

Direct and compound nucleus reaction

Introduction to Nuclear Science

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- 2 Direct reactions

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- 3 The neutron stripping (d, p) reaction

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- 5 Boltzmann's distribution

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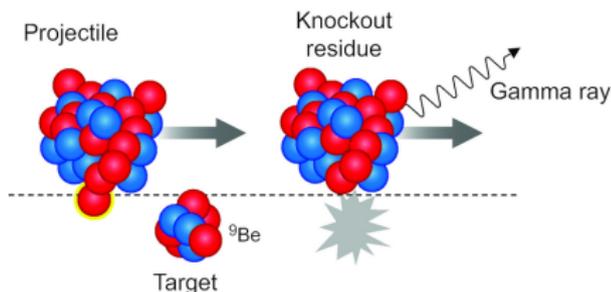
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- 4 Compound nucleus reactions
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- 6 Resonant compound nuclear reactions

The number of interactions

- There are two extreme scenarios for nuclear reactions:
 - Projectile and target are within the range of nuclear forces for the very short time allowing for an interaction of a single nucleon only. These type of reactions are called the direct reactions.
 - Projectile and target are within the range of nuclear forces for the time allowing for a large number of interactions between nucleons. These type of reactions are called the compound nucleus reactions.
- Interaction time is critical for defining the reaction mechanism.
- Direct reactions are well described as a one-step transition from the initial state in the entrance to the final state in the exit channel.
- The opposite is true for the compound nucleus reactions. In fact the number of steps involved in these types of reactions is so large that can only be handled using statistical methods.

Types of direct reactions

- Direct reactions include:
 - stripping, for example neutron stripping in the (d, p) reaction,
 - pickup, for example neutron pick up in the (p, d) reaction,
 - break-up: $A + B \rightarrow A + C + D$ (note that this is not a binary reaction),
 - knock-out, a reaction in which a single nucleon or a light cluster is removed from the projectile by a collision with the target.



Significance of direct reactions

- Direct reactions are widely used as a tool to study nuclear interactions and nuclear reaction dynamics.
- A proper quantum mechanical treatment has been developed for single-step reactions.
- Comparison between measured and calculated observables such as total and differential cross sections as well as angular distributions of the products is used to deduce information about the properties of the reactions and interacting nuclei.
- The nuclear structure of nuclei in the entrance and exit channel has a direct impact on direct reaction dynamics.
- Predictive power for direct reactions is of importance for understanding nucleosynthesis and production of elements in stars.

When is a reaction direct?

- Note that transfer leads to the same products as stripping or pick-up, but this by itself does not implicate that the transfer is direct.
- For the reaction to be direct, it is imperative to involve a single nucleon interaction.
- This implicates reactions at high energy to limit the time available for multiple internuclear interactions.
- Truly, there is always some non-direct component in reactions studied experimentally, the goal is to limit these as much as possible.
- Quantum mechanics can study (limited) number of multi-nucleon interactions via so called “coupled channel method” but the predictive power decreases quickly with the number of channels involved.

Characteristics of direct reactions

- Direct reactions are fast (time scale to be estimated in the next slide).
- Direct reactions require projectile energy larger than $\sim 5 \text{ MeV}/A_p$.
- Direct reactions happen on the surface rather than in the volume of interacting nuclei.
- Direct reaction products have highly anisotropic, forward focused, angular distributions in the centre of mass reference frame.
- Angular distributions of direct reaction products are sensitive to the momentum transfer and parity change during the reactions.
- Based on the selection rules from angular momentum and parity conservation the angular distribution measurements in direct reactions yield spin and parities of states populated in the exit channel.

The time scale of direct reactions

- The time scale of direct reaction is equal to the range of nuclear force in the interaction divided by the relative speed of the projectile and the target.
- For the range of nuclear forces let us use the sum of the nuclear radii plus $\Delta = 2$ fm:

$$R = R_p + R_T + \Delta R = 1.2 \cdot (A_p^{1/3} + A_T^{1/3}) + 2 \text{ [fm]} \quad (1)$$

- From the centre of mass transformation we know that the relative speed is the same in the lab and in the centre of mass frame thus we can use the initial lab speed

$$v = \sqrt{\frac{2E}{m_p}} = \sqrt{\frac{2E}{A_p u c^2}} c \quad (2)$$

The time scale of the neutron stripping

- Let us consider the $^{56}\text{Fe}(d, p)^{57}\text{Fe}$ neutron stripping reaction at the beam energy of 15 MeV.
- The relative speed is

$$v = \sqrt{\frac{2E}{A_p u c^2}} c = \sqrt{\frac{2 * 15}{2 * 931.5}} c = 0.127c \quad (3)$$

- The range of nuclear forces is

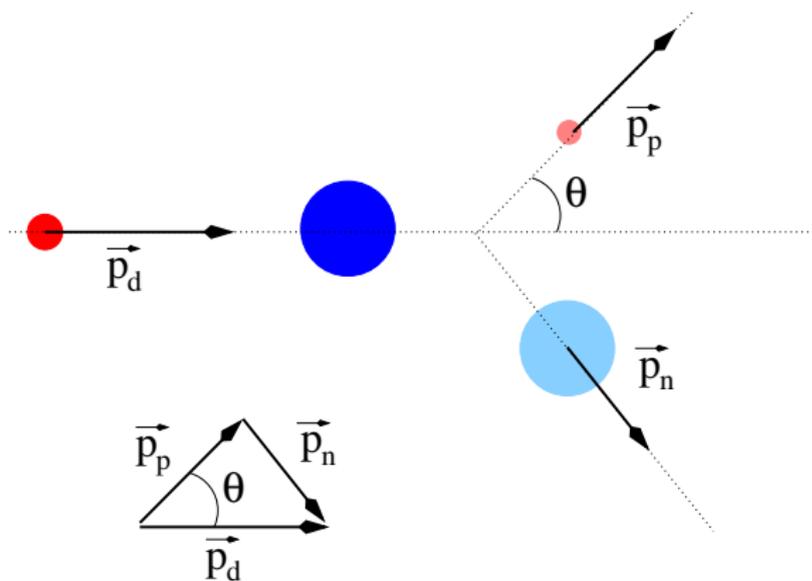
$$R = R_p + R_T + \Delta R = 1.2 \cdot (2^{1/3} + 56^{1/3}) + 2 = 8.1 \text{ [fm]} \quad (4)$$

- The speed of light is

$$c = 3 \times 10^8 \text{ [m/s]} = 3 \times 10^8 \text{ [fm/fs]} = 3 \times 10^{23} \text{ [fm/s]} \quad (5)$$

- The time for the reaction is

$$t = \frac{R}{v} = \frac{8.1}{0.127 * 3 \times 10^{23}} = 2.1 \times 10^{-22} \text{ [s]} \quad (6)$$

The kinematic of the neutron stripping (d, p) reaction

$$p_n^2 = p_d^2 + p_p^2 - 2p_d p_p \cos \theta \quad (7)$$

The kinematic of the neutron stripping (d, p) reaction

- From the law of cosines and reaction kinematics

$$p_n^2 = p_d^2 + p_p^2 - 2p_d p_p \cos \theta \quad (8)$$

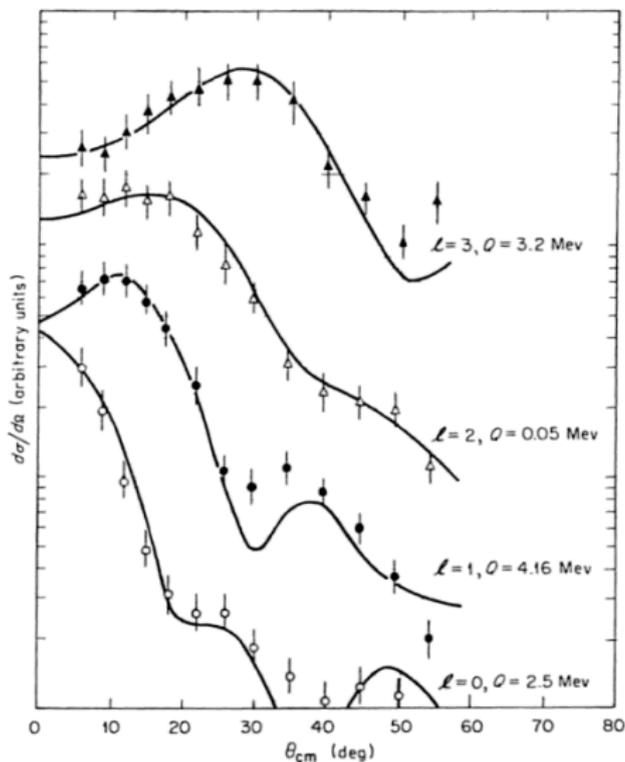
- From the conservation of angular momentum and fact that the reaction occurs at the surface

$$L^2 = l(l+1)\hbar^2 = R^2 p_n^2 \quad (9)$$

with R being the radius of the interaction.

- Above equations imply a relationship between the angular momentum l transferred in the reaction and the angle of emission θ of the products

$$\frac{l(l+1)\hbar^2}{R^2} = p_d^2 + p_p^2 - 2p_d p_p \cos \theta \quad (10)$$

Angular distribution in the neutron stripping (d, p) reaction

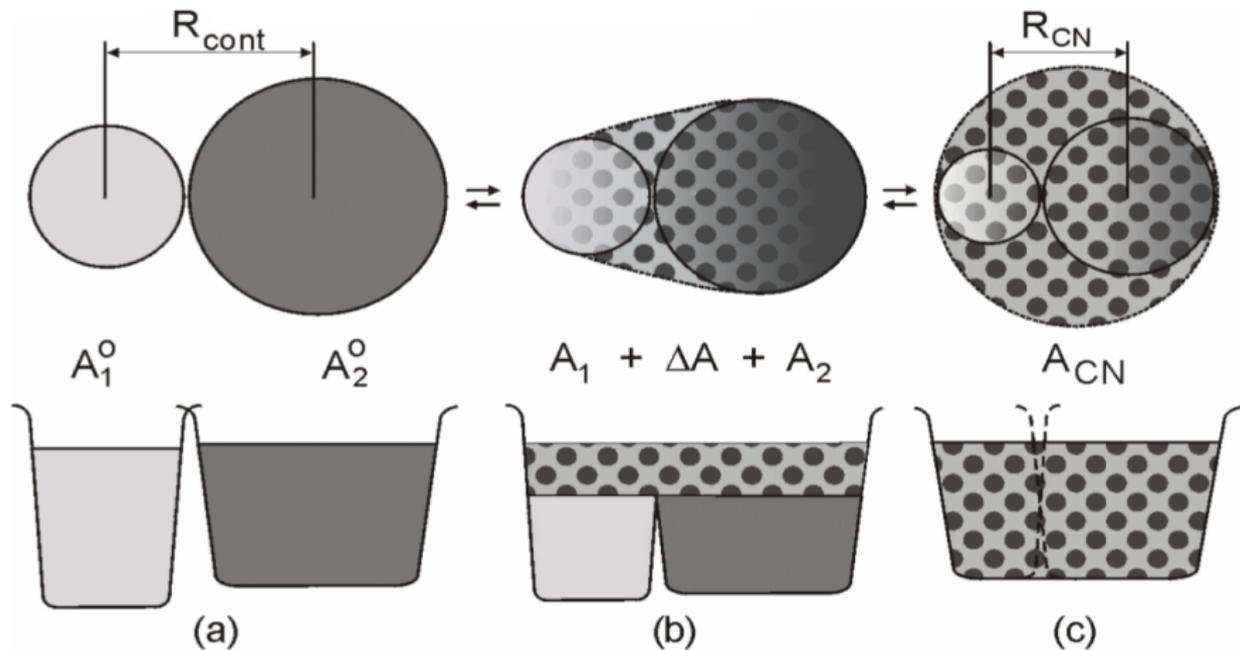
Characteristics of compound nucleus reaction

- The direct reactions involve a single-nucleon interaction and are fast.
- In contrast, compound nucleus reaction involve many nucleon-nucleon interactions, in fact very many so these collisions lead to a complete thermal equilibrium (equal energy partition between nucleons) inside the compound nucleus.
- Since energy equilibration require time, the compound nucleus reaction are significantly slower than direct reactions.
- The compound system releases energy by emission of neutrons, protons, α particles and γ -rays, but has a lifetime on the order of 10^{-19} s.
- The 10^{-19} s may seem short but it is ~ 1000 times longer than the characteristic time for direct reactions.

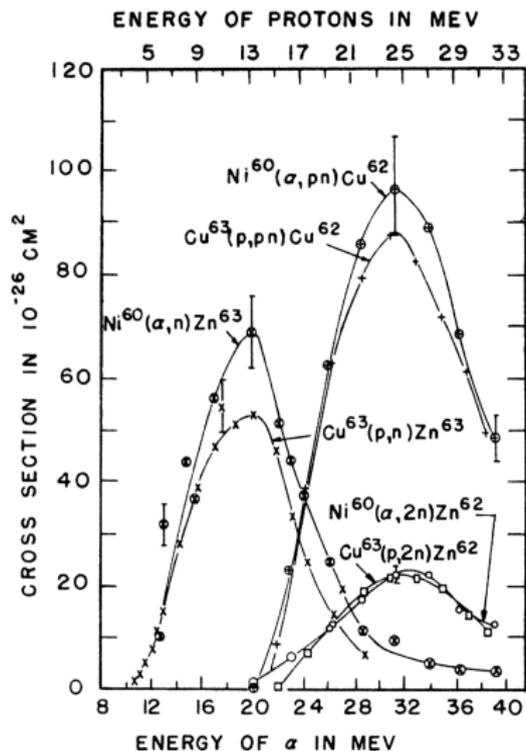
Independence hypothesis

- The idea of compound nucleus formation comes from Niels Bohr, who postulated equilibration of energy.
- A direct consequence of the equilibration is in the fact that the decay of compound nucleus should not depend on the way the compound nucleus is formed.
- A way to think about it is to realize that with the large number of collisions between the nucleons in the compound the information on the entrance channel is completely lost from the system.
- This idea is sometimes expressed by a statement that the compound nucleus loses memory on the way of its formation.
- Consequently, the decay of the compound depends only on the mass and atomic numbers, excitation energy and angular momentum.

Compound nucleus formation



Compound nucleus excitation functions



Thermal equilibrium in compound nucleus reactions

- Equilibration of energy is one of the key hypothesis for the compound nucleus reactions.
- Thermal equilibration implies that particles emitted from the compound nucleus have Maxwell-Boltzmann distribution, very much like vapour molecules escaping from a liquid.
- This hypothesis can be tested experimentally.
- Moreover, if confirmed, the fit to measured Maxwell-Boltzmann distribution of particles provides information on the temperature and density of state in the compound system.

Maxwell distribution

The Maxwell distribution describes a probability of an particle in ideal gas to move with a speed v . The properly normalized distribution is

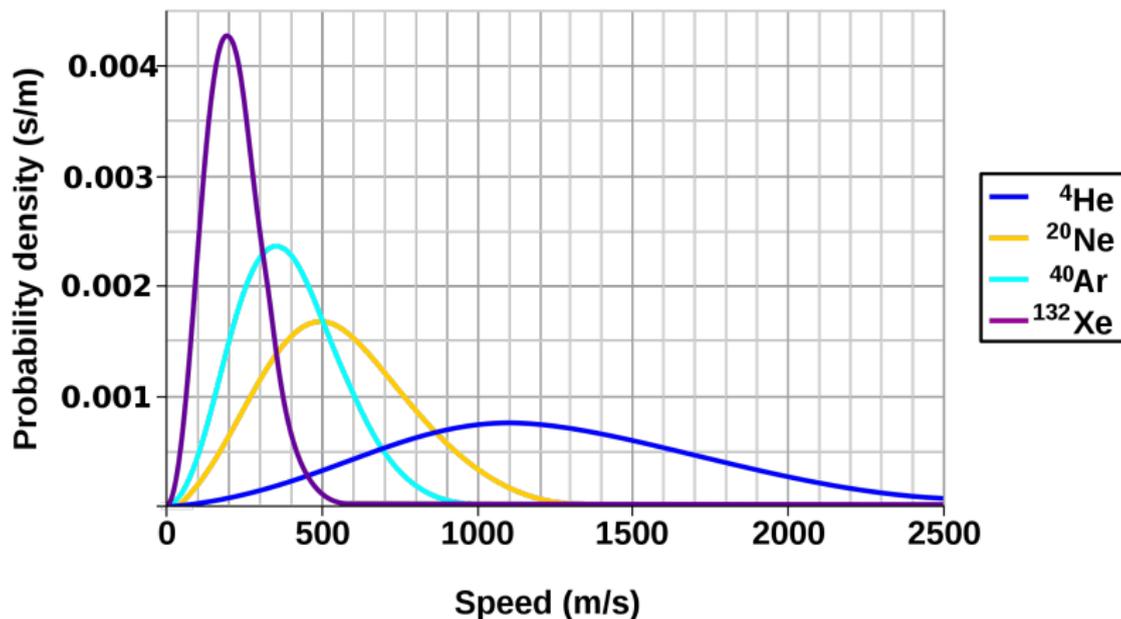
$$\Phi(v)dv = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi v^2 dv.$$

with

$$\int_0^{\infty} \Phi(v)dv = 1.$$

Maxwell-Boltzmann distribution

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases



Maxwell distribution

For particles in ideal gas kinetic energy dominates over all other types of energies

$$E = K = \frac{mv^2}{2}$$

The Maxwell distribution as a function of energy is:

$$\Phi(E)dE = 2\sqrt{\frac{E}{\pi(kT)^3}} \exp\left(-\frac{E}{kT}\right) dE,$$

with

$$\int_0^{\infty} \Phi(E)dE = 1$$

Boltzmann distribution

Boltzmann generalized Maxwell's distribution by proposing that for any system the probability of existing in a state of energy E is given by

$$\Phi(E)dE = Ng(E) \exp\left(-\frac{E}{kT}\right) dE,$$

with $g(E)$ being a factor defining number or density of states at energy E and N being the normalization constant defined by the condition

$$\int_0^{\infty} \Phi(E)dE = 1$$

Boltzmann distribution is valid for a large number of systems, especially at large temperatures. It is invalid if quantum properties of the system start to dominate, which usually happens at low temperatures.

Thermal equilibrium in compound nucleus reactions

- Another indication of thermal equilibrium comes from angular distribution of the products.
- If the system is equilibrated, the information about the beam, or the projectile direction is lost.
- Consequently, the only direction which can be defined is the direction of the angular momentum of the compound system.
- Angular distributions of compound reaction products are symmetric with respect to the direction of the spin of the compound.
- This should be contrasted with the direct reactions which show highly anisotropic angular distributions.

Cross section for compound nucleus reactions

- Based on the independence hypothesis the cross section for a compound nucleus reaction



can be factorized into

$$\sigma(A, a, B, b, E_a) = \sigma_C(A, a, E_C, I_C) P_C(B, b, E_C, I_C) \quad (12)$$

with $\sigma_C(A, a, E_C, I_C)$ being the cross section for formation of the compound nucleus C with the excitation energy E_C and angular momentum I_C by the entrance channel $A + a$ and $P_{B,b}(E_C, I_C)$ being the probability for the decay of the compound C at the excitation energy E_C and angular momentum I_C into the exit channel $B + b$.

Resonant formation of a compound system

