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Study of the lowest 2^+ excitations and $B(E2)$ transition strengths in relativistic QRPA for Sn-, and Pb-isotopes

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Abstract

In a relativistic quasiparticle random phase approximation (RQRPA) we have calculated the excitation energies of the first 2^+ excited states and reduced $B(E2; 0^+ \to 2^+)$ transition rates for very neutron-defficient to neutron-rich long chain of Sn-isotopes with even mass numbers $A = 100$–136. In RQRPA approach this is the first such attempt and a very satisfactory agreement with the available experimental data is obtained. Furthermore, we predict a substantial rise of the $B(E2)$ values at neutron magic numbers 50 and 82, where the E_2 energy also goes up, rather substantially, compared to that for the immediate neighbours. This feature is quite similar to the already experimentally known behaviour of E_2 and $B(E2)$ around ^{208}Pb, which are also well reproduced in the present calculation.

With progress on experimental measurements towards nuclear drip line, it is of interest to make theoretical specroscopic studies of isotopic chains over large range of neutron and proton numbers. Recently Terasaki et al. [1] have investigated the E_2 and $B(E2)$ properties of some isotopic chains around ^{132}Sn in a non-relativistic QRPA approach using quadrupole + pairing model Hamiltonian. In their scheme of calculations they predict an increase of $B(E2)$ value at $A = 132$ for Sn, but it is found to disappear with the use of Nilsson spherical single particle energies (spe). There are also some shell model studies for a few nuclei treating ^{132}Sn as an inert core and employing empirical single particle energies and two-body matrix elements [2,3]. In the present context we will discuss later on about the shell model results on ^{134}Sn.

Recently Ring et al. [4,5] have derived fully self-consistent (ground and excited state properties calculated in the same framework) relativistic RPA and QRPA equations based on the relativistic Hartree–Bogoliubov (RHB) mean field within the 'no-sea' approximation. For the pairing (particle–particle) channel the finite range non-relativistic Gogny force 'D1S' [6,7] is used. So far these have been applied mainly to the study of the collective multipole excitations in

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some neutron-rich isotopes like that of oxygen and tin etc. For example, Fig. 10 in Ref. [5] shows a good agreement between calculated isovector giant resonance (GDR) energies and the available experimental values for tin isotopes. The greatest advantage of this RHB + RQRPA scheme is that one does not have to assume an inert core and adjust parameters of the Hamiltonian from nucleus to nucleus or region to region of the periodic table. Encouraged with good results on multipole resonances, we now apply it to study the low lying vibrational states in spherical nuclei. Tin isotopes seem to provide the longest chain of isotopes to study the structural properties as a function of the neutron number. So, we have chosen for the present study Sn isotopes with $A = 100$–136, the isotopes with $A > 136$ are expected to be deformed in the ground state.

For the formalism of RQRPA we refer, as mentioned above, to the recent paper by Paar et al. [5]. As in the above reference, here too we have used the NL3 effective interaction for the relativistic mean field Lagrangian, and the pairing part of the Gogny force as a phenomenological pairing interaction. The NL3 set of parameters are particularly suited for Sn and Pb isotopes with improved isospin dependence [8]. For the basis space 20 harmonic oscillator shells are considered, though usually 16 shells are sufficient. The hole–particle (hp) (or two quasiparticle) energy cut-off = 180 MeV and hole–antiparticle (ha) energy cut-off = 1800 MeV have been used, which lead to the full QRPA dimension of about 3000–4000. These values of cut-offs in energy are rather high (see Ref. [5]).

Firts we present our mean field (RHB) results for some representative isotopes of lead with mass number $A = 202$–214 in Table 1, and for some tin isotopes with $A = 112$–132 in Table 2. The binding energy per particle (BE/A) listed under column 'RHB' is compared with the experimental values and that calculated in the non-relativistic approach with the use of Gogny force denoted by 'Gogny'. Clearly RHB results are, throughout, in a good agreement with the experimental data. The pairing energy, E_{pair} (computed as trace of the pairing interaction with the pairing density [5]) corresponding to 'RHB' are compared with the value 'Gogny' obtained in the non-relativistic calculation (here noted from Ref. [7]). Again the 'RHB' numbers are in overall good agreement with the 'Gogny' ones. Similar results are obtained for the tin isotopes regarding the binding energies (see Table 2). The pairing energies show some differences for the lighter tin isotopes. However, it may be noted that the pairing energy is not a directly measureable quantity. Also here we do not want to indulge in playing with parameters.

Table 1
RHB results on binding energy per particle and pairing energy for Pb isotopes. Gogny indicates the values obtained by Gogny in the non-relativistic mean field approach, whereas RHB implies those obtained in the present RHB calculation

A	BE/A [MeV]			E_{pair} [MeV]	
	Expt	Gogny	RHB	Gogny	RHB
202	7.882	7.811	7.895	14.41	10.87
204	7.880	7.813	7.895	10.49	7.32
206	7.876	7.813	7.892	5.74	3.51
208	7.868	7.815	7.886	0.0	0.0
210	7.836	7.769	7.849	4.19	5.54
212	7.805	7.730	7.812	8.64	9.98
214	7.772	7.692	7.775	12.89	13.56

Table 2
As in Table 1, for Sn isotopes

A	BE/A [MeV]			E_{pair} [MeV]	
	Expt	Gogny	RHB	Gogny	RHB
112	8.513	8.419	8.505	19.04	11.49
114	8.523	8.433	8.515	19.29	11.74
116	8.523	8.437	8.516	19.39	12.87
118	8.517	8.431	8.512	19.15	14.09
120	8.505	8.417	8.502	17.92	14.77
122	8.489	8.400	8.488	16.76	14.73
124	8.467	8.378	8.471	14.94	13.86
126	8.444	8.353	8.450	12.50	12.10
128	8.418	8.326	8.426	9.45	9.34
130	8.388	8.300	8.399	5.48	5.41
132	8.355	8.283	8.369	0.00	0.00

Before presenting results on E_2 and $B(\text{E2})$ for Sn isotopes, we have checked it for $^{204\text{–}210}$Pb isotopes for which experimental data are available since a long time [9]. Normally a large value of $B(\text{E2})$ is a measure of collectivity, and at and near a shell closure the collectivity vanishes. However, it is known since long that in case of ^{208}Pb the E_2 energy becomes suddenly large (4.085 MeV) accompanied by an enhancement of $B(\text{E2})$ as well [9]. In order to keep the RPA matrix dimension around 3500 we have used 16 harmonic oscillator shells for the basis space and kept the values of the energy cut-offs as mentioned earlier. In Fig. 1 we see that our calculation reproduces the above men-

Table 3
QRPA results for ^{120}Sn compared with experimental data. The first row represents the experimental data, the second row is from Ref. [10] and the third row gives present RQRPA results

	E_{2+} [MeV]	B(E2)↑$[e^2b^2]$	E_{3-} [MeV]	B(E3)↑$[e^2b^3]$
Expt	1.17	0.202	2.4	0.09
Ref. [10]	2.3	0.105	3.0	0.077
Present	1.27	0.150	1.9	0.165

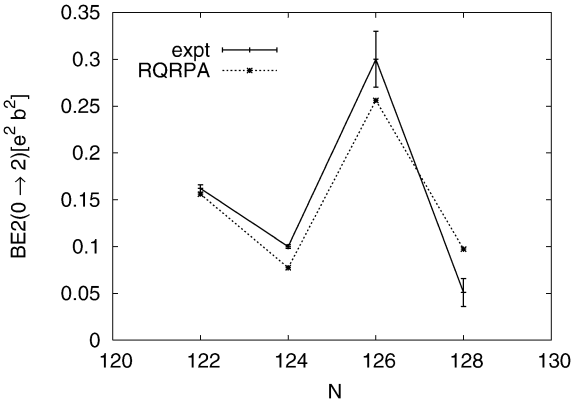

Fig. 1. B(E2)↑ values for $^{204–210}$Pb isotopes as a function of the number of neutrons. A sudden large jump in the B(E2) value for the doubly magic nucleus, ^{208}Pb is observed.

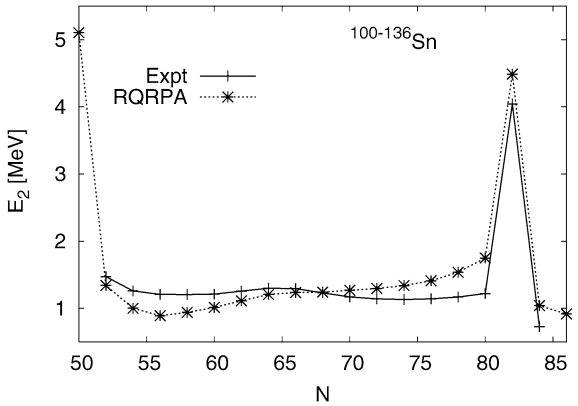

Fig. 2. Dependence of E_2 on the number of neutrons for Sn isotopes.

tioned behaviour of experimental data very well. The calculated values of E_2 energies come out to be 1.09, 1.04, 5.08, and 1.40 MeV as compared to the experimental values of 0.90, 0.80, 4.08, and 0.80 MeV for $^{204–210}$Pb, respectively. As far we are aware, no other theoretical calculation has produced better results.

In Fig. 2 we compare our calculated E_2 values for Sn isotopes with the available experimental data [9]. At $A = 132$ pairing collapses and we obtain $E_2 = 4.487$ MeV as compared to the experimental value of 4.041 MeV. Our predicted value for E_2 at $A = 100$ is 5.1 MeV. To appreciate the quality of these results let us compare, for example, our numbers for ^{120}Sn with recent results of Hagino and Sagawa [10] computed in 'continuum QRPA in coordinate space representation'. In Table 3 such a comparison is made. It is clear that our results are very satisfactory.

Now in Fig. 3 we compare our RQRPA B(E2)↑ results for Sn isotopes with the available experimental data [9,11,12]. Again in view of there being no free adjustable parameters and effective charges the agreement with data is very good. The most important feature is the increase of B(E2) at the magic numbers

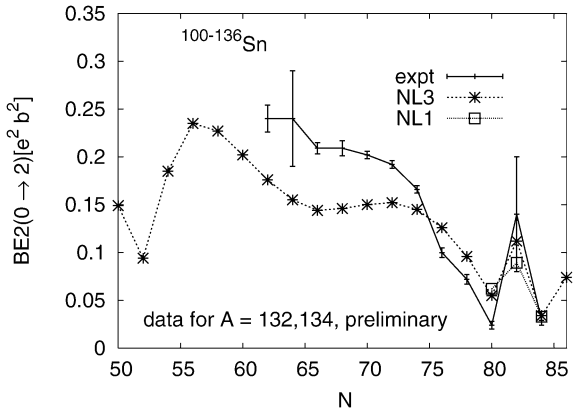

Fig. 3. Dependence of B(E2)↑ on N for Sn isotopes. Experimental data for $A = 132$ and 134 are still preliminary [15]. However, these values are consistent with the theoretical predictions and a large jump at the magic number $N = 82$ is like that for ^{208}Pb. The labels NL1 and NL3 indicate the calculations with the corresponding Lagrangian parameters.

$N = 50$ and 82 compared to that for the immediate neighbouring isotopes. Through private communication we have learned from Beene [15] that their recent preliminary data indicate such a behaviour.

In Fig. 13 of Ref. [1] separate proton and neutron contributions to the total normalization $\sum_{i,j}(X^2_{i,j} - Y^2_{i,j}) = 1$ are plotted to check the variation of these quantities with the mass number. At $A = 132$ a sudden rise in the proton contribution to about 10% of the total is seen, which is much insignificant at other mass numbers. We have analysed it somewhat differently in terms of two-quasiparticle configurations for the lowest value of E_2 in ^{132}Sn. For this purpose a few two quasiparticle configurations (i, j) along with their energies and X, Y components are obtained satisfying the condition $X^2_{ij} - Y^2_{ij} > 0.01$. It is found that corresponding to neutrons there are two hp configurations $(1h_{11/2}, 2f_{7/2})$ and $(1h_{11/2}, 1h_{9/2})$ with hp energies as 6.20 MeV and 7.16 MeV, and (X, Y) components as (0.78, 0.07) and (−0.23, −0.03), respectively. Similarly for protons there are two hp configurations $(1g_{9/2}, 1g_{7/2})$ and $(1g_{9/2}, 2d_{5/2})$ with hp energies as 6.16 MeV and 8.99 MeV and the (X, Y) components as (0.36, 0.03) and (−0.36, −0.07), respectively. Such proton (X, Y) components are much smaller in magnitude in case of the neighbouring isotopes which lead to smaller B(E2) values. In fact, no component for protons satisfies the condition $X^2 - Y^2 > 0.01$ in case of ^{130,134}Sn. On the otherhand, for example, in case of ^{120}Sn two proton configurations $(1g_{9/2}, 1g_{7/2})$ and $(1g_{9/2}, 2d_{5/2})$ satisfy this condition with combined contribution of 0.045 to unity.

Furthermore, in order to check if only NL3 force can produce such good results for the Sn-isotopes it may be worth repeating these calculations with, say, NL1 force parameters. However, at present we have checked it only for ^{130,132,134}Sn. At $A = 132$ the B(E2) value is relatively reduced, but its enhancement compared to the neighbouring isotopes is very much there. We obtain E_2(MeV) = 1.70, 3.72, 1.03 and $B(\mathrm{E2})(e^2b^2)\uparrow = 0.062, 0.089, 0.033$ for $A = 130$, 132, and 134, respectively. These B(E2) values are also depicted in Fig. 3 indicating that at $A = 130$ and 134 the NL3 and NL1 produce essentially same results.

It may be noted that there is no use of effective charges here and so, neutrons do not contribute to the B(E2) decay rates. On the otherhand in the shell model calculation for ^{134}Sn (a simple 2 neutron system) in Ref. [3] protons do not contribute to its B(E2) and a value of $0.0366e^2b^2$ is predicted with the use of neutron effective charge of $e_n = 0.72$. However, this value should be quite reliable as the effective charge e_n has been fitted to reproduce the $6^+ \to 4^+ B$(E2) decay rates of the same nucleus. We also obtain a similar value of $0.0335e^2b^2$ along with $E_2 = 1.04$ MeV which is closer to the experimental value of 0.725 MeV as compared to 1.2 MeV obtained in the shell model calculation [2]. For the shell model calculation the single particle energies are very important ingredients. In RPA calculation also these enter as difference between the particle and hole energies ($e_{ph} = e_p - e_h$) across the Fermi surface. Though, in an RHB calculation there is no handle on the single particle energies, it should be useful to see as to how these compare with the experimental ones. For this purpose we list in Table 4 some experimental [13] and calculated (RHB with NL3 as well as NL1 parameters) spe near the Fermi surface of ^{132}Sn. We notice that the calculated energy spectrum is somewhat spread (low density of states) compared to the experimental one. But for protons the lowest energy e_{ph} corresponding to the $(1g_{7/2}, 1g_{9/2})$ pair of ph states is almost the same, about 6.0 MeV, in experiment as well as theory. On the other hand for the neutron $(2f_{7/2}, 1h_{11/2})$ configuration the experimental value of e_{ph} is 4.97 MeV and 6.31 MeV and 5.15 MeV corresponding to the RHB calculation with NL3 and NL1 parameters, respectively. These energy differences are essentially reflected in the calculated values of E_2. It is 3.72 MeV with the use of NL1 parameters and 4.48 MeV with NL3 parameters.

Table 4
A few important experimental and RHB (with NL3 and NL1 set of parameters) spherical single particle energies (in MeV) near the $Z = 50$ and $N = 82$ shell closures in ^{132}Sn

	Proton				Neutron		
nlj	Expt	NL3	NL1	nlj	Expt	NL3	NL1
$3s_{1/2}$	2.80	5.73	4.69	$2f_{5/2}$	2.00	1.41	1.45
$1h_{11/2}$	2.79	4.62	4.35	$3p_{1/2}$	1.66	1.08	1.11
$2d_{3/2}$	2.44	4.66	3.67	$1h_{9/2}$	1.56	0.87	1.89
$2d_{5/2}$	0.96	2.90	1.93	$3p_{3/2}$	0.85	0.77	0.79
$1g_{7/2}$	0.0	0.0	0.0	$2f_{7/2}$	0.0	0.0	0.0
$1g_{9/2}$	−6.08	−6.16	−6.35	$2d_{3/2}$	−4.73	−7.42	−7.06
$2p_{1/2}$	−6.44	−7.16	−7.88	$1h_{11/2}$	−4.97	−6.31	−5.15
				$3s_{1/2}$	−5.06	−6.99	−6.76
				$2d_{5/2}$	−6.38	−9.14	−8.73
				$1g_{7/2}$	−7.16	−10.97	−9.48

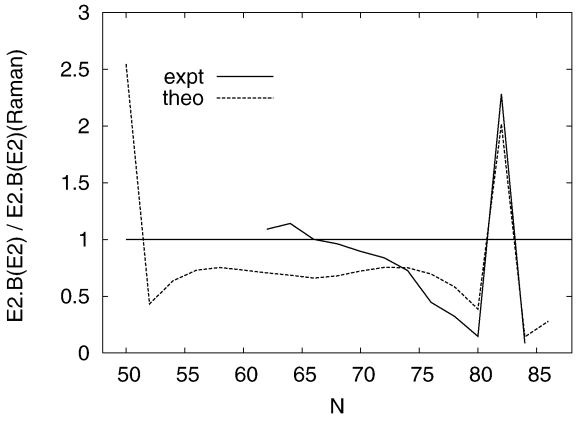

Fig. 4. Grodzins plot for Sn isotopes. Instead of Grodzins empirical formula, somewhat improved Raman's formula is used [9]. The formula has error bars like experimental data, but in the plot error bars have been ignored.

It may also be interesting to examine if the product $E_2(\text{keV}) \cdot B(\text{E2})\uparrow(e^2b^2)$ follows the Grodzins [9,14] empirical formula which could be used to predict $B(\text{E2})$ if E_2 is known. We look at it using, however, an improved version of the formula derived by Raman et al. [9]

$$E_2 \cdot B(\text{E2})\uparrow = 2.57 \pm 0.45 Z^2 A^{-2/3}. \quad (1)$$

Ignoring the error bars we display in Fig. 4 the ratio of experimental values (as well as that of theory) to the right-hand side of Eq. (1) (indicated as Raman's values). We see that the ratio is close to unity only at a few points for the experimental data ($A = 112$–120), and shows gradually strong deviations with increasing neutron number. The theoretical points are not close to unity (calculated $B(\text{E2})$ values are usually smaller than the experimental values), but remain close and parallel to it for a longer range of isotopes (should be a mean field effect) till about ^{124}Sn. Thus, strong deviations are found near and at the shell closures in theory as well as experiment (notwithstanding large error bars at $N = 82$).

Finally from these studies we would like to draw the following conclusions:

(1) For the first time a relativistic QRPA approach has been applied for the calculation of E_2 and $B(\text{E2})$ in spherical nuclei employing a global parameter set (NL3) for the Lagrangian along with Gogny's pairing interaction (D1S). The excitation energies of the lowest 2^+ states and the $B(\text{E2})$ decay rates are quite well reproduced for some Pb isotopes near the doubly closed shell nucleus ^{208}Pb, and for a long isotopic chain of Sn. For the range of Sn nuclei considered here the ratio N/Z varies from 1.0 to 1.7.

(2) From a close scrutiny of the numbers in Table 2, it seems desirable that the pairing strength should be somewhat increased for the lighter Sn isotopes ($A < 120$). But this would be a hard task without increasing the number of parameters of the Lagrangian.

(3) The NL1 and NL3 both parameter sets produce somewhat spread spe spectrum near the Fermi surface of ^{132}Sn as compared to the experimental one (see Table 4). So, there is a need to further adjust the Lagrangian parameters for a better description of the spectroscopic properties, though the results on E_2 and $B(\text{E2})$ presented here are remarkably good.

(4) As Fig. 4 shows, the Grodzins (or Raman's) formula [9] cannot be used to make predictions for the $B(\text{E2})$ values near and at shell closures.

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