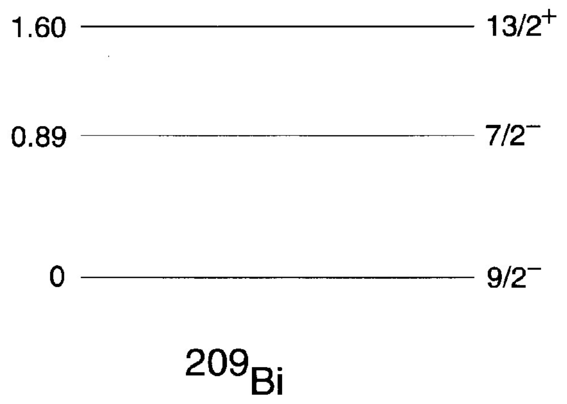
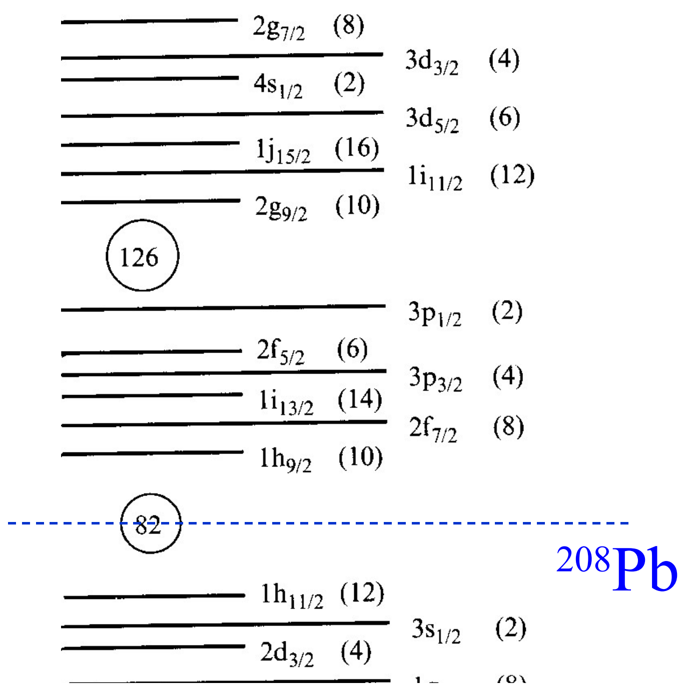
Pairing Correlations

$^{209}_{83}\mathrm{Bi}_{126} = {}^{208}_{82}\mathrm{Pb}_{126} + p$





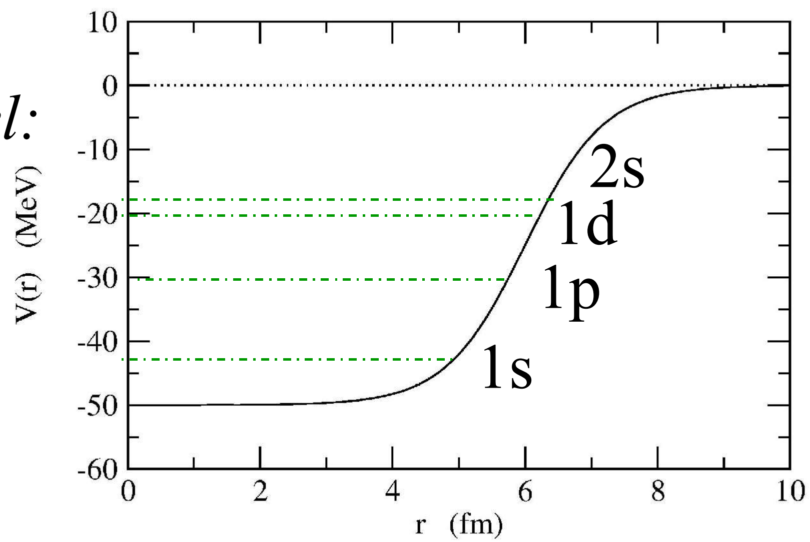
$^{210}_{84}\mathrm{Po}_{126} = {}^{208}_{82}\mathrm{Pb}_{126} + 2p$

expectation of the indep. particle model:

E=0: $[h_{9/2} \otimes h_{9/2}]^I$ (I=0,2,4,6,8)

E=0.89 MeV: $[h_{9/2} \otimes f_{7/2}]^I$ (I=1,2,3,4,5,6,7,8)

→ # of states below 1 MeV: 13



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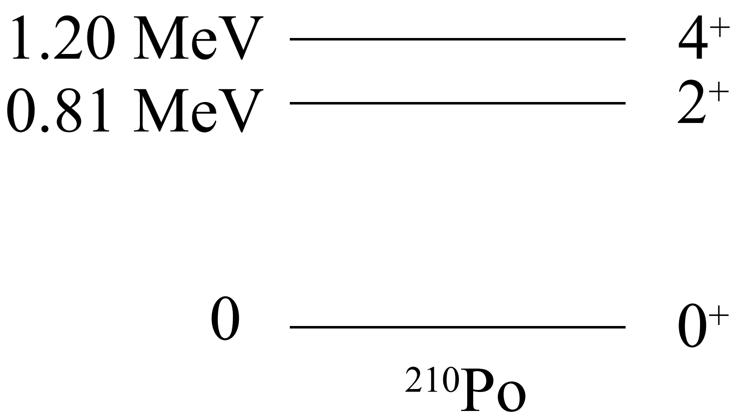
observed spectra:



↔ Effects of the residual interaction

$$H = \sum_{i=1}^{A} \left(-\frac{\hbar^2}{2m} \nabla_i^2 + V_{\mathrm{HF}}(i) \right) + \frac{1}{2} \sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j) - \sum_i V_{\mathrm{HF}}(i)$$

Effects of the residual interaction

$$H = \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\mathrm{HF}}(i)\right) + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j) - \sum_i V_{\mathrm{HF}}(i)$$

$$\sim -g\,\delta(\boldsymbol{r}-\boldsymbol{r}')$$ (short range force)

$$= -g\frac{\delta(r-r')}{rr'}\sum_{\lambda\mu} Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})Y_{\lambda\mu}(\hat{\boldsymbol{r}}')$$

$$\Delta E_I \sim \langle [j \bigotimes j]^I | -g\delta(\boldsymbol{r}-\boldsymbol{r}') | [j \bigotimes j]^I \rangle$$

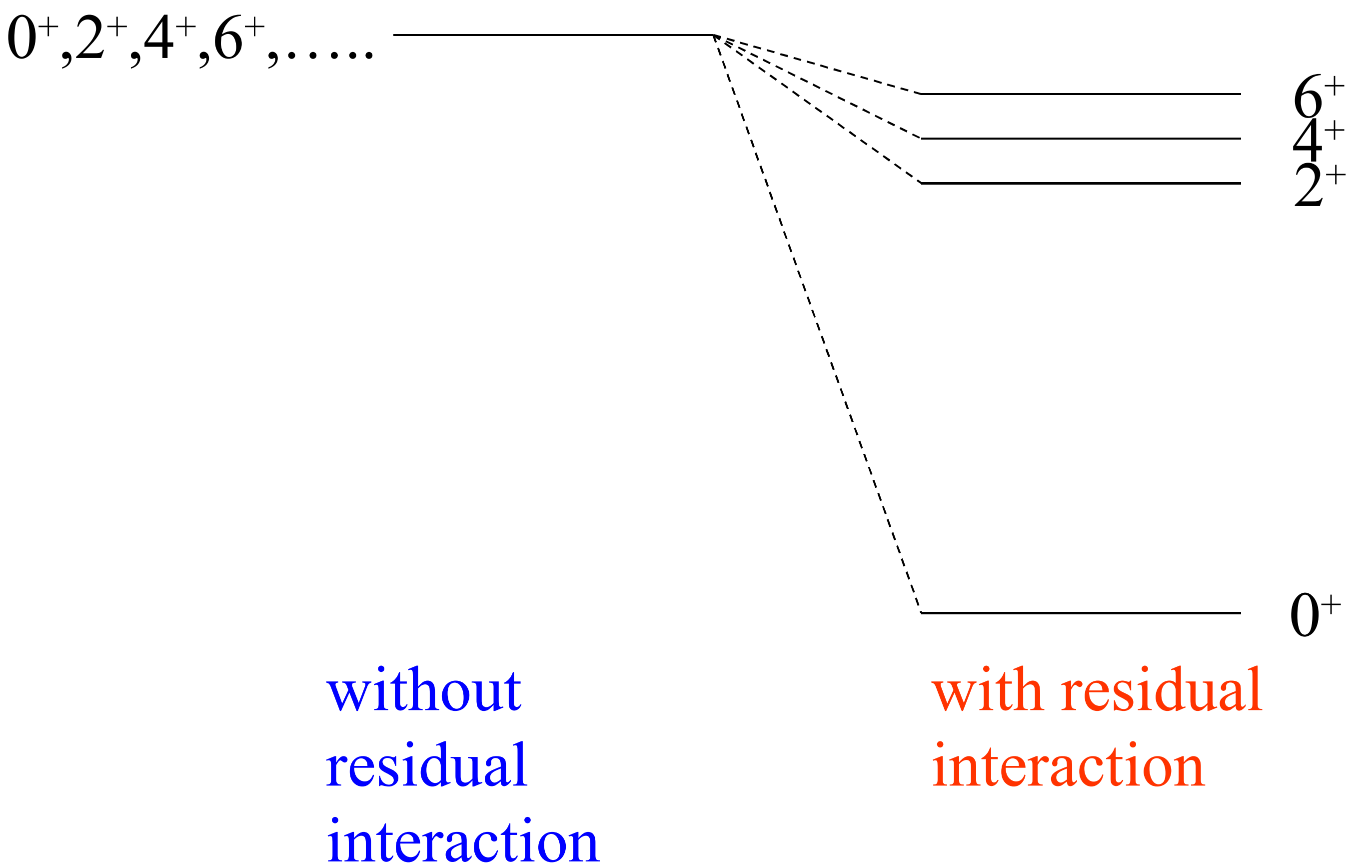
$$= -g\,F_r\,\frac{(2j+1)^2}{2}\begin{pmatrix} j & j & I \\ 1/2 & -1/2 & 0 \end{pmatrix}^2$$

(for even j)

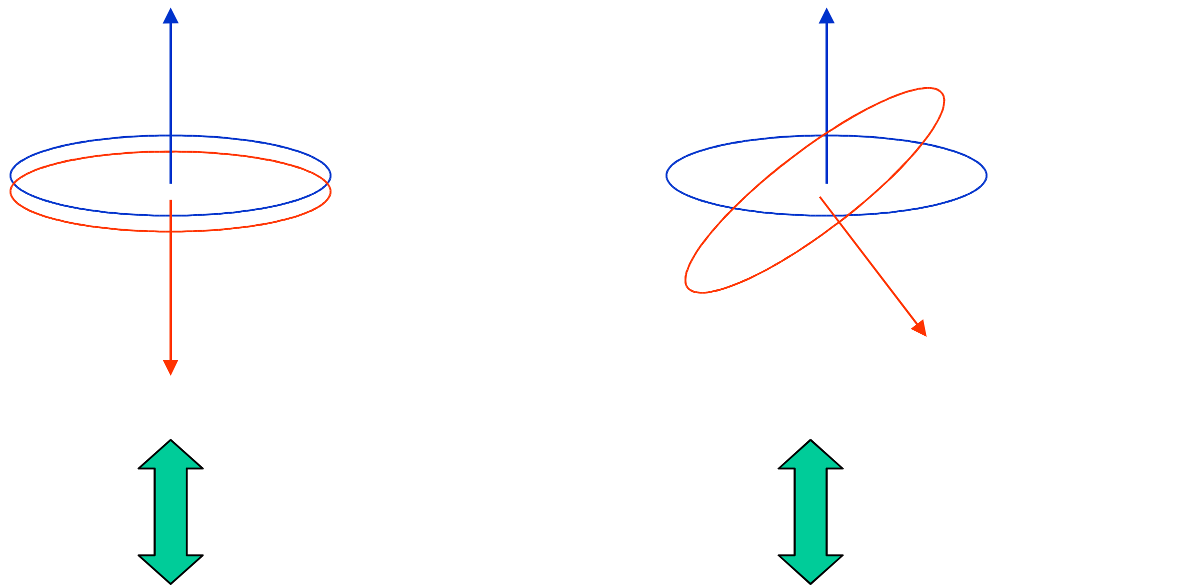
$$F_r = \int dr \frac{u_{jl}^4(r)}{4\pi r^2}$$ (radial integral)

$$\Delta E_I \sim -g\, F_r \frac{(2j+1)^2}{2} \begin{pmatrix} j & j & I \\ 1/2 & -1/2 & 0 \end{pmatrix}^2 \equiv -g\, F_r\, A(jj;I)$$

A(jj;I)	*I*=0	*I*=2	*I*=4	*I*=6
j=5/2	3.00	0.685	0.286	---
j=7/2	4.00	0.95	0.467	0.233



Simple interpretation:



I=0 pair $\quad\quad$ $I \neq 0$ pair

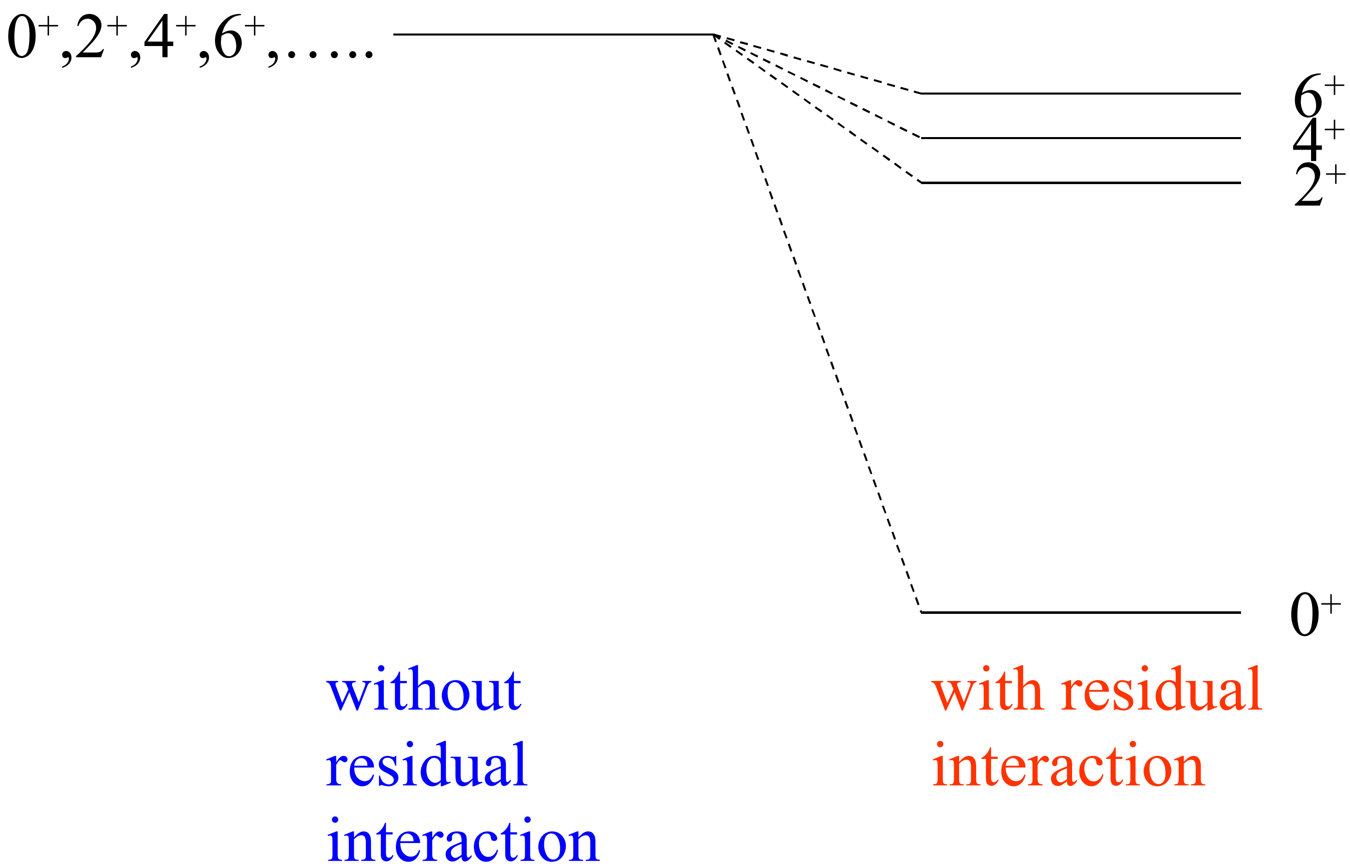
The spatial overlap is the largest for the I=0 pair.

"Pairing Correlation"

(note) The $I=2j$ pair is unfavoured due to the Pauli principle.

(note)

$$\psi(l^2; L=0) = \sum_{\mu} \langle l\mu l-\mu | L=0,0\rangle \, Y_{l\mu}(\hat{\boldsymbol{r}}_1) Y_{l-\mu}(\hat{\boldsymbol{r}}_2) = Y_{l0}(\theta_{12})/\sqrt{4\pi}$$



The ground state spin of nuclei

- Even-even nuclei: 0^+
- Even-odd nuclei: the spin of the valence particle

Mass Formula (Even-odd mass difference)

Extra binding when like nucleons form a spin-zero pair

Example:

	Binding energy (MeV)
${}^{210}_{82}Pb_{128} = {}^{208}_{82}Pb_{126} + 2n$	1646.6
${}^{210}_{83}Bi_{127} = {}^{208}_{82}Pb_{126} + n + p$	1644.8
${}^{209}_{82}Pb_{127} = {}^{208}_{82}Pb_{126} + n$	1640.4
${}^{209}_{83}Bi_{126} = {}^{208}_{82}Pb_{126} + p$	1640.2

$$
\begin{aligned}
B_{\text{pair}} &= \Delta && (\text{for even} - \text{even}) \\
&= 0 && (\text{for even} - \text{odd}) \\
&= -\Delta && (\text{for odd} - \text{odd})
\end{aligned}
$$

More later

The BCS theory

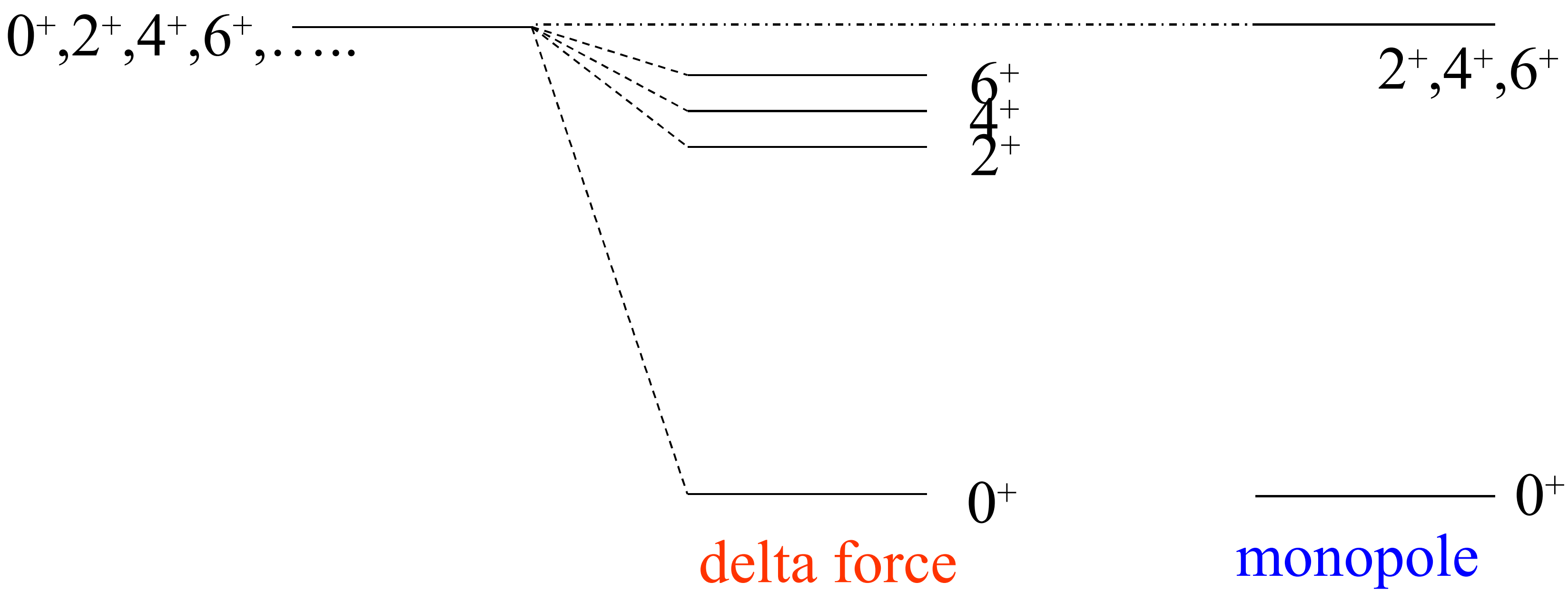
Many-particles in non-degenerate levels
~ mean-field approx. for the pairing channel ~

Simplified pairing interaction

$$V = -G\,P^\dagger P; \quad P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$$

$\bar{\nu}$: the time reversed state of ν

e.g.,
$|\nu\rangle = |njlm\rangle, \quad |\bar{\nu}\rangle = |njl-m\rangle$



Cf. Metallic superconductivity

Solve the pairing Hamiltonian

$$H = \sum_\nu \epsilon_\nu (a_\nu^\dagger a_\nu + a_{\bar{\nu}}^\dagger a_{\bar{\nu}}) - G \left(\sum_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger\right)\left(\sum_{\nu>0} a_{\bar{\nu}} a_\nu\right)$$

in the mean-field approximation

- Mean-field approximation:

$$V = -G\,P^\dagger P \to -G\left(\langle P^\dagger\rangle P + P^\dagger\langle P\rangle\right) = -\Delta(P^\dagger + P)$$

Cf. HF potential

$$V_H(\boldsymbol{r}) = \int v(\boldsymbol{r},\boldsymbol{r}')\rho_{\mathrm{HF}}(\boldsymbol{r}')d\boldsymbol{r}$$

⟺ particle number violation

- The Bardeen, Cooper, Schrieffer (BCS) ansatz

$$|\Psi\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar{\nu}}^\dagger\right)\Big|0\Big\rangle$$

$$|u_\nu|^2 + |v_\nu|^2 = 1 \longleftarrow \text{normalization}$$

(note) $\left\langle a_\nu^\dagger a_\nu\right\rangle = |v_\nu|^2$: occupation probability

- The Bardeen, Cooper, Schrieffer (BCS) anzatz

$$|\Psi\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

$$|u_\nu|^2 + |v_\nu|^2 = 1$$ ← normalization

(note) $\left\langle a^\dagger_\nu a_\nu \right\rangle = |v_\nu|^2$: occupation probability

(note)

BCS convention: $u_{\bar{\nu}} = u_\nu, \quad v_{\bar{\nu}} = -v_\nu$ (real numbers)

(note) $$\left(1 + \frac{v_\nu}{u_\nu}\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle = \exp\left(\frac{v_\nu}{u_\nu}\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

$$|\Psi\rangle \propto \exp\left(\sum_{\nu>0} \frac{v_\nu}{u_\nu}\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$ (pair condensed wave function)

(note)

$$|\Psi\rangle \propto \prod_{\nu>0} \alpha_\nu \alpha_{\bar{\nu}} |0\rangle$$

$$\alpha^\dagger_\nu = u_\nu a^\dagger_\nu - v_\nu a_{\bar{\nu}}$$

$$\alpha^\dagger_{\bar{\nu}} = u_\nu a^\dagger_{\bar{\nu}} + v_\nu a_\nu$$

$$H=\sum_\nu \epsilon_\nu (a^\dagger_\nu a_\nu + a^\dagger_{\bar\nu} a_{\bar\nu}) - G\left(\sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar\nu}\right)\left(\sum_{\nu>0} a_{\bar\nu} a_\nu\right)$$

- Mean-field approximation:

$$V=-G\,P^\dagger P \to -G\left(\langle P^\dagger\rangle P + P^\dagger\langle P\rangle\right) = -\Delta(P^\dagger + P)$$

- The Bardeen, Cooper, Schrieffer (BCS) anzatz

$$|\Psi\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar\nu}\right)\big|0\big\rangle \qquad |u_\nu|^2 + |v_\nu|^2 = 1$$

(note) $\left\langle a^\dagger_\nu a_\nu \right\rangle = |v_\nu|^2, \qquad \Delta = G\langle P^\dagger\rangle = G\sum_{\nu>0} u_\nu v_\nu$

<u>Minimize</u> $\langle H'\rangle = \left\langle \sum_\nu \epsilon_\nu (a^\dagger_\nu a_\nu + a^\dagger_{\bar\nu} a_{\bar\nu}) - GP^\dagger P - \lambda\hat N \right\rangle$

with $\langle\Psi|\hat N|\Psi\rangle = 2\sum_{\nu>0} v_\nu^2 = N$

$$\hat N = \sum_\nu (a^\dagger_\nu a_\nu + a^\dagger_{\bar\nu} a_{\bar\nu})$$

$$E' = \langle\Psi|H'|\Psi\rangle \sim 2\sum_{\nu>0}(\epsilon_\nu - \lambda)v_\nu^2 - \Delta^2/G$$

$$E' = 2\sum_{\nu>0}(\epsilon_\nu - \lambda)v_\nu^2 - \left(G\sum_{\nu>0} u_\nu v_\nu\right)^2 /G$$

Minimization:

$$\begin{aligned} 0 &= \left(\frac{\partial}{\partial v_\nu} + \frac{\partial u_\nu}{\partial v_\nu}\frac{\partial}{\partial u_\nu}\right)E' \\ &= 2(\epsilon_\nu - \lambda)u_\nu v_\nu + \Delta(v_\nu^2 - u_\nu^2) \end{aligned}$$

$$u_\nu^2 + v_\nu^2 = 1$$

$$\begin{aligned} u_\nu^2 &= \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{E_\nu}\right) \\ v_\nu^2 &= \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{E_\nu}\right) \end{aligned}$$

$$E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}$$

$$\Delta = \frac{G}{2}\sum_{\nu>0}\frac{\Delta}{E_\nu}$$

(Gap equation)

Gap Equation

$$\Delta = \frac{G}{2}\sum_{\nu>0}\frac{\Delta}{\sqrt{(\epsilon_\nu-\lambda)^2+\Delta^2}}$$

i) Trivial solution: always exists

$$\Delta = 0$$

$$v_\nu^2 = \frac{1}{2}\left(1+\frac{\epsilon_\nu-\lambda}{\sqrt{(\epsilon_\nu-\lambda)^2}}\right) = 1 \quad (\epsilon_\nu \leq \lambda)$$

$$= 0 \quad (\epsilon_\nu > \lambda)$$

G a/o *N* ⟶ large

ii) Superfluid solution

$$1 = \frac{G}{2}\sum_{\nu>0}\frac{1}{\sqrt{(\epsilon_\nu-\lambda)^2+\Delta^2}}$$

⇐ (Note) obviously this equation cannot be satisfied for G=0

$$v_\nu^2 = \frac{1}{2}\left(1+\frac{\epsilon_\nu-\lambda}{\sqrt{(\epsilon_\nu-\lambda)^2+\Delta^2}}\right) < 1$$

i) Trivial solution: always exists

$$\Delta = 0$$

$$v_\nu^2 = 1 \quad (\epsilon_\nu \leq \lambda)$$
$$= 0 \quad (\epsilon_\nu > \lambda)$$

$$|\Psi\rangle = \prod_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger |0\rangle$$

G a/o $N \longrightarrow$ large

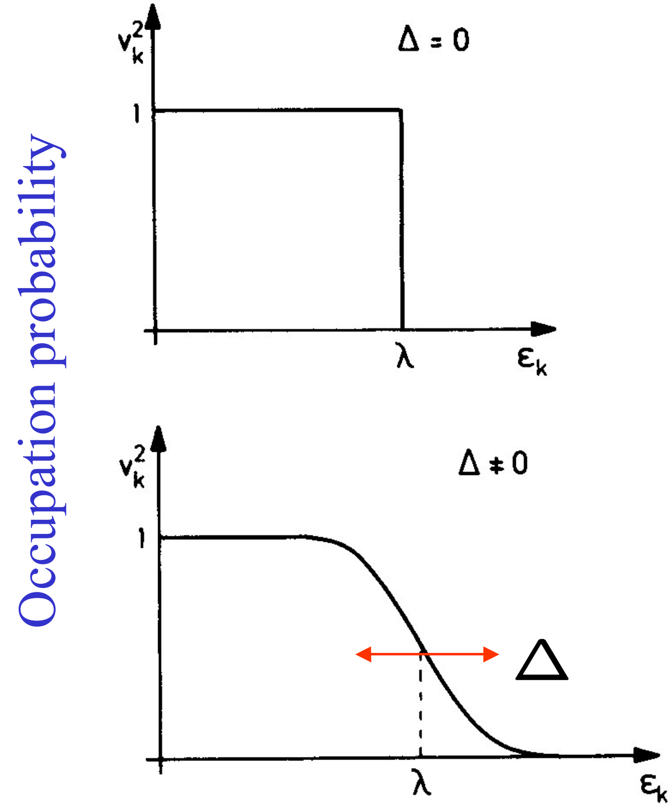
ii) Superfluid solution

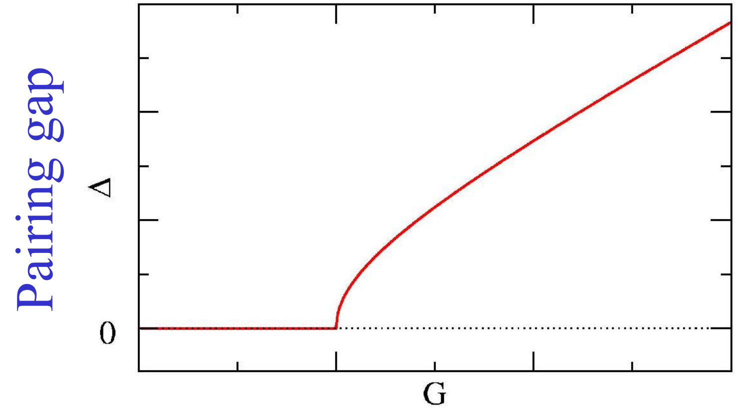
$$\Delta \neq 0$$

$$v_\nu^2 < 1$$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu a_\nu^\dagger a_{\bar{\nu}}^\dagger\right) \Big| 0 \Big\rangle$$

Number fluctuation





Norma-Superfulid phase transition

Particle Number Projection

$$|BCS\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar\nu}\right)\big|0\big\rangle$$: violation of the particle number

→ Particle number projection

Cf. Violation of the rot. symmetry for def. nuclei
and the angular momentum projection

Projection operator:

$$\hat{P}_N = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{i\varphi(\hat{N}-N)}$$

$$\begin{aligned}(\Delta N)^2 &= \langle(\hat{N}-N)^2\rangle \\ &= 4\sum_{\nu>0} u_\nu^2 v_\nu^2\end{aligned}$$

(note) $$|BCS\rangle = \sum_{N'} C_{N'}|N'\rangle$$

$$\to |\mathrm{proj}\rangle = \hat{P}_N|BCS\rangle = C_N|N\rangle$$

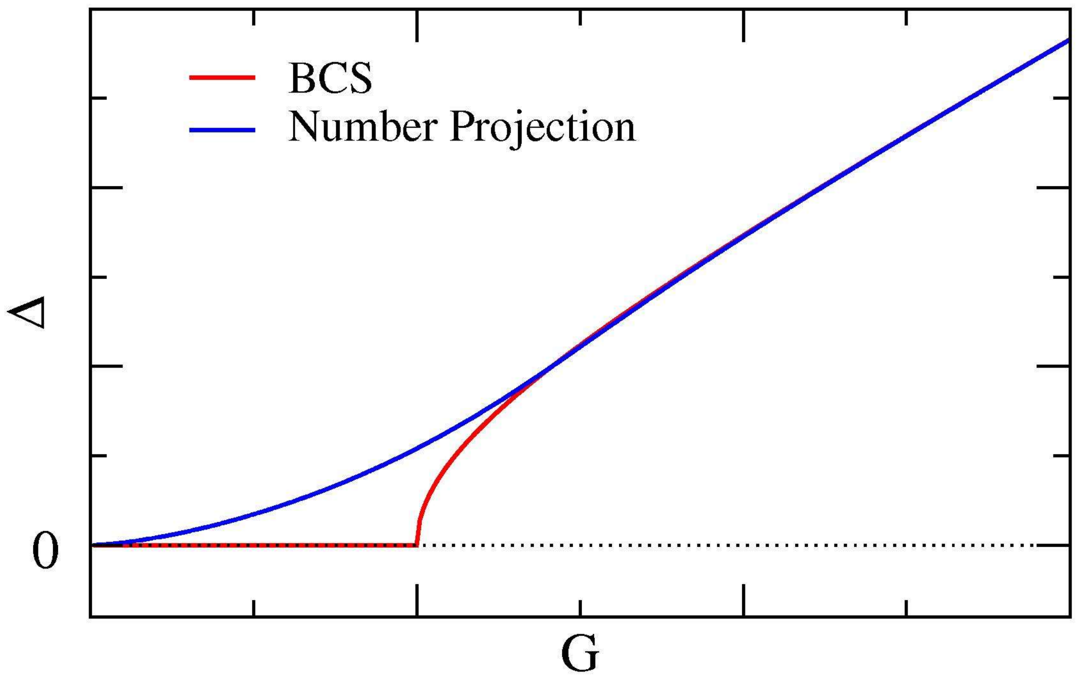
(note) $$e^{i\hat{N}\varphi}|BCS\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, e^{2i\varphi}\, a^\dagger_\nu a^\dagger_{\bar\nu}\right)\big|0\big\rangle$$ ⇐ degenerate with $|BCS\rangle$

Variation After Projection: determine v_ν by minimizing

$$E'_{\text{proj}} = \frac{\langle BCS|\hat{P}_N(\hat{H} - \lambda\hat{N})\hat{P}_N|BCS\rangle}{\langle BCS|\hat{P}_N\hat{P}_N|BCS\rangle}$$

$$\Longrightarrow \left(\frac{\partial}{\partial v_\nu} + \frac{\partial u_\nu}{\partial v_\nu}\frac{\partial}{\partial u_\nu}\right) E'_{\text{proj}} = 0$$

$$\Longrightarrow \Delta = G \sum_{\nu>0} u_\nu v_\nu$$



Bogoliubov Transformation

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar\nu}\right)\Big|0\Big\rangle \propto \prod_{\nu>0} \alpha_\nu \alpha_{\bar\nu}|0\rangle$$

Bogoliubov transformation

$$\alpha^\dagger_\nu = u_\nu a^\dagger_\nu - v_\nu a_{\bar\nu}, \qquad \alpha^\dagger_{\bar\nu} = u_\nu a^\dagger_{\bar\nu} + v_\nu a_\nu$$

(Quasi-particle operator)

(note)

$$\{\alpha_\nu, \alpha_{\nu'}\} = 0, \qquad \{\alpha_\nu, \alpha^\dagger_{\nu'}\} = \delta_{\nu,\nu'}$$

(note)

$$\alpha_\nu|BCS\rangle = 0$$

$$
\begin{aligned}
H &= \sum_{\nu>0} \epsilon_\nu (a^\dagger_\nu a_\nu + a^\dagger_{\bar{\nu}} a_{\bar{\nu}}) - G \left(\sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}} \right) \left(\sum_{\nu>0} a_{\bar{\nu}} a_\nu \right) \\
&= E_{BCS} + H_{11} + H_{20} + V_{res}
\end{aligned}
$$

$$
\begin{aligned}
E_{BCS} &= const. = \langle BCS | H | BCS \rangle \\
H_{11} &\sim \alpha^\dagger \alpha \\
H_{20} &\sim \alpha^\dagger \alpha^\dagger + \alpha\alpha \\
V_{res} &\sim \alpha^\dagger \alpha^\dagger \alpha^\dagger \alpha^\dagger + \alpha^\dagger \alpha^\dagger \alpha^\dagger \alpha + \cdots + \alpha\alpha\alpha\alpha
\end{aligned}
$$

BCS solution

$$
\begin{aligned}
H_{11} &= \sum_\nu E_\nu \alpha^\dagger_\nu \alpha_\nu \\
H_{20} &= 0
\end{aligned}
$$

$$
E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}
$$

The meaning of the Bogoliubov transformation

$$H' = \sum_\nu (\epsilon_\nu - \lambda)(a^\dagger_\nu a_\nu + a^\dagger_{\bar\nu} a_{\bar\nu}) - G\left(\sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar\nu}\right)\left(\sum_{\nu>0} a_{\bar\nu} a_\nu\right)$$

Mean-field approximation:

$$V = -G\,P^\dagger P \to -G\left(\langle P^\dagger\rangle P + P^\dagger\langle P\rangle\right) = -\Delta(P^\dagger + P)$$

(note) the quadratic form can be diagonalized with the Bogoliubov transformation:

$$h = \xi\, c^\dagger c + \eta\,(c^\dagger c^\dagger + cc)$$

Bogoliubov transformation $c^\dagger = u\,\alpha^\dagger - v\,\alpha$

$$h = \xi(u^2 - v^2)\alpha^\dagger\alpha + (\eta - \xi uv)(\alpha^\dagger\alpha^\dagger + \alpha\alpha)$$

Choose $uv = \eta/\xi$

$$h = \xi(u^2 - v^2)\alpha^\dagger\alpha$$

Quasi-particle excitations

$$H \sim E_{BCS} + \sum_\nu E_\nu\, \alpha^\dagger_\nu \alpha_\nu$$

- g.s. of even-even nuclei: $|BCS\rangle$
- One quasi-particle states:

$$|\nu_1\rangle = \alpha^\dagger_{\nu_1}|BCS\rangle = a^\dagger_{\nu_1} \prod_{\nu\neq\nu_1} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

Wave function for odd-mass nuclei

$$\langle\nu_1|H|\nu_1\rangle = \langle H\rangle + E_{\nu_1}$$

- Two quasi-particle states:

$$|\nu_1\nu_2\rangle = \alpha^\dagger_{\nu_1}\alpha^\dagger_{\nu_2}|BCS\rangle$$

Excited state of the even-even nuclei

$$\langle\nu_1\nu_2|H|\nu_1\nu_2\rangle - \langle H\rangle \;=\; E_{\nu_1} + E_{\nu_2}$$

$$\geq\; 2\Delta$$ ⟵ Energy gap

(note) no pairing limit:

$$\alpha^\dagger_p\alpha^\dagger_h \to a^\dagger_p a_h, \qquad E_p + E_h \to (\epsilon_p - \lambda) + (\lambda - \epsilon_h)$$

(particle-hole excitation)

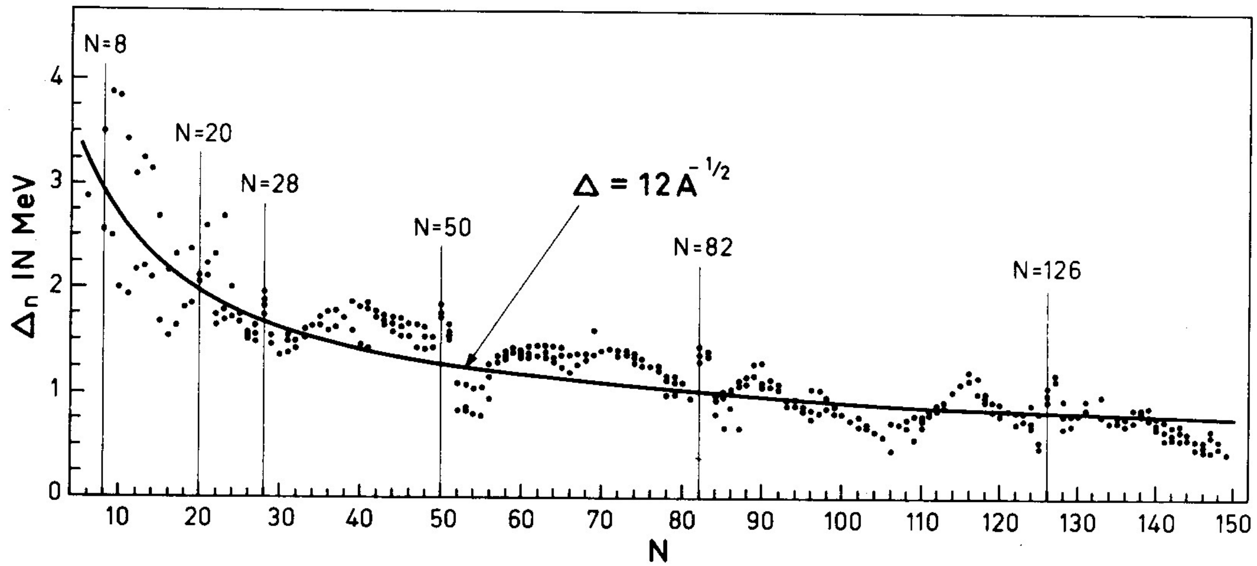
Even-odd mass difference and pairing gap

$$B_{\text{pair}} = \Delta \quad \text{(for even – even)}$$
$$= 0 \quad \text{(for even – odd)}$$
$$= -\Delta \quad \text{(for odd – odd)}$$

$$E(N+2, Z) = E(N,Z) + 2\lambda$$
$$E(N+1, Z) = E(N,Z) + \lambda + \Delta$$

$$-\Delta_n \sim [E(N+2,Z) - 2E(N+1,Z) + E(N,Z)]/2$$

Or $$\Delta_n \sim (\Delta_n(N) + \Delta_n(N-1))/2$$



Bohr-Mottelson ('69)

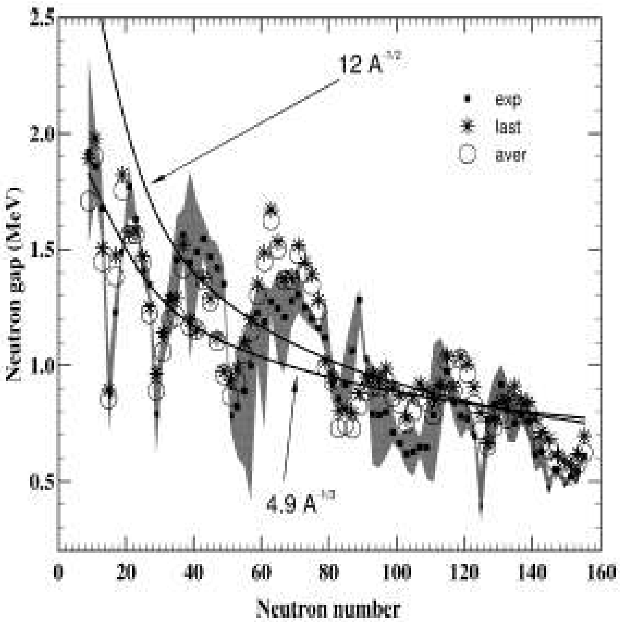
(note) weaker A-dependence?

$$\Delta(N) = \frac{(-)^N}{4}[3B(N-1) - 3B(N) - B(N-2) + B(N+1)]$$

→ $\Delta \sim 12A^{-1/2}$ (Bohr-Mottelson)

$$\Delta(N) = \frac{(-)^N}{2}[B(N-1) - 2B(N) + B(N+1)]$$

→ Weaker A-dependence



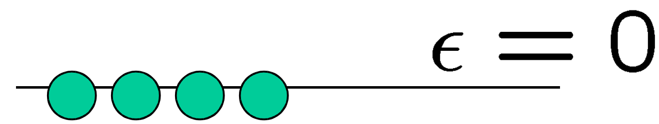
- Isolation of the pairing effect from the deformation effect
- Consistent with Gogny HFB

W. Satula, J. Dobaczewski, and W. Nazarewicz, PRL81('98)3599

S. Hilaire, et al., PLB531('02)61

Seniority Scheme

Particles in a single degenerate level



Degeneracy: 2Ω

$$\begin{aligned} H &= -G\,P^\dagger P; \quad P^\dagger = \sum_{m>0} a^\dagger_m a^\dagger_{-m} \\ &= -G\Omega A^\dagger A; \quad A^\dagger = P^\dagger/\sqrt{\Omega} \end{aligned}$$

- BCS approximation

$$2\Omega\, v^2 = N$$

$$\begin{aligned} v^2 &= N/2\Omega \\ u^2 &= 1 - N/2\Omega \end{aligned}$$

$$\Delta = \Omega\, u\, v = G\Omega\sqrt{\frac{N}{2\Omega}\left(1-\frac{N}{2\Omega}\right)}$$

$$E_{\mathrm{BCS}} = \langle H \rangle = -\Delta^2/G = -\frac{GN\Omega}{2}(1 - N/2\Omega)$$

$$
\begin{aligned}
H &= -G\,P^\dagger P; \qquad P^\dagger = \sum_{m>0} a_m^\dagger a_{-m}^\dagger \\
&= -G\Omega A^\dagger A; \qquad A^\dagger = P^\dagger/\sqrt{\Omega}
\end{aligned}
$$

•Exact solution (Seniority scheme)

(note)

$$[A, A^\dagger] = 1 - \frac{\hat{N}}{\Omega}, \qquad A|0\rangle = 0, \quad \hat{N}|0\rangle = 0$$

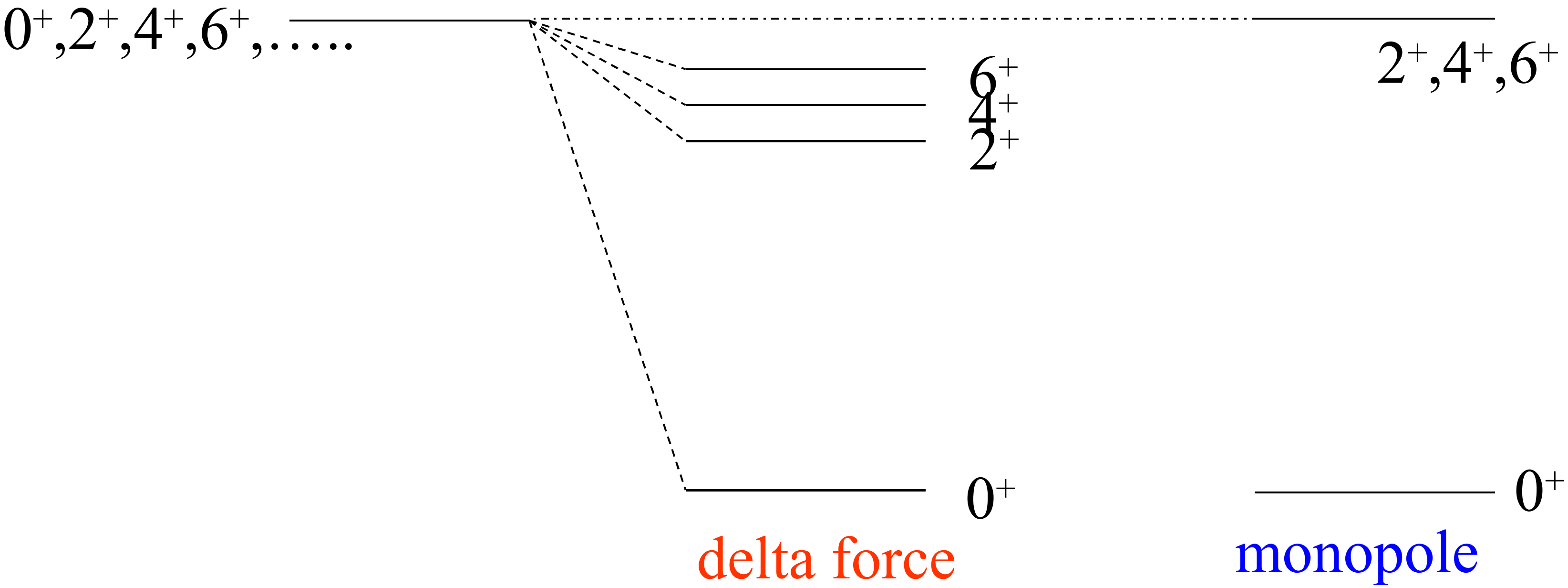
$$
\begin{aligned}
H\,A^\dagger|0\rangle &= -G\Omega\,A^\dagger|0\rangle \\
H(A^\dagger)^2|0\rangle &= -2G(\Omega-1)\,(A^\dagger)^2|0\rangle \\
&\cdots \\
H(A^\dagger)^{N/2}|0\rangle &= -GN/4\cdot(2\Omega - N + 2)\,(A^\dagger)^{N/2}|0\rangle
\end{aligned}
$$

$\Updownarrow$

$$E_{\text{BCS}} = -\frac{GN\Omega}{2}(1 - N/2\Omega)$$

The BCS approximation is good for large N.

BCS approximation with a delta interaction



delta force

$$V = -\sum_{\nu,\nu'>0} G_{\nu\nu'} \times \sum_{\lambda,\mu} (-)^{\mu} [a^\dagger_{\nu} a^\dagger_{\bar{\nu}}]^{(\lambda\mu)} \, [a_{\bar{\nu}'} a_{\nu'}]^{(\lambda-\mu)}$$

$$V = -G\, P^\dagger P$$

$$P^\dagger = \sum_{\nu>0} a^\dagger_{\nu} a^\dagger_{\bar{\nu}}$$

State dependent pairing gap Δ_{ν}

Application to weakly bound nuclei

Divergence problem associated with a contact force

Gap equation in the momentum space:

$$\Delta_{\boldsymbol{k}} = -\sum_{\boldsymbol{k}'} \frac{\langle \boldsymbol{k}|v|\boldsymbol{k}'\rangle}{2} \frac{\Delta_{\boldsymbol{k}'}}{\sqrt{(\epsilon_{\boldsymbol{k}'} - \epsilon_F)^2 - \Delta^2_{\boldsymbol{k}'}}}$$

(note)

$$\langle \boldsymbol{k}|v|\boldsymbol{k}'\rangle = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}\, e^{-i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}}\, v(\boldsymbol{r}) = \frac{g}{(2\pi)^3}$$

for $v(\boldsymbol{r}) = g\,\delta(\boldsymbol{r})$

The gap equation diverges if the whole ***k*** space is included.

Gap equation in a truncated space

$$\int d\boldsymbol{k} \to \int_{\epsilon_k \leq E_C} d\boldsymbol{k}$$

Refs. G.F. Bertsch and H. Esbensen, Ann. of Phys. 209('91)327
H. Esbensen, G.F. Bertsch, K. Hencken, Phys. Rev. C56('97)3054

(note) phase shift for nn scattering with a contact interaction

$$k \cot \delta = -\frac{2}{\alpha\pi}\left[1 + \alpha k_c + \frac{\alpha k}{2} \ln\left(\frac{k_c - k}{k_c + k}\right)\right]$$

$$\alpha = \frac{m\,g}{2\pi^2\hbar^2}$$

Effective range expansion:

$$k \cot \delta \sim -1/a + rk^2/2$$

a: scattering length, r: effective range

$$g = 2\pi^2\frac{\hbar^2}{m}\frac{2a}{\pi - 2k_c a} \sim -2\pi^2\frac{\hbar^2}{mk_c}$$

(note) $a_{nn} = -18.5 \pm 0.5$ (fm)

Gap equation in a truncated space

$$\int d\boldsymbol{k} \to \int_{\epsilon_k \leq E_C} d\boldsymbol{k}$$

Refs. G.F. Bertsch and H. Esbensen,
Ann. of Phys. 209('91)327
H. Esbensen, G.F. Bertsch, K. Hencken,
Phys. Rev. C56('97)3054

$$g = 2\pi^2 \frac{\hbar^2}{m} \frac{2a}{\pi - 2k_c\, a} \sim -2\pi^2 \frac{\hbar^2}{mk_c}$$

(note) another dimensional regularization scheme
Ref. A. Bulgac and Y. Yu, PRL88('02)042504

(note) Application of a density-dependent delta interaction

$$v(\boldsymbol{r}, \boldsymbol{r}') = g \left[1 - \left(\frac{\rho(\bar{\boldsymbol{r}})}{\rho_0}\right)^{\alpha}\right] \delta(\boldsymbol{r} - \boldsymbol{r}')$$

→ In the next lecture (in connection to the Hartree-Fock-Bogoliubov theory)