3. Particle accelerators

3.1 Relativistic particles
3.2 Electrostatic accelerators
3.3 Ring accelerators
    Betatron // Cyclotron // Synchrotron
3.4 Linear accelerators
3.5 Collider
Van-de-Graaf accelerator

- particle source
- accelerating electrodes
- beam
- target

$E = ZeU$

$E_{\text{max}} = 15\ \text{MeV/nucleon}$

Particle Current = 100 $\mu$A

$E = ZeU$
Fig. A.1. Sketch of a tandem Van de Graaff accelerator. Negative ions are accelerated from the left towards the terminal where some of their electrons are stripped off and they become positively charged. This causes them to now be accelerated away from the terminal and the potential difference between the terminal and the tank is traversed for a second time.
RING ACCELERATORS

1) The Betatron

\[ qv \times B_0 = -mv \times \omega \]

\[ \omega_C \text{ Cyclotron Frequency such that: } qB_0 = -mw_c = -mv/r = -|p|/r \]

If we have a time dependent \( B \) field, this induces an electric field that can be used for acceleration.

\[ \nabla \times E = -\frac{\partial B}{\partial t}, \quad \text{Stoke Theorem:} \]

\[ \Rightarrow 2\pi r E = -\frac{d}{dt}(B_0 r^2), \quad r = r_0 \Rightarrow F = \frac{d|p|}{dt} = qE = -\frac{qr_0}{2} \frac{dB}{dt} \]

\[ \Rightarrow \Delta p = -\frac{qr_0}{2} \Delta B \quad \text{and} \quad \Delta|p| = -qr_0 \Delta B_0 \]

\[ \Rightarrow \Delta B = 2\Delta B_0 \]

*The* particle can be accelerated only once until the field reaches its maximum value \( B_0 \).
How it looks like:

- vacuum chamber
- beam
- field coils
- correction coils
- iron yoke

Graph shows the variation of B(t) over time with peaks at 0.01, 0.02, 0.03, and 0.04 seconds.
Axial stability

The magnetic field forces the particle back to the medium plane. The restoring force is provided by the magnetic field gradient.

$E_{\text{max}} = 300 \text{ MeV for } e^-$
2) **Cyclotron**

Constant Magnetic Field, Acceleration happens via a oscillating electric field between the dees $\omega_{HF}=|\omega_Z|$ angular velocity of the particle (10 MHz)

$$m \cdot \frac{v^2}{r} = q \cdot (v \times B) \rightarrow \frac{v}{r} = \omega_Z = \frac{q}{m} B$$

- Independent of the radius!!!
- Maximal Energy does not depend on E!!

$E_{\text{MAX proton}} = 20$ MeV

Schematic from the top
When the particles become relativistic $m \rightarrow m\gamma$ with

Hence the particle becomes heavier and the $\omega_C$ diminishes.

One can overcome this problem reducing the HF frequency while the particle travels (Syncrocyclotron, only possible in bunch mode) or one can increase the magnetic field such that the radius stays constant (Isocyclotron, possible in continuous mode)

$$\omega_z = \frac{q}{m(E)} B(r(E))$$

Not suited for the acceleration of electrons!
One of the first Cyclotrons...
... and a little bit later
The isochrone-cyclotron at PSI
Synchrotron

For relativistic particles (\(v \approx c\)):

\[
\frac{v}{r} = \frac{q}{m} B \rightarrow r = \frac{mv}{qB} = \frac{p}{qB} \approx \frac{E}{qcB}
\]

\[p = eBr \quad \text{bzw.} \quad p(\text{GeV} / c) = 0.3 \cdot B(T) \cdot r(\text{m})\]

The orbit radius increases with the Energy and this can be compensated only by higher magnetic fields. Maximal \(B = 5-10\) T!!!! Moreover big jokes are very expensive.

The new idea is to keep the orbit constant and oblige the particle to run along the circle via dipole magnets. Along the path there are different acceleration gaps such that \(E/B\) stays constant. This means that the magnetic field has to be risen synchronic to the \(E\) field.
Synchrotron

(Wille, Teilchenbeschleuniger)

accelerating gap

injection magnet

ejection magnet

linac

magnet for beam deviation

focusing magnet
Dipole and Quadropole

Dipolmagnet

Quadrupolmagnet

Graue Pfeile: Feldlinien
Blaue Pfeile: Kraft auf Teilchen
Since the quadrupole is focusing the beam in one direction and defocusing in the other, there are placed couple-wise after one another and turned of 90 deg.
Synchrotron Radiation

Since the particles are accelerated on a circular orbit, they radiate energy. For each circle we have the following energy loss:

$$\Delta E_{\text{synchr}} = \frac{e^2}{3\epsilon_0(m_0c^2)} \frac{E^4}{R}$$

Such energy loss is $10^{13}$ times larger for electrons than for protons. Despite of the fact that large radii can reduce such loss this implies a maximal reachable energy for electron of 100 GeV

$$\Delta E_{\text{synch},e}(\text{keV}) = 88,5 \cdot \frac{E^4(\text{GeV}^4)}{R(m)}$$

$$\Delta E_{\text{synch},p}(\text{eV}) = 7,79 \cdot 10^{-9} \frac{E^4(\text{GeV}^4)}{R(m)}$$

The limit in the acceleration of the protons is given by the steering magnets. Furthermore particles have to be pre-accelerated before entering the synchrotron, since the magnets cannot deflect particles with energy close to 0.
Phase Diagram of the Synchrotron

\[ \omega = \frac{|qB_0|}{\gamma m} = \frac{|qB_0c^2|}{E} \]

Stability only for bunches in the orbit!
Lear (CERN)
Proton-Linac

(Wille, Teilchenbeschleuniger)

Good also for electron acceleration!

\[ E_{\text{MAX proton}} = 100 \text{MeV}, \] used as injector for ring accelerators

TESLA: 30 km electron LINAC for 500 GeV electrons
Phase Focusing

A particle that is faster and arrives earlier sees a smaller $V$ and hence will be slowed down in the next cycle. This is again only possible for a BUNCHEd beam.
LIKE THE WAVE PROPELS THE SURFER
ELECTROMAGNETIC WAVES ACCELERATE
PARTICLES

THE USE OF SUPRACONDUCTIVITY TO INCREASE
PERFORMANCES AND CONSIDERABLY REDUCE
ELECTRICITY CONSUMPTION

ACCELERATING
ELECTROMAGNETIC WAVE

LIQUID HELIUM
COOLED TO - 269°C

SUPERCONDUCTING
ACCELERATING CAVITY
MADE OF NIOBium
Linacs at CERN

Largest LINAC at SLAC (Stanford Linear Accelerator Center)

$L = 3\text{km}$, $E_{\text{MAX electron}} = 50\text{ GeV}$
Collider

(Wille, Teilchenbeschleuniger)
1. Energy:
in the cm system in terms of momentum 4-vectors \( s = (p_1 + p_2)^2 \)

fixed target: \[ s = (m_1 c^2)^2 + 2\gamma_1 m_1 c^2 m_2 c^2 + (m_2 c^2)^2 \]
special case of equal masses \[ \sqrt{s} = mc^2 \sqrt{(2+2\gamma)} \]
high energy limit \[ \sqrt{s} = mc^2 \sqrt{2\gamma} \]

colliding beams: \[ s = (m_1 c^2)^2 + (m_2 c^2)^2 + 2\gamma_1 \gamma_2 m_1 c^2 m_2 c^2 (1 + \beta_1 \beta_2) \]
high energy limit \[ s = 4E_1 E_2 \]
special case of equal mass and energy \[ \sqrt{s} = 2E = mc^2 \sqrt{2\gamma} \]

note: linear in \( \gamma \)
Luminosity

2. Luminosity:

**fixed target:** \( \mathcal{L} = N_b[1/s] N_t [1/cm^2] \) beam rate times target thickness

\[ N_t = \rho t N_A/M \quad \text{e.g. for 1m liquid hydrogen} \quad N_t = 2 \times 10^{24}/cm^2 \]

typical for protons \( N_b = 10^{13}/s \quad \rightarrow \quad \mathcal{L} = 2 \times 10^{37}/cm^2/s \)

**colliding beams:** \( \mathcal{L} = f n N_1 N_2 / A \)

- \( f \) frequency
- \( n \) number of bunches in either beam around ring
- \( N_{1,2} \) particles per bunch
- \( A \) cross sectional area of beam

- typical \( e^+ e^- \) collider \( \mathcal{L} = 10^{31}/cm^2/s \)
- ppbar collider \( \mathcal{L} = 10^{30}/cm^2/s \)
- pp collider \( \mathcal{L} = 10^{33}/cm^2/s \)
Large Electron-Positron Collider (LEP) am CERN

Betrieben von 1989-2000
Maximalenergie 100 GeV \( \rightarrow \sqrt{s} = 200 \text{ GeV} \)
Umfang 26.7 km, zwischen 40 und 150 m unter der Erde, 1,4% Neigung
3368 Magnete
272 Beschleunigerkavitäten
4 Kollisionspunkte mit Experimenten
Aleph Detector at CERN LEP

\[ e^+ + e^- \rightarrow Z^0 \rightarrow q + \bar{q} + g \rightarrow \text{Hadrons} \]
Accellatorator Evolution: Fixed target Experiment
Accelarator Evolution: Colliders
<table>
<thead>
<tr>
<th>Accellarator</th>
<th>Energy, GeV</th>
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<td><strong>Proton Synchrotrons</strong></td>
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<td><strong>Colliding-beam machines</strong></td>
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<tr>
<td>PETRA</td>
<td>$e^+e^-$ 22+22</td>
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<tr>
<td>LEP II</td>
<td>$e^+e^-$ 100+100</td>
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<tr>
<td>HERA</td>
<td>ep 30e+820p</td>
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<tr>
<td>LHC</td>
<td>pp 7000+7000</td>
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</tbody>
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Electron Beam Cooling at ESR GSI

Cooling in the ESR counteracts the beam heating effect of the experimental insertions.
Momentum spread due to the thermal motion. Cooling should reduce the spread and hence increase the phase-space density.

**Principle of the stochastic Cooling**

- Measure beam center by pick-ups
- Correction signal to opposite kicker

S. van der Meer included in Nobel prize for W,Z