Gamma-ray decay

Introduction to Nuclear Science

Simon Fraser University
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Outline

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1. Electromagnetic spectroscopy

2. Atomic spectroscopy

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3. Nuclear spectroscopy

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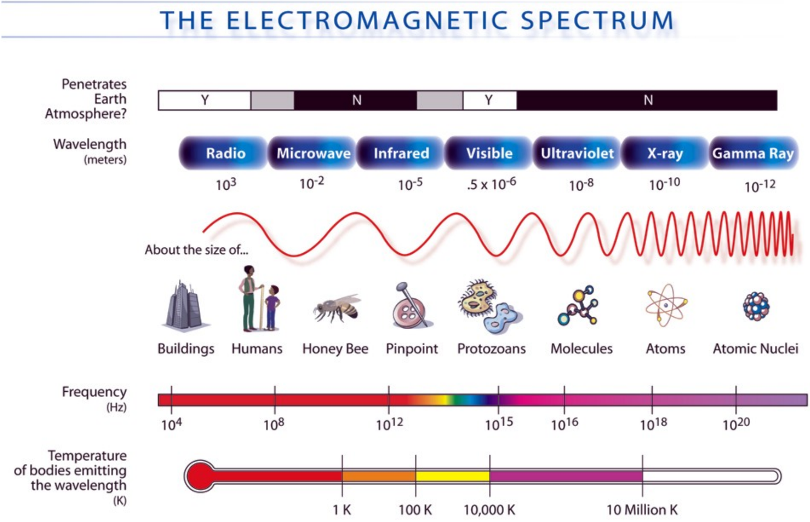
2. Atomic spectroscopy
3. Nuclear spectroscopy
4. Relativistic Doppler effect
5. Energetics of γ-ray decay

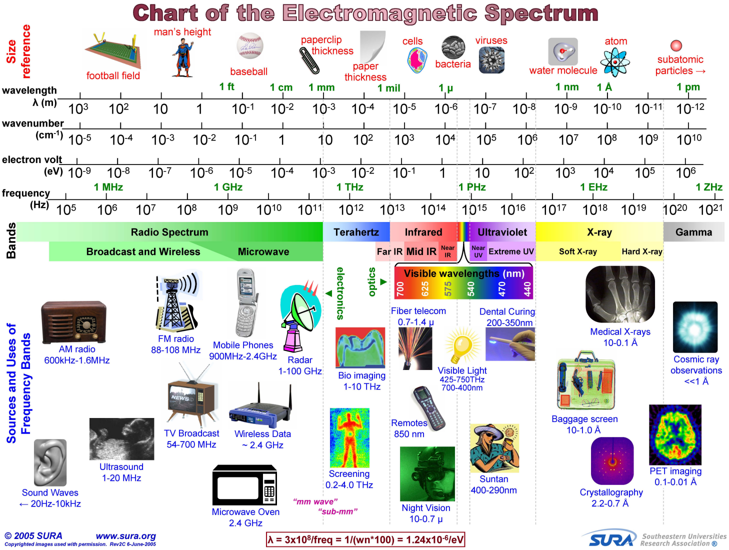
Photo de-excitation

- The γ-ray decay is electromagnetic deexcitation by emission of photons.
- As such it is analogues to the photo-deexcitation in atoms.
- The difference is in the energy of the photon.
- Conservation of energy implies that the energy of an emitted photon corresponds to the energy difference between quantized initial and final states.
- In atoms and molecules bound by the electromagnetic force the energies of excited states are on the order of eV. Photon energies are on the same scale.
- In nuclei bound by the strong force the energies of the excited states are on the order of MeV. Photon energies are on the same scale.

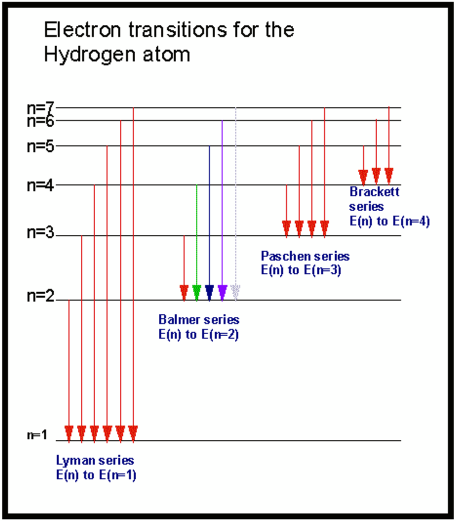
Electromagnetic spectrum



Electromagnetic spectrum

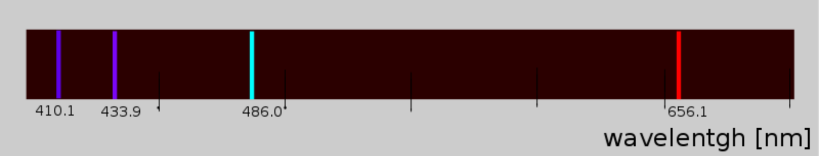


Line spectra for the Hydrogen atom



Visible line spectra for the Hydrogen atom

Part of the Balmer series falls into the visible spectrum.



Atomic spectroscopy

- Johann Balmer in 1885 derived an empirical formula describing relationship between wavelengths observed in the Balmer series of the Hydrogen atom

$$\lambda = 364.56 \frac{m^2}{m^2 - 2^2} \quad [\mathrm{nm}] \tag{1}$$

  with $m > 2$ is an integer number.

- In 1888 Johannes Rydberg propose a very similar formula which was applicable to spectral lines of many chemical elements

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \tag{2}$$

  with R_∞ being the Rydberg constant and $n_2 > n_1$ were integer numbers.

Atomic spectroscopy

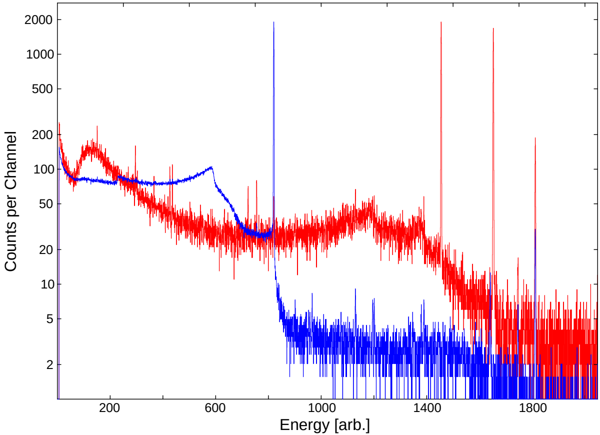
- Rydberg formula worked for the Balmer series as well as other series being observed in Hydrogen atom. Different series were identified with different n_1, for the Balmer series $n_1 = 2$.
- In 1911 E. Rutherford discovers atomic nucleus.
- In 1913 Niels Bohr (Rutherford's student at the time) proposes a model of an atom which explains (after 25 years from the discovery) the Balmer and Rydberg laws.
- In 1926 Schrödinger proposes "the equation".
- The development of quantum mechanics and proper model for hydrogen atom follows.
- Atomic spectroscopy was crucial to disentangle the structure of atom and molecules and led to quantitative and computational chemistry from the first principles.

Nuclear spectroscopy

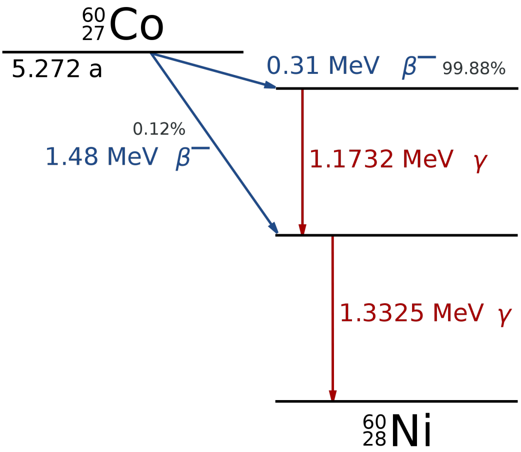
- Gamma rays are nuclear line spectra.
- In contrast to atomic spectroscopy in which every element has its characteristic line spectrum, in nuclear spectroscopy every isotope has its characteristic γ-ray spectrum.
- As a consequence nuclear spectroscopy can provide information on elemental and isotopic composition of a sample.
- Gamma-rays are penetrating and can emerge in significant quantities from dense and extended samples.
- Thus combination of activation techniques (such as, for example, neutron activation) with nuclear spectroscopy is a powerful analytical technique sensitive to parts per billion in concentration.
- The same is true for applications of nuclear spectroscopy to radio-traces and radio-labelling.

Gamma-ray line spectra

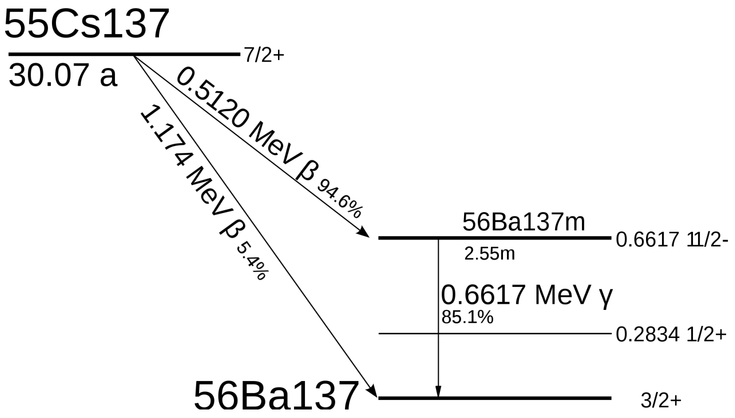
Red: ^{60}Ni from the decay of ^{60}Co
Blue: ^{137}Ba from the decay of ^{137}Cs



Decay of ^{60}Co



Decay of ^{137}Cs



Gamma-ray line spectra

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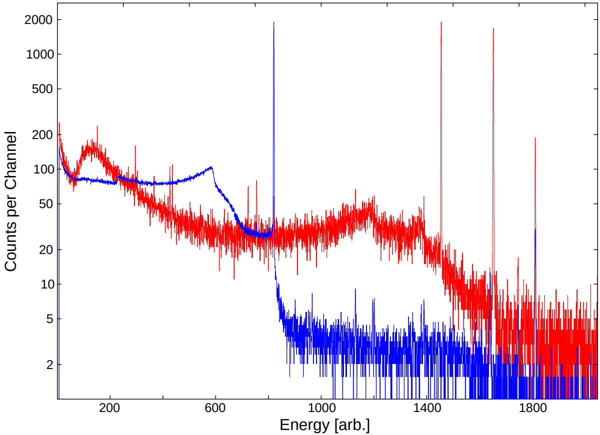
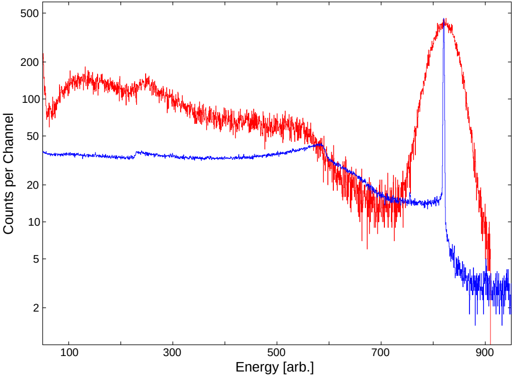


Low vs. high resolution detection

^{137}Cs decay detected in
red: NaI scintillator
blue: HpGe (high purity Ge semiconductor)



The relativistic Doppler effect

- For an applet demonstrating relativistic Doppler effect see this link.
- Photons emitted from a moving source have different frequency ν, energy $E = h\nu$, and, in visible spectrum the colour, as compared to the frequency ν_0, energy $E_0 = h\nu_0$ and the colour of the light emitted from the same source at rest. Frequency, energy and colour depend on the angle θ between the source direction of motion and the direction of observation:

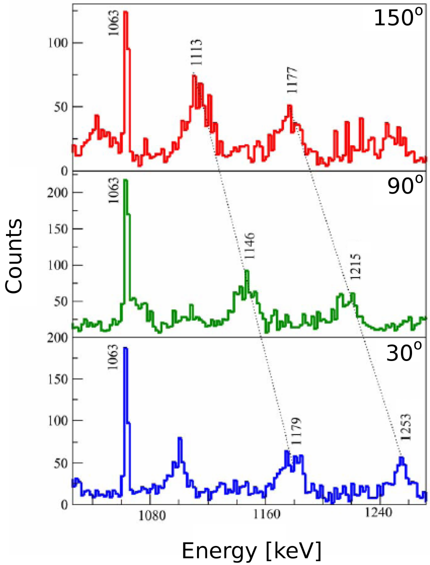
$$\nu = \nu_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta}$$

$$E = h\nu = h\nu_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} = E_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta}$$

Relativistic Doppler Effect

$$E = E_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta}$$

While the 1063 keV transition is emitted at rest, the other transitions are emitted in flight.



Energy and momentum conservation

- Gamma-ray decay is a binary decay.
- Gamma-rays are massless but they carry (relatively small) momentum.

$$E_\gamma = p_\gamma c \implies p_\gamma = \frac{E_\gamma}{c} \tag{3}$$

- For the parent at rest conservation of momentum implies that the daughter nucleus recoils with the momentum of the same magnitude but the opposite direction

$$\vec{p_d} + \vec{p_\gamma} = 0 \implies \vec{p_d} = -\vec{p_\gamma} \tag{4}$$

- The recoil energy is small thus it can be calculated non-relativistically.
- The energy conservation for the initial state of mass M_i and the final state of mass M_f in the γ-decay is

$$M_i c^2 = M_f c^2 + E_\gamma + T_d \tag{5}$$

Energy and momentum conservation

- The energy conservation can be rewritten as

$$M_i c^2 - M_f c^2 = E_i - E_f = \Delta E = E_\gamma + T_d \quad (6)$$

  with ΔE being the energy difference between the initial and the final state.

- Note that the final state does not have to be the ground state. The decay can very well go between excited states.
- If the initial state is high-energy cascades of several γ rays are emitted before the ground state is reached (the reason will be explained in the next lecture).
- Note that the energy of the γ ray is smaller than the energy difference between excited states since part of the energy is in the recoil of the daughter. This is significant and crucial for understanding the Mössbauer effect.

Energy and momentum conservation

- The recoil energy of the daughter is

$$T_d = \frac{p_d^2}{2M_d} = \frac{p_\gamma^2}{2M_d} = \frac{E_\gamma^2}{2M_d c^2} \tag{7}$$

- The energy of the γ-ray can be calculated from

$$\Delta E = E_\gamma + T_d = E_\gamma + \frac{E_\gamma^2}{2M_d c^2} \tag{8}$$

- The γ-ray energy is

$$E_\gamma = \left(\sqrt{\frac{2\Delta E}{M_d c^2} + 1} - 1\right) M_d c^2 \approx \Delta E \left(1 - \frac{1}{2}\frac{\Delta E}{M_d c^2}\right) \tag{9}$$

Recoil on emission

- The above reasoning implicates that the γ ray energy is lower than the energy difference between excited states by

$$E_\gamma = \Delta E - \delta \implies \delta = \Delta E \frac{1}{2} \frac{\Delta E}{M_d c^2} \tag{10}$$

- For the 0.439 MeV energy difference between an excited state in ^{69}Zn and the ground state

$$\Delta E = 0.439 \text{ [MeV]} \quad M_{^{64}\mathrm{Zn}} c^2 = 68.297u$$
$$\delta = 0.439 \frac{0.439}{2 * 68.297 * 931.5} = 1.5 \times 10^{-6} \text{ [MeV]} = 1.5 \text{[eV]} \tag{11}$$

- $\delta = 1.5$ eV is small comparing to the ΔE energy difference and in most cases insignificant, except one important comparison.

The natural line width

- You may recall the Heisenberg uncertainty principle

$$\Delta p_x \Delta x \geq \hbar \tag{12}$$

- It implicates that at a given time both momentum and position can not be defined exactly.
- Uncertainty principles of a similar kind hold between other, so called, conjugated variables. For example between the time and the energy

$$\Delta E \Delta t \geq \hbar \tag{13}$$

  Note that in the above equation ΔE represents the uncertainty in the energy, not the energy difference between the initial and the final state as in the previous slides.
- If we would have sufficient resolving power we would see that the state does not have a single energy, but a distribution of energies with the width defined by Γ.

The natural line width

- In nuclear processes the statistical nature of radioactive decay implicates an uncertainty in the existence of a quantum state which is on the order of the lifetime. This results in uncertainty of the energy, or the natural width of the state on the order of

$$\Gamma \approx \Delta E = \frac{\hbar}{\Delta t} = \frac{\hbar}{\tau} \tag{14}$$

- Typical lifetimes of γ-ray decaying states are on the order of a picoseconds.
- Thus typical natural width for such states is

$$\Gamma \approx \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22}}{10^{-12}} \text{ [MeV]} = 6.58 \times 10^{-10} \text{ [MeV]} = 0.7 \text{ [meV]}$$

- Note, that the natural line width $\Gamma \sim 6.58 \times 10^{-4}$ [eV] is small comparing to the γ-ray energy shift due to the recoil of $\delta = 1.5$ eV.

Recoil on absorption

- Let us examine the energy and momentum conditions on absorption of a γ-ray. It should be now clear that the γ-ray energy has to be slightly larger than the energy difference between the initial and final state to compensate for the recoil of the daughter.

- The energy conservation for the initial state of mass M_i and the final state of mass M_f in the γ-ray absorption is

$$\begin{aligned} M_i c^2 + E_\gamma &= M_f c^2 + T_d \\ \Delta E &= -E_\gamma + T_d \implies E_\gamma = -\Delta E + T_d \end{aligned} \tag{15}$$

  if we use the same labelling convention as for Eq. 6.

- Note that for absorption $E_f > E_i$ and

$$\Delta E = E_i - E_f < 0 \tag{16}$$

Recoil on absorption

- Calculations similar to these performed for the γ-ray emission lead to the following expression for the γ-ray absorption

$$\Delta E = -E_\gamma + T_d = -E_\gamma + \frac{E_\gamma^2}{2M_d c^2} \tag{17}$$

- The γ-ray energy is

$$\begin{aligned} E_\gamma &= \left(1 - \sqrt{1 - \frac{2 \mid \Delta E \mid}{M_d c^2}}\right) M_d c^2 \approx \mid \Delta E \mid \left(1 + \frac{1}{2}\frac{\mid \Delta E \mid}{M_d c^2}\right) \\ &= \mid \Delta E \mid + \delta \end{aligned} \tag{18}$$

with the same shift but now up in energy

$$\delta = \mid \Delta E \mid \frac{1}{2}\frac{\mid \Delta E \mid}{M_d c^2} \tag{19}$$

The recoil on emission and absorption

- We observed that the natural line width of γ-ray emitting states are on the order of 1 meV.
- We calculated the recoil shifts of the γ-ray energy on absorption or emission as being on the order of 1 eV.
- Base on that we should conclude that the re-absorption of an emitted γ ray by the nucleus of the same isotope is impossible as the energy shifts are not compatible.
- This was the understanding until $\sim$ 1957
- But things have changed. We will talk about it next week, but before we go let us consider one more issue.

Doppler broadening due to the thermal motion

- Since atoms are in thermal motion nuclei are never at rest.
- Since nuclei are moving the emitted γ rays are Doppler shifted.
- Due to the same thermal motion the range of energies of γ rays for absorption is broader than the natural width.
- The impact of the Doppler effect resulting from the thermal motion can be taken into account by an increase of the natural line width for a state up to the temperature-dependent Doppler width Γ_D for a state.
- An important fact is that in a normal thermal conditions the Doppler width Γ_D is on the order of a fraction of eV, smaller than the δ recoil under absorption or emission.

The recoil on emission and absorption