Gamma-ray decay

Introduction to Nuclear Science

Simon Fraser University
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Outline

1. Gamma-ray decay and nuclear structure

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2. Angular distribution of electromagnetic radiation

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3. The $1^+ \to 0^+$ decay

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4. The coordinate system

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5. Un-oriented states
6. Oriented states

Importance of γ-ray decay

- Gamma-ray decay becomes in the last 50 years one of the most frequently used tools in nuclear science.
- There are several experimental and theoretical reasons for that
- The experimental advantages are:
  - high (better than particle per thousand) accuracy of γ-ray energy measurement
  - γ-ray ability to penetrate and emerge nearly unimpeded from dense samples
- The theoretical advantages are:
  - electromagnetic interactions which lead to a photon emission are well understood and are calculable
  - the electromagnetic interactions are a small perturbation of the nuclear system bound by the strong force, photon emission only weakly disturbs the nucleus.

Information on nuclear structure

- Gamma-ray decay provides an insight into the nuclear structure.
- It provides information on energies, spins and parities of excited states from energies, multipolarities and selection rules for gamma-ray transitions.
- It provides information on nuclear deformation (in particular the quadrupole moments) from lifetimes and branching ratios measurements for excited states.
- It provides information on nuclear magnetic moment from perturbations in magnetic fields.
- It provides information on nuclear electric quadrupole moment from perturbations in electric fields.

Radiation angular distribution

- Nuclear excitation of various type de-excite via emission of a photo of a multipolarity which is directly related to the excitation mode.
- Multipolarity of the photon can be distinguished from the angular distribution. Click here for the angular distribution pattern for the quadrupole field.
- The nuclear excitation modes can be thus identified from gamma-ray multipolarities and angular distributions.

Radiation fields for a vibrating sphere

- click here for the quadrupole vibrations
- click here for the octupole vibrations
- click here for the hexadecuple vibrations

Radiation fields for an oscillating current

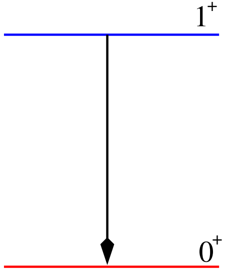
- click here for the dipole oscillations

Radiation fields for a quadrupole rotor and vibrator

- click here for the quadrupole rotor
- click here for the quadrupole vibrator

Our toy model: the $1^+ \to 0^+$ decay

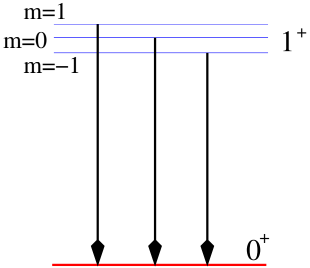
- To demonstrate the way the angular distribution effects arise we are going to investigate a hypothetical $1^+ \to 0^+$ transition.



- This is the transition of $M1$ multipolarity.

Magnetic substates

- The initial 1^+ state has three degenerate magnetic substates which differ by the magnetic quantum number m.
- The final 0^+ state has a single magnetic substate.
- The γ-ray transition can proceed from any of the three magnetic substates of the initial state to the single magnetic substate of the final state. The degeneracy of the substates implies the same energy for each transition.



Angular distributions

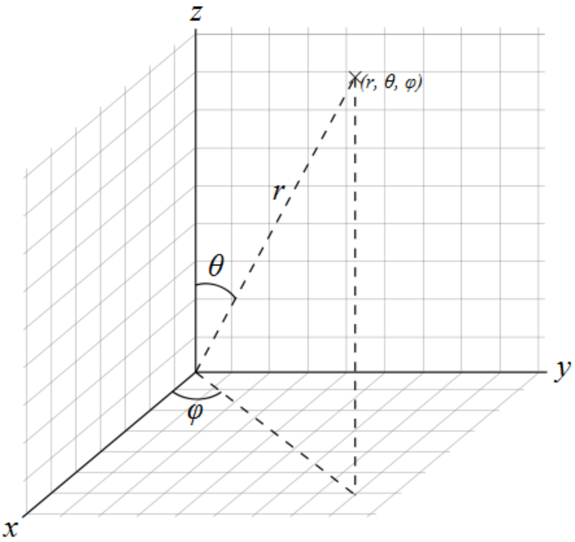
- While the energy of the transitions between different substates are the same the radiation angular distributions are different.
- The angular distribution of the radiation does depend on two factors: the multipolarity L of the transition and the change of the magnetic quantum number $\Delta m = m_i - m_f$.
- For the $M1$ case of our interest the angular distributions $W(\theta)$ are

$$\begin{aligned} W_{M1,\Delta m=1}(\theta) &= \frac{3}{16\pi}(1+\cos^2\theta) \\ W_{M1,\Delta m=0}(\theta) &= \frac{3}{8\pi}\sin^2\theta \\ W_{M1,\Delta m=-1}(\theta) &= \frac{3}{16\pi}(1+\cos^2\theta) \end{aligned} \tag{1}$$

The coordinate system

- Note that angular distributions depend only on the azimuthal angle θ, but not on the polar angle ϕ.
- For the angle θ to be used, however, we need to know the orientation of the spherical coordinate system in use.
- So far we did not explicitly talked about orientation of the coordinate system, which could prompt you to wondering how to use the polar angles for angular distribution.
- The answer to that question is in the recognition of the fact that the definition of the substates and the magnetic quantum number m for the initial and the final state requires a spherical coordinate system.
- For the angular distribution we use the same spherical coordinate system in which the magnetic quantum number m is defined.

The spherical coordinates



$$
\begin{aligned}
r &= \sqrt{x^2+y^2+z^2} \\
\cos\theta &= \frac{z}{\sqrt{x^2+y^2}} \\
\tan\phi &= \frac{y}{x}
\end{aligned}
$$

$$
\begin{aligned}
x &= r\sin\theta\cos\phi \\
y &= r\sin\theta\sin\phi \\
z &= r\cos\theta
\end{aligned} \tag{2}
$$

The orientation of the z-axis

- Note that the magnetic quantum number m is the projection of the total angular momentum of the state on the quantization z axis.
- Let us consider the $m =$ substate.
- In this substate the projection of the total angular momentum of the state on the quantization z axis is zero.
- This implies that in the $m = 0$ substate the total angular momentum of the state is perpendicular to the quantization z axis
- Thus is we can define the $m = 0$ substate and its orientation in space we can also define the orientation of the quantization axis in space.

Un-oriented states

- The key concept to recognize in the angular distribution studies is the fact that the magnetic quantum number m defines the orientation of the total angular momentum vector in space.
- We have demonstrated that for $m = 0$ which correspond to the total angular momentum perpendicular to the quantization axis.
- If the population of the initial magnetic substate is completely random, each of the substate will have the same population probability.
- Since the total probability of occupation of the initial state is one, the above implies probability of $1/3$ for the population of any particular substate of the initial state.
- Such a population is refereed to as an un-oriented state since it implies that the angular momentum vector of the initial state can point out with equal probability into any direction in space.

Angular distribution from un-oriented states

- If the initial 1^+ is unoriented we will observe a transition from any magnetic substate to the final state with an equal probability.
- As each of these transitions has its angular distribution we will observe the probability-weighted sum of the angular distribution. The sum is

$$\begin{aligned} W_{M1} &= \frac{1}{3} W_{M1,\Delta m=1} + \frac{1}{3} W_{M1,\Delta m=0} + \frac{1}{3} W_{M1,\Delta m=-11} = \\ &= \frac{1}{3}\frac{3}{16\pi}(1+\cos^2\theta) + \frac{1}{3}\frac{3}{8\pi}\sin^2\theta + \frac{1}{3}\frac{3}{16\pi}(1+\cos^2\theta) \\ &= \frac{1}{8\pi}(1+\cos^2\theta) + \frac{1}{8\pi}\sin^2\theta) \\ &= \frac{1}{8\pi}(1+\cos^2\theta+\sin^2\theta) = \frac{1}{8\pi}2 = \frac{1}{4\pi} \end{aligned} \tag{3}$$

- This implies that the angular distribution $W_{M1}(\theta)$ does not depend on the azimuthal angle θ or the polar angle ϕ or in other words, probability for emitting a γ-ray is the same in all directions.

Oriented states

- Let us consider now a possibility of selective population the $m = 0$ substates of the initial state.
- In such a case the probability of the occupation of the initial substate with $m = 0$ is 1, while the other probabilities are zero.
- As each of the transitions has its angular distribution we will observe the probability-weighted sum of the angular distribution. The sum is

$$\begin{aligned} W_{M1} &= 0W_{M1,\Delta m=1} + 1W_{M1,\Delta m=0} + 0W_{M1,\Delta m=-11} = \\ &= \frac{3}{8\pi}\sin^2\theta \end{aligned} \quad (4)$$

- This implies that the angular distribution $W_{M1}(\theta)$ does depend on the azimuthal angle θ and the probability for emitting a γ-ray is large in the direction perpendicular to the z axis and zero in the direction along the z axis.

Oriented states

- Let us consider now a possibility of selective population the $m = 1$ substates of the initial state.
- In such a case the probability of the occupation of the initial substate with $m = 1$ is 1, while the other probabilities are zero.
- As each of the transitions has its angular distribution we will observe the probability-weighted sum of the angular distribution. The sum is

$$\begin{aligned} W_{M1} &= 1W_{M1,\Delta m=1} + 0W_{M1,\Delta m=0} + 0W_{M1,\Delta m=-11} = \\ &= \frac{3}{8\pi}(1+\cos^2\theta) \end{aligned} \quad (5)$$

- This implies that the angular distribution $W_{M1}(\theta)$ does depend on the azimuthal angle θ and the probability for emitting a γ-ray is large in the direction along the z axis and small in the direction perpendicular to the z axis.

Oriented states

- States with unequal population of the magnetic substates are referred to as oriented states.
- The name is justified by the fact that the unequal population of magnetic substates implies a preferential orientation of the angular momentum vector in space.
- Gamma-ray emitted from oriented states show multipolarity -dependent angular distributions.
- Angular distributions of gamma-rays emitted from unoriented states are isotropic and independent from multipolarity.
- Several processes have been developed to orient the nuclear states (see next slide).

Methods to orient nuclear states

- There are predominantly three methods used to orient the nuclear states

  1. Nuclear reactions.
  2. Zeeman splitting combined with the cryogenic cooling to sub Kelvin temperatures (this method was used in the Wu experiment)
  3. Radioactive decay.

Oriented states from nuclear fusion reactions

- In nuclear reaction the beam axis provides the quantization z axis.
- The momentum of the beam is along the z axis.
- The angular momentum of the beam is

$$\vec{L} = \vec{r} \times \vec{p} \tag{6}$$

  and has to be perpendicular to the beam axis from the property of the cross product.
- For the fusion of the spinless beam and the spinless target the angular momentum of the beam is the total angular momentum in the reaction thus the total angular momentum of the compound system.
- Since the total angular momentum is perpendicular to the quantization axis only $m = 0$ substate is populated in the reaction.

Zeeman splitting plus cooling

- The Zeeman effect in the external magnetic field splits the energy of magnetic substates according to the interaction energy given by the dot product of the magnetic field and the magnetic moment.
- Following the Zeeman effect an energy gap develops between the magnetic substates.
- The system favours occupation of the lowest energy state, however, the thermal motion can result in occupation of the higher energy states.
- If the system is cooled to the temperatures at which the thermal energy kT is small compared to the energy gap from the Zeeman splitting the system gets oriented with the lowest energy substate populated preferentially.