Coulomb excitation of the K\(^{π}=8^{-}\) isomer in \(^{178}\)Hf

Abstract

The K\(^{π}=8^{-}\) isomer at 1147.4 keV in \(^{178}\)Hf was populated in a Coulomb excitation experiment with \(^{130}\)Te ions. The measured cross section for this isomer was 2.7±0.2, 4.3±0.3 and 7.5±0.4 mb for three beam energies below the Coulomb barrier, namely 560, 590 and 620 MeV. If one assumes that the projection of the total angular momentum on the symmetry axis, K, is only an approximate quantum number, the population of this isomer can be reproduced via direct E3 excitations of odd spin members of the isomeric band from the ground state band.

1 Introduction

In the region of well deformed nuclei between A=160 to A=180, \(^{178}\)Hf is unusual in that a number of high-K isomers have been reported [1, 2, 3, 4, 5, 6, 7], one is the K\(^{π}=8^{-}\) isomer with \(I_{1/2}=4.0\) s. It is a mixed two-quasiparticle state formed by about 36% of the \{\(μ[404]7/2+\nu[514]9/2\}\) configuration and about 64% of the \{\(n[624]9/2+n[514]7/2\}\) configuration. The 8\(^{-}\) isomer at an excitation energy of 1147.4 keV is known to decay dominantly to the 8\(^{+}\) state in the ground state band (K=0) via a hindered 88.9 keV E1 transition. The population of this isomer can be identified if the delayed γ-ray cascade of 8\(^{+}\) → 6\(^{+}\) → 4\(^{+}\) → 2\(^{+}\) in the ground state band is observed (the 2\(^{+}\) → 0\(^{+}\) and the 8\(^{-}\) → 8\(^{+}\) transitions are highly converted). In deformed nuclei the electromagnetic transitions between states of different rotational bands are governed by both the usual spin and parity selection rules and by a supplementary selection rule for the quantum number K, which is the projection of the total nuclear angular momentum I on the symmetry axis. If there were no K-admixtures, transitions of multipole order \(λ\) between states with \(|ΔK| > λ\) could not occur. The K selection rule is strictly fulfilled only when the internal and rotational motions are independent of each other. Actually there is always some coupling between these two forms of motion, and therefore the K selection rule results in small transition probabilities rather than complete forbiddeness.

2 Electromagnetic excitation of the K\(^{π}=8^{-}\) isomer

2.1 Review of recent experimental results

In a Coulomb excitation process one starts from the ground state and populates nuclear levels which are related to the ground state band by collective E2 and E3 transitions, and these may or may not be yrast levels. Surprisingly, one observes the population of the lowest K\(^{π}=8^{-}\) isomer at 1147.4 keV in \(^{178}\)Hf following Coulomb excitation. First indications for a population of this isomer were reported by Hamilton et al. [8]. However, the reaction mechanism for the population of this isomer remained a puzzle. To investigate the reaction mechanism for populating the K\(^{π}=8^{-}\) isomer in \(^{178}\)Hf, a Coulomb excitation experiment was performed using the Darmstadt-Heidelberg Crystal Ball, by bombarding an enriched \(^{178}\)Hf target of 0.5 mg/cm\(^2\) thickness with a pulsed \(^{130}\)Te beam provided by the accelerator facility at the MPI für Kernphysik [9]. Three beam energies of 560, 590 and 620 MeV below the Coulomb barrier [9] (\(V_C=634\) MeV) were used in the experiment. Both prompt and delayed γ-rays were recorded with the Crystal Ball. A delayed time window with a width of 45 ns was set on the time spectra of the NaI detectors in between the beam pulses to distinguish the delayed isomer decay from the prompt 8\(^{+}\) → 6\(^{+}\) → 4\(^{+}\) → 2\(^{+}\) γ-rays. Fig. 1 shows
the \(\gamma\)-ray spectrum of the Crystal Ball, where the three peaks of 426.4, 325.6, and 213.4 keV, respectively, correspond to the delayed \(\gamma\)-rays depopulating the isomer.

![Graph of \(E_{cb}^{\text{del}}\) vs Counts](image)

**Figure 1:** Delayed \(\gamma\)-ray spectrum of the Crystal Ball obtained at a bombarding energy of 590 MeV, demanding \(850 \text{ keV} \leq E_{\text{sum}}^{\text{delay}} \leq 1100 \text{ keV}\) and \(2 \leq M_I^2 \leq 5\). In the right corner the partial level scheme of the \(\gamma\)-ray cascade depopulating the isomer is displayed.

The excitation cross section \(\sigma_{\text{g-}}\) of the isomer can be derived by

\[
\sigma_{\text{g-}} = \frac{Y_{\text{delay}}}{Y_{\text{prompt}}} \sigma_{\text{prompt}}
\]

(1)

where \(Y_{\text{delay}}\) and \(Y_{\text{prompt}}\) are the delayed and prompt \(\gamma\)-ray yields and \(\sigma_{\text{prompt}}\) is the excitation cross section producing the observed prompt yield. The excitation cross sections of states in the ground state band in \(^{178}\)Hf, \(\sigma_{\text{prompt}}\), were calculated with the semiclassical Coulomb excitation code [11] using the reduced matrix element [12] \(<2||M(E2)||0> = 2.20 \text{ eb}\) and the rigid rotor model for the other matrix elements in the ground state band. The E2-matrix elements within the ground state band (\(K=0\)) are given by

\[
< I_f || M(\lambda \Lambda) || I_i > = \sqrt{2I_i+1} < I_i \lambda K || I_f K > M_{\lambda 0}
\]

(2)

for \(\lambda=2\) and the matrix element \(M_{\lambda 0} = < K || M(E2,0) || K > = \sqrt{8 \rho_{0e}} Q_0 e\) can be related to the intrinsic electric quadrupole moment \(Q_0\). The Clebsch-Gordan coefficient \(< I_f 2K0 || I_f K >\) represents the coupling of the angular momenta in the intrinsic frame. Some coefficients, relevant for the discussion, are given in the appendix. Using Eq.(1) we obtained the excitation cross sections for
the isomer at three bombarding energies shown in fig. 2. Systematic uncertainty in the absolute values of the cross sections amount to about +70%/-45%.

![Graph showing the excitation cross sections for the isomer at different incident beam energies compared with semi-classical Coulomb excitation calculations with various intrinsic E3 matrix elements, M_{30} [e b^{3/2}]. The error bars correspond to statistical uncertainties only (see text).](image)

Figure 2: Experimental excitation cross sections for the isomer at different incident beam energies compared with semi-classical Coulomb excitation calculations for various intrinsic E3 matrix elements, M_{30} [e b^{3/2}]. The error bars correspond to statistical uncertainties only (see text).

2.2 Transitions between different rotational bands

If one assumes that the projection of the total angular momentum I on the symmetry axis, K, is a good quantum number, a direct transition from the K=0 ground state band to the K=8 isomeric state and to the rotational states built on it is forbidden. However, an admixture of different K-components in the nuclear wave functions could allow for direct transitions to the isomeric band in ^{178}Hf. Indeed, measurements of log ft values [5], E2/M1 mixing ratios [1, 2, 4] and g-factors [7] all suggest a strong mixing in the wave function of the $8^-$ isomeric state. One can distinguish two basic cases in which the wave function of one rotational band is always assumed to be pure. The resulting coupling schemes are depicted in fig. 3.

If one includes a small K=8 admixture in the wave function of the ground state band, the E3 matrix elements can be calculated from the Alaga-rule Eq.(2) [13] with K=8. From the measured lifetime ($\tau_{1/2}=4$ s) of the $8^-$ state, we can estimate an upper limit of the reduced intrinsic E3 matrix element of $M_{30} \leq 0.01 \ e b^{3/2}$ leading to an excitation cross section, which is too small to account for the observed population of the $8^-$ isomer in the mb range.
Figure 3: Different coupling schemes relevant for the excitation of the $8^-$ isomer in $^{176}$Hf. The E3 interband transitions are shown for a K=8 admixture in the wave function of the ground state band (left) and for a K=0 admixture in the wave function of the isomeric band (right).

It is interesting to note, however, that in case of a K=0 admixture in the wave function of the isomeric band the E3 matrix elements (using Eq.(2) with K=0) for transitions from the ground state band to the even-spin members of the isomeric band vanish, while it is possible to excite the odd-spin members in the isomeric band directly from the ground state band via E3 transitions, i.e. $6^+ \rightarrow 9^-$, $8^+ \rightarrow 11^-$, etc. Responsible for the E3 matrix elements are the Clebsch-Gordan coefficients given in the appendix. Hence, the isomeric $8^-$ state can be populated by M1 and E2 transitions from higher lying levels in the band. Free from the constraint set by the known 4 s half-life of the $8^-$ level, one can determine in this case an intrinsic E3 matrix element in the limit of the Alaga-rule. Fig. 2 shows a comparison of the calculated cross sections with the measured ones for the $8^-$ isomer in $^{176}$Hf. Good agreement is obtained assuming an intrinsic E3 matrix element of $M_{30} = 0.18_{-0.03}^{+0.03} eb^2$ (including systematic uncertainties). This matrix element also explains the result of a Coulomb excitation experiment with $^{16}$O ions, mentioned in ref.[8], in which no feeding of the isomer was observed. The population of the $8^-$ state in this reaction is a factor of 40 smaller than that of the highest excited state ($10^+$) in the ground state band.

Although the value of the obtained matrix element is compatible with known E3 matrix elements in this mass region, alternative excitation scenarios such as through highly excited intermediate states may be present. Our interpretation mainly shows that there are reasonable ways to explain the observed yield of the $8^-$ isomer by a pure Coulomb excitation process, but the role of other likely K-admixtures in the isomeric band has to be considered. In future experiments with EUROBALL arrays one may have a chance to observe the direct population of the rotational states built on the $8^-$ isomer. This will enable a model-independent determination of individual E3 matrix elements which would allow to shed more light on the excitation path of the $8^-$ isomer.
A Clebsch-Gordan-Coefficients

\[<I1K0|(I - 1)K> = -\sqrt{\frac{(I + K)(I - K)}{I(I + 1)}} \]  
(A.1)

\[<I1K0|IK> = \frac{K}{\sqrt{I(I + 1)}} \]  
(A.2)

\[<I1K0|(I + 1)K> = \sqrt{\frac{2(I + K + 1)(I - K + 1)}{(2I + 1)(2I + 2)}} \]  
(A.3)

\[<I2K0|(I - 2)K> = \sqrt{\frac{3(I + K - 1)(I + K)(I - K - 1)(I - K)}{2(I - 1)(2I + 1)}} \]  
(A.4)

\[<I2K0|(I - 1)K> = -\sqrt{\frac{3(I + K)(I - K)}{(I - 1)(I + 1)(2I + 1)}} * K \]  
(A.5)

\[<I2K0|IK> = -\sqrt{\frac{1}{I(I + 1)(2I - 1)(2I + 3)}} * (I^2 + I - 3K^2) \]  
(A.6)

\[<I2K0|(I + 1)K> = \sqrt{\frac{3(I + K + 1)(I - K + 1)}{I(I + 1)(2I + 1)}} * K \]  
(A.7)

\[<I2K0|(I + 2)K> = \sqrt{\frac{3(I + K + 1)(I + K + 2)(I - K + 1)(I - K + 2)}{2(I + 1)(2I + 1)(2I + 2)(2I + 3)}} \]  
(A.8)

(A.9)

\[<I3K0|(I - 2)K> = \sqrt{\frac{15(I + K - 1)(I + K)(I - K - 1)(I - K)}{(I - 2)(I - 1)(2I - 1)(2I + 1)(2I + 2)}} * K \]  
(A.10)

\[<I3K0|(I - 1)K> = -\sqrt{\frac{3(I + K)(I - K)}{(I - 1)(2I - 3)(2I + 1)(2I + 2)(2I + 3)}} * (5K^2 - I^2 + 1) \]  
(A.11)

\[<I3K0|IK> = \frac{5K^2 - 3I^2 - 3I + 1}{\sqrt{(I - 1)(I + 1)(I + 2)(2I - 1)(2I + 3)}} * K \]  
(A.12)

\[<I3K0|(I + 1)K> = \sqrt{\frac{3(I + K + 1)(I - K + 1)}{I(I + 1)(2I - 1)(2I + 1)(2I + 4)(2I + 5)}} * (5K^2 - I^2 - 2I) \]  
(A.13)

\[<I3K0|(I + 2)K> = \sqrt{\frac{15(I + K + 1)(I + K + 2)(I - K + 1)(I - K + 2)}{I(I + 1)(2I + 1)(2I + 3)(2I + 6)}} * K \]  
(A.14)

\[<I3K0|(I + 3)K> = \sqrt{\frac{5(I + K + 1)(I + K + 2)(I + K + 3)(I - K + 1)(I - K + 2)(I - K + 3)}{2(I + 1)(2I + 1)(2I + 3)(2I + 5)(2I + 7)}} \]  
(A.15)
References