A NUCLEAR SQUID: DIABOLIC PAIR TRANSFER IN ROTATING NUCLEI☆

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A new unexpected behavior of pair transfer matrix elements in superfluid rotating nuclei is predicted. With increasing angular velocity they drop to zero, change their sign and in some cases even oscillate between positive and negative values. This effect is related to diabolical points in rotating quasiparticle spectra and is closely analogous to the DC-Josephson effect in superconductors in the presence of a magnetic field.

As early as 1960, Mottelson and Valatin [1] predicted a sharp collapse of the pairing correlations in nuclei at high angular velocities. Theoretical investigations going beyond the mean field approximation, taking into account fluctuations, show that in real nuclei such an effect is probably completely smeared out [2,3], and to date it has not yet been experimentally observed. We have to distinguish between the order parameter for this phase transition, the parameter Δ, and the gap in the spectrum of the rotating quasiparticles. At high angular momenta we have alignment processes, which reduce the gap in the quasiparticle spectrum and often cause gapless superconductivity [4]. In these cases it is very difficult to deduce information about the order parameter Δ from the spectra.

Within the last few years pair transfer matrix elements in rotating nuclei have become an object of great interest [5,6], in particular those for the transfer of a pair of particles coupled to angular momentum zero $[S^{\dagger}=(c^{\dagger}c^{\dagger})_{I=0}]$. At a first glance they seem to be especially suitable for measurement of pairing correlations, because they are defined as

$$P=\langle A+2|\, S^{\dagger}\, |A\rangle\,, \tag{1}$$

and are therefore related to the order parameter

$$\Delta/G=\langle \Phi|\, S^{\dagger}\, |\Phi\rangle \tag{2}$$

in an analogous way as the "spectroscopic" quadrupole moment in deformed nuclei is to the "intrinsic" deformation parameter. In the ground state of superfluid nuclei these matrix elements are strongly enhanced by pairing correlations and therefore provide direct information on the degree of nuclear superfluidity. At finite spins, however, as we have already recently reported in a short note [7], they behave in some cases rather independently of the pairing parameter Δ: they can vanish at relatively low angular velocities, can change their sign and may even sometimes oscillate as a function of the angular velocity, an effect which we have called "diabolic pair transfer". In this letter we study this effect in more detail. In particular, we investigate its dependence on the strength of the pairing correlations, i.e., on the size of the order parameter Δ.

In the simple BCS approximation, which is valid

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for time reversal invariant wave functions, i e , at spin zero, the pair transfer matrix element (1) is directly proportional to the gap parameter Δ In the rotating nucleus, however, time reversal symmetry is broken, and we have to use full Hartree–Fock–Bogoliubov (HFB) theory Besides the occupation probabilities we have to consider the influence of the Coriolis field on the single particle wave functions in the form of alignment processes

For our investigations we use number projected HFB theory in the rotating frame, a theory which has since many years turned out to be very successful for the description of high spin phenomena [8–11] We work with the Kumar–Baranger hamiltonian [12] and study the nucleus ^{168}Hf

As a first step we solve the cranked number projected HFB equations for this hamiltonian (we also carry out a calculation without number projection in order to study the difference) These calculations are identical to those reported in ref [2] and details are given there For each angular momentum I we thus have a self-consistently determined gap parameter $\Delta(I)$ The cranking frequency ω and chemical potential are adjusted to the average angular momentum and the average particle numbers For the case without number projection (fig 1a) we find a pairing collapse at spin $24\hbar$, however, with number projection the parameter Δ decreases rather smoothly

In the second step these wave functions are used to calculate the number projected transfer matrix elements (1) using the techniques described in Appendix E of ref [13] in some detail

In fig 1 we present the surprising result, that with increasing angular momentum the "spectroscopic" pair transfer matrix element P and the "intrinsic" order parameter Δ/G behave very differently In both calculations the pair transfer matrix elements decrease more rapidly than the order parameter, they even go through zero for critical angular momenta (regions of "diabolic pair transfer") changing their sign afterwards In contrast to expectation this behavior is more dramatic in the case of a variation after projection, where the parameter Δ/G behaves very smoothly In both cases the sign changes occur considerably earlier than the pairing collapse in the simple HFB calculation

The vanishing of a similar quantity, namely the matrix element for particles transferred from the

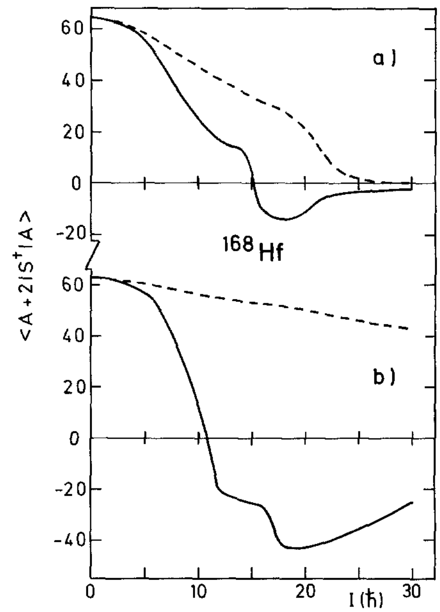

Fig 1 Pair transfer matrix elements P as defined in eq (1) (full lines) and intrinsic gap parameters Δ/G (dashed lines) in the nucleus ^{168}Hf as function of the angular momentum The gap parameter is taken from the self-consistent calculation without (a) and with (b) number projection in ref [2]

intruder shell to the remaining orbits of the core, has already been observed and discussed by Faessler [14] in 1980 Another related quantity, the matrix element of the time reversal operator between two rotating orbits with opposite signature, also shows zeros This quantity had been investigated in ref [15] for entirely different reasons, however, the zeros had been overlooked, and only oscillations around a value different from zero were discussed.

In order to reach a better understanding of this strange behavior, we discuss in fig 2 a number of cases with a constant gap parameter (2) obtained from cranked shell model calculations in the same model with properly adjusted pairing fields Only unprojected results will be shown, because it turns out, that for the calculation of the pair transfer matrix elements (1) number projection is not important as long as we take into account the change in the chemical potential between the nucleus $|A\rangle$ and $|A+2\rangle$ For a constant gap parameter Δ we find that the absolute size of the transfer matrix element P stays essentially constant as a function of the angular momentum, and, as in the case of spin zero, it is pro-

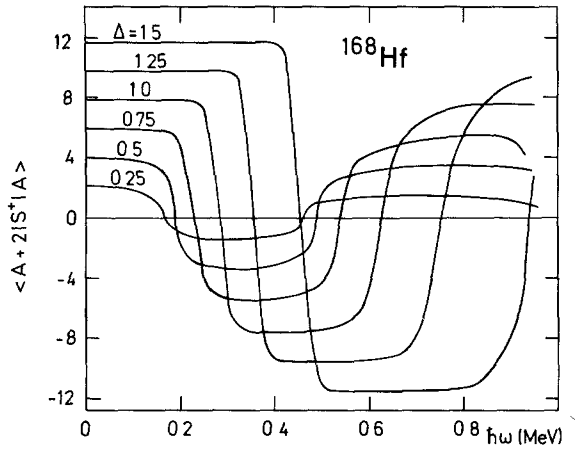

Fig 2 Pair transfer matrix elements in the nucleus ^{168}Hf as a function of the angular velocity ω Constant values of the gap parameters $\Delta = G\langle \Phi | S^{\dagger} | \Phi \rangle$ are used The matrix elements are calculated without number projection

portional to the gap parameter There are, however, certain exceptional regions of "diabolic pair transfer" In these relatively narrow regions the matrix elements change rapidly, their sign going through zero at critical angular velocities With increasing spin we observe several such sign changes, i e , we find an oscillating behavior

As we have now seen the behavior of the realistic calculation in terms of simulations with constant gap parameters, we would like to go a step further and try to clarify the diabolic behavior itself It turns out that pairing correlations increase the absolute size of the transfer matrix element by up to an order of magnitude in the rare earth region (more precisely a factor $1/G = A/22$) and they also affect the diabolic frequencies, as more detailed calculations [16] show However, they do not change the qualitative structure It therefore becomes interesting to study the behavior of the pair transfer matrix elements for $\Delta = 0$

It turns out that the intruder $i_{13/2}$ shell plays an important role in the understanding of this effect Actually the effect occurs only in those realistic cases where the transferred pair occupies orbits in the intruder shell [16] with a large probability In fig 3 we therefore present a calculation for a prolate $i_{13/2}$ shell with and without pairing The different curves numbered by $\nu = 1, 2, 3$ correspond to a situation, where we have $(\nu - 1)$ pairs in the $i_{13/2}$ shell of the

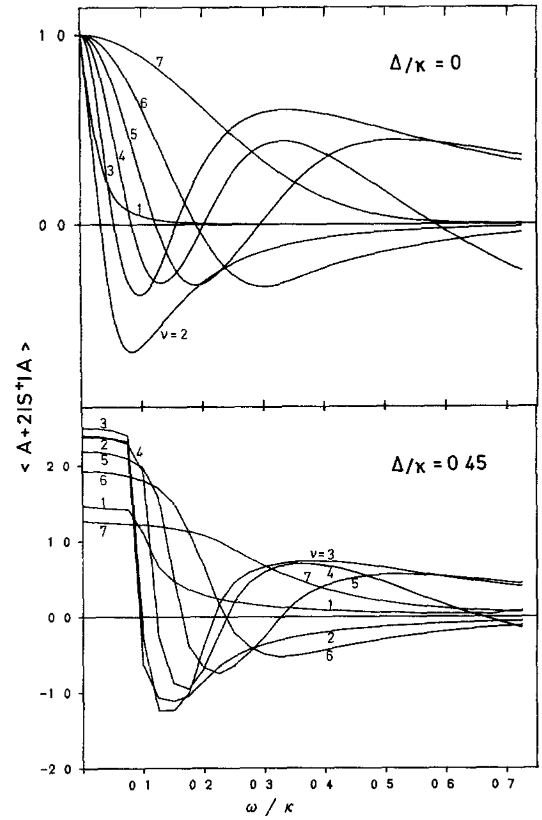

Fig 3 Pair transfer matrix in a prolate $i_{13/2}$ shell with and without pairing as a function of the angular velocity ω (in units of the parameter κ of ref [17]) Further details are given in the text

nucleus having particle number A and ν pairs for the particle number $A+2$

Let us first discuss the case without pairing Here the difference between the two nuclei is a pair of nucleons in the $K=1/2$ level for $\nu=1$, $K=3/2$ level for $\nu=2$, and so on For each pair the individual nucleons have different signature We find their single particle wave functions $D^{\pm}$ by diagonalizing the matrix $\epsilon_k \mp \omega j_x$ The corresponding "rotating particles" are created by the operators $a_{\bar{K}}^{\dagger}$ and $a_K^{\dagger}$ The wave function $|A+2\rangle$ is in this case obtained from the wave function $|A\rangle$ by adding two particles in the orbits K and $\bar{K}$ The pair transfer matrix element P turns out to be the spatial overlap of the two wave functions with different signature

$$P = \langle A+2 | S^{\dagger} | A \rangle = \langle A | a_{\bar{K}} a_K S^{\dagger} A | \rangle = \sum_m D^{-}_{m\bar{K}} D^{+}_{mK} , \quad (3)$$

a quantity which has been calculated in a different context in ref [15] As we increase the angular velocity this spatial overlap changes and it depends on special properties of the orbit, whether it vanishes at some critical velocities and how many times it changes sign

The case $\nu=1$ corresponds to $K=1/2$ orbits In this case we have no "diabolic region" With increasing angular velocity this overlap is only reduced, it vanishes in the limit $\omega \to \infty$ For the case $\nu=2$, which corresponds to $K=3/2$ orbits, we observe a rapid decrease and one sign change, i e , one diabolic region This goes on up to the middle of the shell ($K=7/2$) with three diabolic regions From there on the situation is reversed

This behavior can be understood (for details see ref [16]) in the simpler model of a one-dimensional harmonic oscillator with a pushing operator vp_x instead of ωj_x in our three-dimensional case This operator causes a shift in momentum space, and therefore the pair transfer matrix element is just the overlap of two wave functions with the same oscillator quantum number in two oscillator wells shifted in momentum space The ground state ($n=0$) has no node and the overlap decreases monotonically with increasing pushing velocity The first excited state ($n=1$) has one node and therefore we find one critical velocity where the overlap vanishes In general, we have for quantum number n n nodes and n points More detailed investigations [16] show that the state with $n=0$ in the oscillator case corresponds to the $K=13/2$ orbit and increasing n corresponds to decreasing K-quantum numbers Up to $K=7/2$ we indeed observe an increasing number of nodes However, in the lower half of the shell this simple picture no longer applies Here there is an increasing amount of alignment, which causes the orbits to rapidly approach eigenstates of j_x with different quantum numbers, so that the spatial overlap rapidly decays to zero before the oscillations manifest themselves

The second part of fig 3 shows the calculations in the $i_{13/2}$ model with a constant pairing potential Here we find the same behavior as in the case of $\Delta=0$ The pair transfer is, as expected, enhanced, and the diabolic regions are shifted to somewhat higher angular velocities This indicates, that the effect of diabolic pair transfer is not washed out by pairing correlations, which is also evident from the calculations in realistic situations in figs 1 and 2

The oscillating behavior of the pair transfer matrix element as a function of the angular velocity has a close analogy to the oscillating behavior of the electric current in Superconducting Quantum Interference Devices (SQUID) as a function of the magnetic field, the DC-Josephson effect [18] In fact, it is well known that the Coriolis field and the magnetic field have the same underlying mathematical structure

It had already been observed in earlier investigations that the *oscillating behavior of backbending* [17] was connected with the vanishing of the interaction matrix element between particles in the intruder orbits and particles in the core [14] It turned out that the node of this matrix element corresponds precisely to the first zero in the pair transfer matrix elements discussed in the previous section Since we now understand why these zeros occur, we are in a position to give a simple interpretation of this effect

We recall that the oscillating behavior of backbending is connected with the vanishing of the interaction between the ground state band and the s-band of two aligning particles for certain particle numbers, ie , for certain chemical potentials λ In the cranking approximation the s-band is described as a two-quasiparticle band and for each particle number, i e , for each value of the chemical potential λ, the interaction $V(\lambda)$ between this band and the ground state band is obtained as the minimum of the two-quasiparticle energy E_1+E_2 It turns out that for certain values of λ there are sharp level crossings in the spectrum of the cranked shell model hamiltonian

$$\begin{pmatrix} \epsilon_K-\lambda-\omega j_x & \Delta \\ \Delta & -(\epsilon_K-\lambda-\omega j_x) \end{pmatrix} \qquad (4)$$

whose eigenvalues are the quasiparticle energies E_K These sharp level crossings are called *diabolical points* [19]

In order to understand the connection of these *diabolical points* to the vanishing of our transfer matrix elements more clearly and in order to show why we call the regions where the transfer matrix elements vanish *regions of diabolic pair transfer*, we decompose the diagonalization of the cranked shell model hamiltonian (4) into two steps We first diagonalize the submatrices $\epsilon_K - \lambda^{\mp}\omega j_x$ and find the eigenvalues $\epsilon_K^{\pm}(\omega) - \lambda$ and the eigenfunctions $D^{\pm}(\omega)$ In the second step we work in this basis, i e , in a basis of a deformed rotating model without pairing In this basis the upper left and the lower right corner of the matrix (4) are diagonal, but the upper right and the lower left corner, which were originally diagonal, are no longer diagonal They contain a multiple of the transfer matrix $T = (D^-)^{T} \cdot (D^+)$, whose diagonal matrix elements are the pair transfer matrix elements (3) For small values of ω this matrix is still close to diagonal, and since the first zeros of the pair transfer matrix elements in fig 3 lie at relatively small ω-values, we adopt in the following the approximation of neglecting the off-diagonal matrix elements of this matrix This approximation has already been used in the literature [20], it is certainly only an approximation and it fails for large ω-values, but it gives us a simple possibility to understand the connection between the oscillating behavior of backbending and the first zero of our transfer matrix elements, since it gives us an analytic expression for the eigenvalues of the matrix (4), i e , for the quasiparticle energies We find for the sum of the two lowest eigenvalues

$$E_1 + E_2 = 2\{[(\epsilon^+ + \epsilon^-)/2 - \lambda]^2 + \Delta^2 T_{\bar{K}K}^2(\omega)\}^{1/2} \qquad (5)$$

In order to find the *diabolical points* in this approximation the square root has to vanish, i e , for fixed λ we have to look for the minimum of this square root It lies at the points where the transfer matrix element $T_{\bar{K}K}$ vanishes for the first time For these minima we have to vary the chemical potential λ until the first part vanishes This occurs for

$$\lambda = [\epsilon^+(\omega) + \epsilon^-(\omega)]/2 \qquad (6)$$

In table 1 we give a list of the diabolical points found in this approximation and compare them with the exact diabolical points obtained from an exact diagonalization of eq (4) We find good agreement, which means a close connection to *diabolic pair transfer* discussed in this paper and the well-known oscillating behavior of backbending

It also explains the well-known fact that for an arbitrary j-shell there are $j - 3/2$ points, where the interaction matrix element $V(\lambda)$ vanishes Since these points correspond precisely to the first diabolic points of the pair transfer matrix elements without pairing, we cleary understand this number There are $j + 1/2$ pairs with the quantum numbers $K = 1/2, \ \ , j$ The lowest level has no diabolic point, because there the large matrix element $\langle 1/2 | j_x | -1/2 \rangle$ causes immediate alignment and no diabolical points are possible The highest level $K = j$ has in j_x-space a wave function close to a gaussian with no node The corresponding operlap has no zero We therefore have only diabolic points for the $j - 3/2$ levels with $K = 3/2, \ \ , j - 1$

Table 1
Diabolical points in the cranked shell model hamiltonian (4) for an exact solution and an approximate solution of eq (6)

K	Cranking frequency ω/κ		Chemical potential λ/κ	
	exact	approx	exact	approx
3/2	0 085	0 033	−0 936	−0 824
5/2	0 097	0 055	−0 534	−0 556
7/2	0 112	0 084	−0 151	−0 170
9/2	0 158	0 124	0 388	0 355
11/2	0 230	0 190	1 122	1 057

The question whether this nuclear SQUID can be observed experimentally, is certainly very interesting Since, as we have seen, the diabolic pair transfer is closely connected with the oscillating behavior of backbending, which has been clearly seen in experiment, there now already is indirect experimental evidence for this effect On the other hand, pair transfer matrix elements in rotating nuclei can be measured directly in Coulomb excitation with heavy ions In order to obtain not only absolute values but also relative phases of these matrix elements, one should exploit the interference between a transfer before and after the diabolic region in a similar way as interference has been used to determine the sign of the quadrupole moment by the reorientation effect [21] In this case it would be a "reorientation in gauge space"

So far, measurements of the pair transfer matrix elements at high spins has had the purpose of detecting the often predicted pairing collapse In this article we have shown that in rotating nuclei this matrix element is no longer proportional to the gap parameter Δ and that it can vanish at angular velocities considerably lower than that at which the pairing collapses In fact, a new effect in its own right has been found, which has nothing to do with the pairing phase transition an oscillating behavior of the pair transfer matrix element, which is analogous to the DC-Josephson effect in condensed matter physics It will be of importance, therefore, to explore it experimentally

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