Alpha decay

Introduction to Nuclear Science

Simon Fraser University SPRING 2011

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1 The decay processes

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2 The energetics

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1 The decay processes

2 The energetics

3 The observed decay rates

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2 The energetics

- The observed decay rates
- 4 The tunneling process

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- 2 The energetics
- 3 The observed decay rates
- 4 The tunneling process
- 5 α -decay rate calculations for even-even nuclei

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Decay processes

- The classification of decay processes into the α , β and γ decay originates comes from the early XX-century studies of decay processes using nuclear emulsion.
- Nuclear emulsion is a photographic plate (as used in analog cameras for black and white photography) with a particularly thick emulsion layer and with a very uniform grain size.
- Nuclear emulsions exposed to radiation showed tracks which were classified into the three groups
 - class α : short and thick
 - class β : longer and thin
 - class γ : very long and very thin.
- Subsequent studies revealed the origin of the processes resulting in the observation of these different tracks, however, the names stick to the processes.

The α -decay process

- Ernest Rutherford pioneered application of α -decay process into nuclear science experimental studies.
- He correctly identified α particles as nuclei of helium atoms.
- This has been achieved using atomic spectroscopy. α particles from a source were collected in a discharge tube and then observed to show line spectra identical with these from helium reference gas.
- Rutherford experiments on scattering of α -particles led to the discovery of atomic nucleus in 1911 (100 years from now).
- Thus α decay process has been recognized as emission of energetic helium nucleus from a parent nucleus.

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The α -decay process

• In the most general way the α -decay process can be written as

$${}^{A}_{Z}Y^{0}_{N} \rightarrow {}^{A-4}_{Z-2}X^{2-}_{N-2} + {}^{4}_{2}He^{2+}_{2} + Q_{\alpha}$$
 (1)

• The Q_{lpha} value is

$$Q_{\alpha} = (m_Y - m_X - m_{\alpha})c^2 \tag{2}$$

- The Q_α value has to be positive for nuclei to undergo a spontaneous (exothermic) decay.
- The Q_{α} values can be calculated from measured masses, or estimated using the Liquid Drop Model.
- In the Q_{α} calculations binding energies of electrons can be neglected as small and masses of neutral atoms can be used.
- For know α emitters the Q_{α} values vary between ~ 3 and ~ 11 MeV.

The Q_{α} values



The energetics

The S_{α} values

Note that Q_α values correspond to the negative separation energy S_α of an α particle from the parent nucleus.

$${}^{A}_{Z}Y_{N} + S_{\alpha} \quad \rightarrow \quad {}^{A-4}_{Z-2}X_{N-2} + {}^{4}_{2}He_{2} \tag{3}$$

Thus

$$S_{\alpha} = (m_X + m_{\alpha} - m_Y)c^2 = -Q_{\alpha} \tag{4}$$

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• α separation energies can be calculated using binding energies

$$S_{\alpha} = B(A, Z) - B(A - 2, Z - 2) - B(4, 2)$$

= $B(A, Z) - B(A - 2, Z - 2) - 28.3 \text{ MeV}$ (5)

taking into account the binding energy of 28.3 MeV for the α particle.

• Negative S_{α} (or positive Q_{α}) is indicative of nuclei which undergo spontaneous α decay.

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The S_{α} values



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The S_{α} values

- The known masses as well as the Liquid Drop model indicate that all nuclei with mass number A > 150 have negative S_{α} , positive Q_{α} and could undergo a spontaneous α decay.
- Experimentally, the α decay dominates in heavy nuclei with mass number A > 210
- For these heavy nuclei the emission of an α particle increases binding energy per nucleon for the system.
- This is a consequence of two factors:
 - α particle is very tightly bound with binding energy per nucleon of $B/A \sim 7.1$ MeV comparable to these of mid- and heavy-nuclei. Thus, there is only a little reduction of binding associated with emission of α .
 - The α decay process reduces the charge of the nucleus, thus increases the binding significantly due to reduction of Coulomb repulsion of protons inside a nucleus.

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Kinetic energy of α particles

• The α decay process conserves momentum.

$$0 = \vec{p}_X + \vec{p}_\alpha \tag{6}$$

with zero being the initial momentum of the parent and \vec{p}_X and \vec{p}_α being the final momenta of the daughter and the α particle.

• The zero initial momentum of the parent implicates that momenta of the daughter and the α particle have the opposite direction along the same line

$$\vec{p}_X = -\vec{p}_\alpha \tag{7}$$

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• Conservation of energy implies that the sum of kinetic energy of the daughter and the α particle is equal to the Q_{α} value

$$K_X + K_\alpha = \frac{\vec{p}_X^2}{2m_X} + \frac{\vec{p}_\alpha^2}{2m_\alpha} = Q_\alpha \tag{8}$$

Kinetic energy of α particles

• Since, from the conservation of momentum

$$\vec{p}_X = -\vec{p}_\alpha \tag{9}$$

for the kinetic energy of α particle one can write

$$\frac{\vec{p}_X^2}{2m_X} + \frac{\vec{p}_\alpha^2}{2m_\alpha} = Q_\alpha$$

$$\frac{\vec{p}_\alpha^2}{2m_X} + \frac{\vec{p}_\alpha^2}{2m_\alpha} = Q_\alpha$$

$$\frac{\vec{p}_\alpha^2}{2m_\alpha} (\frac{m_\alpha}{m_X} + 1) = K_\alpha (1 + \frac{m_\alpha}{m_X}) = Q_\alpha$$
(10)

• This leads to

$$K_{\alpha} = Q_{\alpha} \frac{1}{1 + \frac{m_{\alpha}}{m_{\chi}}} = Q_{\alpha} \frac{m_{\chi}}{m_{\chi} + m_{\alpha}} \approx Q_{\alpha} \frac{m_{\chi}}{m_{Y}}$$
(11)

Image: A matrix

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Kinetic energy of α particles

• The kinetic energy of the α particle is slightly smaller than the ${\it Q}_{\alpha}$ value

$$\mathcal{K}_{lpha} pprox Q_{lpha} rac{m_X}{m_Y} = Q_{lpha} rac{m_Y - m_{lpha}}{m_Y} = Q_{lpha} (1 - rac{m_{lpha}}{m_Y}) pprox Q_{lpha} (1 - rac{4}{A})$$
 (12)

with A >> 4 being the mass number of the parent.

• The recoil energy of the daughter is

$$K_X = Q_\alpha - K_\alpha = Q_\alpha \frac{m_\alpha}{m_Y} \approx Q_\alpha \frac{4}{A}$$
 (13)

 For nuclei with mass number around A = 200 the kinetic energy of the α is ~98% of the Q_α value, while the kinetic energy of the daughter is ~ 2% of the Q_α value.

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Comparison with the Coulomb barrier

- Let us compare the observed kinetic energies of α particles with the height of the Coulomb repulsion between the daughter and the ⁴He.
- The energy of the Coulomb repulsion at a distance R is

$$V = \frac{e^2}{4\pi\epsilon_0 R} \frac{Z_{\alpha}(Z-2)}{R} = 1.44 \; [\text{MeV fm}] \frac{Z_{\alpha}(Z-2)}{R} \qquad (14)$$

• The height of the Coulomb barrier is the energy of the Coulomb repulsion at the distance corresponding to the sum of the radii for the daughter and the α -particle

$$V_{C} = 1.44 \frac{Z_{\alpha}(Z-2)}{1.2(\sqrt[3]{A-4} + \sqrt[3]{4})} = 2.4 \frac{Z-2}{\sqrt[3]{A-4} + 1.59} \left[MeV \right]$$
(15)

with A and Z being mass and atomic number of the parent.

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Comparison with the Coulomb barrier

• For
238
U $A = 238$, $Z = 92$ and

$$V_C = 2.4 \frac{Z - 2}{\sqrt[3]{A - 4} + 1.59} = 27.9 \ [MeV]$$
 (16)

• The measured Q_{α} value is 4.3 MeV.

- Eq. 12 implies that the kinetic energy of α particles is 4.2 MeV.
- Thus the observed kinetic energy of α particles is by a significant factor (\sim 7) smaller than the Coulomb barrier.
- One consequence of that fact is that α particles from naturally occurring radioactive sources are in general not energetic enough to induce nuclear reactions or nuclear transmutations.
- The exceptions are reactions on very light nuclei.

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Pu-Be neutron source

• Let us calculate the Coulomb barrier for the reaction of α particles on beryllium

$$V_C = 1.44 \frac{2 \cdot 4}{1.2(\sqrt[3]{9} + \sqrt[3]{4})} = 2.6 \,[\text{MeV}]$$
 (17)

- This implies that for most of naturally occurring α emitters the energy of α particles is high enough to induce nuclear transmutation of beryllium.
- Such mixtures are useful neutron sources since

$${}^{9}Be + \alpha \rightarrow {}^{13}C \rightarrow {}^{12}C + n \tag{18}$$

- A popular source is a mixture of ${}^{241}Pu$ with ${}^{9}Be$.
- It produces 30 neutrons per million of emitted α particles.

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The energetics

$^{235}\textit{U}$ and $^{238}\textit{U}$ decay chains

proto	Chart of the Nuclides Showing both the U235 and U238 Series														es								
Ins –	(The color of each nuclide indicate the half-life. For each color the half-life is measured in:) (less than a second, seconds, minutes, hours, days, years, over 10,000 years, Non Radioactive)																						
92											U 226	U 227	U 228	U 229	U 230	U 231	U 232	U 233	U 234	235	U 236	U 237	
91										Pa 224	Pa 225	Ра 226	Pa 227	Pa 228	Pa 229	Pa 230	Pa 231	P .32	P-	Pa 234	Pa 235	P-	Pa 237
90										Th 223	Th 224	Th 225	Th 226	Th 227	Th 228	29	(Th 230)	(III) 231)	Th 232	Th 233	Th 234	Th 235	Th 236
89	Ac 213	Ac 214	Ac 215					Ac 220	Ac 221	Ac 222	Ac 223	Ac 224	A	Ac 226	AC 227	A7 28	Ac 229	Ac 230	Ac 231	Ac 232	Ac 233	Ac 234	Ac 235
88	Ra 212	Ra 213	Ra 214	Ra 215				Ra 219	Ra 220	Ra 221	Ra 222	Ra 223	Ra 224	P	Ra 226	Ra 227	Ra 228	Ra 229	Ra 230	Ra 231	Ra 232	Ra 233	Ra 234
87	Fr 211	Fr 212	Fr 213	Fr 214	Fr 215			Fr 218	Fr 219	Fr 220	21	Fr 222	Er 223	Fr 24	Fr 225	Fr 226	Fr 227	Fr 228	Fr 229	Fr 230	Fr 231	Fr 232	Fr 233
86	Rn 210	Rn 211	Rn 212	Rn 213	Rn 214	Rn 215	Rn 216	Rn 217	Rn 218	Rn 219	Rn 220	Rn 221	Rn 222	Rn 223	Rn 224	Rn 225	Rn 226	Rn 227	Rn 228	Rn 229	Rn 230	Rn 231	Rn 232
85	At 209	At 210	At 211	At 212	At 213	At 214	At 215	At 216	A7	At 218	At 219	At 20	At 221	At 222	At 223	At 224	At 225	At 226	At 227	At 228	At 229		
84	Po 208	Po 209	Po 210	Po 211	Po 212	Po 213	Po 214	(P0) 215	Pr 16	Po 217	Po 218	Po 219	Po 220	Po 221	Po 222	Po 223	Po 224	Po 225	Po 226				
83	Bi 207	P 108	Bi 209	Bi 210	(Bi 211)	B'	P/ 13	Bi 214	Bi 215	B'	Bi 217	Bi 218	Bi 219	Bi 220	Bi 221	Bi 222	Bi 223	Bi 224					
82	5 8		7	209	Pb 210	Pb 211	Pr 12	Pb 213	Pb 214	Pb 215	Pb 216	Pb 217	Pb 218	Pb 219	Pb 220								
81	205	TT 206	207	TI 208	TI 209	TI 210	TI 211	TI 212	TI 213	TI 214	TI 215	TI 216	TI 217										
	124 neu	125 strons	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146

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^{235}U decay chain sequence



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Correlation between Q_{α} and α decay half-life

• Investigation of α decay by Rutherford's students and collaborators Hans Geiger and John M. Nuttall lead to the observation of correlation between the Q_{α} value and α decay half-life

$$\ln(\tau_{\alpha}) = a - \frac{b}{\sqrt{Q_{\alpha}}} \tag{19}$$

- Initially, this correlation was observed as a correlation between the half-life and the track length (range) of α particles emitted from various natural sources of α radioactivity.
- The track length (range) is proportional to the initial momentum which is proportional to the square root of the initial kinetic energy, and thus proportional to the square root of the Q_{α} value.
- The constants A and B depend on the Z of the parent.

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The Geiger-Nuttall plot



Classical turning point

 Particles at energy lower than the Coulomb barrier should start moving away from the interaction centre at the classical turning point.



 On the graph the red line is the Coulomb energy as the function of radius for the *p*+*p* reaction, green line represents zero energy, while the blue line represents centre of mass energy of 30 keV. The classical turning points for these conditions are at *R* ~ 5 and *R* ~ 47 [fm].

Tunnelling

• Quantum mechanics allow particles to tunnel through the barrier.



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Tunnelling

• The probability of finding a particle across the barrier is given by the transmission coefficient defined as the ratio of amplitude squared of the incoming and transmitted wave functions

$$T = \frac{|\Psi(R_N)|^2}{|\Psi(R_C)|^2},$$
(20)

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with R_N and R_C being the nuclear radius and the distance to the classical turning point, respectively.

- The barrier suppresses the amplitude of the wave function, but not completely.
- The degree of suppression is calculable for the Coulomb potential and is given by

$$T = \exp\left(-2KR_{C}\left[\frac{\arctan\sqrt{\frac{R_{C}}{R_{N}}-1}}{\sqrt{\frac{R_{C}}{R_{N}}-1}} - \frac{R_{N}}{R_{C}}\right]\right), \quad K = \sqrt{\frac{2\mu}{\hbar^{2}}(E_{C}-E)}$$

The approximation for P

• If the distance to the classical turning point is much larger than the nuclear radius $R_C >> R_N$ which is equivalent to $E << E_C$ the tunnelling probability is well approximated by

$$T = \exp\left(-b\frac{1}{\sqrt{E}}\right),\tag{21}$$

with

$$b = \frac{\sqrt{2\mu}}{4\epsilon_0} \frac{Z_1 Z_2 e^2}{\hbar} \tag{22}$$

 $\bullet\,$ Symbol μ represents the reduced mass of the system

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \approx \frac{A_1 A_2}{A_1 + A_2}$$
(23)

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• P is a very rapidly changing function of the energy.

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The Gamow factor

 The transmission coefficient for α tunnelling with the energy equal to the Q_α value is expressed in terms of the Gamow factor

$$T = \exp^{-2G} \implies 2G = b \frac{1}{\sqrt{Q_{\alpha}}} = 2\sqrt{\frac{2\mu}{\hbar^2 Q_{\alpha}}} Z_{\alpha}(Z-2) \frac{e^2}{4\pi\epsilon_0} \frac{\pi}{2}$$
 (24)

- Above $Z_{\alpha} = 2$ and Z and Z 2 are the atomic numbers of the α , the parent, and the daughter nucleus.
- Note that the above equation is equivalent to Eq. 7.17 in the textbook except for the difference in the units of charge.
- The textbook uses for the Coulomb energy

$$V_C = \frac{Z_1 Z_2 e^2}{r} \tag{25}$$

while in the SI units

$$V_C = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r} \tag{26}$$

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The α -decay rate

• The decay rate, given by the inverse lifetime,

$$\lambda_{\alpha} = \frac{1}{\tau_{\alpha}} \tag{27}$$

represents average number of α decays per second.

- Let us assume that the decay rate is completely determined by the quantum mechanical tunnelling process. This idea was originated by Gorge Gamow in late \sim 1920.
- Gamow postulated that the decay rate is a product of the frequency f at which the α particle impinges on the barrier and the transmission coefficient T through the barrier

$$\lambda_{\alpha} = fT = f \exp^{-2G} \tag{28}$$

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The frequency factor

• The average time at which the α particles trapped inside the parent potential well impinges on the barrier is given by the diameter of the well D = 2R divided by the velocity of the α -particle

$$t = \frac{2R}{v} \tag{29}$$

• The frequency is the inverse of that time

$$f = \frac{v}{2R} \tag{30}$$

• The velocity can be calculated from the kinetic energy of the α particle in the well

$$\frac{\mu v^2}{2} = V_0 + Q_\alpha \tag{31}$$

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with V_0 representing the depth of the well.

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The decay rate

• With the frequency factor

$$f = \frac{v}{2R} = \sqrt{\frac{2(Q_{\alpha} + V_0)}{\mu} \frac{1}{2R}}$$
(32)

and the transmission factor

$$T = \exp^{-2G} = \exp\left(-\frac{b}{Q_{\alpha}}\right) \tag{33}$$

the rate is

$$\lambda_{\alpha} = fT = \sqrt{\frac{2(Q_{\alpha} + V_0)}{\mu}} \frac{1}{2R} \exp\left(-\frac{b}{Q_{\alpha}}\right)$$
(34)

with Z_D being the atomic number of the daughter.

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The Geiger-Nuttall law

• The above result from the tunnelling model yields

$$\ln(\lambda_{\alpha}) = \ln(\frac{1}{\tau_{\alpha}}) = -\ln(\tau_{\alpha}) =$$

$$\ln\left(\sqrt{\frac{2(Q_{\alpha} + V_{0})}{\mu}}\frac{1}{2R}\exp\left(-\frac{b}{\sqrt{Q_{\alpha}}}\right)\right)$$

$$= -b\frac{1}{\sqrt{Q_{\alpha}}} + \ln\left(\sqrt{\frac{2(Q_{\alpha} + V_{0})}{\mu}}\frac{1}{2R}\right)$$
(35)

• Since usually $Q_{\alpha} < V_0$ the last term in the above can be approximated as a constant

$$-a = \ln\left(\sqrt{\frac{2(Q_{\alpha} + V_0)}{\mu}}\frac{1}{2R}\right)$$
(36)

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The Geiger-Nuttall law

Half-life 11/2, sec

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• With the above substitution we derive the Geiger-Nuttall law

$$\ln(\tau_{\alpha}) = \frac{b}{\sqrt{Q_{\alpha}}} + a$$
(37)

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6 Alpha-decay energy Q_{α} , Mev

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