Compound and heavy-ion reactions

Introduction to Nuclear Science

Simon Fraser University
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Outline

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1. Density of states in a compound nucleus
2. Resonant compound nuclear reactions

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4. Semi-classical cross section for compound nucleus formation
5. Decay of highly excited compound
6. Heavy-ion collisions

Density of nuclear states

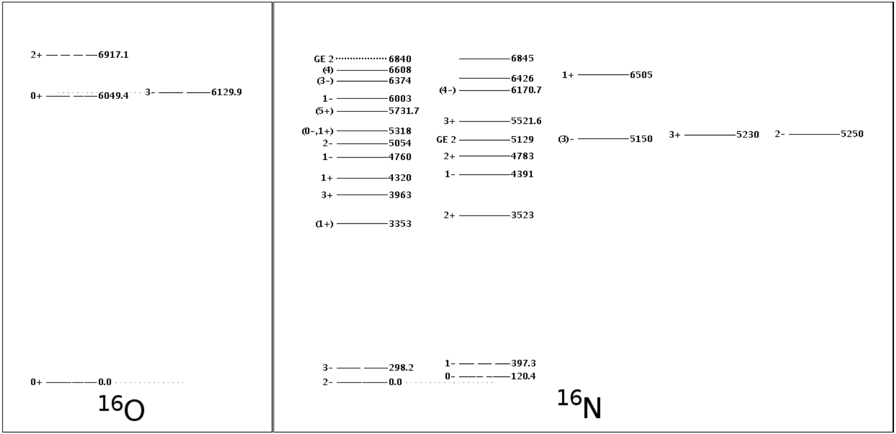
- The number of quantized states per unit energy, referred to as the density of states, plays important role in nuclear reactions and decays.
- The reason is in quantum-mechanical behaviour of the nuclear reactions or decay.
- Reactions and decays can proceed only if there exist a final state with excitation energy, angular momentum and parity the system can populate in the exit channel.
- High density of states increases the rate, low density of state reduces the rate (potentially up to zero if no final state exists).
- An example is provided by the Fermi's golden rule and its application in β decay.
- In this lecture we examine the impact of the density of states on the compound nucleus formation and decay.

Density of nuclear states at low energy

- The data indicates that density of states is small near the ground state, with gaps on the order of a few hundred keV observed in some nuclei.
- Near the ground state pairing interactions have a significant impact on the density of states.
- The density of states near the ground states is smaller in even-even nuclei, moderate in odd-even nuclei and large in odd-odd nuclei.
- This is a consequence of the fact that unpaired nucleons can couple to various spins at comparable energy, which increase the number of states, while paired nucleons couple to spin 0 forming a single state.

Density of nuclear states at low energy

Number of states below 7 MeV in ^{16}O (left) and ^{16}N (right)



Density of nuclear states at low energy

- Density of states at low excitation energy depends on nuclear structure.
- Density of states is low in doubly magic nuclei, larger in singly magic nuclei and high in nuclei far from the magic numbers.
- The reason comes from the fact that in doubly and semi-magic nuclei energy larger than the shell gap is needed to form a state from the excitation across the shell gap.
- For nuclei away from the magic numbers states are made up by excitation withing the shell without crossing the shell gap.

Density of nuclear states at high energy

- Experimental data indicates that the number of excited states, thus the density of nuclear states increases with the excitation energy.
- Let us recall, that every excited state has a natural width

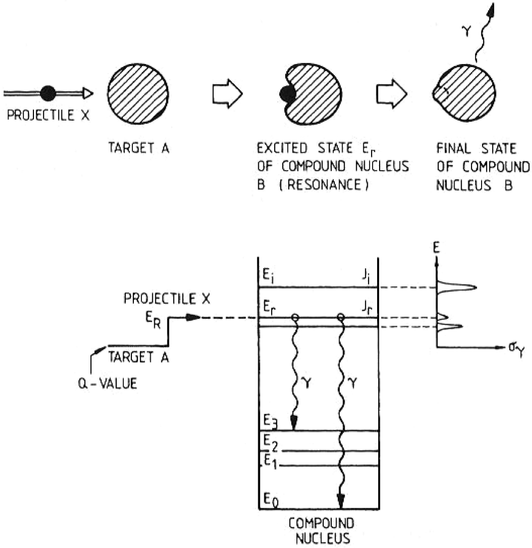
$$\Gamma = \frac{\hbar}{\tau} \quad (1)$$

- High energy excited states have lifetimes in the range of femtoseconds (10^{-15}s) and the natural width is 0.7 eV.
- At increasing excitation energies the number of states becomes so high that average spacing between levels is comparable or smaller than the natural line width ($\sim 10^6$ states per MeV).
- In such case the levels start to overlap and the nuclear system ceases to be quantized, there are no gaps between states, and the excited nucleus behaves as a classical system with continuous excitation energy spectrum.

Resonant and non-resonant compound nuclear reactions

- Below, we will consider two scenarios for compound nuclear reactions depending on the excitation energy and the density of states in the compound system.
- Let us denote by D the average spacing between levels.
- D is inversely proportional to the level density.
- If the average spacing is large comparing to the natural line width of the states $D >> \Gamma$ the compound nucleus can be formed through capture to well defined resonances and quantum mechanical treatment is needed.
- If the average spacing is small comparing to the natural line width of the states $D >> \Gamma$ the formation of the compound nucleus can be described using classical method.

Resonant formation of a compound system



The cross section for a single resonance

- The resonant reaction in question is a two-step process.
- The cross section, therefore has to separate into a product of two terms, one representing a formation of a compound system, the other the decay

$$\sigma_\gamma \propto |< B + b \mid H_n \mid E_r >|^2 |< E_r \mid H_n \mid A + a >|^2 \tag{2}$$

- The decay term is proportional to the decay width

$$\Gamma_{bB} \propto |< B + b \mid H_n \mid E_r >|^2 \tag{3}$$

- What about the formation term? To approach the calculations of the formation term let us look into the time reversal transformation.

Time reversal

- Time reversal T is a transformation of the equations of motion which exchanges time with negative time $T(t) = -t$.
- Let us consider Newton's second law:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2}. \tag{4}$$

- The time reversal does not change the force $T(\vec{F}) = \vec{F}$ as it does not explicitly involve time.
- The same is true for the position vector $T(\vec{r}) = \vec{r}$.
- However, the time reversal changes the direction of the velocity vector

$$T(\vec{v}) = T(\frac{d\vec{r}}{dt}) = \frac{T(d\vec{r})}{T(dt)} = \frac{d\vec{r}}{-dt} = -\frac{d\vec{r}}{dt} = -\vec{v}. \tag{5}$$

Time reversal

- But, time reversal does not change the direction of the acceleration vector, since it involves the second derivative of the position vector with respect to time; the time is inverted twice, therefore it is not inverted.

$$T(\vec{a}) = T(\frac{d^2\vec{r}}{dt^2}) = \frac{T(d^2\vec{r})}{T(dt^2)} = \frac{d^2\vec{r}}{(-dt)(-dt)} = \frac{d^2\vec{r}}{dt^2} = \vec{a}. \tag{6}$$

- Because of all of the above the Newton's equations are invariant under time reversal.

- This implies that we can not distinguish between processes proceeding forward and backward in time based on Newton's equations alone.

Time reversal in nuclear processes

- Invariance under time reversal applies also to quantum mechanics.
- Quantum mechanics defines the rates for nuclear reactions.
- The consequence of time reversal is that there is no distinction between rates for formation or decay of a state, in particular the partial widths for both processes are the same $\Gamma^{\mathrm{f}} = \Gamma^{\mathrm{d}}$.
- Note, that the time reversal implies equivalence of partial decay and formation widths for the same process. Usually we form a system in a particular way, and it can decay in many other ways, thus the formation width is smaller than the total width.
- The formation, decay and total widths are equal only in the case of a state which can be formed and decay in one and only one way.

The cross section for a single resonance

- The resonant reaction in question is a two-step process.
- The cross section, therefore has to separate into a product of two terms, one representing a formation of a compound system, the other the decay

$$\sigma_\gamma \propto |< B + b \mid H_n \mid E_r >|^2 |< E_r \mid H_n \mid A + a >|^2 \quad (7)$$

- The decay term is proportional to the decay width

$$\Gamma_{bB} \propto |< B + b \mid H_n \mid E_r >|^2 \quad (8)$$

- The formation term is proportional to the decay width

$$\Gamma_{aA} \propto |< A + a \mid H_n \mid E_r >|^2 = |< E_r \mid H_n \mid A + a >|^2 \quad (9)$$

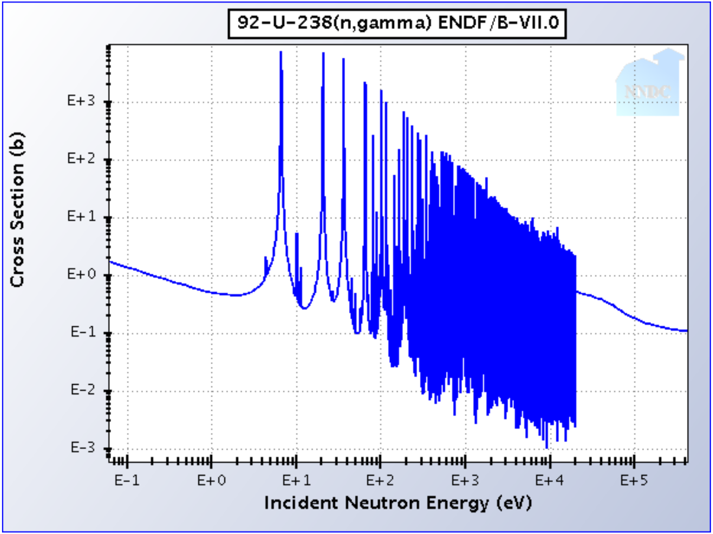
Breit-Wigner cross section

- Quantum mechanical analysis yields the following dependence on energy for the cross section of a resonant reaction:

$$\sigma_{BW}(E) = \pi\lambda\!\!\!^{-2}\frac{2J+1}{(2J_A+1)(2J_a+1)}(1+\delta_{aA})\frac{\Gamma_{aA}\Gamma_{bB}}{(E-E_R)^2+(\Gamma/2)^2} \quad (10)$$

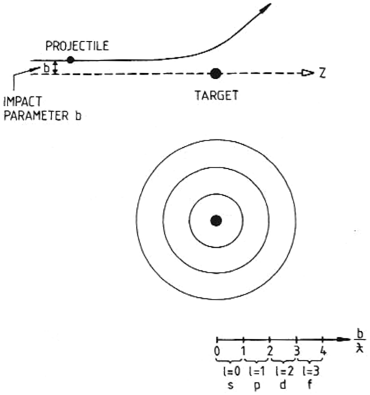
  with
  - $\lambda\!\!\!^{-} = \frac{\lambda}{2\pi}$ representing reduced wavelength in the centre of mass frame,
  - J_a, J_A and J representing spins of the projectile, target and the state in the compound system,
  - δ_{aA} representing the effect of identical particles,
  - $E_R = E_r - Q$ representing the beam energy at the resonance,
  - Γ_{aA}, Γ_{bB} and Γ representing partial width for formation of the compound through $a + A$, partial width for the decay of the compound through $b + B$ and the total width $\Gamma = \Gamma_{aA} + \Gamma_{bB} + ...$

^{238}U(n,γ) compound nucleus resonances



Semi-classical cross section and angular momentum

- Let us look closer to the relation between the cross section σ, impact parameter b and the angular momentum $L = bp$ at high excitation of the compound system when semi-classical treatment is valid.



- Momentum in the centre of mass is related to the wavelength $p = \frac{h}{\lambda} = \sqrt{2\mu E_{CM}}$.
- Angular momentum is quantized $L^2 = l(l+1)\hbar^2$.
- The black disc representing the geometric cross section can be split into concentric zones which correspond to different value of angular momentum.
- The cross section for the zone l is

$$\sigma_l = \pi b_{l+1}^2 - \pi b_l^2 = \pi(b_{l+1}^2 - b_l^2) \qquad (11)$$

Cross section for formation of a compound system

- The cross section for the zone l is

$$\sigma_l = \pi(b_{l+1}^2 - b_l^2) = \pi\frac{\hbar^2}{p^2}\left((l+1)(l+2) - l(l+1)\right) = \pi\lambda\!\!\!^{-2}2(l+1). \tag{12}$$

- The total cross section is the sum over the partial cross sections σ_l up to the maximum value of angular momentum in the reaction:

$$\sigma = \sum \sigma_l T_l \tag{13}$$

- T_l called transmission coefficients are real numbers between 0 and 1 representing a probability that the channel with angular momentum l will result in the reaction of interest.
- Semi-classically all T_l below the maximum angular momentum l_{max} are 1, all above 0.

Cross section for formation of a compound system

- The sum over partial cross sections with semi-classical transmission coefficients T_l can be calculated analytically since it forms an arithmetic series.

- For the $L^2 = l(l+1)\hbar^2$

$$\begin{aligned} \sigma &= \pi\lambda\!\!\!^{-2} 2 \sum_0^{l_{\max}} (l+1) = \pi\lambda\!\!\!^{-2} 2 \left(\sum_0^{l_{\max}} l + \sum_0^{l_{\max}} 1 \right) = \\ &= \pi\lambda\!\!\!^{-2} 2 \left(\frac{l_{\max}(l_{\max}-1)}{2} + l_{\max} \right) = \\ &= \pi\lambda\!\!\!^{-2} l_{\max}(l_{\max}+1) \end{aligned} \tag{14}$$

- With the above understanding the semi-classical cross section depends on the l_{max} value of the maximum angular momentum allowed in the reaction.

The maximum angular momentum transfer

- The l_{max} can be calculated semi-classically as the angular momentum in the centre of mass at the maximal inter-nuclear separation R_{max} which still leads to reaction

$$l_{max}(l_{max}+1)\hbar^2 = R_{max}^2 p^2(R_{max}) \tag{15}$$

- The momentum at this separation depends on the kinetic energy in the centre of mass available above the Coulomb barrier $V_C(R_{max})$

$$\begin{aligned} p^2(R_{max}) &= 2\mu(E_{CM} - V_C(R_{max})) = 2\mu E_{CM}\left(1 - \frac{V_C(R_{max})}{E_{CM}}\right) \\ &= \frac{\hbar^2}{\lambda\!\!\!^-{}^2}\left(1 - \frac{V_C(R_{max})}{E_{CM}}\right) \end{aligned} \tag{16}$$

- The maximum momentum transfer is

$$l_{max}(l_{max}+1) = \frac{R_{max}^2}{\lambda\!\!\!^-{}^2}\left(1 - \frac{V_C(R_{max})}{E_{CM}}\right) \tag{17}$$

The semi-classical formation cross section

- With the above expression for the l_{max} the semi-classical formation cross section becomes

$$\sigma = \pi\lambda\!\!\!^{-2} l_{\max}(l_{\max}+1) = \pi R_{max}^2\left(1 - \frac{V_C(R_{max})}{E_{CM}}\right) \quad (18)$$

- The maximum interaction radius can be taken approximately as the sum of radii plus ΔR=2 fm

$$R_{max} = R_a + R_A + \Delta R = 1.2(A_A^{1/3} + A_a^{1/3}) + 2 \text{ [fm]} \quad (19)$$

- The Coulomb barrier is

$$V_C(R_{max}) = \frac{1}{4\pi\varepsilon_0}\frac{Z_a Z_A e^2}{R_{max}} \quad (20)$$

- The energy in the centre of mass is

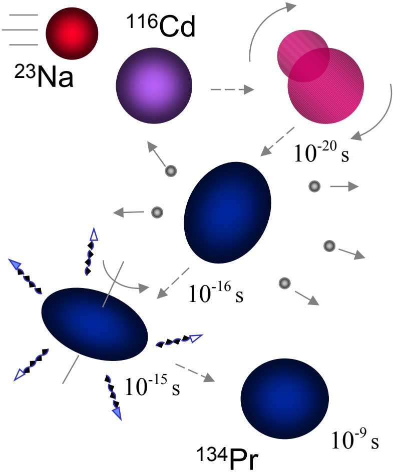
$$E_{CM} = \frac{\mu v^2}{2} \quad \mu = \frac{A_a A_A}{A_a + A_A} \quad (21)$$

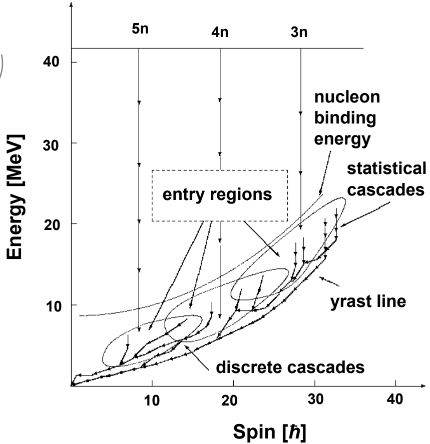
and v is the initial speed of the projectile in the lab.

Decay of a highly excited compound system

- The decay of the compound system does not depend on the way the compound was formed but only on the excitation energy and angular momentum of the compound.
- The decay of the highly excited compound system proceeds first by emission of the nucleons or α particles on the time scale of 10^{-19}s defined by the strong interactions involved.
- When the energy is not sufficient to continue particle emission the decay proceed via the γ-ray emission with the timescale 10^{-15} – 10^{-9} s defined by the electromagnetic interactions involved.
- This cooling process described by thermal equilibrium and statistical methods with decay probabilities which depend strongly on the densities of initial and final states.

Decay of a highly excited compound system





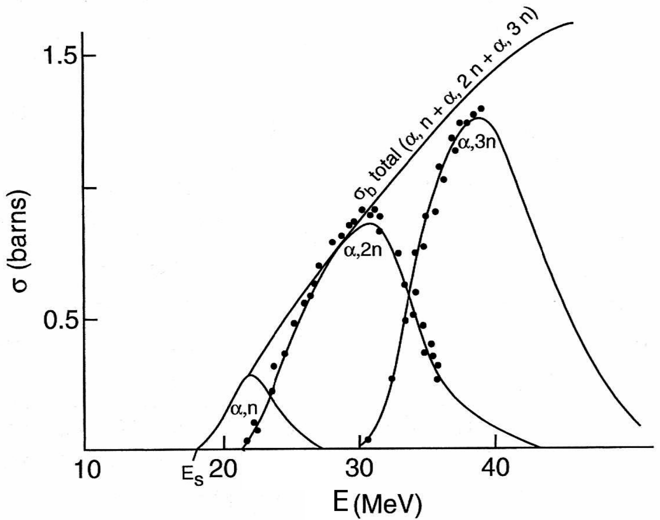
Excitation function of highly excited compound

- The semi-classical formation cross section is

$$\sigma = \pi R_{max}^2 \left(1 - \frac{V_C(R_{max})}{E_{CM}}\right) \quad (22)$$

- Formation of compound system is highly suppressed below the Coulomb barrier.
- The formation cross section increases swiftly as a function of the beam energy available above the Coulomb barrier asymptotically approaching the geometric cross section $\sigma = \pi R_{max}^2$.
- As the excitation energy increases the decay channels change, with higher excitation energy favouring evaporation of larger number of light particles from the compound.

Excitation function in ^{209}Bi(α,xn) reaction



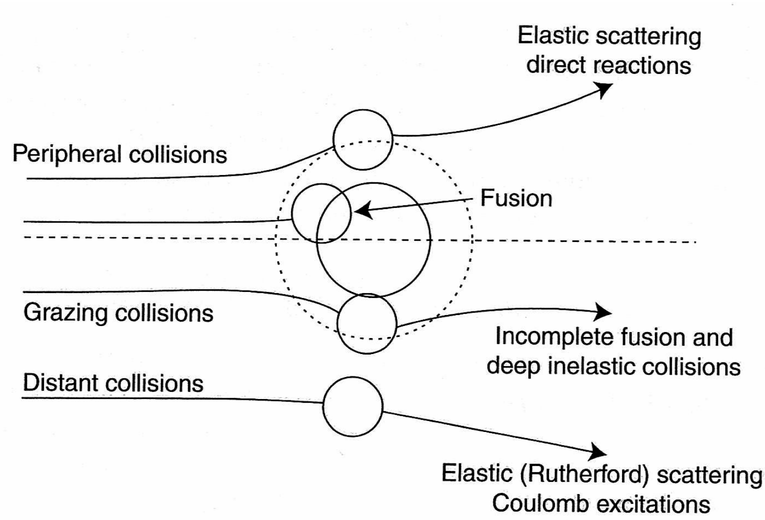
Heavy-ion collisions

- Heavy ions are ions heavier than the α particle.
- Heavy-ion reactions require substantial energies, around 5 MeV/A for the projectile in the lab frame, to overcome the Coulomb repulsion.
- At these energies the momentum in the centre of mass is large, the reduced wavelength $\lambda\!\!\!^{-}$ is short compared to the size of the ions and therefore a classical description is valid.
- In particular, heavy ion trajectories can be defined in a collisions.
- The classical analogue of a heavy ion collision is a collision of two charged liquid droplets.
- The name is intended to emphasize the contrast with light ions (proton, neutron, deuteron, triton, α, ^{3}He) for which quantum mechanical description of collisions is required.

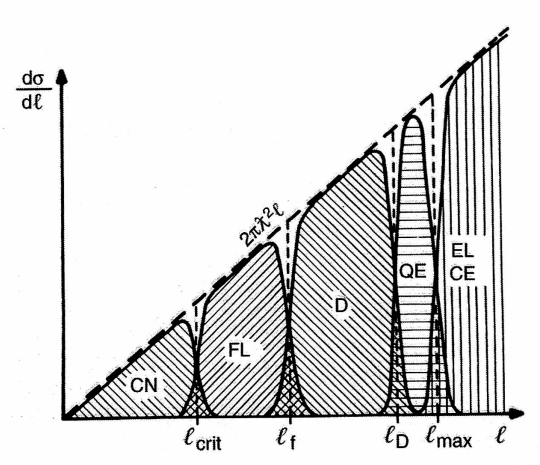
Heavy ion collisions

- Heavy ion collisions are characterized by
  - large number of nucleons participating in a collision,
  - significant Coulomb barrier,
  - large angular momenta, up to $\sim 150\hbar$ in peripheral collisions.
- The outcome of the collision is very strongly correlated to the impact parameter.
- Only relatively central collisions (small impact parameter) lead to complete fusion and compound nucleus formation.
- At large impact parameters the centrifugal barrier associated with large angular momentum prevent colliding ions from fusion.

Classification of heavy ion collisions



Partial cross sections vs. angular momentum



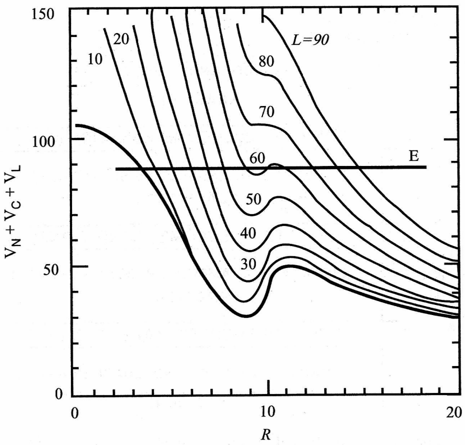
- CN: compound nucleus
- FL: fusion-like
- D: deep inelastic
- QE: quasi elastic
- CE: Coulomb excitation
- EL: elastic

Interaction potentials

- Semi-classical analysis of heavy-ion collisions is based on the analysis of interaction potential V as a function of the radial distance R between ions.
- The interaction potential V is the sum of the nuclear, Coulomb, and centrifugal potential
$$V = V_N + V_C + V_L \tag{23}$$
- For large impact parameters angular momentum is large and the repulsive centrifugal potential dominates the interaction potential.
- For smaller impact parameters and smaller angular momenta the attractive nuclear potential results in a potential well developing at the radial distance comparable with sum of the radii.
- If the interacting ions are trapped in this potential well they will fuse and form a compound system.

Interaction potentials

Sum of the nuclear V_N, Coulomb V_C and centrifugal V_L potentials in heavy ion collisions as a function of radial distance R (in [fm]).



The significance of heavy ion collisions

- Heavy ion collisions are used for
  - produce nuclei at very high angular momentum/excitation energy
  - produce nuclei with large neutron deficiency
  - produce nuclei along the $N = Z$ line
  - produce trans uranium elements.