Beta decay

Introduction to Nuclear Science

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Outline

1. The transition rate per fixed electron momentum

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2. The decay rate

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Outline

The transition rate per fixed electron momentum

- Based on the Fermi's golden rule the probability for the decay with electron momentum between p_e and $p_e + dp_e$ taking into account distorted waves and Coulomb interactions is

$$\lambda(p_e)dp_e = \frac{1}{2\pi^3\hbar^7 c^3} g^2 \mid M_{if} \mid^2 F(Z_d, p_e)(Q - T_e)^2 p_e^2 dp_e \quad (1)$$

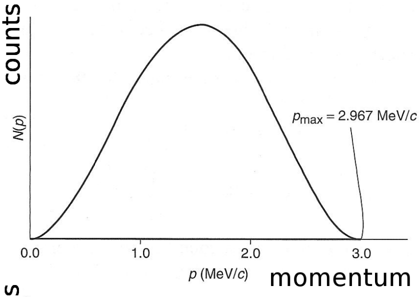
- The electron/positron spectra as a function of momentum for undistorted waves are symmetric with respect to

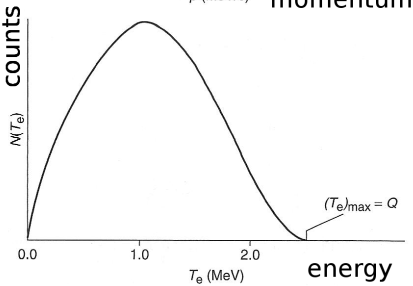
$$p_0 = \frac{1}{2}\sqrt{2m_e Q} \quad (2)$$

- The electron spectra as a function of momentum for distorted waves are shifted to lower momenta by Coulomb attraction.

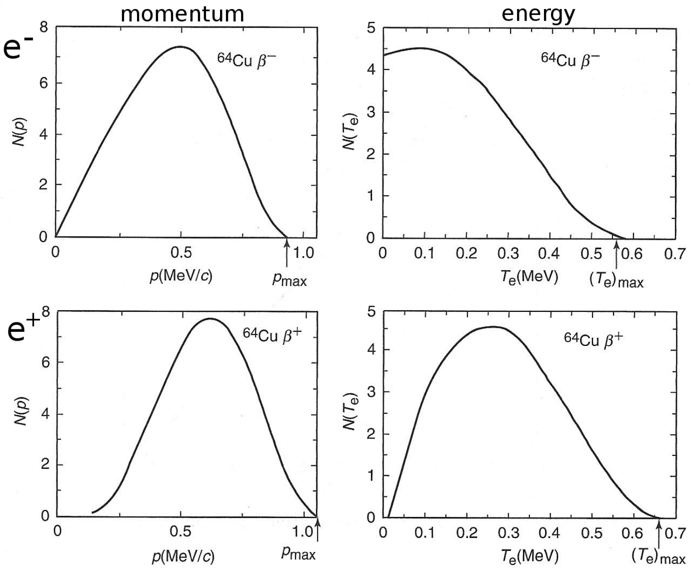
- The positron spectra as a function of momentum for distorted waves are shifted to higher momenta by Coulomb repulsion.

The statistical phase space factor for the three body decay





Distorted electron/positron spectra



The total transition rate

- The β-decay rate is equal to the integral of the transition rate per fixed electron momentum over the entire electron spectrum.

$$\begin{aligned}\lambda &= \int_0^\infty \frac{1}{2\pi^3\hbar^7 c^3} g^2 \mid M_{if} \mid^2 F(Z_d, p_e)(Q - T_e)^2 p_e^2 dp_e \\ &= \frac{1}{2\pi^3\hbar^7 c^3} g^2 \mid M_{if} \mid^2 \int_0^\infty F(Z_d, p_e)(Q - T_e)^2 p_e^2 dp_e \end{aligned} \quad (3)$$

- Since the integral is a sum this is consistent with λ being a sum of the $\lambda(p_e)$ over the continuous distribution of electron/positron momenta.

- The integral of the product of the Fermi function $F(Z_d, p_e)$ and terms depending on electron energy/momentum is called the f function

$$f(Z_d, Q) = \frac{1}{m_e^5 c^7} \int_0^\infty F(Z_d, p_e)(Q - T_e)^2 p_e^2 dp_e \quad (4)$$

The f-function

- The $f(Z_d, Q)$ function depends on the atomic number of the daughter Z_d and the Q-value for the decay.
- It represents the integrated number of states available for to the electron and the neutrino which share the energy equal to the Q-value of the decay.
- Note that we neglected recoil of the daughter and neutrino mass, these correction are available from more sophisticated theories.
- The $f(Z_d, Q)$ function is calculable and tabulated.
- Using the $f(Z_d, Q)$ function the decay rate is given by

$$\lambda = \frac{ln(2)}{t_{1/2}} = \frac{m_e^5 c^4}{2\pi^3 \hbar^7} g^2 \mid M_{if} \mid^2 f(Z_d, Q) \tag{5}$$

The ft value

- Let us look at the product of the f function and the β-decay half life. For a decay, this product is refereed to as the ft-value

$$ft = f(Z_d, Q)t_{1/2} = \frac{2\pi^3\hbar^7}{m_e^5 c^4} ln(2) \frac{1}{g^2 \mid M_{if} \mid^2} \propto \frac{1}{g^2 \mid M_{if} \mid^2} \quad (6)$$

- Note that the ft value is inversely proportional to the squared strength of the weak interactions g^2, thus the β decay can be used a tool to study the weak force.

- Note also that the ft value is inversely proportional to the square of the nuclear matrix element $\mid M_{if} \mid^2$ thus the β decay can be used as a tool to study the nuclear structure.

- We are going to set aside the study of the weak interactions but we will discuss the impact of the nuclear structure.

Conservation of angular momentum

- The conservation of angular momentum in the β-decay requires that

$$\vec{J_p} = \vec{J_d} + \vec{L} + \vec{S} \tag{7}$$

with $\vec{J_p}$ and $\vec{J_d}$ being the spin of the parent and the daughter and

$$\vec{L} = \vec{L_e} + \vec{L_\nu} \tag{8}$$

being the orbital angular momentum of the leptons and

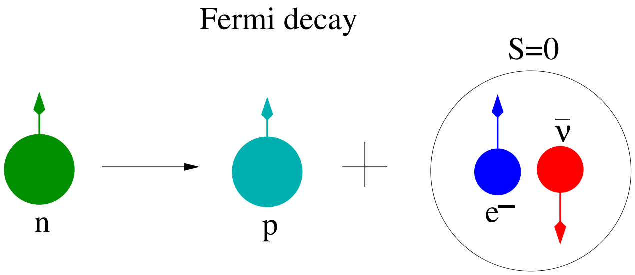
$$\vec{S} = \vec{S_e} + \vec{S_\nu} \tag{9}$$

being the spin angular momentum of the leptons.

- We distinguished two types of β decay, the Fermi decay with $\vec{S} = \vec{0}$ and the Gamow-Teller decay with $\vec{S} = \vec{1}$

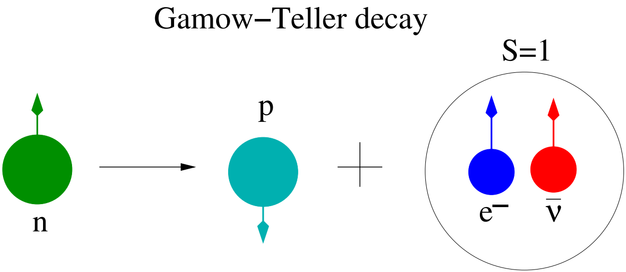
Fermi β decay

- In the Fermi decay mode the conservation of angular momentum requires that the spin of the baryons to point in the same direction before and after the decay.



Gamow-Teller β decay

- In the Gamow-Teller decay mode the conservation of angular momentum requires that the spin of the baryons to point in the opposite direction before and after the decay. This decay mode is sometimes called the spin-flip mode.



The impact of small mass of the leptons

- Let us investigate the implication of the small mass of the electron and neutrino in the following way:
  1. let us assume a decay with angular momentum L of $1\hbar$
  2. let us assume that the decay occurs at the nuclear surface at $r = 5$ fm
  3. let us see what is the implication for the energy of the leptons.
- For the maximum angular momentum for the emission from the surface

$$L = rp \implies p = \frac{L}{r} \implies E = pc = \frac{Lc}{r} \tag{10}$$

- For $L = 1\hbar$ and $r = 5$ fm

$$E = \frac{\hbar c}{r} = \frac{197\ [\text{MeV fm}]}{5\ [\text{fm}]} \approx 20\ [\text{MeV}] \tag{11}$$

- The above implies that leptons are emitted preferentially with no orbital angular momentum.

The impact of small mass of the leptons

- Leptons (electron and neutrino) are light
- The lepton orbital angular momentum L can be different than zero if:
  1. Leptons are emitted at high energy, however, the energy available to the leptons can not exceed Q which imposes the limit on angular momentum,
  2. Leptons are emitted at large radii, this is possible as the nuclear density distribution extends beyond the radius, however, the probability of large radius emission is suppressed.
- As a consequence, the larges β-decay rates are for the processes with $L = 0$, the processes with $L = 1$, $L = 2$ and $L = 3$ are possible but have rapidly fast decreasing probabilities or increasing lifetimes.

The parity

- The conservation of angular momentum in the β-decay

$$\vec{J}_p = \vec{J}_d + \vec{L} + \vec{S} \implies \Delta\vec{J} = \vec{J}_p - \vec{J}_d = \vec{L} + \vec{S} \tag{12}$$

imposes the following condition on the parity of the initial and final state

$$\pi_p = \pi_d(-1)^L \implies \Delta\pi = \pi_p\pi_d = (-1)^L \tag{13}$$

with L being the magnitude of the lepton angular momentum $\vec{L}$.

- Based on these conservation laws the observed β decay process are classified into groups.

- Various groups are observed to correspond to different values of ft.

Classification of β-decay processes

Type	L	$\Delta\pi$	$\Delta\vec{J}$ $\vec{S}=\vec{0}$ Fermi	$\Delta\vec{J}$ $\vec{S}=\vec{1}$ Gam-Tel
super-allowed	0	+	0	0
allowed	0	+	0	0,1
first forbidden	1	-	0,1	0,1,2
second forbidden	2	+	1,2	1,2,3
third forbidden	3	-	2,3	2,3,4

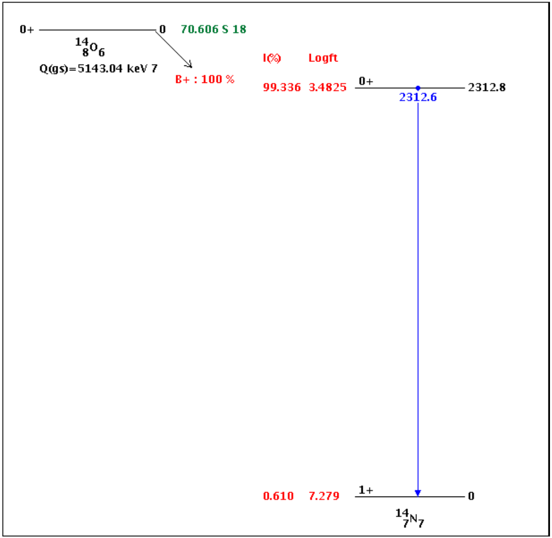
Super-allowed Fermi β-decay

- Let us consider a few examples.
- First, let us consider the decay between the parent 0^+ state and the daughter 0^+ state, for example

$$^{14}O \rightarrow ^{14} N(0^+, E = 2.313 \text{ MeV}) + e^- + \bar{\nu} \tag{14}$$

- Note that the Gamow-Teller decay between the 0^+ and 0^+ state is forbidden by the conservation of angular momentum and parity.
- Indeed the $\Delta\pi{=}{+}$, which implies that L is even.
- If $L = 2$ and $S = 1$, $\vec{L} + \vec{S}$ can not add to zero, therefore, can not connect the parent 0^+ state with the daughter 0^+ state.
- The only allowed combination is $L = 0$ and $S = 0$ which is the super-allowed Fermi transition.

Super-allowed Fermi β-decay



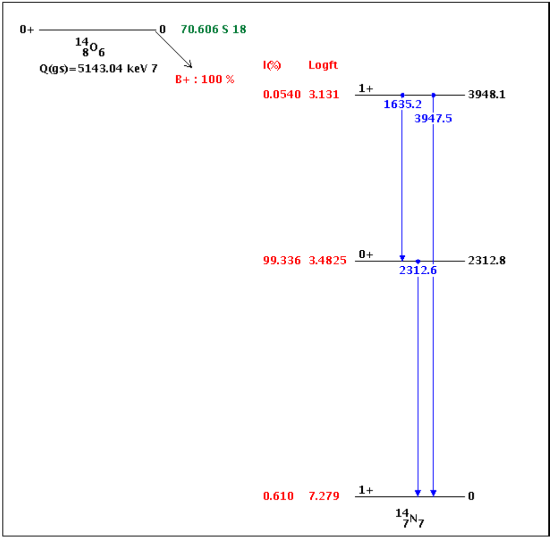
Allowed Gamow-Teller β-decay

- But ^{14}O is observed to decay to other states in ^{14}N as well.
- Let us consider the decay between the parent 0^+ state and the daughter 1^+ states

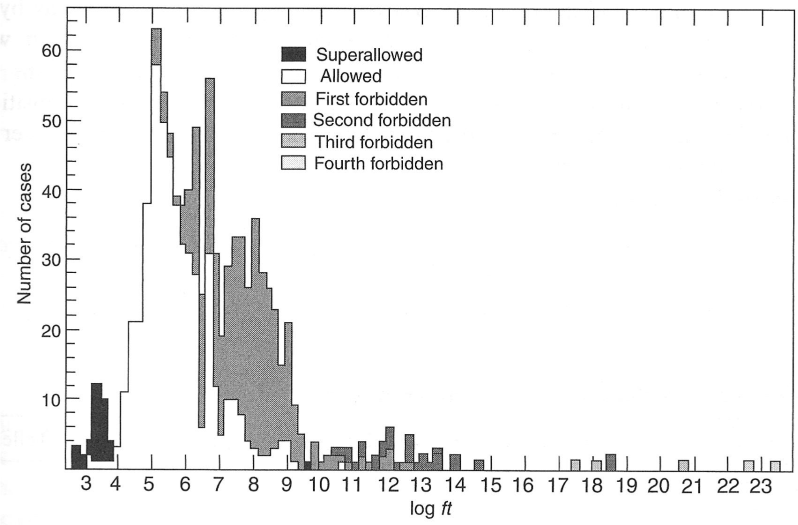
$$
\begin{aligned}
^{14}O \rightarrow ^{14} N(1^+, E = 0 \text{ MeV}) + e^- + \bar{\nu} \\
^{14}O \rightarrow ^{14} N(1^+, E = 3.948 \text{ MeV}) + e^- + \bar{\nu}
\end{aligned}
\tag{15}
$$

- Note that the Fermi decay between the 0^+ and 1^+ state is forbidden by the conservation of angular momentum and parity.
- Indeed the $\Delta\pi{=}{+}$, which implies that L is even.
- If $L = 0$ and $S = 0$, $\vec{L} + \vec{S}$ add to zero, therefore, can not connect the parent 0^+ state with the daughter 1^+ state.
- The allowed combination is $L = 0$ and $S = 1$ which is the allowed Gamow-Teller transition.
- The $L = 2$ and $S = 0$ (second forbidden Fermi decay) allowed but significantly less probable.

Allowed β-decay of ^{14}O



Classification of β-decay processes



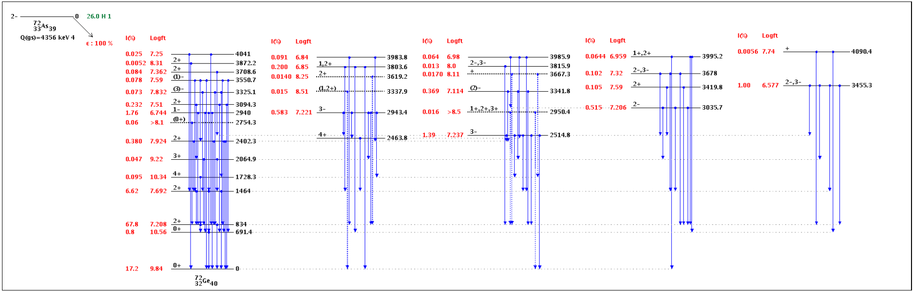
Classification of β-decay processes

Type	log(ft)	L	$\Delta\pi$	$\Delta\vec{J}$ $\vec{S}=\vec{0}$ Fermi	$\Delta\vec{J}$ $\vec{S}=\vec{1}$ Gam-Tel
super-allowed	2.9-3.7	0	$+$	0	0
allowed	4.4-6.0	0	$+$	0	0,1
first forbidden	6-10	1	-	0,1	0,1,2
second forbidden	10-13	2	$+$	1,2	1,2,3
third forbidden	> 15	3	-	2,3	2,3,4

Branching ratios in β decay

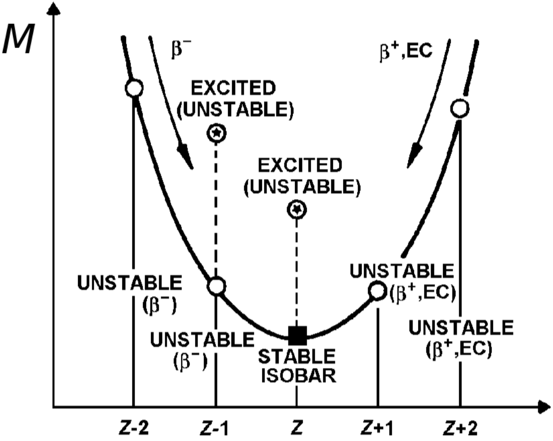
- As we observed for ^{14}O the β decay can populate several states.
- This is true for majority of β decays.
- This fact is often referred to as the branching in β decay.
- The relative population of the branches (called branching ratios) is determined by the Q-value and the selection rules.
- The number of populated branches is especially large if a high-spin, negative-parity ground state of an odd-odd nucleus decays to an even-even nucleus.
- The branching ratios can be used to define $\log(ft)$ values for individual states.
- Based on these $\log(ft)$ values and knowing the spin of the parent state spins and parities can be assigned in the daughter nucleus.

Branching ratios in the decay of ^{72}As



beta-decay far from stability

- The Q value for the decay increases far from stability, one way to see that is to examine the mass parabolas.



β-delayed particle emission

- For nuclei far from the line of stability the β decay may populate states in the daughter which are above the particle separation energy.
- This happens when the particle separation energy is small as compared to the Q-value.
- This happens near the proton dripline for the protons or near the neutron driplines for the neutrons.
- In such cases the β decay is followed by particle emission giving rise to either β-delayed proton or β-delayed neutron radioactivity.
- While neutrons are emitted promptly, the proton emission is further delayed by the Coulomb barrier.
- We also observed β-delayed α emission in the decay of naturally ocuring radioactive chains of U and Th.

The parity non conservation in β-decay

- The parity conservation in fundamental interactions was assumed as a law of nature until late 1950.
- In a Nobel prize experiment performed on β-decay of ^{60}Co by Madame Wu and collaborators the weak interaction were shown not to conserve parity.
- For a great article describing the pathway for this experiment please check this link
- For now the weak interactions are the only fundamental interactions showing non conservation of parity.

Wu experiment