ON THE POSSIBILITY OF OBSERVING EXPERIMENTALLY DIABOLIC PAIR TRANSFER IN ROTATING NUCLEI \star

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Received 28 January 1987

We investigate the cross section for two-nucleon transfer reactions to rotational states by heavy projectiles. In particular we study the influence of the recently predicted unexpected behavior of the diabolic pair transfer amplitude. Dramatic reduction of the probabilities for two-nucleon transfer in connection with inelastic Coulomb excitation are found in the angular momentum region, where the pair transfer matrix element changes its sign.

Within the last few years the pairing degree of freedom in rotating nuclei has found renewed interest. Since at high spins it is very difficult to gain information on the strength of pairing correlations from experimental spectra [1], it has been proposed to investigate transfer reactions of nucleon pairs between rotating superfluid nuclei [2,3]. In the ground state they are strongly enhanced by pairing correlations, because in this case the spectroscopic amplitude $\langle A+2|(a^{\dagger}a^{\dagger})_{L=0}|A\rangle$ is proportional to the gap parameter Δ .

Recently the behavior of this amplitude $\langle A+2, I|(a^{\dagger}a^{\dagger})_{L=0}|A, I\rangle$ has been investigated as a function of the angular momentum and an entirely new and unexpected phenomenon has been predicted [4]: "diabolic pair transfer", which means that this matrix element is no longer always proportional to the pairing gap parameter, but shows, in special cases, an oscillating behavior around zero. This phenomenon is in close analogy to the DC-Josephson effect in solid superconductors in the presence of a magnetic field. It is also closely connected to the oscillating behavior of backbending in rotating nuclei [5]).

So far diabolic pair transfer has not been observed experimentally. We therefore investigate in this paper its influence on the probability of a pair transfer in heavy ion reactions, where the deformed target nucleus is excited rotationally via inelastic Coulomb excitation and the transfer takes place at a relatively high spin. So far one has assumed [6] that the spectroscopic amplitude

$$a_{\text{spec}}(I) = \langle A+2, I | (a^{\dagger}a^{\dagger})_{L=0} | A, I \rangle \tag{1}$$

is proportional to the gap parameter \varDelta even at high spins, and has used a phenomenological ansatz for its angular momentum dependence. We now use microscopically calculated spectroscopic amplitudes. Our investigations are therefore split into two parts, a structure part, where these amplitudes are calculated in number projected cranked Hartree –Fock–Bogoliubov theory and a reaction part, where the transfer probability connected with Coulomb excitation to a certain spin value is calculated in the sudden limit of backward scattering. As an example we use the deformed target nucleus ¹⁶⁰Dy and the spherical projectile ¹¹⁸Sn.

Following the arguments of ref. [2] we assume no excitation of the spherical projectile and restrict our calculations to head-on collisions, which correspond to near-backward scattering in the center-of-mass

0370-2693/87/\$ 03.50 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

^{*} Work supported by the Alexander von Humboldt Stiftung, the Bundesministerium f
ür Forschung und Technologie, the CNPq and FINEP (Brasil).

system. In this case the rotation of the deformed nucleus and the relative motion between target and projectile occur in the same plane. Considering pure Rutherford scattering and neglecting the rotation of the target during the collision (sudden approximation) we obtain the probability for Coulomb excitation of the target nucleus to the state with angular momentum I in coincidence with backward projectile scattering as

$$P(I) \approx |S_I|^2 , \qquad (2)$$

where the S-matrix element is given by [7]

$$S_{I} = \sqrt{2I+1} \int_{0}^{1} \mathrm{d}x F(x) P_{I}(x) \exp[-\mathrm{i}\frac{4}{3}q_{2}P_{2}(x)] .$$
(3)

 q_2 is the quadrupole coupling strength, $P_I(x)$ are Legendre polynomials and F(x) is the form factor. It depends on $x = \cos \chi$, where the angle χ describes the orientation of the deformed target with respect to the incoming particle. For simple Coulomb excitation without transfer we have F(x) = 1 and the expression (3) is quantum mechanically exact [8] in the limit we are considering here.

The problem of transfer in connection with Coulomb excitation is in principle much more complicated, because we have different nuclei in the entrance and the exit channels. One also has to consider absorption due to the nuclear optical potential. However, in this letter we are more concerned with the effect of diabolic pair transfer. We therefore neglect in analogy to ref. [2] recoil effects and use a transfer form factor F(x) based to a large extent on semiclassical arguments. Only the spectroscopic amplitude (1) is calculated microscopically.

We assume that the transfer occurs at the distance of closest approach $r_c = Z_p Z_t e^2 / E_{cm}$. The form factor F(x) for transfer is in this approximation a product of three factors:

$$F(x) = a_{\text{tunl}}(x)a_{\text{abs}}(x)a_{\text{spec}}(I) .$$
(4)

The factor $a_{tunl}(x)$ is the probability amplitude for a pair of neutrons found at the nuclear surface to tunnel through the potential barrier associated with the orientation χ . If the particle transfer is a simple barrier penetration this amplitude is expected to depend exponentially on the distance between the two surfaces,

$$a_{\text{tunl}}(x) \propto \exp\{-\alpha [r_{\text{c}} - R_{\text{p}} - R_{\text{t}}(x)]\},\qquad(5)$$

where R_p is the radius of the projectile and $R_t(x)$ is the radius of the deformed target, which depends on the orientation. The slope parameter $\sigma = 0.41$ (fm⁻¹) for this exponential dependence is taken from the recent experimental values for the ¹⁶²Dy(¹¹⁶Sn,¹¹⁸Sn)¹⁶⁰Dy reaction [9].

The factor $a_{abs}(x)$ is associated with the absorption by the imaginary part of the nucleus-nucleus potential, and is also related to the orientation of the system. From ref. [10] we find

$$a_{\rm abs} = \exp\left(-\frac{1}{\hbar} \int_{-\infty}^{+\infty} W(t) \, \mathrm{d}t\right), \tag{6}$$

with

$$W(t) = W_0 \exp\{[R_t(x) - r(t)]/a_W\}.$$
 (7)

We approximate the Coulomb trajectory around the point of closest approach r_c up to second order in the time t and obtain

$$a_{\rm abs}(x) = \exp\{-(W_0/\hbar)(2\pi\mu r_c^2 a_W/Z_p Z_t e_1^2)^{1/2} \\ \times \exp[(R_p + R_t(x) - r_c)/a_W]\}, \qquad (8)$$

where μ is the reduced mass and where we used the values $W_0 = 25$ (MeV) and $a_w = 0.54$ (fm) of ref.[10].

The factor a_{spec} in eq.(4), whose influence on the transfer probability we want to investigate, is the spectroscopic amplitude which is proportional to the pair transfer matrix element defined in eq. (1). In our approximation it does not depend on the orientation of the target nucleus and can therefore be taken out of the integral (3). It depends, however, on the angular momentum I_c at which the transfer takes place. For the head-on case and in the sudden limit we are considering, we find, that the transfer takes place at the spin $I_c = I/2$. This means that on the incoming part of the trajectory the projectile transfers the same amount of angular momentum to the target as on the outgoing part. We therefore find for the full transfer probability

$$P_{\rm tr}(I) = P_0(I) |a_{\rm spec}(I/2)|^2 , \qquad (9)$$

where $P_0(I)$ is the transfer probability calculated without the inclusion of the new matrix element.

The numerical evaluation of the amplitude a_{spec} in eq. (1) is done in cranked and number projected Hartree-Fock-Bogoliubov theory, which has been proven to be a very reliable tool for a microscopic description of high spin phenomena [11]. Since diabolic pair transfer depends very sensitively on the single-particle structure in the neighborhood of the Fermi surface, it is essential to use two different intrinsic wave functions for the nuclei with the particle numbers A and A+2, i.e. in our case for the nuclei ¹⁶⁰Dy and ¹⁶²Dy.

In a first step we calculated the dependence of the crancking frequency $\omega(I)$ and the gap parameter $\Delta(I)$ for the two nuclei ¹⁶⁰Dy and ¹⁶²Dy (see table 1). We used the configuration space and the residual interaction of Kumar and Baranger [12], i.e. a pairing-plus-quadrupole force acting in a valence space of two major shells for each type of particle. The deformation parameters $\beta = 0.310$ for ¹⁶⁰Dy and $\beta = 0.320$ for ¹⁶²Dy were kept fixed and the cranked Hartree–Fock–Bogoliubov equations were solved after exact number projection by the gradient method [13, 14] as described in detail in ref. [1].

In the second step these wave functions are used for the calculation of the transfer amplitudes a_{spec} in eq. (1). Details of these rather lengthy calculations

are given in ref. [15]. In fig. 1 these amplitudes are presented as a function of angular momentum. Diabolic pair transfer is observed between spin I = 10 and 12 \hbar , where the amplitude vanishes and changes sign. The gap parameter $\Delta(I)$, which is also given in fig. 1 shows in this region only a small reduction. Clearly diabolic pair transfer has nothing to do with the pairing collapse predicted by Mottelson and Valatin [16] at much higher angular momenta, which is probably smeared out completely by fluctuations [1,17]. As discussed in ref. [4]; the pair transfer here shows a diabolic behavior, because the pair is transferred to a large part to the Nilsson level $5/2^{+}[642]$, which belongs to the $\nu i_{13/2}$ intruder orbit, and which is expected to show sign changes of the spectroscopic amplitude as a function of the angular momentum I, i.e. two diabolic regions. The second diabolic region is not shown in fig. 1, because it lies at relatively large angular momenta and there is presently no hope to observe it experimentally.

In fig. 2 we show the probabilities P(I) to transfer a pair of neutrons from the projectile nucleus ¹¹⁸Sn and to the nucleus ¹⁶⁰Dy and to excite the nucleus ¹⁶²Dy obtained in this way up to the final spin *I* at various center-of-mass energies. The dotted line corresponds to the simple Coulomb excitation without transfer. It shows the well-known interference pattern. In the dashed line absorption and transfer by

Table 1

The cranking frequency $\omega(I)$ and the gap parameter $\Delta(I)$ for the nuclei ¹⁶⁰Dy and ¹⁶²Dy obtained from a self-consistent solution of the cranked Hartree-Fock-Bogoliubov equations after exact number projection and the spectroscopic amplitude $a_{spec}(I)$ defined in eq. (1).

$I(\hbar)$	¹⁶⁰ Dy		¹⁶² Dy		$a_{\rm spec}(I)$
	$\omega(I)$	$\Delta(I)$	$\omega(I)$	$\Delta(I)$	
0	0.000	0.871	0.000	0.827	6.307
2	0.075	0.861	0.071	0.817	6.219
4	0.131	0.835	0.125	0.794	5.997
6	0.170	0.790	0.170	0.756	5.566
8	0.189	0.731	0.205	0.706	4.851
10	0.196	0.669	0.231	0.653	3.991
12	0.202	0.611	0.250	0.595	- 1.472
14	0.209	0.559	0.261	0.543	-2.359
16	0.223	0.517	0.266	0.500	-2.775
18	0.242	0.484	0.271	0.470	-2.945
20	0.264	0.454	0.278	0.440	-2.917
22	0.280	0.421	0.285	0.298	-2.754
24	0.291	0.391	0.292	0.381	-2.626
26	0.301	0.359	0.301	0.350	-2.436



Fig. 1. Pair transfer matrix elements as defined in eq. (1) for the nucleus ¹⁶⁰Dy (full line) and the gap parameter $\Delta(I) = G\langle A | (a^{\dagger}a^{\dagger})_0 (aa)_0 | A \rangle^{1/2}$ (dashed line) as a function of the angular momentum *I*.

tunneling is included in the conventional way and in the full line the spectroscopic amplitude of fig. 1 is taken into account.

At the lowest center-of-mass energy $(E_{\rm cm}=310$ MeV in fig. 2a) absorption and tunneling changes the probability distribution only very little. There is however a big effect coming from the spectroscopic amplitude at angular momenta larger than 20 \hbar . This can easily be understood by considering that at spin values of 10 \hbar the spectroscopic amplitude is considerably reduced by the effect of diabolic pair.

In the other parts of fig. 2 we show the same reaction at higher center-of-mass energies. In pure Coulomb excitation we observe that the interference pattern and in particular the angular momentum cut off is shifted to larger values of *I*. In addition, with increasing energy, the absorption becomes more and more important. The absolute probability is considerably reduced and the Coulomb interference pattern is smeared out. Diabolic pair transfer in all cases leads to a step-like reduction of considerable magnitude. In contrast to the interference pattern of Coulomb excitation, this reduction due to diabolic pair transfer does not depend on the centre-of-mass energy. This fact should allow a clear-cut distinction between a probable interference minima produced by the Coulomb excitation and the reduction produced by diabolic pair transfer.

Of course the experiments proposed in this letter can reveal only a reduction of the transfer probability, i.e. they will not be able to provide direct evidence of the sign change of the spectroscopic amplitude in the diabolic region. In order to observe not only absolute values, but also relative signs, one needs an interference effect. One such possibility would be to observe pair transfer coming coherently from two different trajectories. Investigations in this directions are in progress.

We would like to express our gratitude for illuminating discussions with J. de Boer, Ch. Lauterbach, H. Massman. Three of us (L.F.C., R.D. and P.R.) would like to thank D.R. Bes and the Comision Nacional de Energia Atomica in Buenos Aires, for having invited us to the IX Workshop of Nuclear Physics in Buenos Aires, where part of this work was done.



Fig. 2. Probabilities for pair transfer from the projectile nucleus ¹¹⁸Sn to the target nucleus ¹⁶⁰Dy in connection with Coulomb excitation at a scattering angle of 180° to the level with angular momentum *I* on the yrast line of the nucleus ¹⁶²Dy at various center-of-mass energies E_{cm} . The different steps of the calculation ar shown; the dotted line gives simply inelastic Coulomb excitation (eq. (3), F(x) = 1), the dashed line gives the conventional results for transfer with inclusion of absorption and tunneling ($F(x) = a_{tunl}(x)a_{absp}(x)$) and the full line includes the microscopic spectroscopic amplitude $a_{spec}(I/2)$ as defined in eq. (1). The probabilities are normalized to 1 at angular momentum I=0.

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