Beta decay

Introduction to Nuclear Science

Simon Fraser University
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Outline

1. Fundamental Forces

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2. The Standard Model of Elementary Particles

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6. β-decay in nuclei

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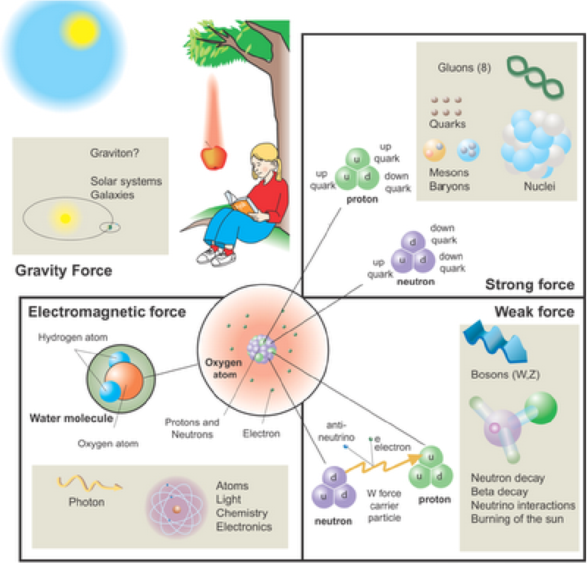
4. Conservation laws in the β-decay
5. Angular momentum in β-decay
6. β-decay in nuclei
7. Stability of nuclei

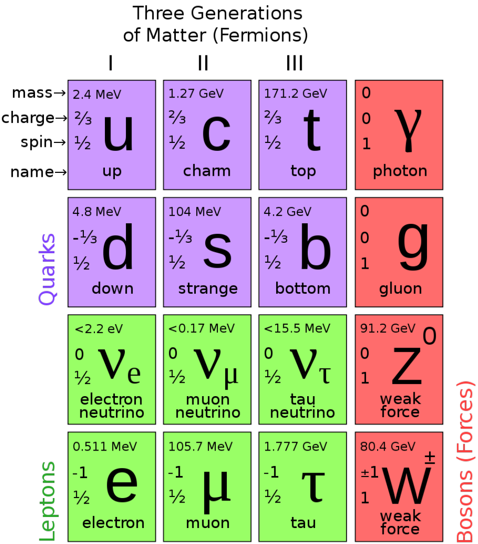
Fundamental Forces



Fundamental Forces

Force	Range [m]	Relative strength		Impact
		within	beyond	
		nucleus		
Strong	10^{-15}	100	0	Nuclei
Electromagnetic	∞	1	1	Chem/Bio
Weak	10^{-18}	10^{-5}	0	Nuclei
Gravity	∞	10^{-43}	10^{-43}	Universe

The Standard Model of Elementary Particles



Three Generations of Matter (Fermions)

	I	II	III	Bosons (Forces)
mass→ charge→ spin→ name→ Quarks	2.4 MeV ⅔ ½ u up	1.27 GeV ⅔ ½ c charm	171.2 GeV ⅔ ½ t top	0 0 1 γ photon
	4.8 MeV -⅓ ½ d down	104 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom	0 0 1 g gluon
Leptons	<2.2 eV 0 ½ ν_e electron neutrino	<0.17 MeV 0 ½ ν_μ muon neutrino	<15.5 MeV 0 ½ ν_τ tau neutrino	91.2 GeV 0 1 Z^0 weak force
	0.511 MeV -1 ½ e electron	105.7 MeV -1 ½ μ muon	1.777 GeV -1 ½ τ tau	80.4 GeV ±1 1 $W^\pm$ weak force

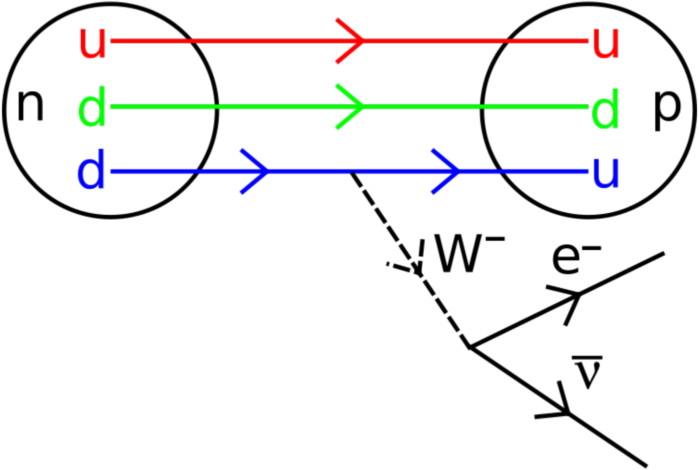
The decay of the down quark

- In the fundamental level the β^--decay process is a decay of the down quark by emission of the W^- boson.
- The W^- boson decays to an electron and anti-neutrino.
- This is the process which defines the decay of a neutron into proton

$$n \rightarrow p + e^- + \bar{\nu_e} \tag{1}$$

- The down quark is heavier than the up quark, the neutron is heavier than the proton, thus the decay is allowed by conservation of energy.
- The interactions involved in the decay are weak interactions.
- Note that the decay has three particles in the final state, unlike the α decay which resulted in two particles in the final state.

The decay of the down quark



Three body β decay

- The β-decay process involves three particles in the final state, for example:
$$n \rightarrow p + e^- + \bar{\nu}_e \tag{2}$$
- The energy released in the decay of the neutron as given by the Q value is shared between the recoil of the proton, and the energy of the electron and anti-neutrino.
- The decay conserves momentum and energy.
- The recoil of proton is small since it is significantly heavier that the electron and neutrino, most of the energy is shared between the positron and the neutrino.
- Because of that sharing the energy spectrum observed for electrons is continuous.

Three body β decay

- In experiment on β decay only the final nuclei and electrons are observed. Neutrinos escape detection because of the small interaction cross section.
- In early studies (∼1920) the three-body nature of the β decay has not been recognized and the process was treated as a two body process.
- Since the conservation of energy and momentum in a binary process leads to a discrete energies for the final products the observed continuous energy spectrum for electrons seemed highly confusing.
- Some prominent scientists, including Niels Bohr, were ready to give up conservation of momentum in the microscopic world.
- In the attempt to save the conservation laws Wolfgang Pauli proposed in December of 1930 that the β decay is a three body process and that the third emitted particle is neutral and escapes detection.

Three body β decay

- From the kinematics of the β decay and proximity of the highest observed electron energies to the decay Q value it is clear that the neutral particle postulated by Pauli has a very low mass.
- Pauli called this neutral particle the neutron.
- In 1932 James Chadwick reports discovery of a neutral particle with mass similar to the mass of a proton.
- The name neutron fits better to the Chadwick's particle rather than the Pauli's particle.
- To make everyone happy Enrico Fermi in mid 1930 develops a theory of the β-decay based on the Pauli's idea, calls the Pauli's particle a neutrino which translates from Italian as "a little neutron". Name calling is over and conservation laws are saved.
- Direct observation of neutrinos is reported from reactor experiments in mid 1950.

Baryons and leptons

- Elementary particles can be classified as baryons which are made of quarks and leptons which are the fundamental particles of the standard model.
- Proton and neutron are baryons.
- Electron, muon, the tau particle and all neutrinos are leptons.
- In years of study of nuclear and high energy reactions the laws of conservation of baryons and leptons were established.
- These laws state that the net number of baryons or leptons are the same through the reaction, no matter what happens.

Matter and antimatter

- According to current understanding every particle has its anti-particle, which is a particle of the same mass, same spin, and the opposite charge.
- Positron e^+ from the β^+ decay is an anti-particle of electron e^-.
- High energy collisions of protons can produce anti proton in the reaction
$$p + p \rightarrow p + p + p + \bar{p} \tag{3}$$
- Matter and anti-matter particles annihilate when in contact turning itself into pure energy in the form of photons.
- Late last year international collaboration called *Alpha* at CERN managed to produce anti-hydrogen (a bound system of a anti-proton and anti-electron) and trap it in electromagnetic fields. Anti-hydrogen annihilates on contact with matter.

Baryon and lepton number conservation

- The neutrino hypothesis solves the problem of lepton number conservation which would exist in the β-decay otherwise.
- Without the anti-neutrino in the decay

  $$n \rightarrow p + e^{-} \tag{4}$$

  there is one baryon on the left hand side, one baryon on the right hand side and one lepton on the right hand side. The conservation of the lepton number does not hold.
- With the anti-neutrino in the decay

  $$n \rightarrow p + e^{-} + \bar{\nu}_e \tag{5}$$

  there is one baryon on the left hand side, one baryon on the right hand side, one lepton on the right hand side and one anti-lepton on the right hand side. The conservation of the lepton number does hold since with one lepton and one anti-lepton the net number of leptons on the left and the right hand side is zero.

The angular momentum

- Another problematic issue which has been confusing before the neutrino hypothesis was related to spins and angular momenta.
- Neutron, proton, and electron are fermions with half integer spin.
- Without the anti-neutrino in the decay

  $$n \rightarrow p + e^- \tag{6}$$

  the angular momentum on the left hand side is half integer, while the coupling of two angular momenta on the right hand side has to be integer. In ∼1920 this has been already recognized as inconsistent with Quantum Mechanics.
- Following this argument, the neutrino has to be a fermion with a half integer spin. Only in this way the angular momentum conservation can be fulfilled.

Fermi and Gamow-Teller β-decay

- Let us consider the coupling of lepton spins in a β-decay process

$$n \rightarrow p + e^{-} + \bar{\nu_e} \tag{7}$$

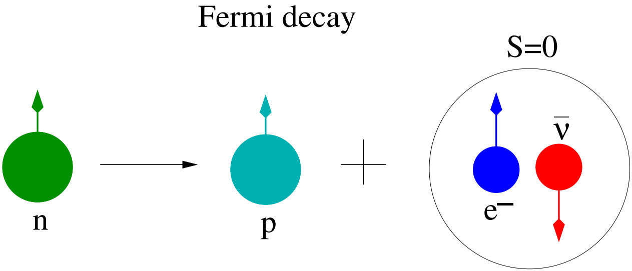
- Since both leptons, electron and anti-neutrino have half integer spins there are two possible couplings

  1. Net lepton spin is zero, which implies that the spins of the electron and the anti-neutrino are anti-parallel. This mode is called the Fermi decay.

  2. Net lepton spin is one, which implies that the spins of the electron and the anti-neutrino are parallel. This mode is called the Gamow-Teller decay.

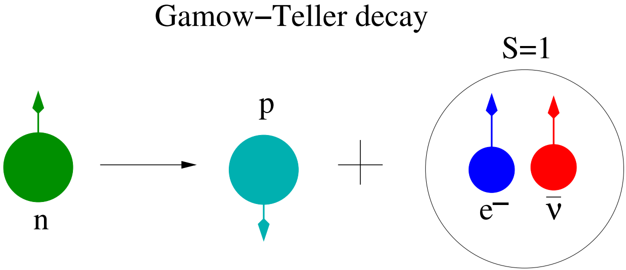
Fermi β decay

- In the Fermi decay mode the conservation of angular momentum requires that the spin of the baryons to point in the same direction before and after the decay.



Gamow-Teller β decay

- In the Gamow-Teller decay mode the conservation of angular momentum requires that the spin of the baryons to point in the opposite direction before and after the decay. This decay mode is sometimes called the spin-flip mode.



β^- decay of a nucleus

- β^- decay can convert a neutron in a nucleus into a proton.
- The process proceeds with emission of an electron e^- and electron anti-neutrino in a similar way as the β^- decay of a neutron. For example
$$^{14}C \rightarrow ^{14}N + e^- + \bar{\nu_e} \quad \tau = 5730 \text{ [y]} \tag{8}$$
- Nuclear β^- decay can take place if the energy balance allows it, meaning, that the sum of the masses in the final state is smaller than the mass in the initial state.
- Let us examine carefully the energy balance for the example case.

β^- decay energetics for ^{14}C decay

- The energy Q turned into the kinetic energy of ^{14}N, electron, and anti-neutrino in the final state is the difference between the mass of ^{14}C and the mass of ^{14}N, electron and anti-neutrino (times c^2).

$$Q = (m_{^{14}C} - m_{^{14}N} - m_e - m_{\bar{\nu}_e})c^2 \tag{9}$$

- We drop the mass of anit-neutrino.
- The nuclear mass difference between ^{14}C and ^{14}N can be calculated from mass defects realizing that $A_i = A_f = 14$, $Z_i = 6$, $Z_f = 7$. Thus

$$\begin{aligned} Q &= (\Delta M(6,14) - \Delta M(7,14) + (7-6)m_e - m_e)c^2 = \\ &= (\Delta M(6,14) - \Delta M(7,14))c^2 = 3.020 - 2.863 = 0.157 \text{ [MeV]} \end{aligned} \tag{10}$$

Forbidden β^+ decay of a proton

- β^+ decay turns a proton into a neutron in a nucleus.

$$p \rightarrow n + e^+ + \nu_e \tag{11}$$

- Note that compared to a neutron decay the positron e^+ is an anti-particle of an electron, and the neutrino ν_e is an anti-particle of an anti-neutrino $\bar{\nu}_e$. Consequently the mass of a positron is the same as for an electron, and the mass of an anti-neutrino is the same as for a neutrino. The baryon and lepton numbers are conserved.
- This process of a conversion of a free proton into a free neutron by β^+ decay is forbidden by energetics. Let us examine the energy balance.

Energetics of a forbidden β^+ decay of a proton

- The energy Q is the difference between the mass of a proton and the mass of a neutron , positron and neutrino (multiplied by c^2). We use the fact that the mass of a positron is the same as electron.

$$Q = (m_p - m_n - m_e - m_{\nu_e})c^2 \tag{12}$$

- Mass of the neutrino is very small and can be dropped from the equation without any significant loss of accuracy.
- The nuclear mass difference between a proton and a neutron can be calculated from mass defects realizing that $A_i = A_f = 1$, $Z_i = 1$, $Z_f = 0$:

$$\begin{aligned} Q &= (\Delta M(1,1) - \Delta M(0,1) + (0-1)m_e - m_e)c^2 = \quad (13)\\ &= (\Delta M(1,1) - \Delta M(0,1) - 2m_e)c^2 = \\ &= 7.289 - 8.071 - 2 * 0.511 = -1.804 \text{ [MeV]} \end{aligned}$$

- Negative Q indicates that process does not release energy but rather requires energy to proceed.

β^+ decay of a nucleus

- β^+ decay can convert a neutron in a nucleus into a proton if the energy balance allows it. This is the case for a large number of nuclei which have excess of protons as compared to neutrons.
- The process proceeds with emission of a positron e^+ and electron neutrino. For example

$$^{64}Cu \rightarrow ^{64}Ni + e^+ + \nu_e \quad \tau = 12.7\ [\mathrm{h}] \tag{14}$$

- The decay of ^{64}Cu is only partially (39% cases) through the nuclear β^+ decay, the remaining (61%) is through the electron capture (to be discussed later).
- Let us examine carefully the energy balance for the example case of β^+ decay.

β^+ decay energetics for ^{64}Cu decay

- The energy Q turned into the kinetic energy of ^{64}Ni, positron, and neutrino in the final state is the difference between the mass of ^{64}Cu and the mass of ^{64}Ni, positron and neutrino (times c^2). Mass of the positron is the same as electron.

$$Q = (m_{^{64}Cu} - m_{^{64}Ni} - m_e - m_{\nu_e})c^2 \tag{15}$$

- We drop the mass of neutrino again.
- The nuclear mass difference between ^{64}Cu and ^{64}Ni can be calculated from mass defects realizing that $A_i = A_f = 64$, $Z_i = 29$, $Z_f = 28$. Thus

$$\begin{aligned} Q &= (\Delta M(29,64) - \Delta M(28,64) + (28-29)m_e - m_e)c^2 = \\ &= (\Delta M(29,64) - \Delta M(28,64) - 2m_e)c^2 = \\ &-65.421 - (-67.096) - 2*0.511 = 0.653 \text{ [MeV]} \end{aligned} \tag{16}$$

- Note, that in general the atomic mass difference has to be at least $2*m_ec^2$=1.022 MeV for the β^+ decay to proceed.

Forbidden electron capture in a hydrogen atom

- Electron capture decay turns a proton into a neutron in a nucleus via capture of an electron from an atomic orbit.

$$p + e^- \rightarrow n + \nu_e \tag{17}$$

- Note that compared to a neutron decay we deal with an electron again and that the neutrino ν_e is an anti-particle of an anti-neutrino $\bar{\nu}_e$.
- This process of a conversion of a proton in a hydrogen atom into a neutron by electron capture decay is forbidden by energetics. Let us examine the energy balance.

Forbidden electron capture in hydrogen

- The energy Q is the difference between the mass of a proton plus mass of the electron and the mass of a neutron plus mass of the neutrino (multiplied by c^2). Note that the mass of a proton plus mass of an electron is effectively mass of hydrogen, thus the electron capture would be allowed if hydrogen atom is heavier than neutron.

$$Q = (m_p + m_e - m_n - m_{\nu_e})c^2 \tag{18}$$

- We drop the neutrino mass.
- The nuclear mass difference between a proton and a neutron can be calculated from mass defects realizing that $A_i = A_f = 1$, $Z_i = 1$, $Z_f = 0$:

$$\begin{aligned} Q &= (\Delta M(1,1) - \Delta M(0,1) + (0-1)m_e + m_e)c^2 = \\ &= (\Delta M(1,1) - \Delta M(0,1))c^2 = \\ &= 7.289 - 8.071 = -0.782 \text{ [MeV]} < 0 \end{aligned} \tag{19}$$

Electron capture energetics for ^{64}Cu decay

- The energy Q turned into the kinetic energy of ^{64}Ni and neutrino in the final state is the difference between the mass of ^{64}Cu plus an electron and the mass of ^{64}Ni and neutrino (times c^2).

$$Q = (m_{^{64}Cu} + m_e - m_{^{64}Ni} - m_{\nu_e})c^2 \tag{20}$$

- We drop the mass of neutrino again.
- The nuclear mass difference between ^{64}Cu and ^{64}Ni can be calculated from mass defects realizing that $A_i = A_f = 64$, $Z_i = 29$, $Z_f = 28$. Thus

$$\begin{aligned} Q &= (\Delta M(29,64) - \Delta M(28,64) + (28-29)m_e + m_e)c^2 = \\ &= (\Delta M(29,64) - \Delta M(28,64) - m_e + m_e)c^2 = \\ &-65.421 - (-67.096) = 1.675 \text{ [MeV]} \end{aligned} \tag{21}$$

- Electron capture is allowed if β^+ decay is allowed and results in larger energy release (since it does not need to create a positron).

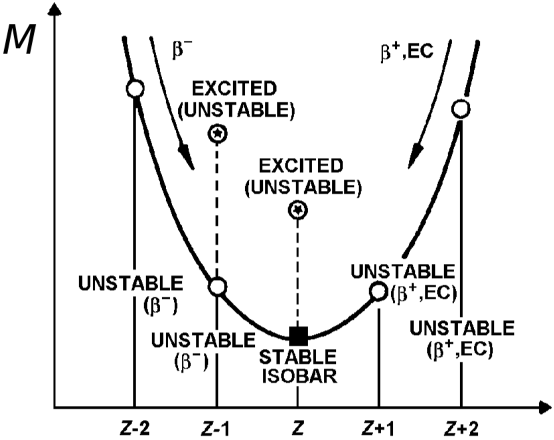
β-decay and stability of nuclei

- We used nuclear masses and the Liquid Drop Model to define the proton and neutron drip line.
- The nuclei beyond proton/neutron drip line have negative separation energy for a proton/neutron thus should decay by proton/neutron emission on a timescale of 10^{-20} s.
- This is indeed the case for the neutron dripline, however, the proton decays are hindered by tunnelling through the Coulomb barrier and are on the microsecond time scale.
- The nuclei within the drip line are stable with respect to particle emission. This is the stability with respect to the decays governed by the strong force.
- Stability with respect to particle emission does not imply the absolute stability.

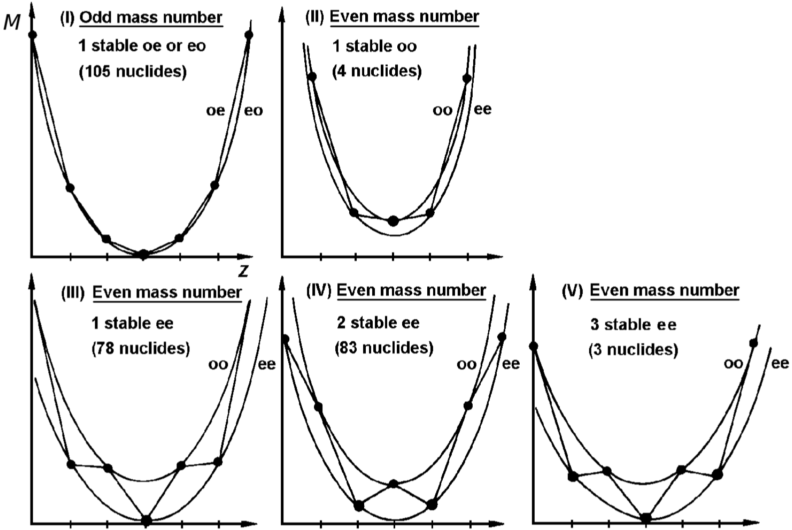
β-decay and stability of nuclei

- Nuclei within the driplines do not emit particles thus they have a constant mass number A.
- We examined nuclear masses which for a fixed A show a parabolic dependence as a function of the atomic mass number Z or the neutron number N.
- This parabolic dependence is well accounted for by the Liquid Drop Model as discussed at the beginning of the class.
- Nuclei at the bottom of this parabolic mass dependence form the true line of stability.
- β decay is the process in nuclei which at the fixed mass number A changes the proton and neutron numbers Z and N until the line of stability is reached.

Mass parabolas in odd-mass nuclei



Observed mass parabolas



Nuclear decay modes