Control of the fission chain reaction

Introduction to Nuclear Science

Simon Fraser University
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Outline

1. Fission chain reaction
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2. Energy in nuclear fission
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3. Self sustained chain reaction
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4. Moderation and thermalization of neutrons
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2. Energy in nuclear fission
3. Self sustained chain reaction
4. Moderation and thermalization of neutrons
5. Steady state in nuclear reactors
Significance of fission chain reaction

- Fission chain reaction is the key to:
  - conversion of nuclear energy to electrical energy,
  - harvesting isotopes for medical and industrial applications,
  - production of neutron beams for basic and applied research,
  - production of neutrino beams for basic research,
  - nuclear weapon applications.

- The control of the parameters of the fission reaction is the key to above applications.

- It is possible, however, not necessarily easy, to control the fission chain reaction.
Nuclear fission

neutron → uranium nucleus → uranium nucleus plus neutron → nucleus splitting → two daughter nuclei

fast neutrons
Energy balance in $^{235}$U fission

- Let us investigate the energy balance in a single fission of $^{235}$U
  - average kinetic energy of fission fragments is 168 MeV
  - there are 2.5 neutrons emitted on average with average energy of 2 MeV energy per neutron
  - thus the average energy of neutrons is 5 MeV
  - average energy of prompt $\gamma$-rays after fission is 8 MeV
  - average energy of $\beta^-$ particles from fission fragment decay is 8 MeV
  - average energy of $\gamma$ rays following $\beta^-$ decay of fission fragments is 7 MeV
  - average energy of electron antineutrinos following fission fragment $\beta^-$ decay is 12 MeV.

The total average energy is 208 MeV

Since antineutrinos escape 196 MeV per fission is recoverable.
The energy perspective

Let us calculate the energy release from fissioning of 1 g of $^{235}$U.

There number of $^{235}$U atoms in 1 g is

$$n = \frac{m}{\mu} N_A = \frac{1}{235} \times 6.02 \times 10^{23} = 2.56 \times 10^{21}$$ (1)

The conversion of MeV to Jules is

$$196 \times 10^6 \text{ [eV]} \times 1.6 \times 10^{-19} \text{ [J/ev]} = 3.1 \times 10^{-11} \text{ [J]}$$ (2)

The energy content in 1 g of $^{235}$U is

$$2.56 \times 10^{21} \times 3.1 \times 10^{-11} \text{ [J]} \approx 8 \times 10^{10} \text{ [J]} = 80 \text{ [GJ]}$$ (3)

An energy content of 1 g of coal is at most 35 kJ, 2.3 million less.
The energy perspective

- So we have noted that energy content of 1 g of $^{235}\text{U}$ is equivalent to the energy content of 2.3 tons of coal.

- What about radiation? Burning coal produces quite a lot, since long-lived radioactive isotopes are dug out from the crest of the Earth and released in burning.

- What about solar power?

- Average solar energy delivered to 1 m$^2$ on the surface of the earth is 1360 W.

- Let us assume that we have access to solar panels which convert solar light into electric energy 12 h per day.

- Let us calculate surface area of panels delivering energy equivalent to 1 g of $^{235}\text{U}$. 
The energy perspective

- Let us calculate area of panels delivering energy of 80 GJ in 12 h.

- The power is energy over time which results in

\[
P = \frac{E}{t} = \frac{80 \times 10^9}{12 \times 60 \times 60} = 1.85 \times 10^6 \text{ [W]} = 1.85 \text{ [MW]} \quad (4)
\]

- The solar power is 1360 W per meter square.

- Thus to get the equivalent energy one needs

\[
A = \frac{1.85 \times 10^6}{1360} = 1.36 \times 10^3 \text{ m}^2 \quad (5)
\]

- This corresponds to a square of 37 m side.

- Current maximum efficiency of panels is 40%, typical will be smaller.

- Efficiency of 10% gives a square of a 117 m side.
So in principle, nuclear chain reaction should be self sustained.

There are 2.5 neutrons per fission, 5 neutrons per 2 fission, so naively every 2 fissions should produce 5 fissions with 196 MeV energy release.

But life is not so simple, predominantly for three reasons

- Fission cross sections are energy-dependent and rather small for 2 MeV neutrons.
- Neutrons can be lost in reactions which do not produce fission.
- Neutrons can leak out from the volume of the reactor without inducing fission.

For the above reasons, building and controlling nuclear reactor is an art, even more so for nuclear weapon.
Nuclear chain reaction
The critical mass

- The critical mass is the smallest amount of fissile material needed for a sustained nuclear chain reaction.

- The critical mass of a fissionable material depends on
  - nuclear cross section thus the type of material
  - density of the material
  - shape of the material
  - enrichment (or content of non-fissile impurities)
  - the temperature of the material
  - surroundings of the material.

- For a self-sustaining fission reaction in a critical mass of a material here is no increase or decrease in power, temperature or neutron population.
The critical mass

Left: subcritical
Middle: critical
Right: subcritical with a reflector
The critical mass

- The critical mass can be changed by
  - varying the amount of fuel
  - varying the shape of the fuel
  - varying the temperature (hot fuel is less reactive)
  - varying the density of the fuel
  - application of neutron moderators
  - application of neutron reflectors

- Critical mass for a bare sphere of $^{239}$Pu is 10 kg (radius of 10 cm).
- Critical mass for a bare sphere of $^{235}$U is 52 kg (radius of 17 cm).
- Critical mass of 20% enriched $^{235}$U is 400 kg.
Nuclear reactors

- Nuclear reactor is a device to initiate and control a self-sustained fission chain reaction.
- Nuclear reactors can operate with the fuel mass significantly below the critical mass of a bare sphere.
- This is achieved by changing the property of the neutron flux.
- A moderator in a reactor shifts the intensity of the neutron flux to the thermal region where cross sections for fission are high.
- Moderators allow low enriched uranium which is a mixture of 3% $^{235}\text{U}$ and 97% of $^{238}\text{U}$ to be used as a reactor fuel.
- Note, that natural uranium contains 0.7% of $^{235}\text{U}$ and 99.3% of $^{238}\text{U}$. 
Fission neutron spectra

Red: $^{235}\text{U}$  Blue: $^{239}\text{Pu}$
$^{235}\text{U}$ fission neutron spectra

$n(0.0 \text{ MeV}) + ^{235}\text{U}$

Compare experimental spectra in peak region

![Graph showing neutron spectrum](image-url)

Neutron-induced fission cross section for $^{235}$U
The need for moderation

A comparison between the neutron energy spectra from fission of $^{235}\text{U}$ and the cross section for $^{235}\text{U}$ fission indicates the need for moderation of neutrons.

Indeed, the most probable energy for neutrons is 2 MeV.

The fission cross section at this energy is 1.3 b.

If neutrons are moderated and brought to thermal equilibrium with the surroundings they will have temperature of 20 deg. C or 293 deg. K, the speed of 2200 m/s and the energy of 0.025 eV.

The cross section at this energy is 584 b.

For $^{23}\text{U}$ moderation increases the cross section by a factor of 450.

Some neutrons will be lost during the moderation.
The way to moderate

- The moderation of neutrons should proceed predominantly through the elastic scattering process since other processes, like reactions, will reduce neutron flux.
- Elastic collision conserves momentum and energy.
- Let us consider a central elastic collision between a neutron and a nucleus of mass $A$. Conservation of energy and momentum yields:

$$\vec{p}_i = \vec{p}_f + \vec{p}_A$$

$$\frac{p_i^2}{2m} = \frac{p_f^2}{2m} + \frac{p_A^2}{2Am} \quad (6)$$

- A bit of algebra leads to

$$A(p_i^2 - p_f^2) = (p_i - p_f)^2 = p_A^2 \quad (7)$$
The way to moderate

The solution to the equation

$$A(p_i^2 - p_f^2) = (p_i - p_f)^2$$  \hspace{1cm} (8)

is provided by

$$A(p_i^2 - p_f^2) = A(p_i + p_f)(p_i - p_f) = (p_i - p_f)^2$$
$$A(p_i + p_f) = p_i - p_f$$
$$A\rho_i + A\rho_f = p_i - p_f$$

$$p_f = -p_i \frac{A - 1}{A + 1}, \quad E_f = E_i \left(\frac{A - 1}{A + 1}\right)^2$$  \hspace{1cm} (9)

Note that for $A = 1$ the momentum after collision is $p_f = 0$

For $A >> 1$ the momentum after collision is $p_f = -p_i$. 
**The way to moderate**

- We have observed that the moderation via elastic scattering works best for moderators of small $A$.

- From the collision point of view hydrogen at $A = 1$ seems best.

- Water, containing significant quantities of hydrogen is used as a moderator in Light Water Reactors.

- A good moderator will reduce energy of neutrons without reducing the neutron flux by absorption.

- Hydrogen captures neutrons and forms deuterium.

- Deuterium at $A = 2$ is a reasonably good moderator with low cross section for a neutron capture.

- Heavy water containing deuterium instead of hydrogen is used as a moderator in Heavy Water Reactors.
Light Water moderator of SIMON

![Graph showing moderation and thermalization of neutrons in Water]

<table>
<thead>
<tr>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entries</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>RMS</td>
</tr>
</tbody>
</table>

Counts vs. Time in the moderator [ms]
Heavy Water moderator of SIMON

### Heavy Water

- **Entries**: 9998
- **Mean**: 4.572
- **RMS**: 4.211

### Time in the moderator [ms]

<table>
<thead>
<tr>
<th>Time (ms)</th>
<th>Counts</th>
</tr>
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<tbody>
<tr>
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<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>210</td>
</tr>
<tr>
<td>15</td>
<td>310</td>
</tr>
<tr>
<td>20</td>
<td>410</td>
</tr>
</tbody>
</table>

### Chart

- X-axis: Time in the moderator [ms]
- Y-axis: Counts
- Logarithmic scale with base 10
Neutron reflectors

- Analysis of elastic collision with a nucleus of large mass $A \gg 1$ implies that the momentum of the incoming neutron is reversed with nearly unchanged magnitude.

- This implies that materials of $A \gg 1$ and low neutron absorption cross section can be used to reflect neutrons.

- Materials used for reflectors are graphite, beryllium, lead, steel, tungsten, carbide and other.

- A reflector can make nuclear chain reaction which is not self-sustaining to become self-sustaining.

- In 1946 a 6.2 kg core of $^{239}$Pu become critical (developed self-sustained chain reaction) after a brick made of neutron reflecting material accidentally fell on it. Brick was removed promptly but neutron irradiation resulted in death of a worker 25 days later.
Neutron multiplication factor

- The critical parameter for operating nuclear reactor is the neutron multiplication factor denoted by $k$ which is defined as the average number of neutrons from one fission which cause another fission.

- The neutrons which do not cause fission are either absorbed or escape from the reactor.

- If the time scale for neutron multiplication by a factor of $k$ is denoted by $\tau$ the change of the neutron flux $N$ as a function of time is defined by

$$ \frac{dN}{dt} = \frac{kN - N}{\tau} = \frac{k - 1}{\tau}N $$

(10)

- The solution of this equation is define the operating condition of a reactor.
Reactor in the critical mode

- The neutron multiplication factor $k = 1$ is the special case corresponding to reactor operating in the critical mode.

- The equation for the flux becomes in this case

  \[
  \frac{dN}{dt} = \frac{k - 1}{\tau} N = 0 \quad (11)
  \]

- The solution of this equation gives a constant flux

  \[
  N(t) = N(0) \quad (12)
  \]

- This is the solution for a steady state of the reactor without any growth or reduction in the number of neutrons.

- The energy or heat generation in the steady state is the flux $N$ times the recoverable energy per fission.
Reactor in the subcritical mode

- The neutron multiplication factor $k < 1$ corresponds to reactor operating in the subcritical mode.

- The equation for the flux becomes in this case

  $$\frac{dN}{dt} = \frac{kN - N}{\tau} = \frac{k - 1}{\tau} N = -\frac{1 - k}{\tau} N \quad (13)$$

- The solution of this equation is

  $$N(t) = N(0) \exp\left(-\left(1 - k\right)\frac{t}{\tau}\right) \quad (14)$$

- In the subcritical mode the flux decreases which leads to a shutdown of the reactor within the timescale defined by the effective lifetime of

  $$\tau/(1 - k). \quad (15)$$
Reactor in the supercritical mode

- The neutron multiplication factor $k > 1$ corresponds to reactor operating in the supercritical mode.

- The equation for the flux becomes in this case
  \[
  \frac{dN}{dt} = kN - N = \frac{k - 1}{\tau}N
  \]  

  (16)

- The solution of this equation is
  \[
  N(t) = N(0) \exp\left( (k - 1) \frac{t}{\tau} \right)
  \]  

  (17)

- In the supercritical mode the flux increases within the timescale defined by the effective lifetime of
  \[
  \frac{\tau}{(k - 1)}.
  \]  

  (18)
The control of a reactor

- The goal of the control of a reactor is to maintain a steady state at the criticality with the multiplication factor $k = 1$.

- This is not an easy task since it is practically impossible to maintain a constant $k$ without even slight deviations.

- What helps to control the reactor is the fact that increased temperatures reduce reactivity of the neutrons as cross sections decrease with increased energy.

- This feature provides a negative feedback needed for control: supercriticality leads to decreased reactivity which lowers the temperature giving subcriticality which increases reactivity etc.

- As a consequence the multiplication factor $k$ oscillates around the critical value $k = 1$. 