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HOW CAN WE TELL THE TRANSITION FROM THE SUPERCONDUCTING TO THE NORMAL PHASE?

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Abstract: The importance of the pairing correlation in rapidly rotating nuclei is reviewed by making use of the theoretical method which takes higher order correlations into account. The characteristic difference of the properties of the pairing correlation in the "static" and "dynamic" pairing regimes, which are distinguished by the dominance of the mean-field contributions, is emphasized. Through the investigation of the relative energy spectra the concept of the pairing phase transition is re-examined. Interesting phenomena related to the pair transfer reactions are also mentioned briefly.

1. Introduction

I must confess that the title of my talk given by the organizer is a subtle question which is beyond my ability to give a definite answer. The difficulty lies in the fact that the concept of the phase transition cannot be so easily extended to finite systems such as atomic nuclei. A standard point of view is to *define* the phase transition within the mean-field approximation: The problem is then of qualitative nature, namely the different phases are clearly distinguished by the order parameter, e.g. the (static) pairing gap in the case of the superconducting-to-normal transition. It is, however, well known that the many body correlations induced by the residual nucleon-nucleon interactions always exist and are often non-negligible in nuclei. The problem is now of quantitative nature: In order to correctly describe the physical phenomena, the concept of the "phase transition" should be re-examined depending on how weak or strong the correlation is.

The pairing correlation is known to be rather weak compared to, e.g., the quadrupole particle-hole correlation which leads to the spherical-to-deformed transition. This means that the mean-field (referred to by "static") and the higher-order ("dynamic") contributions are comparative even in the ground state regions in the case of pairing correlations. However, the effects of the static and the dynamic correlations on physical observables are qualitatively different, and when the nucleus is rotated rapidly the importance of two effects interchanges. Here let me use the terms "static pairing regime" and "dynamic pairing regime" if the static and dynamic correlations, respectively, play dominant roles.

The main purpose of this talk is to clarify the basic features of two regimes and to discuss whether it is reasonable or not to say "the normal phase" is realized at higher spin regions. I would like also to make some comments on the similarity

and the difference of the two commonly used approaches to take dynamic correlations into account, i.e. the random phase approximation (RPA) and the variation after number projection (NP). The theoretical analysis in this talk is based on the investigations of energy spectra or routhians. It is, however, well known that the transition amplitudes, the pair transfer matrix elements, reflect more sensitively the effect of correlations, although it is much more difficult to extract their information from the experimental data. I also briefly mention the interesting phenomena related to the pair transfers in rapidly rotating nuclei.

2. Basic quantities

Here I would like to summarize basic ingredients for the theoretical analysis, see refs. [1-3]) for details. As for the model of pairing correlations, only the monopole pairing interaction is considered for simplicity, although the quadrupole pairing is known to be important for realistic analysis of experimental data especially in the static pairing regime. Thus the model hamiltonian is

$$H' = h_{\rm def} - \omega J_x + V, \tag{2.1}$$

with

$$V = -\tfrac{1}{2}G(P^\dagger P + PP^\dagger), \tag{2.2}$$

where $h_{\rm def}$ is the deformed mean-field potential, $-\omega J_x$ the cranking term and $P^\dagger$ is the monopole pair transfer operator.

The static correlation is typically measured by the correlation energy evaluated within the (selfconsistent) mean-field approximation,

$$E_{\rm stat} = E'_{\rm HFB} - E'_{\rm HF}, \tag{2.3}$$

with

$$E'_{\rm HFB} = \langle \Phi_{\rm HFB}|H'|\Phi_{\rm HFB}\rangle, \quad E'_{\rm HF} = \langle \Phi_{\rm HF}|H'|\Phi_{\rm HF}\rangle, \tag{2.4}$$

where $|\Phi_{\rm HFB}\rangle$ is the wave function determined by the Hartree-Fock-Bogoliubov (HFB) procedure (in the superconducting phase), i.e. the usual BCS treatment at $\omega = 0$, and $|\Phi_{\rm HF}\rangle$ by the Hartree-Fock (HF) procedure (in the normal phase), namely the simple product state (Slater determinant). The dynamic (higher order) correlation energy is obtained by going beyond the mean-field approximation [4]), e.g. the NP or the RPA approach,

$$E^{\rm (NP)}_{\rm dyn} = E^{\rm (NP)}_{\rm corr} - E_{\rm stat}, \tag{2.5}$$

with

$$E^{\rm (NP)}_{\rm corr} = E'_{\rm NP} - E'_{\rm HF}, \quad E'_{\rm NP} = \langle \Psi_{\rm NP}|H'|\Psi_{\rm NP}\rangle, \tag{2.6}$$

or $E^{\rm (RPA)}_{\rm dyn}$ is defined in the same way by replacing the NP wave function $|\Psi_{\rm NP}\rangle$ by the RPA wave function $|\Psi_{\rm RPA}\rangle$. Note that V is the two body operator so that

in eqs. (2.4) and (2.6) the expectation value of H' contains the exchange (Fock) contributions even in the mean-field level*. They are not included in the definition of the correlation energies, eqs. (2.3) and (2.5), which are measured from

$$E'_{\rm HF} = \sum_{i<F} \varepsilon'_i + E_{\rm exch}. \tag{2.7}$$

This is because the exchange energy $E_{\rm exch}$ only weakly depends either on the rotational frequency or on the configurations. For the total routhian,

$$E'_{\rm tot} = E_{\rm dyn} + E_{\rm stat} + E'_{\rm HF}, \tag{2.8}$$

however, it is not negligible and reflects the spreading of the pair transfer strength which is caused by the rapid rotation.

Another quantity to measure the pairing correlation is the pairing gap parameter. Corresponding to the level of approximations defined above, various kinds of gap parameters are considered:

$$\langle\Phi_{\rm HFB}|\tfrac{1}{2}(P^\dagger P + PP^\dagger)|\Phi_{\rm HFB}\rangle = (\Delta_{\rm stat}/G)^2 + (\text{exchange contribution}), \tag{2.9}$$

$$\langle\Phi_{\rm NP}|\tfrac{1}{2}(P^\dagger P + PP^\dagger)|\Phi_{\rm NP}\rangle = (\Delta_{\rm NP}/G)^2 + (\text{exchange contribution}), \tag{2.10}$$

etc. Note again the exchange contributions are not included in the definitions. Among them the static pairing gap, $\Delta \equiv \Delta_{\rm stat} = \langle\Phi_{\rm HFB}|P^\dagger|\Phi_{\rm HFB}\rangle$ is the parameter used for the cranked shell model quasiparticle routhian diagram and has the meaning of the order parameter because it vanishes when the normal phase is realized. In contrast, the NP pairing gap, $\Delta_{\rm NP}$, and also the RPA gap, $\Delta_{\rm RPA}$, never quenched up to highest spin regions [3,5]) (this is true also for the exchange contributions). This fact is in a sense a trivial matter: As long as the two-body interactions are acting, the many body correlations always remain and are non-negligible in the finite system though weakened considerably by the Coriolis and centrifugal force. The important is how the correlations affect the observable quantities and it will be discussed in sect. 4.

Finally let me make comments on the RPA versus NP treatment of pairing correlation, for which rather detailed comparisons have been done in ref. [3]). The RPA is the next order treatment in the (in principle) exact boson expansion theory (BET) [6]) with the HB approximation regarded as the lowest. All the eigenmodes which take into account the two-body correlations up to infinity (the ring diagram summations) are included. Thus the results of the RPA seems to contain more correlations than the NP method. Actually the absolute value of the correlation energy calculated in the RPA is always larger than that in the NP. It should, however, be emphasized that the observables in the experimental data are the rotational frequency dependences of physical quantities and for those both treatments often give quite similar results [3]). Therefore, it may be fair to say the essential part of the

* Note that $E^{\rm (RPA)}_{\rm dyn} + E_{\rm exch}$ is called the RPA correlation energy in ref. [1]).

pairing correlation is well accounted for by both approaches. One clear drawback of the RPA is that it is basically the small amplitude approximation which breaks down near the transition points and results in singularities of the physical quantities at those points. Next higher order treatment (e.g. the fourth order BET) is usually enough to overcome the drawback [7]) but it still requires a considerable numerical task. Therefore a simple interpolation procedure across the transition point is required in the RPA calculations in order to obtain physically meaningful smooth results, thus introducing undesired ambiguities which are usually not essential. In this respect the NP treatment is safer since it is free from the small amplitude approximation. Another notice I would like to address for the NP approach is that the full variation of trial wave function [3–5]) is important for taking the higher order pairing correlations into account. A simplified treatment [8]), which is often used and called cranked FBCS, underestimates the correlation energy and overestimates the de-alignment effect [3]).

3. Effects on superdeformed bands

In order to see the effect of pairing correlations on the observable quantities, the rotational frequency dependence of the routhians and the angular momenta are investigated systematically for normal deformed nuclei in ref. [1]). There the role played by the pairing correlation in the dynamic pairing regime is emphasized: the angular momenta are reduced considerably by the dynamic pairing fluctuations, the de-alignment effect. It has been already noticed for some time that the $\mathcal{J}^{(2)}$ moment of inertia, which corresponds to the second derivative of the routhian, may be a sensitive measure of the change of pairing correlations [9,10]). However, $\mathcal{J}^{(2)}$ is sensitive to any changes of the internal structure of the rotational band, e.g. the deformation change and especially the quasiparticle alignments. It is, therefore, very difficult to extract independently the information on the pairing correlation.

In this respect the observation of the superdeformed (SD) rotational band [11,12]) gives us an unique possibility to study the effects of pairing correlations, since the SD bands usually keep their identities over a wide range of spin or rotational frequency. Actually the systematic comparison of the calculations and the experimental data for the SD bands in the $A \approx 150$ region in refs. [2,13]) show clearly the importance of pairing correlations, dominantly of dynamic nature in this region. The results are summarized as follows: The $\mathcal{J}^{(1)}$ moments of inertia are reduced typically by about 15–5% in the observed rotational frequency range, $0.3 \lesssim \hbar\omega \lesssim 0.7$ MeV, whereas the $\mathcal{J}^{(2)}$ are increased by about 10–5% compared to the results without pairing correlations.

These basic trends can be naturally understood by the rotational frequency dependence of the correlation energy, and well explains the systematic discrepancies between the standard Strutinsky calculations (without pairing) and the experimental data. As is shown in fig. 1, the correlation energy (the lowest panel) has generally an inflection point at a frequency ω^*, where $\mathrm{d}^2 E_{\mathrm{corr}}/\mathrm{d}\omega^2 = 0$, and thus its contribution to $\mathcal{J}^{(2)}$ changes sign at this point while that to $\mathcal{J}^{(1)}$ is always negative. For

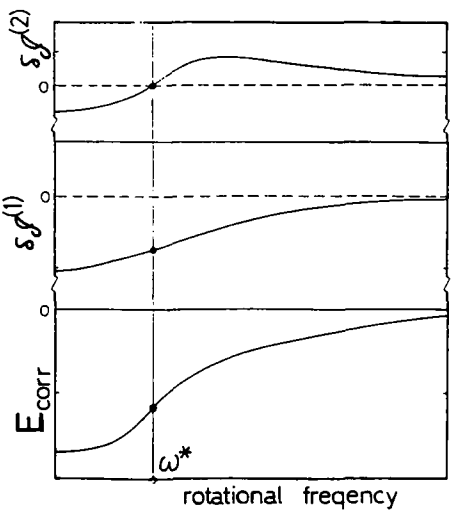

Fig. 1. Schematic figure depicting the influence of pairing correlations on the moments of inertia.

the SD band in the $A \approx 150$ region the inflection point is usually lies in the lower frequency than observed, reflecting the minor role of the static pairing.

Recently a new region of the SD bands in $A \approx 190$ nuclei (Hg isotopes) with a typical deformation [14]) $\varepsilon_2 \approx 0.45$–$0.50$ and $\gamma \approx 0$ is discovered [15–17]). The SD shell structure of the $A \approx 190$ region is not so pronounced, especially for neutrons, as that of the $A \approx 150$ region where the shell gap based on the 2:1 deformation is so large that the static pairing is generally quenched already at lowest spins. Examples of the calculations for ^{190,191}Hg are shown in fig. 2, which are obtained by using the cranked Nilsson-Strutinsky method as in the same way as ref. [2]) but here followed by the full NP method. As is clearly seen from the figure the modifications caused by the static and dynamic pairing correlation is larger than those in the $A \approx 150$ region. In fact the static pairing remains up to considerably higher spins and affects both $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ significantly. Especially the rapid increase of $\mathcal{J}^{(2)}$ is understood by the fact that the inflection point in fig. 1 lies in the middle of the observed frequency range for the SD band in the $A \approx 190$ region. The amount of the correction coming from the pairing correlation is in between of those observed in the normal deformed nuclei (typically about factor two reduction) and in the $A \approx 150$ SD nuclei (typically about 10%) at lower spins.

One of the interesting features in the $A \approx 190$ region is that the trends of all the observed $\mathcal{J}^{(2)}$ moments of inertia are quite similar [16]). This fact suggests the same internal structure of the observed SD bands, e.g. $\pi 6^4 \nu 7^4$ [ref. [17])]. Since the static pairing correlation is generally very different for different configurations, e.g. between even and odd nuclei (blocking effects), we naturally expect considerable differences. Further investigations may reveal a new insight for the property of pairing correlations in strongly deformed and rapidly rotating nuclei.

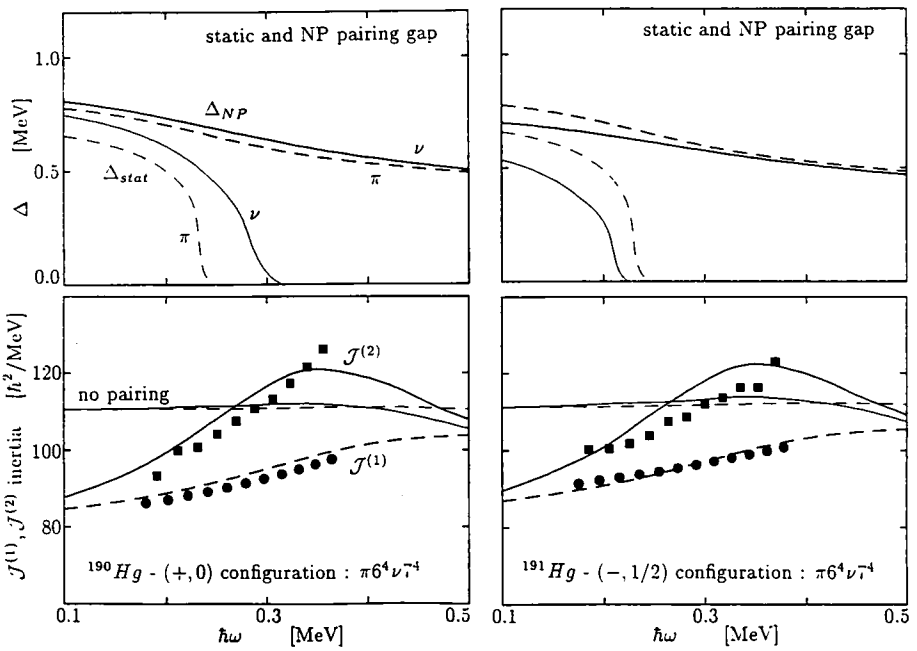

Fig. 2. The static and the NP pairing gaps (the upper panel) and the resultant $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ moments of inertia as functions of the rotational frequency for the $(+,0)$ configuration in ^{190}Hg (left) and for the $(-,1/2)$ configuration in ^{191}Hg. The thick (thin) solid and dashed curves, respectively, are used for $\mathcal{J}^{(2)}$ and $\mathcal{J}^{(1)}$ with (without) the pairing correlations, while the square and circle symbols are for the experimental ones. Here for $\mathcal{J}^{(1)}$ in ^{190}Hg the lowest spin of the observed spectra is assigned to be $14\hbar$ which fits the calculation. In ^{191}Hg the $(-,\pm 1/2)$ signature pair configurations are almost degenerate, and I assigned the observed one to be $\alpha = 1/2$ according to the lowest spin $29/2\,\hbar$ which fits the theoretical prediction.

4. Relative energy spectra as an indication of the super-to-normal transition: A theoretical simulation

As is shown in the previous section and systematic calculations both for superdeformed [2,13]) and normal deformed rotational bands [1]), the pairing correlations play important roles up to highest spins observed. Especially the higher order (dynamic) correlation is so large that the signatures of the superconducting-to-normal phase transition existing in the mean-field level ($\Delta_{\rm stat} \to 0$) are mostly washed out for physical observables [5]). Actually the fact that the pairing correlation remains at high spins is clearly indicated by the NP (or RPA) pairing gap which keeps unquenched, and it is sometimes said that there exists no super-to-normal transition in atomic nuclei. I think this statement is fairly true as long as it is concerned with the property of an *individual* rotational sequence: For the energy spectrum (the routhian) or the rotational frequency dependence of $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ moments

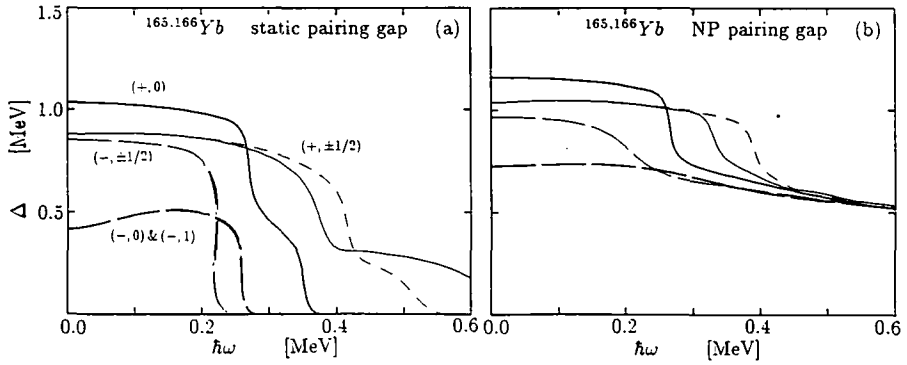

Fig. 3. The neutron static and NP pairing gap parameters as functions of the rotational frequency, which are defined by eqs. (2.9) and (2.10), for various configurations of ^{165}Yb and ^{166}Yb. The thick solid, one-point dash and long dash curves are used for $(\pi, \alpha) = (+,0)$, $(-,0)$ and $(-,1)$ configurations, and solid, dash, one-point dash and long dash for $(+,+1/2)$, $(+,-1/2)$, $(-,+1/2)$ and $(-,-1/2)$, respectively. The results for signature partners, $(-,0)$ and $(-,1)$, and $(-,+1/2)$ and $(-,-1/2)$, are almost degenerate for both $\Delta_{\rm stat}$ and $\Delta_{\rm NP}$.

of inertia for each rotational band, there are no definite indications of the phase transition.

It should, however, be remembered that one of the evidences that the superconducting phase is realized in the ground state region come from the analysis of relative energy spectra in even nuclei and of those between even and odd nuclei. In the case of rotating nuclei, the energy gap present in the relative energy spectra at $\hbar\omega = 0$ decreases as $\hbar\omega$ increases, especially for highly alignable orbits (gapless superconductivity). Although it is not so definite as in the ground state region because of this fact, there are still rather clear differences in the characteristic features of the relative energy spectra in the superconducting (i.e. the static pairing regime) and the normal (i.e. the dynamic pairing regime) phases. Actually the speculation that the normal phase is realized at higher spins (say, $I \gtrsim 30\hbar$) have been done [18,19]) for some time by careful analysis of experimental routhians in comparison to those obtained by the simple cranked shell model (CSM) with $\Delta = 0$. In the following, I would like to support this analysis by a theoretical simulation by taking the higher order pairing correlations into account, i.e. by making use of the NP approach with full variation. The results obtained by the RPA approach are more or less the same.

I selected the nuclei ^{166,165}Yb as examples of the analysis without any special intentions. They are typical stable deformed nuclei and experimental data are available up to high spin states, $I \lesssim 40\hbar$. Since I would like to focus an attention to the pairing correlation, the deformation parameters are kept constant, $\varepsilon_2 = 0.25$ and $\gamma = 0°$ (appropriate for the ground state of ^{166}Yb), both for ^{165}Yb and ^{166}Yb. Thus the detailed comparison with experimental data is not aimed here. The calculational procedure is the same as ref. [3]) [see also refs. [1,2])], but the Strutinsky renormalized rotational frequency, $\omega_{\rm ren} = \omega_{\rm crank}\,(\mathcal{J}_{\rm Strut}/\mathcal{J}_{\rm rig})$, is used [20]) for showing results.

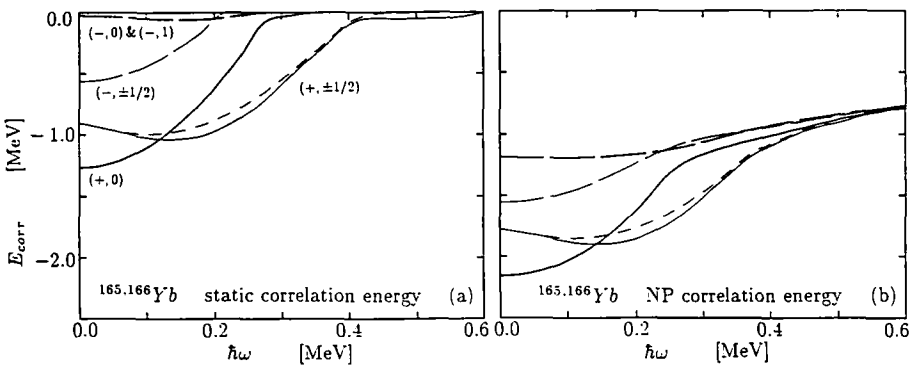

Fig. 4. The neutron static and NP correlation energies as functions of the rotational frequency, which are defined in eqs. (2.3) and (2.6). The legends are the same as in fig. 3.

First, in fig. 3 measures of the pairing correlations, the static and the NP pairing gaps, for neutron configurations with the parity and signature $(\pi, \alpha) = (+,0), (-,0)$ and $(-,1)$ in ^{166}Yb and $(+,\pm 1/2)$ and $(-,\pm 1/2)$ in ^{165}Yb are shown as functions of the rotational frequency. As is clear from the figure the NP gaps never quench while the static gaps do. The more important is, however, that $\Delta_{\rm NP}$'s for various configurations take quite different values, similarly to $\Delta_{\rm stat}$'s, in the static pairing regime (say, $\hbar\omega \lesssim 0.3$ MeV) while in the dynamic pairing regime (say, $\hbar\omega \gtrsim 0.4$ MeV) their values converge to, so to say, a "unified" value. This feature of the pairing correlation is general: It is sensitive to the change of configurations, known as the strong blocking effects seen in the mean-field approximation, in the static pairing regime and becomes almost configuration-independent in the dynamic pairing regime. The reason why the dynamic correlations are insensitive to the differences of configurations is understood, in terms of the RPA: They come from the accumulation of many, but small in each, contributions of excited (non-collective) eigenmodes for which the rearrangement of the vacuum configuration play a minor role. This important characteristic difference in the static and dynamic pairing regime can be seen also in the correlation energies in fig. 4, where again the static and the NP ones are shown for each neutron configuration as functions of $\hbar\omega$. Comparing the NP with the static correlation energies in the static regime, the dynamic correlation energies, which are the differences of the two, eq. (2.5), are rather constant (≈ 1 MeV) and almost configuration-independent, i.e. their properties are mainly governed by the static contributions. On the other hand, in the dynamic regime, they are also independent of configurations but decrease as $\hbar\omega$, resulting considerable de-alignment effects. These basic features of correlation energies have been already discussed in ref. [1]).

Now let me discuss on the "single-particle" routhians in ^{165}Yb relative to the yrast state of ^{166}Yb calculated by making use of the NP results above. They are obtained, as in the same way as done experimentally, by taking the differences between the total routhians of various configurations in ^{165}Yb and that of $(+,0)$ yrast configuration in ^{166}Yb with constant shift corresponding to the chemical

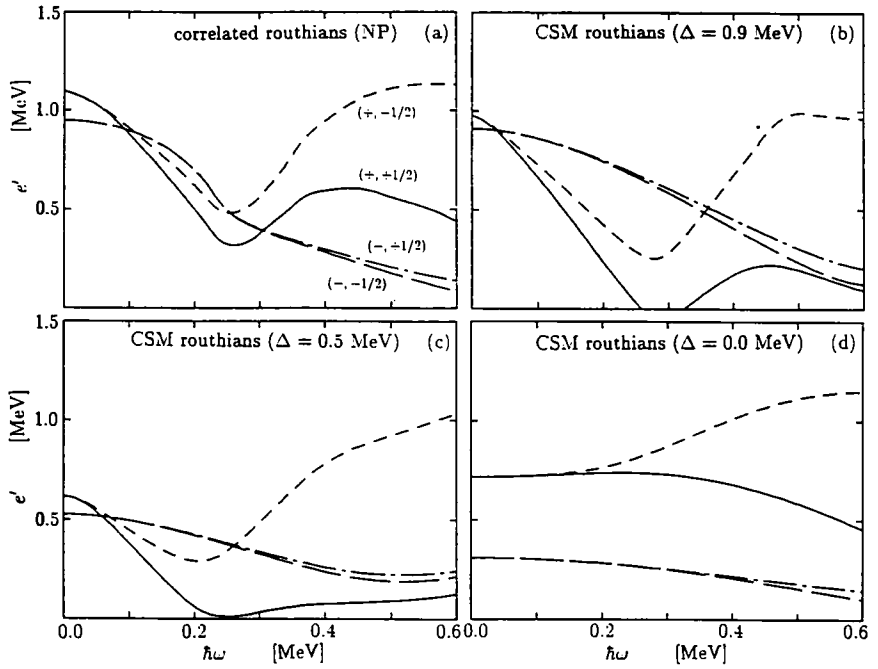

Fig. 5. The correlated neutron single-particle routhians of ^{165}Yb in (a), which are obtained by $E'_{\rm tot}(^{165}{\rm Yb}) - E'_{\rm tot}(^{166}{\rm Yb};(+,0)) + {\rm const}$, where the total routhians are calculated by the NP approach. For comparison, in (b), (c) and (d) are shown the corresponding cranked shell model quasiparticle routhians with $\Delta = 0.9$, 0.5 and 0.0 MeV. Since the details of the rotational frequency dependence of the reference band, $(+,0)$ band of ^{166}Yb, cannot be described by the simple CSM approximation (constant Δ), the frequency dependence of each routhian in (b), (c) and (d) does not precisely correspond to that of the correlated routhian in (a), rather the relative spectrum is meaningful.

potential. Note that the total routhians, taking the higher order pairing correlation into account, is calculated by adding the correlation energies shown in fig. 4 to the normal phase HF routhians. Since the exchange energies are almost the same for all the configurations, the deviations between the correlated and uncorrelated (CSM with $\Delta = 0$) single-particle spectra are estimated by the differences of the correlation energies in fig. 4. The result of the correlated routhians is shown in fig. 5 (a), in addition to the simple CSM routhians with $\Delta = 0.9$ MeV (b) which corresponds roughly to the value of the static pairing gap at $\hbar\omega = 0$, those with $\Delta = 0.5$ MeV (c) which corresponds to the "unified" value of the NP gap at $\hbar\omega \approx 0.6$ MeV, and finally the normal phase routhians, i.e. those with $\Delta = 0$ (d). As it is expected, in the static pairing regime where the static gaps take sizable values, $\hbar\omega \lesssim 0.2$ MeV for $(-,\pm 1/2)$ and $\hbar\omega \lesssim 0.3$ MeV for $(+,\pm 1/2)$, the quasiparticle routhians with $\Delta = 0.9$ MeV are rather good. On the other hand, the normal phase routhians are much better in the dynamic regime, $\hbar\omega \gtrsim 0.4$ MeV: In fact they are almost "exact" because the correlation energies of various configurations

converge to an "unified" value as it is shown in fig. 4. In between the two regimes, $0.2 \lesssim \hbar\omega \lesssim 0.4$ MeV, where the static gaps change dramatically, all the simple CSM spectra fail to reproduce the basic trends of the correlated spectra. Thus, from these results, I think it is fair to say that the normal phase is realized in the dynamic pairing regime in the sense that relative energy spectra are well described by the simple CSM routhians with $\Delta = 0$, although the pairing correlations are never negligible in order to account for the properties of each rotational band.

It is worthwhile to notice that the quasiparticle routhians with $\Delta = 0.5$ MeV do not correspond to the correlated routhians in the whole range of rotational frequency, especially they do not correctly describe the level ordering at higher frequencies, $\hbar\omega \gtrsim 0.4$, where the $(-, \pm 1/2)$ routhians are lower than the $(+, +1/2)$ one in the correlated spectra. This result clearly indicates quite different roles played by the static and the NP pairing gap: $\Delta_{\rm NP}$, which never quenches but converges to an "unified" value (≈ 0.5 MeV) at highest frequency, cannot be used as an input of the quasiparticle energy, while $\Delta_{\rm stat}$ can as a lowest order approximation.

5. Pair transfer amplitude

It is certainly true that more direct information on the pairing correlation can be obtained by the measurement of the transition rates of nucleon transfer reactions. Actually it is well known that the one nucleon pick-up or stripping reaction in the ground states of deformed nuclei clearly revealed the pair condensate [21]) through the BCS-type spectroscopic factor in the DWBA analysis. For rotating nuclei, however, the analysis of experimental data is much more involved because of the complexity of various reaction mechanisms, and the Coulomb excitation followed by nucleon transfers [22,23]) is considered to be best suited in order to minimize the complexity. See refs. [1,24]) for the basic picture expected in such experiments [25,26]).

In contrast to the energy spectra the non-diagonal matrix elements should be considered in the case of the transition rate, i.e.,

$$\mathcal{P}(i \to f) = \langle f|P^\dagger|i\rangle, \tag{5.1}$$

where the initial and final states, $|i\rangle$ and $|f\rangle$, are evaluated in the consistent approximation scheme to the energy spectra, and this makes the analyses more difficult both experimentally and theoretically. In the static pairing regime the yrast-to-yrast transition dominates and the transition amplitude can be well approximated by

$$|\mathcal{P}(y \to y)|_{\rm HFB} \approx |\langle \Phi_{\rm HFB}|P^\dagger|\Phi_{\rm HFB}\rangle| \approx \Delta_{\rm stat}/G. \tag{5.2}$$

In the dynamic pairing regime, however, the yrast-to-excited states transitions can equally well occur [1,27]), and then the identification of the initial and final states is very important in order to extract the precise information on the pairing correlation. Another complication to analyze the experimental data is that we should distinguish the one step pair transfer from the sequential two step single nucleon transfer processes which are known to be non-negligible. The detailed understanding of

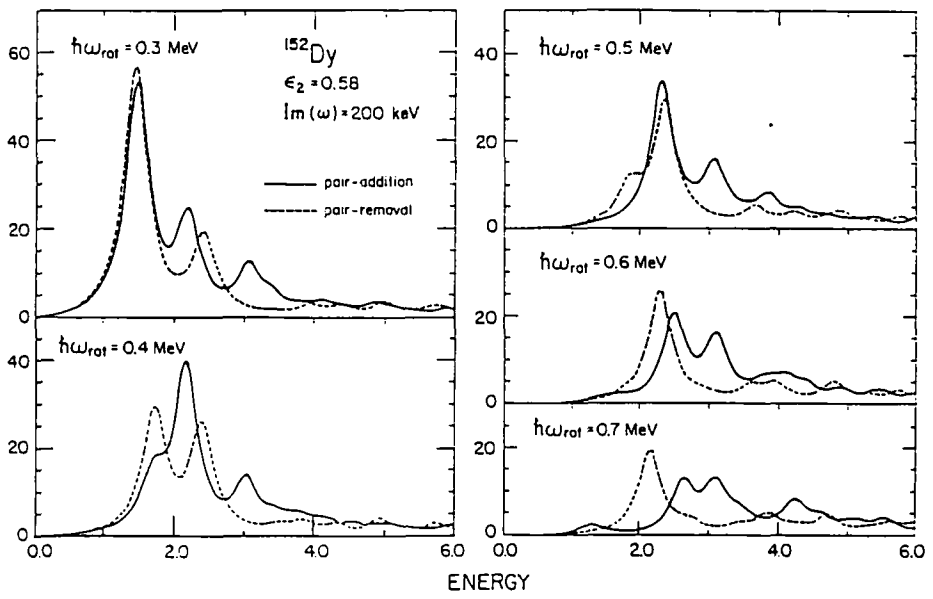

Fig. 6. The neutron pair transfer strength for the SD yrast band of ^{152}Dy. the solid (dashed) curve represents the one for the pair addition (removal) reaction.

the reaction mechanism is, then, also crucial for extracting the information on the pair transfer amplitudes.

In spite of the difficulties mentioned above, experimental data of good quality are eagerly hoped to study the pair transfer in rotating nuclei. Recently it was theoretically pointed out [28]) that the pair transfer amplitude between the yrast states changes the sign at the band crossing point ("diabolic" pair transfer) irrespective of the approximation scheme used, e.g. the HFB, NP [ref. [29])] or "exact" [30,31]) treatments. This phase-change of the amplitude can be naturally interpreted as a manifestation of the Berry's phase (Nuclear SQUID) [28,29]). If the sign-change exists it is predicted that the probabilities of the pair transfer above the crossing frequency are considerably reduced [32,33]). Note that this reduction of the probability is caused by the interference between transfers below and above the crossing frequency and it occurs even if the absolute value of the transfer amplitude is assumed to be constant [34]). In realistic situation both the decrease of the absolute value and the sign-change are expected so that it is important to precisely estimate the effects of both of them.

Another interesting prediction is the enhancements of the pair transfer probability associated with the collective pair vibrations in the superdeformed rotational bands [35]). As it is well known in the region of ^{208}Pb the ground states of neighbouring nuclei with neutron or proton numbers differing by two unit can be interpreted as the states which are built by exciting the collective pair vibrational modes on the "vacuum" of ^{208}Pb [ref. [36])]. The pair transfer rates to such states are greatly

enhanced because of the collectivity. The large shell gap associated with the 2:1 deformation shell closure makes the situation in the SD rotational band quite similar to the case of ^{208}Pb, although at higher spins the diffusion of the pairing matrix elements reduces the collectivity. In fig. 6 is shown the result of theoretical calculations for ^{152}Dy by making use of the RPA [ref. [35])], which predicts considerable enhancements of pair transfer for $\hbar\omega \lesssim 0.5$ MeV. Since the intensity to produce the SD states is small it is certainly difficult to study the pair transfer in the SD band experimentally. I hope the new generation crystal ball, e.g. the GAMMASPHERE, will open the possibility to perform such experiments.

Finally let me mention the new data coming from the NORDBALL collaboration at NBI [ref. [37])]. Recently the Japanese group have done a test experiment of the Coulomb excitations with full use of the post accelerator and the small silicon ball inside the NORDBALL which is arranged for particle detections. Though the data analysis is not yet enough to draw a definite conclusion, the preliminary analysis seems to indicate surprisingly strong intensity of the pair transfer at high spins, which is promising to study the detailed properties of the pairing correlation.

6. Summary

In the this talk I have tried to review the present status of our understanding on the pairing correlation in rapidly rotating nuclei. Especially I focused an attention to the characteristic difference of the properties of pairing correlations in the static and dynamic pairing regimes. Namely, the correlation energies are sensitive to the change of internal structures of the individual rotational bands in the static regime, which results from the dominance of the mean-field contributions, while in the dynamic regime they are rather insensitive and take an "unified" value but leads to the important de-alignment effect at higher spin region. In the course of the discussion the concept of the pairing phase transition is re-examined, and it is pointed out that the statement that the normal phase is realized at observed highest spin regions may be meaningful in a sense that the corresponding single-particle (CSM) routhians can be used for describing the relative energy spectra.

Moreover, the important roles played by the pairing correlations in the superdeformed bands both in the $A \approx 150$ and the $A \approx 190$ regions are also discussed. In the former region the dynamic correlations are dominant because the pronounced shell gap associated with the 2:1 deformation prevents the static pairing to be effective. Thus the relative spectra and/or relative $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ moments of inertia can be well understood by the simple normal phase routhians. Actually this fact is used to identify the SD configurations. In the latter region, however, the static correlations seem to be also important because of the less pronounced shell gap, although further investigations both theoretically and experimentally are needed to draw a definite conclusion.

The transition rates associated with the pairing correlation, the pair transfer in rotating nuclei, which would give more direct information, are also very briefly mentioned. Again, a qualitative difference in the static and dynamic pairing regime

is expected: In the static regime the transfer of the yrast-to-yrast occurs dominantly, while a considerable amount of the yrast-to-excited states are expected in the dynamic regime. An interesting phenomenon which manifests the effect of the Berry's phase suggested in refs. [28,29]) and the enhancement of the pair transfer in the superdeformed band predicted in ref. [35]) are discussed as examples. Though the experiments to see such phenomena are certainly very difficult, I am sure that they will be done in near futures because experimentalists have been and will be clever enough to overcome many kinds of difficulties.

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References

1) Y.R. Shimizu, J.D. Garrett, R.A. Broglia, M. Gallardo and E. Vigezzi, Rev. Mod. Phys. **61** (1989) 131
2) Y.R. Shimizu, E. Vigezzi and R.A. Broglia, Nucl. Phys. **A509** (1990) 80
3) Y.R. Shimizu and R.A. Broglia, Nucl. Phys. A, to be published
4) P. Ring and P. Schuck, The Nuclear Many Body Problem (Springer-Verlag, Berlin, 1980)
5) J.L. Egido and P. Ring, Nucl. Phys. **A383** (1982) 189; **A388** (1982) 19
6) E.R. Marshalek, Nucl. Phys. **A275** (1977) 416
7) Y.R. Shimizu and R.A. Broglia, Nucl. Phys. **A476** (1988) 228
8) See, e.g., A. Faessler *et al.*, Nucl. Phys. **A256** (1976) 106
9) Å. Bohr and B.R. Mottelson, Phys. Scripta **24** (1981) 71
10) M.J.A. de Voigt, J. Dudek and Z. Szymański, Rev. Mod. Phys. **55** (1983) 949
11) P. Twin, talk given at Conf. on Nuclear Structure in the Nineties, Oak Ridge, Tennessee, 23–27 April 1990
12) S. Åberg, talk given at Conf. on Nuclear Structure in the Nineties, Oak Ridge, Tennessee, 23–27 April 1990
13) W. Nazarewicz, R. Wyss and J. Johnson, Nucl. Phys. **A503** (1989) 285
14) R.R. Chasman, Phys. Lett. **219B** (1989) 227
15) E.F. Moore *et al.*, Phys. Rev. Lett. **63** (1989) 360
16) Report of Workshop B, in Nuclear Structure in the Era of New Spectroscopy, Copenhagen, September–November 1989
17) M.A. Riley *et al.*, Nucl. Phys. A, to be published
18) J.D. Garrett, Nuclear Structure 1985, ed. R.A. Broglia, G.B. Hagemann and B. Herskind (North-Holland, Amsterdam, 1985) p. 111
19) J.D. Garrett, G.B. Hagemann and B. Herskind, Ann. Rev. Nucl. Part. Sci. **36** (1986) 419
20) T. Bengtsson, Nucl. Phys. **A496** (1989) 56
21) Å. Bohr and B.R. Mottelson, Nuclear Structure, vol. 2 (Benjamin, New York, 1975) sect. 5-3, p. 258 ff.
22) D. Cline, talk given at Conf. on Nuclear Structure in the Nineties, Oak Ridge, Tennessee, 23–27 April 1990

23) H. Wollersheim, talk given at Conf. on Nuclear Structure in the Nineties, Oak Ridge, Tennessee, 23–27 April 1990
24) R.A. Broglia and M. Gallardo, Nucl. Phys. **A447** (1985) 489c
25) M.W. Guidry, Conf. on Nuclear Structure with Heavy Ions, ed. R. A. Ricci and C. Villi (Editrice Compositori, Bologna, 1986) p. 99
26) J. Gerl *et al.*, Z. Phys. **A334** (1989) 195
27) J.L. Egido and J.O. Rasmussen, Phys. Rev. **C36** (1987) 316
28) R.S. Nikam and P. Ring, Phys. Rev. Lett. **58** (1987) 980
29) R.S. Nikam, P. Ring and L.F. Canto, Phys. Lett. **185B** (1987) 269
30) N. Rowley, K.F. Pal and M.A. Nagarajan, Phys. Lett. **201B** (1988) 187
31) M. Hasegawa, S. Tazaki and K. Muramatsu, Phys. Lett. **226B** (1989) 1
32) L.F. Canto, R. Donangelo, R.S. Nikam and P. Ring, Phys. Lett. **192B** (1987) 4
33) C. Price, H. Esbensen and S. Landowne, Phys. Lett. **197B** (1987) 15
34) J. de Boer, C.H. Dasso and G. Pollarolo, Z. Phys. **A335** (1990) 199
35) Y.R. Shimizu, E. Vigezzi and R.A. Broglia, Phys. Lett. **198B** (1987) 33
36) R.A. Broglia, O. Hansen and C. Riedel, Adv. Nucl. Phys. **6** (1973) 287
37) N. Kato, private communications