Nuclear deformation and quadrupole moment

Introduction to Nuclear Science

Simon Fraser University
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Outline

1. Nucleon density

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2. Nuclear radii

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Density of a nucleon

- Knowing the radius of a nucleon $r_n \sim 0.9$ fm the volume occupied by a nucleon is
$$v = \frac{4}{3}\pi r^3 = 3.05 \; [\text{fm}^3] \tag{1}$$
- This corresponds to a limit of density of nuclear matter inside a nucleon of
$$\rho_{\text{max}} = \frac{1}{v} = 0.327 \;\; [\text{nucleon}/\text{fm}^3] \approx 5.5 \times 10^{14} \;\; [\text{g}/\text{cm}^3]. \tag{2}$$
- Comparing to the density of water of 1 [g/cm^3] ρ_{max} is hundred thousand billion times larger.
- However, it is not clear that the density of nuclear matter inside a nucleus is the same as inside a nucleon.
- As a matter of fact one may expect less dense packing of the nuclear matter inside a nucleus and the ρ_{max} being the stringent upper limit for nuclear density.

Radii measurements: photons

- Measurement of atomic or nuclear radii require a probe.
- Lets examine if photon is a convenient probe.
- For an object of the size r to be observed using photons the wavelength λ of the photon has to be smaller than the size of the object.
- Setting the wavelength $\lambda < r$, using the relativistic energy-momentum condition for mass less particles $E = pc$ and using de Broglie relation between momentum and wavelength $p = \frac{h}{\lambda}$ gives the following energy condition for the photons

$$E = \frac{hc}{\lambda} > \frac{hc}{r} = \frac{\hbar c}{2\pi r}. \tag{3}$$

Radii measurements: photons

- It is possible to see atoms using photons. Indeed let us set r to the typical size of an atom which is $r = 10^{-10}$ m=0.1 nm.

$$E > \frac{\hbar c}{2\pi r} = \frac{197.33 \text{ [eV nm]}}{6.28 * 0.1 \text{ nm}} = 314 \text{ [ev]} = 0.3 \text{ [keV]} \tag{4}$$

- This implies that any simple X-ray generator (like Roentgen tube) with the voltage potential above 0.3 keV can generate photons capable to scatter on atoms providing information on radii.

- If we use a size of a nucleus $r = 1$ fm we get

$$E > \frac{\hbar c}{2\pi r} = \frac{197.33 \text{ [MeV fm]}}{6.28 * 1 \text{ fm}} = 31.4 \text{ [Mev]} \tag{5}$$

- Currently, free-electron lasers can generate photon beams up to 10 MeV coming close to the nuclear limit. However, direct observation of nuclear radii using photon beams are difficult and rare.

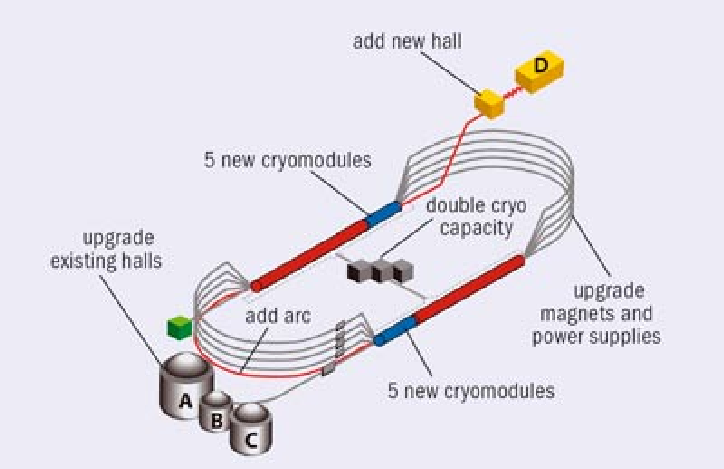
Radii measurements: electrons

- If photons do not work for nuclei, what about measuring radii by electron scattering?
- Great idea! Sources of electrons will shortly be available up to 12 GeV (CEBAF-upgrade, Jefferson Laboratory in North Carolina).
- Let us examine the resolution scale of CEBAF. At 12 GeV electrons are surely relativistic as the energy is mach larger compared to the electron rest energy of 511 keV.
- For relativistic electrons we can use the same energy-momentum relationship as for photons

$$E > \frac{\hbar c}{2\pi r} \implies r > \frac{\hbar c}{2\pi E} = \frac{197.33 \text{ [MeV fm]}}{12000 \text{ [MeV]}} = 0.016 \text{ [fm]} \tag{6}$$

- CEBAF with 0.016 [fm] resolution should see inside a nucleon and observe quarks! Watch the news.

Continues Electron Beam Accelerator Facility



Radii measurements: electrons

- However, here is an issue: what does electron scattering really measure in a nucleus?
- Electrons are charged leptons. As such they do not interact via strong interactions, but rather by electromagnetic and weak interactions.
- Weak interaction is weak. As such, it is dominated by the electromagnetic interaction and can be in the first approximation neglected.
- Neutrons are neutral, apart of small magnetic moment of a neutron which we neglect for now, neutrons in a nucleus do not interact with electrons!
- Protons are charged. Protons do interact with electrons.
- Therefore, electron scattering measures distribution of protons inside a nucleus.

Radii measurements: neutrons

- A question then arises, what would neutron scattering measure?
- Neutrons are neutral hadrons. As such they do interact via strong interactions, but not by electromagnetic interactions (except some small interactions caused by the magnetic moment).
- Weak interaction is still present but it is dominated by the strong interaction and can be in the first approximation neglected.
- Neutron in a low-energy scattering process interacts with both neutrons and protons inside a nucleus.
- Therefore, neutron scattering measures distribution of both type of nucleons inside a nucleus, so called mass distribution.
- At high energies the structure of the neutron starts to play a role as quarks rather than nucleons start to interact.

Radii measurements: protons

- Another question then arises, what would proton scattering measure?
- Protons are charged hadrons. As such they do interact via strong interactions, and also by electromagnetic interactions.
- Weak interaction is still present but it is dominated by the strong/electromagnetic interactions and can be in the first approximation neglected.
- Protons in the scattering process interacts with neutrons inside a nucleus via strong interactions and protons inside a nucleus via strong and electromagnetic interactions.
- Therefore, proton scattering measures distribution of both type of nucleons inside a nucleus which is a superposition of the charge distribution measured with electrons and mass distribution measured with neutrons.

Matter/charge radii

- In summary, distribution of mass (defined by both protons and neutrons) may be different in a nucleus as compared to the distribution of charge (defined by protons).
- Different probes are sensitive to different distributions
  - electrons to charge distribution,
  - neutrons to mass distribution,
  - protons to mass and charge distribution.
- Mass and charge distribution can have different radii, therefore we define the mass radius R_M and the charge radius R_Q.
- Measurements indicate $R_M > R_Q$ and

$$R_M \approx r_M A^{\frac{1}{3}} = 1.4A^{\frac{1}{3}}$$
$$R_Q \approx r_Q A^{\frac{1}{3}} = 1.2A^{\frac{1}{3}}$$

Nuclear density

- Note that the observed proportionality between radii and root-cube of the mass number implies a density inside nucleus which is independent of the number of nucleons

$$\rho = \frac{M}{\frac{4}{3}\pi(R_M)^3} = \frac{Au}{\frac{4}{3}\pi r_M^3(A^{\frac{1}{3}})^3} = \frac{3u}{4\pi r_M^3} \tag{7}$$

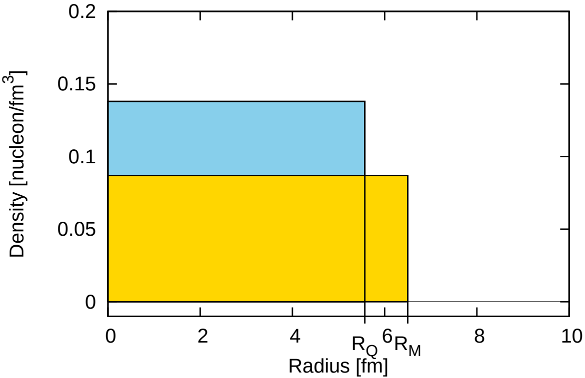
where we neglected the mass defect as small compared to the mass of the nucleus.

- This density can be related to the density inside a nucleon form Eq. 2. For a mass distribution inside a nucleus as compared to the density inside a neutron

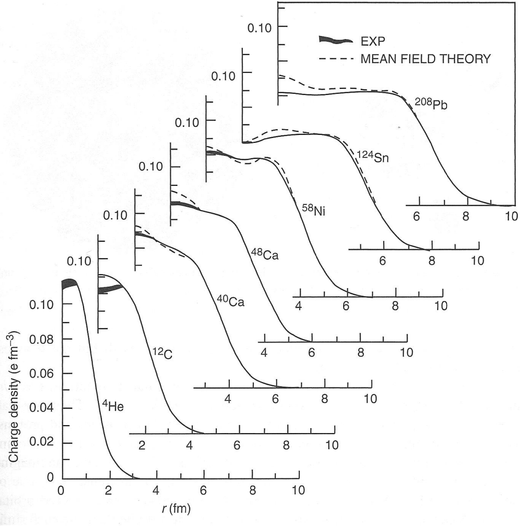
$$\rho = \rho_{\max}\frac{u}{m_n}\left(\frac{r_n}{R_M}\right)^3 = \rho_{\max}\frac{931}{939}\left(\frac{0.9}{1.4}\right)^3 \approx 0.26\rho_{max} \tag{8}$$

Sharp surface density distribution

- So far we assumed the sharp surface cut off, namely constant density until the surface and no density beyond the surface.
- We recognized the difference between the matter (gold) and the charge (blue) radius.



Measured nuclear charge density distribution



Fermi distribution

- Function which approximates well true nuclear density distribution is the Fermi function

$$\rho(r) = \rho_0 \frac{1}{1 + \exp\left(\frac{r - R_{1/2}}{a}\right)} \tag{9}$$

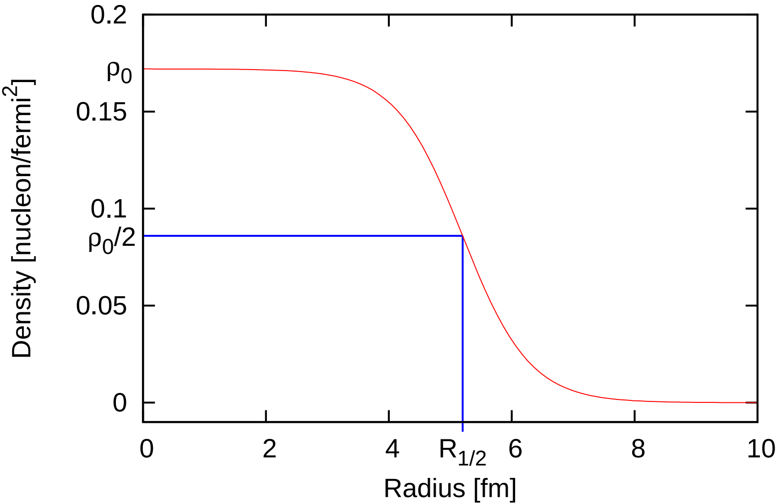
- The radius $R_{1/2}$ corresponds to a point at which density drops to half of that in the centre. Indeed

$$\rho(R_{1/2}) = \rho_0 \frac{1}{1 + \exp\left(\frac{R_{1/2} - R_{1/2}}{a}\right)} = \rho_0 \frac{1}{1+1} = \frac{1}{2}\rho_0 \tag{10}$$

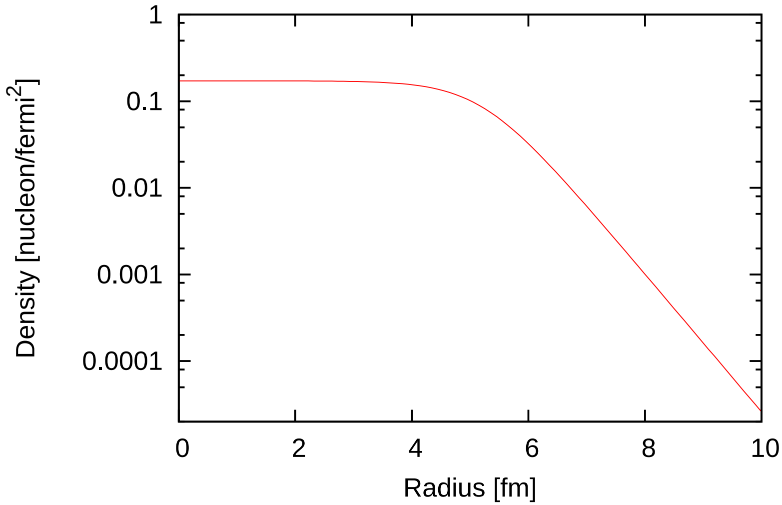
- Experiments indicate that $R_{1/2} = 1.12 A^{\frac{1}{3}}$ fm.

- Note that there is no sharp cut off on the surface, on the contrary, the density distribution extends to infinity!

Fermi distribution

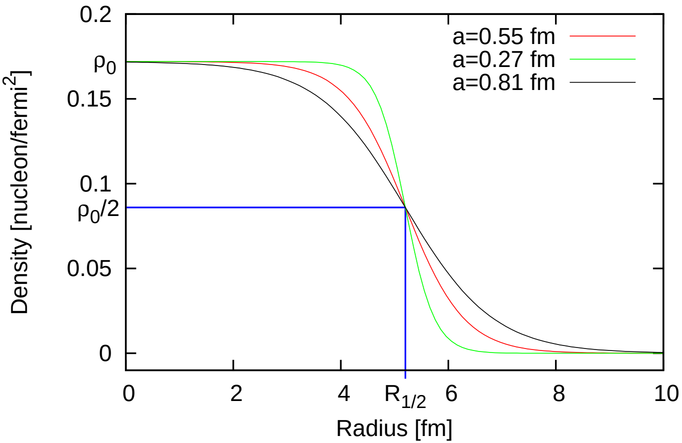


Fermi distribution



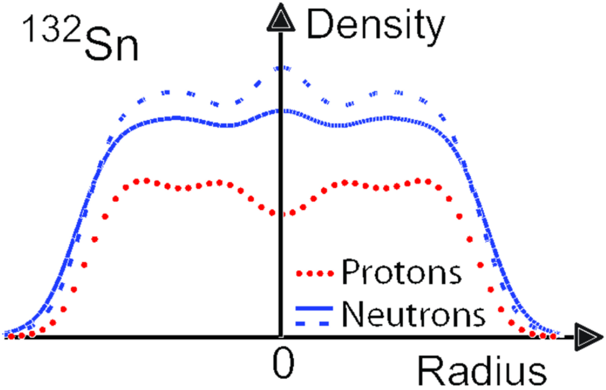
Fermi distribution

- Parameter a in the Fermi function defines the rate of change of density in the vicinity of $R_{1/2}$

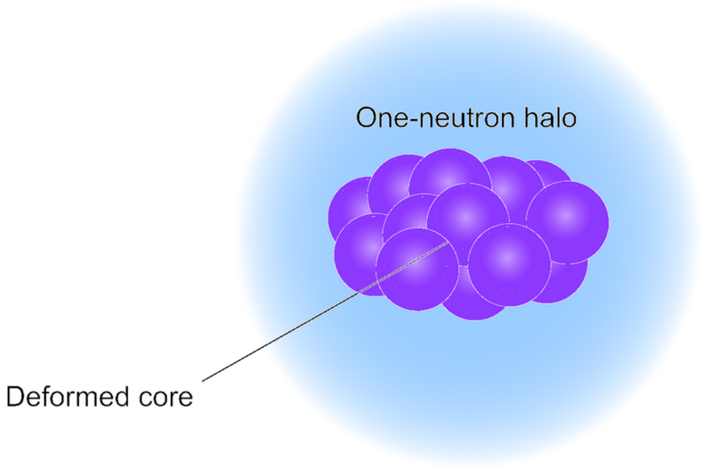


Fermi distribution

- Charge and matter radii are not the same for true nuclear density distributions, especially far from stability.



Nuclear halo



Nuclear deformation

- So far we considered radii which were constant and the same for all direction in space.
- This is applicable to nuclei with spherical distribution of matter and charge.
- However, only a very few nuclei turn out to be spherical! Nuclei, in contrast to atoms, can deform and attain non-spherical shape.
- For deformed nuclei the radius is not the same for all directions, it becomes a function of coordinates

$$R = R(x, y, z) \neq \text{const.} \tag{11}$$

- Moreover, radius can change in time as a function of surface vibration or rotation of the nucleus (we will come back to that later).

Multipole expansion

- When radius is a function of the (x, y, z) coordinates it can be expanded into a polynomial series and terms of increasing order can be grouped starting with the constant term, the linear term, the quadratic term, cubic term, etc.
- The constant term is referred to as the monopole term (orange).
- The linear term is referred to as the dipole term (the bad guy).
- The quadratic term is referred to as the quadrupole term (kiwi).
- The cubic term is referred to as the octupole term (pear).

Volume conservation

- There are two conditions which need to be recognized when considering nuclear deformation.

  1. Nuclei are incompressible. This implies that the nuclear volume has to be conserved. If we transform the radius to a function of (x, y, z) coordinates we need to scale it at the end to ensure that the volume is the same as before deformation.

  2. Transformations which shift the centre of the nucleus without changing its shape are spurious. They are not describing any new phenomena, since we can always shift back the coordinate system into the centre of the nucleus recovering the same system as before the transformation.

Dipole deformation of a circle

- For simplicity let us consider a two dimensional case of a deforming a circle of a unit radius. Before the deformation we have

$$x^2 + y^2 = 1 \tag{12}$$

- Let us try to deform it by adding the term x but keeping the surface area constant. To do that we introduce the scaling term α

$$x^2 + x + y^2 = \alpha \tag{13}$$

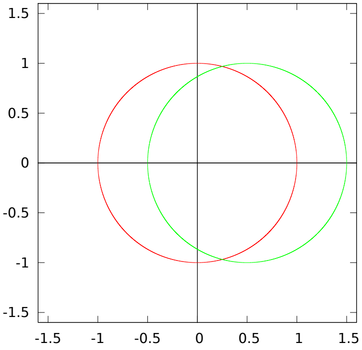
- But with a little math we see that

$$\begin{aligned} x^2 + x + y^2 &= x^2 + x + \frac{1}{4} - \frac{1}{4} + y^2 = \left(x - \frac{1}{2}\right)^2 + y^2 - \frac{1}{4} = \alpha \\ x^2 + x + y^2 &= \alpha \implies \left(x - \frac{1}{2}\right)^2 + y^2 = \alpha - \frac{1}{4} \end{aligned} \tag{14}$$

thus Eq. 13 also defines the circle.

Dipole deformation of a circle

- The consequence of the volume conservation is that the new circle has to have the same area as the old one. This is enforced by proper choice of the α constant $\alpha = 0.75$.
- Thus we did not deform the circle at all! We just shifted the centre.

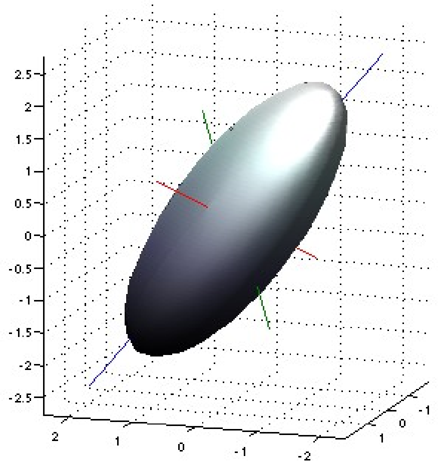


- Red: $x^2 + y^2 = 1$
- Green $x^2 + x + y^2 = 0.75$

Nuclear deformation

- For realistic cases in 3 dimensional space it can be shown as well that the dipole term under the requirement of volume conservation corresponds to the shift of the centre of mass.
- Thus dipole deformation is spurious, does not occur in atomic nuclei.
- It implies that there are no linear terms in expansion of the radius into a multipole series.
- The firs two terms are monopole (spherical:orange) and quadrupole (elliptical:kiwi).

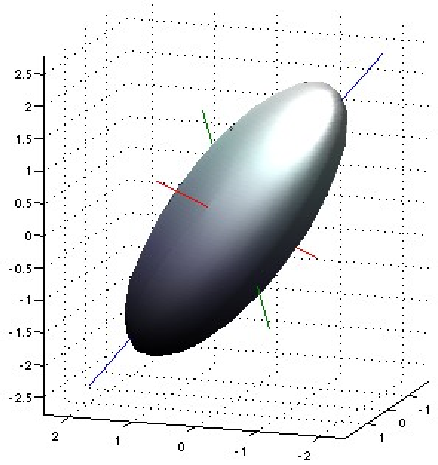
Quadrupole deformation



Principle axes coordinate frame

- For q quadruple-deformed nucleus with elliptical shape we can distinguish a coordinate frame defined by the three axes of deformation. For example we can define the long axis as z, the short axis as x and the intermediate axis as y (other choices are allowed as well).
- This coordinate frame is often referred to as the principle axis reference frame or the intrinsic reference frame.
- Note that the deformed nucleus can have any orientation with respect to the laboratory system.
- Therefore any 3 dimensional orientation of the principle axis reference frame with respect to the laboratory reference frame is allowed.

Quadrupole deformation



Principle axes reference frame

- The mathematical description of an ellipsoidal surface is especially simple in the principle axes reference frame

$$\left(\frac{x}{R_x}\right)^2 + \left(\frac{y}{R_y}\right)^2 + \left(\frac{z}{R_z}\right)^2 = 1 \tag{15}$$

  with R_x, R_y and R_z being the length of the x, y and z axes respectively.

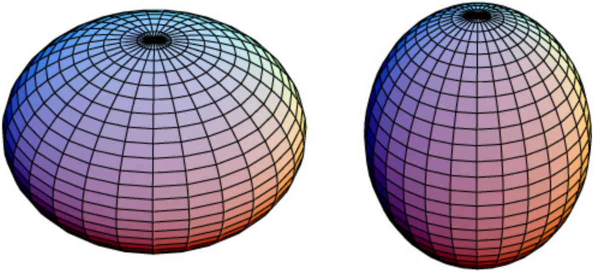
- The special cases are the axial deformation with $R_x = R_y$. This is significant as most of nuclei are believed to be axially symmetric.

- The two possible axial cases $R_x = R_y < R_z$ and $R_x = R_y > R_z$ are referred to as prolate (cigar) and oblate (pancake) deformations.

- Note that $R_x = R_y = R_z$ defines a sphere, or lack of deformation.

Prolate and oblate quadrupole deformation

- Choosing the vertical axis as the z axis the oblate $R_x = R_y > R_z$ and the prolate $R_x = R_y < R_z$ axially-symmetric quadrupole deformations are shown on the left and the right hand side of the figure.



Quadrupole moment

- Quadrupole moment is a measure of the deviation of an elliptical shape from the spherical shape.
- Quadrupole deformation deforms both, the mass and the charge distribution.
- The significance of the electric quadrupole moment of charge distribution is in the fact that this is the quantity which defines interaction of a deformed nucleus with electric field gradient.
- Electric quadrupole moments result in shift of atomic levels through the (hyperfine) interactions between nuclei and atomic electrons.

Quadrupole moment

- The magnitude of a quadrupole moment depends on the choice of the coordinate frame as well as mass/charge distribution in a nucleus.
- For axially symmetric $R_x = R_y$ quadrupole deformation with the z axis chosen as the symmetry axis the magnitude of a electric quadrupole moment is

$$Q = \frac{2}{5} Ze^2 (R_z^2 - R_x^2) = \frac{2}{5} Ze^2 (R_z^2 - R_y^2) \tag{16}$$

- With this choice of coordinates the quadrupole moment for the prolate $R_x = R_y > R_z$ deformation is positive $Q > 0$ and for the oblate $R_x = R_y < R_z$ deformation is negative $Q < 0$.