental data.

The (α, xn) reactions reveal the existence of quasi-ground bands in the doubly even Te isotopes. The two-proton core coupling model describes the structure of the various members of these bands. Since the variable moment of inertia (VMI) model [17]) deals specifically with such bands, a comparison will be given between the results of both models.

In sect. 2 a short review of the formalism is given. Theoretical spectra and spectroscopic factors for a (^{3}He, d) reaction are given in sect. 3, together with a discussion of the results. The calculated wave functions are used in sect. 4 to derive electromagnetic properties. Electric quadrupole moments, reduced E2 transition probabilities and branching ratios are compared with experimental data. An application of the VMI model on a series of Te isotopes is made in sect. 5 and compared to the results of the two-proton coupling model.

2. Formalism

As most of the expressions have been given elsewhere [18, 19]) no detailed derivations will be made. The total Hamiltonian H consists of three parts:

(i) A collective part H_{coll}, which describes the core oscillations. Only quadrupole oscillations are taken into consideration.

(ii) A single-particle contribution $H_{\text{s.p.}}$ which represents the single-particle energies of the two extra protons and their residual interaction, approximated by a SDI.

(iii) A coupling of the two-proton system with the quadrupole vibrations of the core, which is given by

$$H_{\text{int}} = -(\tfrac{1}{5}\pi)^{\frac{1}{2}}\xi\hbar\omega \sum_{i=1}^{2} \sum_{\mu} (b_{2\mu}+(-1)^{\mu}b_{2-\mu}^{+})Y_{2\mu}(\hat{r}_i), \tag{2.1}$$

where the dimensionless parameter,

$$\xi \equiv k\{5/2\pi\hbar\omega C\}^{\frac{1}{2}}, \tag{2.2}$$

has been introduced for the interaction strength. For the notation we refer to ref. [19]). The parameter ξ is related to the coupling constant a, used in many calculations of this type, by the expression

$$a = \xi\hbar\omega/2\sqrt{5}. \tag{2.3}$$

The wave functions for the doubly even nuclei will be expanded in the basis

$$|(j_1\, j_2)J;\, NR;\, IM\rangle \equiv \sum_{M_J,\, M_R} \langle JM_J RM_R|IM\rangle|(j_1\, j_2)JM_J\rangle|NRM_R\rangle, \tag{2.4}$$

where the N-phonon state of the core with total angular momentum R and projection M_R along the z-axis is coupled with the two-proton cluster $|(j_1 j_2)\,JM_J\rangle$ to give a total angular momentum I and projection M along the z-axis. The eigenfunctions at an energy $E^{(\alpha)}$ will be expanded in the basis (2.4) as follows

$$|E^{(\alpha)};\, IM\rangle = \sum_{\substack{(j_1 j_2)J \\ NR}} C_\alpha((j_1\, j_2)J;\, NR;\, I)|(j_1\, j_2)J;\, NR;\, IM\rangle. \tag{2.5}$$

Once the wave functions obtained, one can calculate spectroscopic factors for the stripping $^{A-1}$Sb(^{3}He, d)^A Te reaction. The spectroscopic factor for the stripping of a proton with orbital momentum l is given by

$$S_l^{(\alpha)} = \sum_{j=l\pm\frac{1}{2}} |\langle E^{(\alpha)};\, IM|[\phi_{lj} \otimes \psi^{A-1}(\mathrm{Sb})]^{IM}\rangle|^2, \tag{2.6}$$

where ψ^{A-1} (Sb) and ϕ_{lj} represent the wave function of the $^{A-1}$Sb ground state and the transferred proton respectively. The wave function of the ground state of the Sb target nucleus has been described within an analogous model and is formally given by

$$\psi^{A-1}(\mathrm{Sb}) = \sum_{j'N'R'} \beta^{[j',\, N'R';\, I_{\mathrm{Sb}}]}|j',\, N'R';\, I_{\mathrm{Sb}} M_{\mathrm{Sb}}\rangle. \tag{2.7}$$

with I_{Sb} the total angular momentum of the Sb ground state and M_{Sb} its projection along the z-axis. Introducing eq. (2.7) into (2.6), the spectroscopic factor is given by

$$\begin{aligned} S_l^{(\alpha)} = \sum_{j=l\pm\frac{1}{2}} \Big| \sum_{j'N'R'} \sum_{(j_1 j_2)JNR} & \beta^{[j',\, N'R';\, I_{\mathrm{Sb}}]} C_\alpha((j_1\, j_2)J;\, NR;\, I) \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} \\ & \times \begin{Bmatrix} j' & R & I_{\mathrm{Sb}} \\ I & j & J \end{Bmatrix} \sqrt{(2I_{\mathrm{Sb}}+1)(2J+1)}(-1)^{j'+j+R+I} \\ & \times \delta_{NN'}\delta_{RR'}(\delta_{j_1 j}\delta_{j_2 j'} - (-1)^{j'+j-J}\delta_{j_1 j'}\delta_{j_2 j})\Big|^2. \end{aligned} \tag{2.8}$$

3. Spectra and spectroscopic factors

Calculation of spectra was performed for all Te isotopes with $122 \leqq A \leqq 134$. Only positive-parity states have been considered, since it has been pointed out [6,20]) that the most likely configurations of the observed negative-parity levels are of the two-neutron quasi-particle type $(h_{\frac{11}{2}}, d_{\frac{3}{2}})_J$ and $(h_{\frac{11}{2}}, s_{\frac{1}{2}})_J$, and therefore not covered by the proposed model.

In the present calculation the parameters are chosen so as to obtain the best overall agreement with the experimental spectra of the Te nuclei. The available single-particle shell-model states in the region $50 \leqq Z \leqq 82$ are $3s_{\frac{1}{2}}$, $2d_{\frac{3}{2}}$, $2d_{\frac{5}{2}}$, $1g_{\frac{7}{2}}$ and $1h_{\frac{11}{2}}$. To reduce the size of the configuration space we have taken into account up to three-phonon states of the core and we have neglected several highest-lying two-particle states, since their contribution, estimated by a perturbation calculation seems to be small. The best-fit values for the parameters ξ, $\hbar\omega$, G (coupling strength of the SDI) and $\Delta_{j\pi}$ ($\equiv E_{j\pi} - E_{\frac{7}{2}^+}$), following from the iteration procedure are given in table 1. Values for the phonon energies $\hbar\omega$ of the core vibrations were chosen close to the experimental energies of the first excited 2^+ state in the neighbouring Sn isotopes. The final best values of the pairing strength G were in between the lower and upper limits proposed by Kisslinger and Sorensen [21]). The decrease of ξ with the mass number points to an increase of the stiffness C of the core. A high stiffness (= good shell closure) is also confirmed by the high energy of the first excited state in ^{132}Sn [ref. [22])]. It is evident that the fitted energy differences Δ_j, as obtained in a particle-core coupling calculation for the neighbouring odd-mass Sb isotopes, serve as an important constraint on the analogous parameters in the doubly even Te isotopes. By comparing the results of table 1 with the best-fit values for these parameters as given in ref. [23]), it is clear that the corresponding values are equal within a few percent. It has to be remarked that the level spacing between the $1g_{\frac{7}{2}}$ and $2d_{\frac{5}{2}}$ single-particle states is increasing from isotope to isotope, to reach a value of 963 keV for $A = 134$, an energy difference, which experimentally has been observed between these two single-particle states in the valence nucleus ^{133}Sb [ref. [24])]. The increase of this level spacing with increasing mass number has been explained due to the short-range neutron-proton interaction [23,25]).

Experimental and theoretical level schemes for all considered Te isotopes are compared in figs. 1–5. Although all $I^\pi = 2^+$ levels are reproduced in the considered energy region, it appears that the second 2^+ state lies always higher in energy than expected.

TABLE 1

Best-fit values for the parameters ξ, $\hbar\omega$, Δ_j

A	ξ	$\hbar\omega$ (MeV)	G (MeV)	$\Delta_{\frac{1}{2}}$ (MeV)	$\Delta_{\frac{3}{2}}$ (MeV)	$\Delta_{\frac{5}{2}}$ (MeV)	$\Delta_{\frac{7}{2}}$ (MeV)	$\Delta_{\frac{11}{2}}$ (MeV)
122	2.23	1.171	0.20	1.40	1.29	0.20	0	1.72
124	2.50	1.140	0.21	1.80	1.53	0.30	0	1.98
126	2.25	1.130	0.22	2.20	2.00	0.50	0	2.00
128	2.50	1.200	0.22	2.30	2.19	0.60	0	2.19
130	1.50	1.200	0.22	2.10	2.00	0.70	0	2.00
132	1.00	1.200	0.21	2.10	2.00	0.80	0	2.00
134	0		0.21	1.80	2.00	0.96	0	1.90

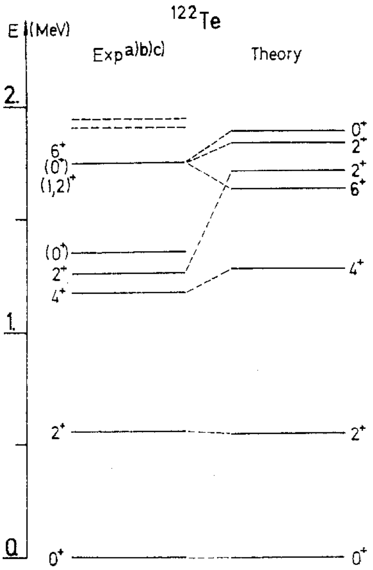

Fig. 1. Experimental and theoretical level schemes of ^{122}Te up to 2 MeV. [a]) Ref. [6]); [b]) ref. [15]); [c]) ref. [26]).

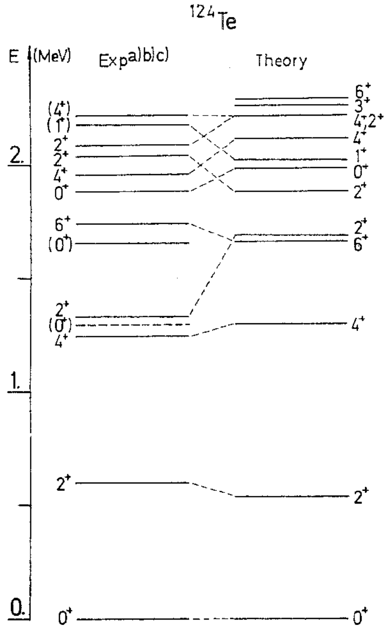

Fig. 2. Experimental and theoretical level schemes of ^{124}Te up to 2.3 MeV. [a]) Ref. [6]); [b]) ref. [15]); [c]) ref. [26]).

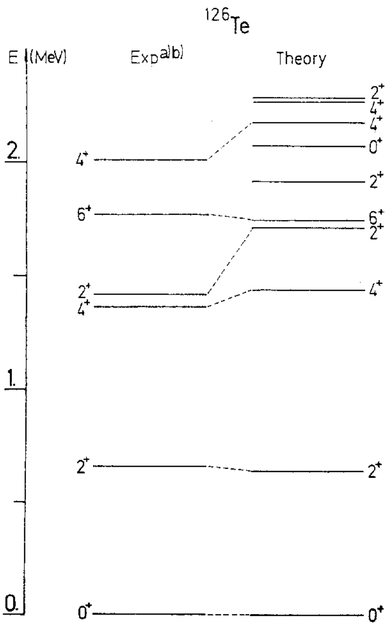

Fig. 3. Experimental and theoretical level schemes of ^{126}Te up to 2.3 MeV. [a]) Ref. [6]); [b]) ref. [2]).

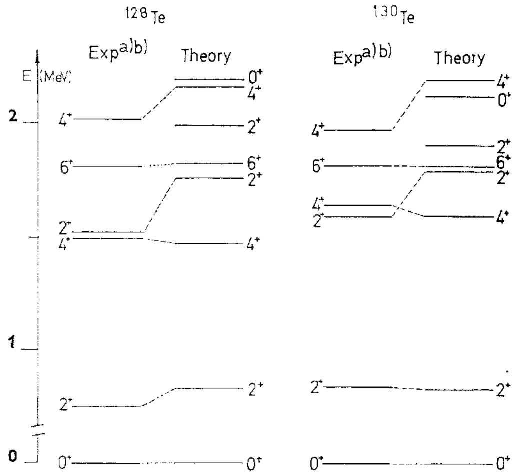

Fig. 4. Experimental and theoretical level schemes of ^{128}Te and ^{130}Te up to 2.2 MeV: [a]) Ref. [1]); [b]) ref. [2]).

On the other hand the position of the various members ($0^+ \leqq I^\pi \leqq 6^+$) of the quasi-ground band are well reproduced and the calculated 6^+–4^+ energy spacing is as experimentally observed in the whole mass region ($122 \leqq A \leqq 134$). Results for higher-spin levels ($I^\pi = 8^+, 10^+$) were not convincing enough to accept a one-to-one correspondence between experiment and theory. This can be explained as a consequence of the truncation of the configuration space and by the neglect of two-quasineutron excitations, which can be expected in that energy region ($E \approx 3$ MeV) [6]).

The components of the wave functions of the levels of the quasi-ground band in ^{126}Te which contribute more than 4 % are listed in table 2. It appears that the ground state as well as the 6^+ level have mainly a two-particle configuration, while on the con-

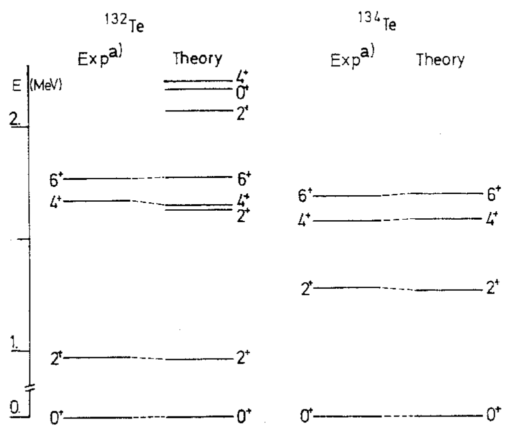

Fig. 5. Experimental and theoretical level schemes of ^{132}Te and ^{134}Te up to 2.2 MeV. [a]) Ref. [1]).

TABLE 2

Calculated wave functions of the members of the quasi-ground band in ^{126}Te

$$
\begin{aligned}
|0_1{}^+; 0\rangle = {} & 0.18|(d_{\frac{3}{2}})^2\, 0; 00\rangle + 0.37|(d_{\frac{5}{2}})^2\, 0; 00\rangle \\
& +0.67|(g_{\frac{7}{2}})^2\, 0; 00\rangle - 0.23|(h_{\frac{11}{2}})^2\, 0; 00\rangle \\
& -0.25|(d_{\frac{3}{2}}, g_{\frac{7}{2}})\, 2; 12\rangle - 0.34|(g_{\frac{7}{2}})^2\, 2; 12\rangle
\end{aligned}
$$

$$
\begin{aligned}
|2_1{}^+; 0.665\rangle = {} & 0.24|(d_{\frac{3}{2}}, g_{\frac{7}{2}})2; 00\rangle + 0.36|(g_{\frac{7}{2}})^2\, 2; 00\rangle \\
& -0.28|(d_{\frac{5}{2}})^2\, 0; 12\rangle - 0.57|(g_{\frac{7}{2}})^2\, 0; 12\rangle
\end{aligned}
$$

$$
\begin{aligned}
|4_1{}^+; 1.361\rangle = {} & 0.20|(d_{\frac{3}{2}}, g_{\frac{7}{2}})\, 4; 00\rangle + 0.20|(d_{\frac{5}{2}}, g_{\frac{7}{2}})\, 4; 00\rangle \\
& +0.43|(g_{\frac{7}{2}})^2 4; 00\rangle - 0.24|(d_{\frac{3}{2}}, g_{\frac{7}{2}})2; 12\rangle \\
& -0.21|(d_{\frac{5}{2}}, g_{\frac{7}{2}})6; 12\rangle - 0.38|(g_{\frac{7}{2}})^2 2; 12\rangle \\
& -0.21|(g_{\frac{7}{2}})^2 6; 12\rangle + 0.32|(g_{\frac{7}{2}})^2 0; 24\rangle
\end{aligned}
$$

$$
\begin{aligned}
|6_1{}^+; 1.777\rangle = {} & 0.51|(d_{\frac{5}{2}}, g_{\frac{7}{2}})6; 00\rangle + 0.48|(g_{\frac{7}{2}})^2 6; 00\rangle \\
& -0.38|(d_{\frac{5}{2}}, g_{\frac{7}{2}})6; 12\rangle - 0.23|(g_{\frac{7}{2}})^2 4; 12\rangle \\
& -0.31|(g_{\frac{7}{2}})^2 6; 12\rangle
\end{aligned}
$$

Only those components which contributed more than 4 % are listed. Each state is denoted by its spin, parity, order of appearance and energy in MeV.

trary the 2^+ state shows principally a collective character. The 4^+ level has mixed characteristics (see also sect. 3). The same situation is found in the analogous wave functions for all considered Te isotopes.

Experimental information for the three lightest considered isotopes can be found in the (α, 2nγ) reaction work of Luukko et *al.* [4]), Warner and Draper [5]) and Kerek [6]).

The nucleus ^{122}*Te.* Additional information is obtained from the (^{3}He, d) and (p, t) reaction work of Auble and Ball [15]). There appear to be several discrepancies in their data regarding spin assignments with those given by earlier ^{122}I decay studies [26]). Jha *et al.* [26]) and Auble and Ball [15]) have given a tentative $I^\pi = 0^+$ spin assignment to the 1.357 MeV level, while Gfoller *et al.* [26]) report a 1.358 MeV gamma connecting that level with the ground state. The existence of this γ-ray would clearly be inconsistent with $I^\pi = 0^+$. A theoretically corresponding level could not be reproduced. Therefore, further experimental investigations are desirable in order to confirm unambiguously the existence or non-existence of that level. A similar situation occurs at the 1.75 MeV region. The (α, 2nγ) measurements [4-6]) indicate unambiguously a level at 1.751 MeV with $I^\pi = 6^+$, which also has been confirmed by the (^{3}He, d) reaction data. The one-nucleon transfer data, together with the (p, t) experiments, suggest $I^\pi = 0^+$ level at the same energy. However decay scheme studies [26]) place a level at 1.753 MeV with probable $I^\pi = 1^+$ or 2^+ from its decay properties. If all the data are correct, this would imply that there is a 6^+, (1^+, 2^+), 0^+ triplet in this region. Confirmation of this assumption is given by the theoretical model, where corresponding 6^+, 2^+ and 0^+ levels have been found at respectively 1.633, 1.844 and 1.897 MeV.

The spectroscopic strengths, as calculated from eq. (2.8), are given in table 3. The experimental data have been extracted from the paper of Auble and Ball [15]). We want to remark here that the quantities, denoted as C^2S', are related to the spectroscopic factors $S_l^{(\alpha)}$) by

$$S_l^{(\alpha)}(\exp) = \frac{2I_i+1}{2I_f+1} C^2 S', \tag{3.1}$$

where I_i and I_f define respectively the spin of the target nucleus and the final state spin in the (^{3}He, d) reaction. Quite good agreement is obtained between the theoretical and experimental strengths. From a comparison of the data it is clear that the two levels, excited in the (^{3}He, d) reaction in the 1.75 MeV region, correspond with the theoretical 6^+ and 0^+ states.

TABLE 3

Comparison of theoretical and experimental spectroscopic factors $S_l(\alpha)$ in ^{122}Te

E_{exp} [a]) (MeV)	l_p	$S_l(\alpha)$(exp) [a])	E_{theor} (MeV)	$S_l(\alpha)$(th)	I^π
0	2	0.66	0	0.45	0^+
0.563	0	0.48	0.552	0.043	2^+
	2	0.18		0.13	
	(4)	(0.48)		0.02	
1.175	2	0.14	1.285	0.13	4^+
1.253	0	0.012	1.711	0.02	2^+
	2	0.08		0.005	
1.353	2	0.24			(0^+)
1.747	4	0.65	1.633	0.51	6^+
	2	1.20	1.897	1.02	0^+
	2		1.844	0.06	2^+

[a]) Ref. [15]).

The nucleus ^{124}Te. Spectroscopic strengths are again available from the reaction work of Auble and Ball [15]). Unique spin assignments for the 2039 and 2092 keV levels were determined by gamma-gamma directional correlation measurements of Baker *et al.* [14]). The two 0^+ states, reported respectively at 1290 keV in a (d, d') measurement by Christensen *et al.* [27]) and at 1656 keV in a β-decay study by Meyer *et al.* [28]), have not been excited in the (^{3}He, d) reaction work. These two states are not reproduced by the proposed model. If they exist, they probably could be explained by introducing into the theory additional degrees of freedom i.e. octupole vibrations, individual neutron and proton excitations,... Auble and Ball suggest at 1.88 MeV a third $I^\pi = 0^+$ level. In that same energy region we theoretically reproduce a $I^\pi = 0^+$ state which could correspond to the experimentally observed level. Theoretical and experimental spectroscopic factors are compared in table 4. For some levels above 2 MeV the influence of the limitation of the configuration space can already be seen by comparing experimental and theoretical spectroscopic factors. Indeed, for the two 2^+ levels, at respectively 2.040

TABLE 4

Comparison of theoretical and experimental spectroscopic factor $S_l^{(\alpha)}$ in ^{124}Te

E_{exp} [a]) (MeV)	l_p	$S_l^{(\alpha)}$(exp) [a])	E_{theor} (MeV)	$S_l^{(\alpha)}$(th)	I^π
0	4	1.6	0	1.12	0_1^+
0.604	2	0.11	0.538	0.16	2_1^+
	4	0.8		0.43	
1.250	0	0.08	1.302	0.05	4_1^+
	2	0.18		0.15	
1.320	4		1.693	0.27	2_2^+
	2			0.03	
1.742	2	0.56	1.666	0.46	6_1^+
	4			0.42	
1.953	0	0.053	2.126	0.07	4_2^+
	2	0.044		0.10	
2.040	2		1.882	0.20	2_3^+
2.085	2	0.43	2.224	0.21	2_4^+
2.175	2	0.93	2.023	0.75	1_1^+
2.220	(0)	(0.017)	2.222	0.04	4_3^+
	(2)	(0.035)		0.22	

[a]) Ref. [15]).

and 2.085 MeV, the complete experimental $l = 2$ spectroscopic strength is found at the highest state, while theoretically this strength is distributed over the two states.

The nuclei $^{126-134}$*Te.* Supplementary data about a $I^\pi = 4^+$ state in the vicinity of 2 MeV ($126 \leqq A \leqq 130$) are given by the inelastic scattering experiments of Mantoba *et al.* [2]). The experimental spectra of the four heavy-mass nuclei have been investigated by β- and γ-spectroscopy measurements [1]). The scarce results are reproduced by the theory, which gives however more information that needs to be confirmed by further experiments. Since no experimental data about odd-spin levels are available, no theoretical analogous levels have been calculated. It has to be remarked, that in the case of ^{134}Te, an exact shell-model calculation has been performed by putting the interaction strength $\xi = 0$.

4. Electromagnetic properties

Having calculated the wave functions of the nuclear levels, it is straightforward to evaluate electromagnetic properties (moments and transition probabilities). The magnetic M1 and electric E2 operators are respectively given by

$$\mathcal{M}^{\mathrm{M}}(\mathrm{M1}) = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \{g_l \sum_{i=1}^{2} (l_\mu)_i + g_s \sum_{i=1}^{2} (s_\mu)_i + g_R R_\mu\} \text{ (n.m.)}, \tag{4.1}$$

$$\mathcal{M}^{\mathrm{E}}(\mathrm{E2}) = \sum_{i=1}^{2} \left(e_i + \frac{Ze}{A^2}\right) r_i^2 Y_{2\mu}(r_i) + \frac{3}{4\pi} ZeR_0^2 \alpha_{2\mu}^* . \tag{4.2}$$

TABLE 5

Experimental and theoretical electric quadrupole moments expressed in units $e \cdot b$

A	$Q(2_1{}^+)$			
	theory	exp [a]	exp [b]	exp [c]
122	−0.35	−0.50±0.22		−050±0.22.
124	−0.38	−0.08±0.11		
126	−0.25	−0.16±0.16	−0.40±0.07	−0.33±0.17
128	−0.19	−0.14±0.13	−0.27±0.07	−0.20±0.20
130	−0.13	−0.19±0.15		−0.15±0.18
132	−0.06			
134	0.03			

[a]) Ref. [10]). [b]) Ref. [8]). [c]) Ref. [29]).

The reduced matrix elements of these operators are calculated using the wave functions obtained by diagonalization. Their explicit form is given in ref. [18]).

Experimental electric quadrupole moments for the first excited state are known for a series of Te nuclei ($122 \leqq A \leqq 130$) [8,10,29]). Theoretical results are compared with the experimental values in table 5. The theoretical values are calculated with an effective proton charge e_p equal to the real proton charge. Although the experimental errors are rather large, there seems to be a tendency for the electric quadrupole moments to decrease with increasing mass number; tendency which is also indicated by the theoretical results. Both the sign and the magnitude of the obtained values are in agreement with the experimental data. The positive quadrupole moment of the first 2^+ state in ^{134}Te agrees with the fact that a nearly pure two-particle state ($|(g_{\frac{7}{2}})^2;2\rangle$) would have a positive quadrupole moment. It has been shown in terms of a perturbation expansion theory [16,30]) that in the mass region $122 \leqq A \leqq 132$ the amplitude as well as the sign of the considered quadrupole moments are critically sensitive to the position of the $2d_{\frac{3}{2}}$ single-particle state. Lowering the $2d_{\frac{3}{2}}$ state gives for the quadrupole moment a somewhat larger negative value. So it is clear that a more accurate determination of these experimental quadrupole moments in the whole mass region would us supply information about the variation of the level spacing between the $2d_{\frac{3}{2}}$ and $1g_{\frac{7}{2}}$ single-particle states from isotope to isotope.

Several reduced E2 transition probabilities are experimentally known [1,8-13]). In table 6 theoretical and experimentally known data are compared. The theoretical values are calculated for proton charges $e_p = e$ and $e_p = 2e$, respectively. The surface stiffness C has been derived from the B(E2) values of the transitions between the first excited 2^+ state and the ground state in the Sn nuclei, on the assumption that those nuclei can be described by a pure harmonic oscillator motion[9]). It appears from table 6 that both theoretical B(E2) values agree with the experimental data within the indicated errors. Although the theoretical second 2^+ state has been found too high in energy, the B(E2) values for the so-called cross-over $2_2^+ \rightarrow 0_1^+$ transition are very well reproduced.

TABLE 6

Experimental and theoretical reduced E2 transition probabilities expressed in units $e^2 \cdot fm^4$

A	$2_1^+ \to 0_1^+$			$2_2^+ \to 2_1^+$			$2_2^+ \to 0_1^+$			$4_1^+ \to 2_1^+$		
	exp	theory		exp	theory		exp	theory		exp	theory	
		$e_p = e$	$e_p = 2e$		$e_p = e$	$e_p = 2e$		$e_p = e$	$e_p = 2e$		$e_p = e$	$e_p = 2e$
122	940±200 [a]) 1300±120 [c,f])	858	1424	3500±1600 [f])	711	1128	39±17 [f])	50	34		900	1612
124	780±160 [a]) 1200±240 [b]) 1054± 14 [c]) 940± 85 [e])	917	1093	3400 [d])	936	1428	33 [d])	50	38		1135	1900
126	940± 40 [c]) 640±128 [a]) 1064± 74 [f]) 974± 10 [g])	881	1390	2000 [d]) 1700 [d])	1117	1700	8 [d])	13	6.2	1275 [g]) ±548	1000	1685

[a]) Ref. [11]). [b]) Ref. [9]). [c]) Ref. [10]). [d]) Experimental values as given in ref. [15]). [e]) Ref. [27]). [f]) Ref. [12]). [g]) Ref. [8]).

The obtained small values can be explained by the fact that the particle and the collective contributions in the theoretical B(E2) values are out of phase, which leads to an appreciable retardation for that transition. In the $2_2^+ \rightarrow 2_1^+$ transition both contributions are in phase, so that much larger B(E2) values are obtained. Although a satisfactory comparison with experiment is quite difficult, because the errors on the experimental values are very large, the theoretical B(E2) values for that transition seem to be systematically too small. This can be explained by the fact that many small components of the initial and final wave functions contribute coherently to the value of this reduced E2 transition probability. Since these small components are not stable enough against small changes in the model parameters, a quantitive comparison with experimental data is not possible. On the other hand the spectroscopic factor for the second 2^+ state is reproduced within the experimental uncertainties, because the value of this spectroscopic factor is mainly determined by a small number of large components of the wave function. The reduced E2 transition probabilities for the transitions between the first excited state and the ground state in the heavy-mass isotopes have been discussed in ref. [19]).

It is also possible to see how well the total wave functions describe the levels by calculating branching ratios of some transitions and half-lives. A detailed discussion was made for the half-life of the isomeric 6^+ levels in the heavy Te isotopes in ref. [19]). It has been shown that the collective contribution to the B(E2) value is mainly responsible for the decrease of the B(E2) value, with increasing mass number. Branching ratios of (M1 + E2) transitions are given as a function of the total transition probabilities, P(M1) and P(E2), as

$$R(I, I', I'') = \frac{P(\mathrm{M1}; \alpha I \rightarrow \beta I') + P(\mathrm{E2}; \alpha I \rightarrow \beta I')}{P(\mathrm{M1}; \alpha I \rightarrow \beta I'') + P(\mathrm{E2}; \alpha I \rightarrow \beta I'')}, \tag{4.3}$$

for several αI. Rather abundant experimental information concerning R is known for ^{124}Te. Experimental and theoretical data are compared in table 7 for that nucleus. An effective g_s factor equal to 2.79 has been used in the calculation of the M1 total transition probabilities. The core g_R and the orbital angular momentum g_l factors are taken equal to Z/A and 1, respectively. In general, good agreement is obtained, except for the transitions from the third 2^+ level. A similar deviation from experimental data has also been mentioned in sect. 3 for the spectroscopic factor of that state. It has been proposed that the wave function describing that 2^+ state is already influenced by the truncation of the configuration space.

5. Quasi-ground bands

The (α, xn) reactions [4-6]) excite the various members of the so-called quasi-ground band in the doubly even Te isotopes. The two-proton core coupling model provides an important support to the existence of quasi-ground bands in the vibrational spectrum. It has been pointed out by Lopac [16]) that in the medium-mass Te isotopes the quadru-

TABLE 7

Calculated M1 and E2 transition probabilities with effective g_s factors and e_p charge for ^{124}Te

$I^\pi \rightarrow I^{\pi\prime}$	E_γ [a]) (keV)	P(M1)(s^{-1})	P(E2)(s^{-1})		Branching ratios R			
			$e_p = e$	$e_p = 2e$	$e_p = e$	$e_p = 2e$	exp [a])	exp [b])
$2_2^+ \rightarrow 2_1^+$	722.8	0.19×10^{12}	0.23×10^{12}	0.35×10^{12}	1	1	1	1
$2_2^+ \rightarrow 0_1^+$	1325.6		0.22×10^{12}	0.29×10^{11}	0.52	0.05	0.14	0.14 ± 0.07
$4_2^+ \rightarrow 2_2^+$	632.4		0.46×10^{11}	0.77×10^{11}	0.04	0.06	0.08	
$4_2^+ \rightarrow 4_1^+$	709.4	0.12×10^{13}	0.45×10^{11}	0.74×10^{11}	1	1	1	1
$4_2^+ \rightarrow 2_1^+$	1355.3		0.90×10^{12}	0.95×10^{12}	0.73	0.75	0.68	1 ± 0.3
$2_3^+ \rightarrow 2_2^+$	713.8	0.96×10^{11}	0.22×10^{10}	0.33×10^{10}	0.07	0.07	1.97	1.64 ± 0.62
$2_3^+ \rightarrow 4_1^+$	790.7		0.44×10^{11}	0.64×10^{11}	0.03	0.04	0.60	0.42 ± 0.24
$2_3^+ \rightarrow 2_1^+$	1436.6	0.11×10^{13}	0.30×10^{12}	0.42×10^{12}	1	1	1	1
$2_3^+ \rightarrow 0_1^+$	2039.6		0.16×10^{13}	0.59×10^{12}	1.14	0.39	0.06	
$2_4^+ \rightarrow 2_2^+$	765.3	0.18×10^{12}	0.43×10^{10}	0.63×10^{10}	0.018	0.019	0.041	
$2_4^+ \rightarrow 2_1^+$	1489.1	0.91×10^{13}	0.76×10^{12}	0.75×10^{12}	1	1	1	1
$2_4^+ \rightarrow 0_1^+$	2091.8		0.90×10^{12}	0.11×10^{11}	0.091	0.001		< 0.16

Total intensity ratios (R) are calculated and compared with experimental values.

a) Ref. [14]). b) Ref. [31]).

pole moments of the states with $I^\pi = 2_1^+$, 4_1^+ and 6_1^+ are large and all have the same negative sign. The transitions between these states are considerably larger than those to the other states. The results obtained in the present calculation are identical with those reported by Lopac [16]), within a few percent. Therefore, it seems to us unnecessary to repeat them here. Since the VMI model [17]) deals specifically with such bands, it was straightforward to study the considered isotopes in that model. This model is based upon two simple empirical laws:

(i) The energy E for a state with spin I is given by

$$E_I = \tfrac{1}{2}C\,(\mathscr{I}_I - \mathscr{I}_0)^2 + \frac{I(I+1)}{2\mathscr{I}_I} \quad \text{(in units } \hbar^2 = 1\text{)}, \tag{5.1}$$

where the variable moment of inertia $\mathscr{I}_I$ is chosen to minimize E_I,

$$\frac{\partial E_I}{\partial \mathscr{I}_I} = 0, \tag{5.2}$$

for each state.

(ii) The second empirical law relates the "transition moments of inertia" $\mathscr{I}_{02} = \frac{1}{2}\,(\mathscr{I}_{I=0} + \mathscr{I}_{I=2})$ to the intrinsic transition quadrupole moment,

$$Q_{02} = [\tfrac{16}{5}\pi B(\mathrm{E2}; 0^+ \to 2^+)]^{\frac{1}{2}}, \tag{5.3}$$

through the relation

$$Q_{02} = k\mathscr{I}_{02}^{\frac{1}{2}}. \tag{5.4}$$

The coefficient k in eq. (5.4) was determined by a least-squares fit to 49 data points [17]) and is given as

$$k = (39.4 \pm 2.6)e \cdot 10^{-24}\,\mathrm{keV}^{\frac{1}{2}} \cdot \mathrm{cm}^2. \tag{5.5}$$

The model has been extended toward magic nuclei by introducing negative values of the parameter $\mathscr{I}_0$ [ref. [17])]. The range of validity of the VMI description for negative $\mathscr{I}_0$ values in terms of the energy ratios $R_I \equiv E_I/E_2$ is given by

$$[\tfrac{1}{6}I(I+1)]^{\frac{1}{2}} \leqq R_I \leqq [\tfrac{1}{6}I(I+1)]^{\frac{2}{3}}. \tag{5.6}$$

All doubly even Te isotopes ($114 \leqq A \leqq 126$) belong to that interval, which has been classified by Scharff-Goldhaber and Goldhaber [17]) as the spherical region. We want to remark here that, although for the isotopes ^{128}Te and ^{130}Te the R_4 values belong to the interval (5.6), all other energy ratios $R_I(I = 6,8)$ are below the lower limit.

The model has been applied to all doubly even Te isotopes with $114 \leqq A \leqq 126$. All these nuclei have at least three known excited states of the ground-state band. The two parameters $\mathscr{I}_0$ and σ (which are in the spherical region parameters no longer related to the physical concept of ground-state moment of inertia and softness, respectively) were determined by means of a least-squares fitting procedure, involving all experimentally known level energies. The values of $\mathscr{I}_0$, C and σ obtained for each nucleus are given in table 8. One can observe that at certain neutron numbers ($N = 64$ and 68) the parameters $\mathscr{I}_0$ and σ present a minimum or maximum value. We suggest that his be-

TABLE 8

Values of the parameters $\mathscr{I}_0$ and C obtained from the least-squares fit and the parameter σ derived from them

Nucleus	$\mathscr{I}_0$ (keV^{-1})	C (10^6 keV^3)	$\sigma = \frac{1}{2C\mathscr{I}_0^3}$
$^{114}Te_{62}$	−0.0071	3.818	−0.363
$^{116}Te_{64}$	−0.0156	1.940	−0.068
$^{118}Te_{66}$	−0.0120	1.828	−0.157
$^{120}Te_{68}$	−0.0042	2.871	−2.294
$^{122}Te_{70}$	−0.0056	2.521	−1.111
$^{124}Te_{72}$	−0.0137	1.647	−0.118
$^{126}Te_{74}$	−0.0450	0.740	−0.007

haviour could be attributed to a shell effect, since in the framework of a single-particle shell model the $2d_{\frac{5}{2}}$, respectively the $2d_{\frac{3}{2}}$, orbital is completely filled for those neutron numbers.

The results are presented in table 9. For each nucleus the first row contains the experimental energies as given by Kerek [6]). The second and third rows give the level energies and moments of inertia, respectively. It is clear from table 9 that these doubly even Te isotopes deviate systematically from the VMI predictions for $I^\pi = 4^+$ or 6^+. It has

TABLE 9

Experimental and calculated energies (in keV) and moments of inertia (in MeV^{-1}) of the levels of the quasi-ground band in the doubly even Te isotopes

Nucleus		$I = 2$	4	6	8	10
^{114}Te	E(exp)	709.0	1484.2	2217.7	3089.3	
	E(th)	711.1	1434.1	2234.9	3100.4	
	$\mathscr{I}$	7.4	11.8	15.6	18.9	
^{116}Te	E(exp)	678.8	1359.4	2002.4	2773.2	
	E(th)	679.4	1326.3	2026.2	2772.1	
	$\mathscr{I}$	8.1	13.3	17.9	22.2	
^{118}Te	E(exp)	605.7	1206.5	1821.0	2574.1	
	E(th)	605.2	1200.6	1852.9	2553.4	
	$\mathscr{I}$	8.9	14.4	19.2	23.5	
^{120}Te	E(exp)	560.3	1161.3	1775.5	2653.0	
	E(th)	559.0	1165.8	1850.6	2598.7	
	$\mathscr{I}$	8.9	13.9	18.1	21.9	
^{122}Te	E(exp)	563.6	1180.5	1750.0	2669.0	3290.7
	E(th)	563.2	1155.9	1829.1	2557.4	3336.3
	$\mathscr{I}$	9.0	14.2	18.6	22.5	26.2
^{124}Te	E(exp)	602.3	1247.9	1746.2	2664.3	3154.2
	E(th)	603.4	1190.3	1830.6	2516.6	3242.4
	$\mathscr{I}$	9.0	14.6	19.6	24.1	28.2
^{126}Te	E(exp)	666.2	1361.3	1776.6	2767.6	2975.2
	E(th)	664.4	1252.6	1866.7	2506.1	3168.7
	$\mathscr{I}$	8.7	14.9	20.7	26.1	31.2

been suggested in ref. [17]) that non-deformed nuclei which do not fit the normal VMI behaviour appear to undergo a phase change between the ground and the higher excited states. The particle core coupling model supplies us the wave function of the various members of the quasi-ground band (see table 2). As one can see, the states building this quasi-ground band are of a rather complicated structure. However, one can observe a variation of the predominant character of the wave functions for the various levels:

(i) The ground state shows a predominant two-particle character, $|(g_{\frac{7}{2}})^2\ 0;\ 00\rangle$ (45 %).

(ii) The predominant component of the 2^+ state is of the collective type, $|(g_{\frac{7}{2}})^2\ 0;\ 12\rangle$ (32 %).

(iii) The 4^+ state has mixed characteristics; the three main components are $|(g_{\frac{7}{2}})^2\ 4;00\rangle$ (19 %), $|(g_{\frac{7}{2}})^2\ 2;12\rangle$ (14 %) and $|(g_{\frac{7}{2}})^2\ 0;24\rangle$ (10 %).

(iv) The 6^+ state is mainly of the two-particle type, $|(d_{\frac{5}{2}}, g_{\frac{7}{2}})\ 6;\ 00\rangle$ (25 %) and $|(g_{\frac{7}{2}})\ 6;\ 00\rangle$ (25 %).

Therefore we suggest that this variation of the predominant character of these wave functions could explain the systematic deviation from the VMI predictions for $I^\pi = 4^+$ or 6^+ in the considered Te isotopes.

TABLE 10

Comparison of the intrinsic transition quadrupole moment Q_{02} and the value of $k\mathscr{I}_{02}^{\frac{1}{2}}$ as defined in eq. (5.4)

A	Q_{02} ($e \cdot$ b)	$\mathscr{I}_2$ (keV^{-1})	$k\mathscr{I}_{02}^{\frac{1}{2}}$ ($e \cdot$ b)
120	2.35±0.24 [a])	0.0089	2.63±0.17
122	2.17±0.23 [a])	0.0090	2.64±0.17
	2.56±0.12 [b,c])		
124	1.98±0.20 [a])	0.0090	2.64±0.17
	2.46±0.25 [c])		
	2.30±0.02 [b])		
	2.17±0.10 [f])		
126	2.17±0.05 [b])	0.0087	2.60±0.17
	1.79±0.18 [a])		
	2.31±0.08 [c])		
	2.21±0.01 [d])		

[a]) Ref. [11]). [b]) Ref. [10]). [c]) Ref. [12]). [d]) Ref. [8]). [e]) Ref. [9]). [f]) Ref. [27]).

We have the possibility to test the second empirical law (5.4) for spherical nuclei. For the spherical region one finds [17]) $\mathscr{I}_{I=0} = 0$ and therefore the transition moment of inertia $\mathscr{I}_{02}$ reduces to $\frac{1}{2}\mathscr{I}_{I=2}$. From table 10 it is clear that the validity of the proposed correlation still holds in the spherical region.

6. Conclusion

The spectra of the doubly even Te isotopes are seen to be well reproduced in the simple two-proton core coupling model. The model explains the absence of the second 0^+ state in the so-called two-phonon region. However if the 0^+ states, which have been observed in the light Te isotopes in the afore mentioned region, could be confirmed by new experiments and if it turns out that they have a predominant two-phonon character, this would be a serious failure of the model. It appears that the second 2^+ state lies always higher in energy than experimentally expected. On the other hand the position of the various members of the quasi-ground band is well reproduced and the calculated $6^+ - 4^+$ energy spacing is as experimentally observed in the whole mass region. It was possible to confirm some suggested spin assignments of several higher-lying levels. The calculated spectroscopic factors for a $^{A-1}$Sb (^{3}He, d)^A Te reaction reproduce the experimental data within the experimental uncertainties.

The negative sign and the magnitude of the electric quadrupole moments of the first excited 2^+ state are well reproduced. A more accurate determination of these experimental quadrupole moments would supply us with information about the level spacing between the $2d_{\frac{5}{2}}$ and $1g_{\frac{7}{2}}$ single-particle states. All experimentally known reduced E2 transition probabilities are explained in a satisfactory way by the proposed model, except the ones for the transition between the second and first 2^+ state. However, since the errors on these last mentioned experimental values are large (of the order of 50 %), it would be highly desirable to perform new experiments to obtain more accurate data. Good agreement is obtained for the other electromagnetic properties, although the limitation of the configuration space influences unfavourably the wave function of some higher-energy levels.

It has been suggested that the variation of the predominant character of the wave functions for the several members of the quasi-ground band could explain the systematic deviation from the VMI predictions for $I^\pi = 4^+$ or 6^+ in the Te isotopes considered.

A generalization of the present model may be attempted. One can include in addition to the quadrupole also octupole vibrational excitations of the core. It will not greatly affect the properties of the low-lying states, but might somewhat influence the position of the higher-lying levels.

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