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Spontaneous fission

- The 1938 discovery of fission followed neutron-induced nuclear reaction.

- In 1940 Petrzhak and Flerov discover (in Soviet Union) spontaneous fission of $^{238}\text{U}$.

- The probability of spontaneous fission of $^{238}\text{U}$ is very small, the dominant decay mode is $\alpha$ decay and fission branch intensity is $5 \times 10^{-7}$ of the $\alpha$ branch intensity.

- Since then over 100 cases of nuclei spontaneously fissioning from the ground state have been observed.

- Spontaneous fission lifetimes vary by a factor of $10^{29}$

- Spontaneous fission is the process which limits the existence of heavy elements, the spontaneous fission lifetimes are observed to decrease with increasing $Z$. 
Spontaneous fission half-lifes

- Large range of variation in spontaneous fission lifetimes points towards the tunnelling effect as the underlying mechanism for spontaneous fission.

- Indeed, a similar range differences in lifetimes were observed in $\alpha$ emitters and shown to be dependent on the $\alpha$-decay $Q$-value by Geiger and Nuttall and further explained by Gamow.

- Following the case of $\alpha$-particle tunnelling the spontaneous fission rate is proportional to the product of the frequency $f$ of impinging on a fission barrier and the barrier penetrability factor $P$

$$\lambda_{SF} = fP \implies T_{1/2}^{SF} = \frac{\ln(2)}{fP}$$

- The frequency factor $f \sim 10^{20}$ is given by the diameter of the fissioning system $\sim 10$ fm over the speed of the fragments in the well $\beta \sim 3\%$. 
Fission penetrability factor

- Fission penetrability factor $P$ is difficult to calculate as it requires detailed understanding of the Liquid Drop and Shell Correction energy in a large parameter space describing shapes of the fissioning system.

- The fission penetrability factor can be calculated in a simplified toy model proposed by Bohr and Wheeler assuming a parabolic shape of the potential well and the barrier.
Fission barriers

**Fission penetrability factor**

- Bohr-Wheeler penetrability factor is

$$P = \frac{1}{1 + \exp\left(\frac{2\pi B_f}{\hbar\omega}\right)}$$  \(\text{(2)}\)

with \(B_f\) being the height of the fission barrier and \(\hbar\omega\) being the curvature of the parabolic potential.

- For typical fissioning systems \(B_f = 5 - 6\ \text{MeV}\) and \(\hbar\omega = 0.5\ \text{MeV}\) thus

$$\exp\left(\frac{2\pi B_f}{\hbar\omega}\right) >> 1$$  \(\text{(3)}\)

- The spontaneous fission lifetime from the Bohr-Wheeler model is

$$T_{1/2}^{SF} = 2.77 \times 10^{-21} \exp\left(\frac{2\pi B_f}{\hbar\omega}\right)$$  \(\text{(4)}\)

and show very strong dependence on the height of the fission barrier.

- This explains a large range of spontaneous fission lifetimes.
Fission isomers

In early 1960 another discovery regarding fission has been made.

Some nuclei were discovered to show small spontaneous fission branches with lifetimes in the nano- to mili-second range.

These lifetimes were $10^{25} – 10^{30}$ times shorter than the corresponding ground state lifetimes.

The fast fissioning states were called the fission isomers.

The strong dependence of the fission rates on the barrier height explains the existence of fission isomers as states fissioning from a different potential minimum comparing to the ground state.

Smaller barrier height for the fission isomers results in significantly shorter lifetimes.
The fission barriers

Diagram showing the potential energy (V) vs. deformation with key features:
- Mass distribution
- Normal fission
- Subbarrier fission
- Isomer fission
- Spontaneous fission

Energies and deformations:
- $E_A$
- $E_B$
- $\varepsilon_I$
- $\varepsilon_{II}$
The impact of the barrier height

- The strong dependence of the fission rates on the barrier height is true also for induced fission.

- In particular it explains the cross section differences in thermal-neutron induced fission for $^{235}$U and $^{238}$U which is of huge interest to fission application in power generation.

- For both $^{235}$U and $^{238}$U the barrier is $B_f \sim 5.7$ MeV.

- The $Q$-values in a neutron capture compound formation are

  $$ Q_{235} = (M_{235} + M_n - M_{236})c^2 = 6.5 \text{ [MeV]} > B_f = 5.7 \text{ [MeV]} $$
  $$ Q_{238} = (M_{238} + M_n - M_{239})c^2 = 4.8 \text{ [MeV]} < B_f = 5.7 \text{ [MeV]} $$

- Thus $^{235}$U fissions following a thermal neutron capture, while $^{238}$U requires a neutron to bring in some threshold energy to allow above-barrier excitation and fission.
Neutron-induced fission cross section for $^{235}$U
Neutron-induced fission cross section for $^{238}\text{U}$
Spontaneous fission half-life systematics

- Recall the role which the empirical Geiger-Nuttal law correlating $\alpha$-decay rates and $Q$-values played in the discovery of the quantum mechanical tunnelling effect.

- Searches of similar correlations have been undertaken in an attempt to understand underlying fission mechanisms.

- One of these attempts was to investigate correlations between the fission half-lifes and the fissionability parameter $x$.

- This correlation, however, while pointing in general to shorter half-lifes for increasing values of $x$, does not show a simple trend.

- In particular, for a given element, small variation of $x$ lead to large, parabolic-like, variations of fission half-lifes.
Spontaneous fission half-life systematics vs. $x$
Spontaneous fission half-life systematics

- Recall that the fissionability parameter $x$ was derived based on the Liquid Drop model (only).

- Poor empirical correlation between spontaneous fission half-lifes and the fissionability parameter $x$ indicates that the Liquid Drop model alone does not account properly for the fission rates.

- This seems obvious in view of our discussion of fission barrier arising due to the superposition of the Liquid Drop and Shell Model energies.

- Swiatecki in early 1970 pointed out an empirical correlation between parameter $x$ and

$$\ln(T_{1/2}) + 5\delta M$$  \hspace{1cm} (5)

with $\Delta M$ being a difference between the actual and the Liquid Drop predicted mass.

- This correlation takes into account the empirical shell corrections.
Spontaneous fission half-life systematics vs. $x$

![Graph showing the relationship between log of half-life (yr) + 5.6m and $x$. The graph includes points for U, Pu, Cm, Cf, and Fm, with a trend line indicating the systematic behavior.]
Interest in fission fragment mass distribution

- Fission produces a large number of fragments with broad mass distribution.
- Significant effort has been devoted to map fission fragment mass distribution for various parent nuclei.
- Fission fragment mass distribution is of interest since:
  - It carries information about the fission process and provides an observable fission models can compare to. Thus it is of great interest to basic research.
  - Fission is a source of isotopes which can be harvest for use in research, medicine and technology.
  - Fission fragment distribution is the key information to understand nuclear fuel waste.
Fission fragment mass distribution

\[ ^{235}\text{U} \text{ fission fragment yields} \]

Number of protons \( Z \)
Number of neutrons \( N \)

\( ^{235}\text{U} \)
\( ^{239}\text{Pu} \)
\( ^{238}\text{U} \)
\( ^{232}\text{Th} \)

NUCS 342 (Lecture 28)
The nuclear chart

- Number of protons $Z$
- Number of neutrons $N$
The Liquid Drop prediction

- The Liquid Drop model can be used to predict the most probable mass split.

- That is done by finding the $A$ and $Z$ for the fragments for which the binding energy per nucleon is the largest with the constrain that the sum of mass and charge for the fragments equals to that of the parent.

- The Liquid Drop model used in this way predicts symmetric fission, meaning splitting into fragments with identical charge and mass.

- Most of observed fission fragment distributions, in particular these for Uranium and Neptunium isotopes are not symmetric.

- This observation underlines again the limitation of the Liquid Drop model in application to fission.
Asymmetric fission

- Experimental data indicates that fission of a parent nucleus at low excitation energies produces two groups of fragments: the heavy group and the light group.

- The centroid mass of the heavy group is almost identical for different parent nuclei.

- The centroid of the light group shifts depending on the mass of the parent; light parents result in lower centroid mass of the light group.

- The solution of the puzzle is in the Shell Corrections.

- But let us first look at the data.
Fission fragment mass distribution

![Graph showing fission fragment mass distribution](image)

- **Mass number, A**
- **Fission yield (per cent)**

- **233U**
- **235U**
- **239Pu**
Fission fragment mass distribution

![Graph showing fission fragment mass distribution]

- **HEAVY GROUP**
- **LIGHT GROUP**

- Mean mass of the fission product group vs. mass of the fissioning nucleus.
- Data points for different types of fission: Spontaneous Fission, Thermal-Neutron-Induced Fission, Reactor-Neutron-Induced Fission.
Why is fission asymmetric at low excitation energies?

- The key to answering this question is in recognition that the heavy group is centred around doubly magic $^{132}$Sn.

- Recall that a closed shell stabilizes a nucleus, doubly closed shell stabilizes it even more.

- Fission favours the most stable split, for the Liquid Drop this is the symmetric split, but when shell corrections are taken into account the split favours emission of a $^{132}$Sn like fragment and a complementary light fragment.

- Thus the asymmetry of fission fragment distribution is a quantum mechanical effect related to shell closures and quantified by shell correction energies.
$^{235}\text{U}$ fission fragment yields
Asymmetry of fission fragment distribution is a quantum mechanical effect related to shell closures and quantified by shell correction energies.

As such it is expected to disappear at high excitation energy of the parent when significant shell breaking is taking place.

Another way to view this fact is to observe that at high excitation energy the system becomes less quantum-like, more classical-like and the Liquid Drop model provides a valid predictions.
Symmetric and Asymmetric fission

Fission fragment mass distribution

![Graph showing fission fragment mass distribution with different neutron energies: 13 MeV, 20 MeV, 27 MeV, 35 MeV, 40 MeV, 45 MeV, and 53 MeV. The graph plots the percentage yield against the pre-neutron emission mass.]
Asymmetric and symmetric fission

- Asymmetry of fission fragment distribution is a quantum mechanical effect related to shell closures and quantified by shell correction energies.

- Shell corrections depend also on mass and charge of the parent.

- In some cases an addition or subtraction of a nucleon can change the deformation of the parent significantly changing in the same way the magnitude of shell corrections.

- This effect explains sometimes dramatic changes in spontaneous fission asymmetry observed for heavy actinides nuclei.
Spontaneous fission fragment mass distribution
Fission fragment charge distribution

![Fission fragment charge distribution chart](image)

- Proton number:
  - 89
  - 90
  - 91
  - 92

- Neutron number:
  - 132
  - 133
  - 134
  - 135
  - 136
  - 137
  - 138
  - 139
  - 140
  - 141
  - 142

- Elements:
  - Ac
  - Th
  - Pa
  - U