Decay Constant, Lifetime and Activity

$$-\frac{dN}{dt} = \lambda N$$

$$\Leftrightarrow -\int_{N(0)}^{N} \frac{dN}{\lambda N} = \int_0^t dt$$

$$\Leftrightarrow \ln N(t) - \ln N(0) = -\lambda t$$

$$\Rightarrow N(t) = N(0)e^{-\lambda t}$$

$$T = 1/\lambda$$

$$\Rightarrow N(t) = N(0)e^{-t/\tau}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \cdot \tau$$

λ = decay constant, probability pro time unit that the decay will happen

$T>4.5 \times 10^{9}$ years = stable nuclei

$$A(t) = \lambda \cdot N(t) \qquad 1\,Bq = 1\,s^{-1}$$

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3$$

$$\dot{N}_1 = -\lambda_1 \cdot N_1$$

$$\dot{N}_2 = -\lambda_2 \cdot N_2 + \lambda_1 \cdot N_1$$

$$N_1 = C_{11} e^{-\lambda_1 t}$$

$$N_2 = C_{21} e^{-\lambda_1 t} N_1 + C_{22} e^{-\lambda_2 t}$$

$$C_{11} = N_1(0) \Rightarrow N_1(t) = N_1(0) e^{-\lambda_1 t}$$

$$N_2(0) = 0 \Rightarrow C_{21} = -C_{22}$$

$$\Rightarrow N_2(t) = -C_{22} e^{-\lambda_1 t} N_1 + C_{22} e^{-\lambda_2 t}$$

$$\Rightarrow$$

$$\lambda_1 C_{22} e^{-\lambda_1 t} - \lambda_2 C_{22} e^{-\lambda_2 t}$$

$$= \lambda_2 C_{22} e^{-\lambda_1 t} - \lambda_2 C_{22} e^{-\lambda_2 t} + \lambda_1 N_1(0) e^{-\lambda_1 t}$$

$$C_{22} = -\frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)$$

$$N_1(t) = N_1(0) e^{-\lambda_1 t}$$

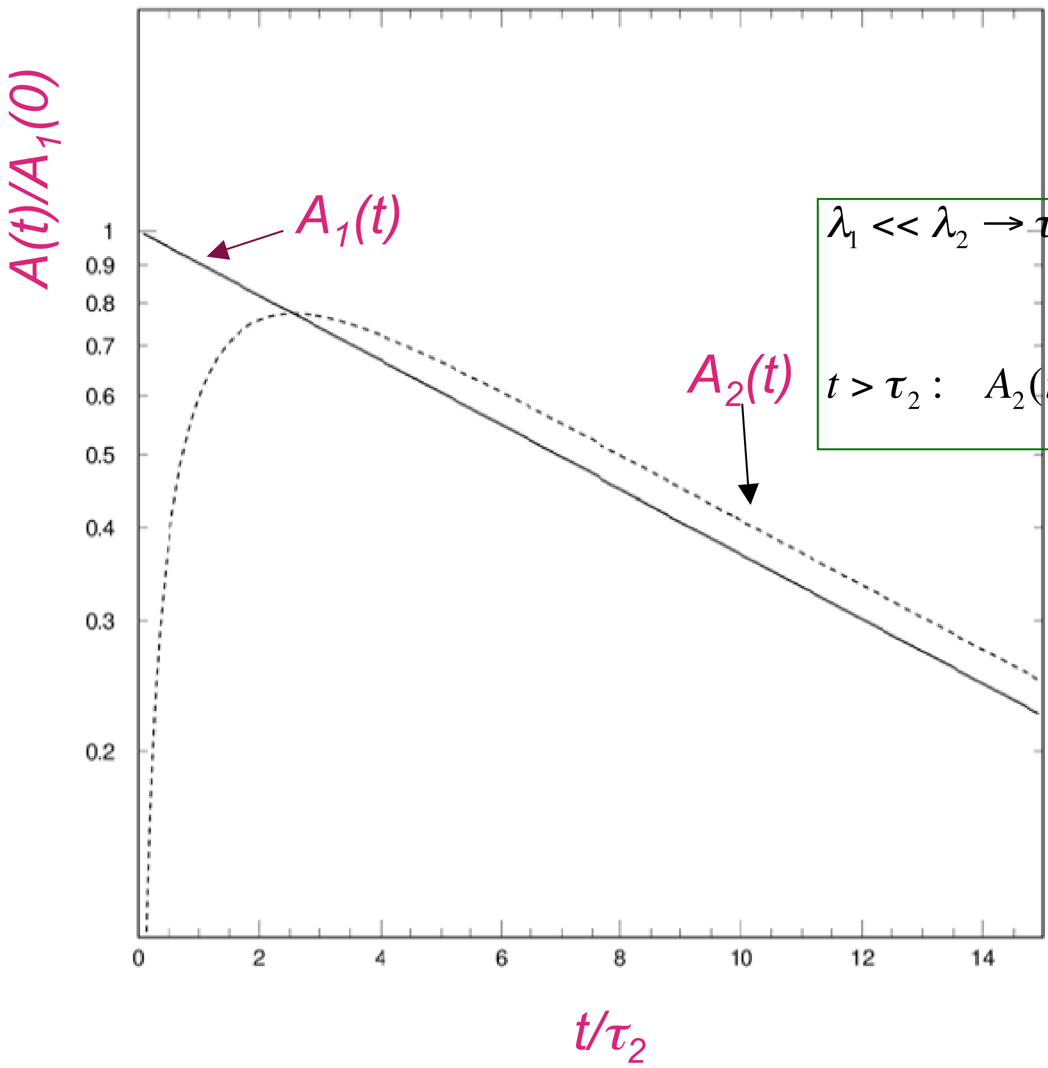
$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0) \cdot \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$A_1 = \lambda_1 N_1 \quad A_2 = \lambda_2 N_2$$

$$\Rightarrow A_1(t) = \lambda_1 N_1(0) \cdot e^{-\lambda_1 t} = A_1(0) \cdot e^{-\lambda_1 t}$$

$$A_2(t) = A_1(0) \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$= A_1(t) \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot \left(1 - e^{-(\lambda_2 - \lambda_1)t}\right)$$



$$\lambda_1 << \lambda_2 \rightarrow \tau_1 >> \tau_2$$

$$t > \tau_2 : \quad A_2(t) \rightarrow A_1(t) \cdot \frac{\lambda_2}{\lambda_2 - \lambda_1} \approx A_1(t)$$

If a nucleus has a life time much larger than its daughter, all daugther nuclei have the same activity as the mother nucleus. *Radioactive Equilibrium*

Decay with instable daughter

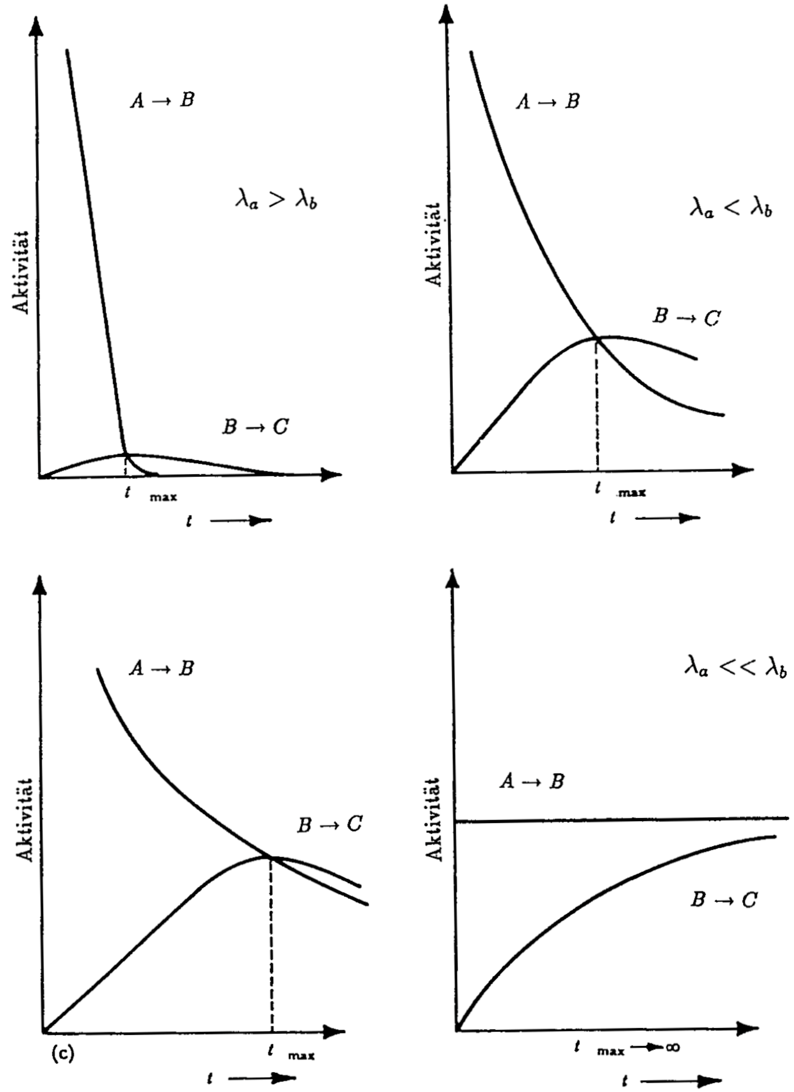
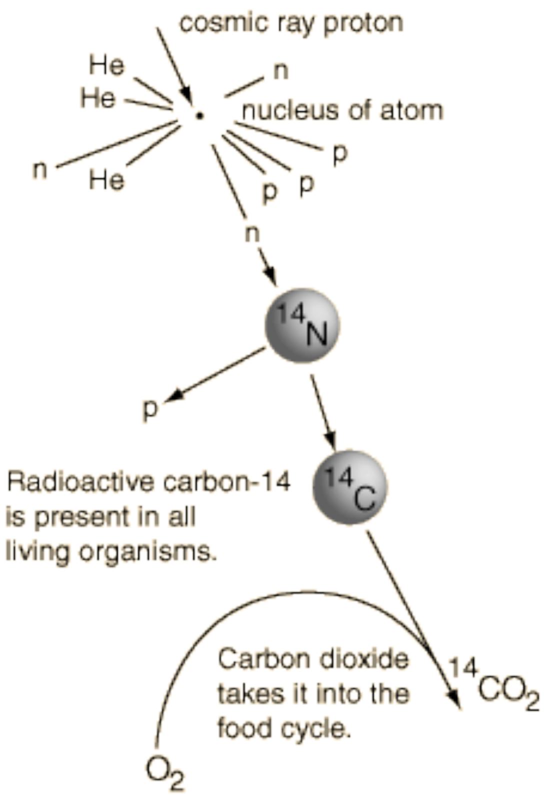


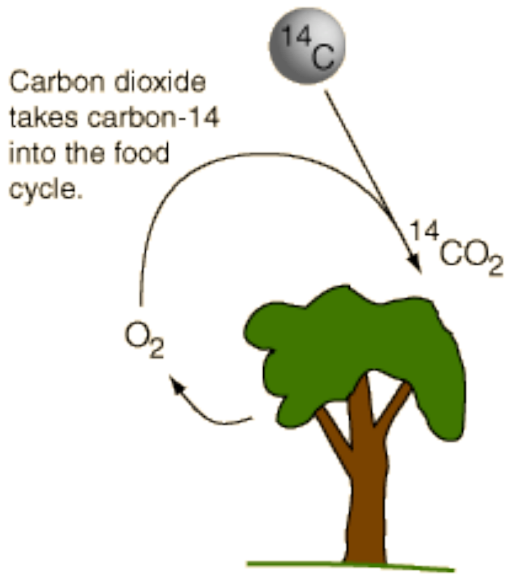
Abb. 1.32: Häufigkeit der Prozesse in einem Zerfall mit Tochter-Aktivität [aus P. Marmier et al. [40]]

Two nuclear decays:

$$A \xrightarrow{\lambda_a} B \xrightarrow{\lambda_b} C$$

Radioactive Dating

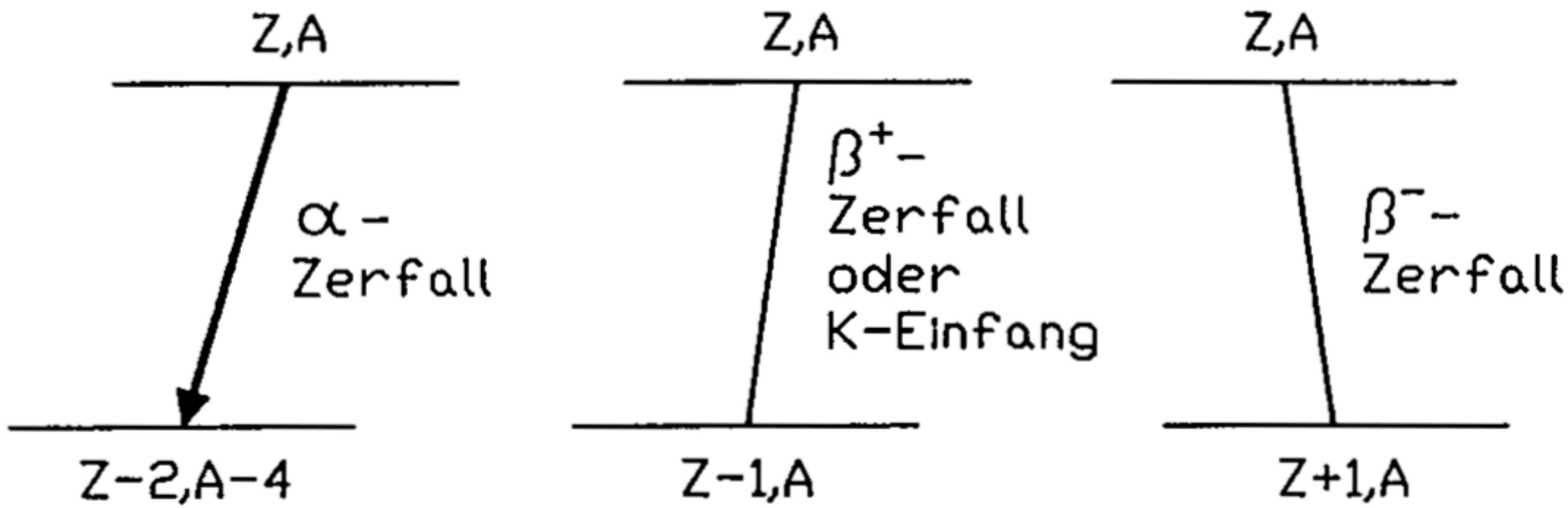




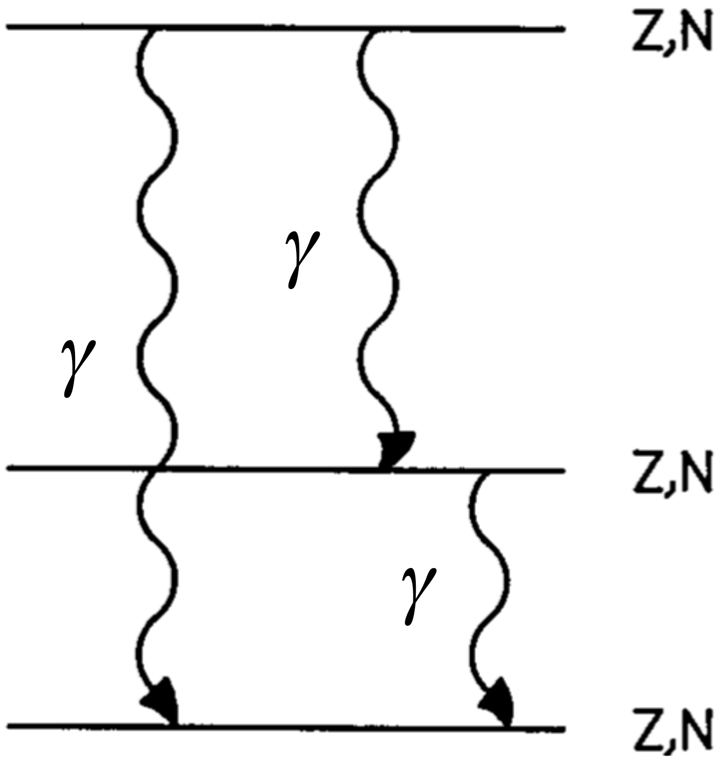
$$^{14}C \dashrightarrow {}^{14}N + e^- + \bar{\nu}$$

Measuring the β^- activity of the probe one can deduce the age

α, β^+ and β^- decay



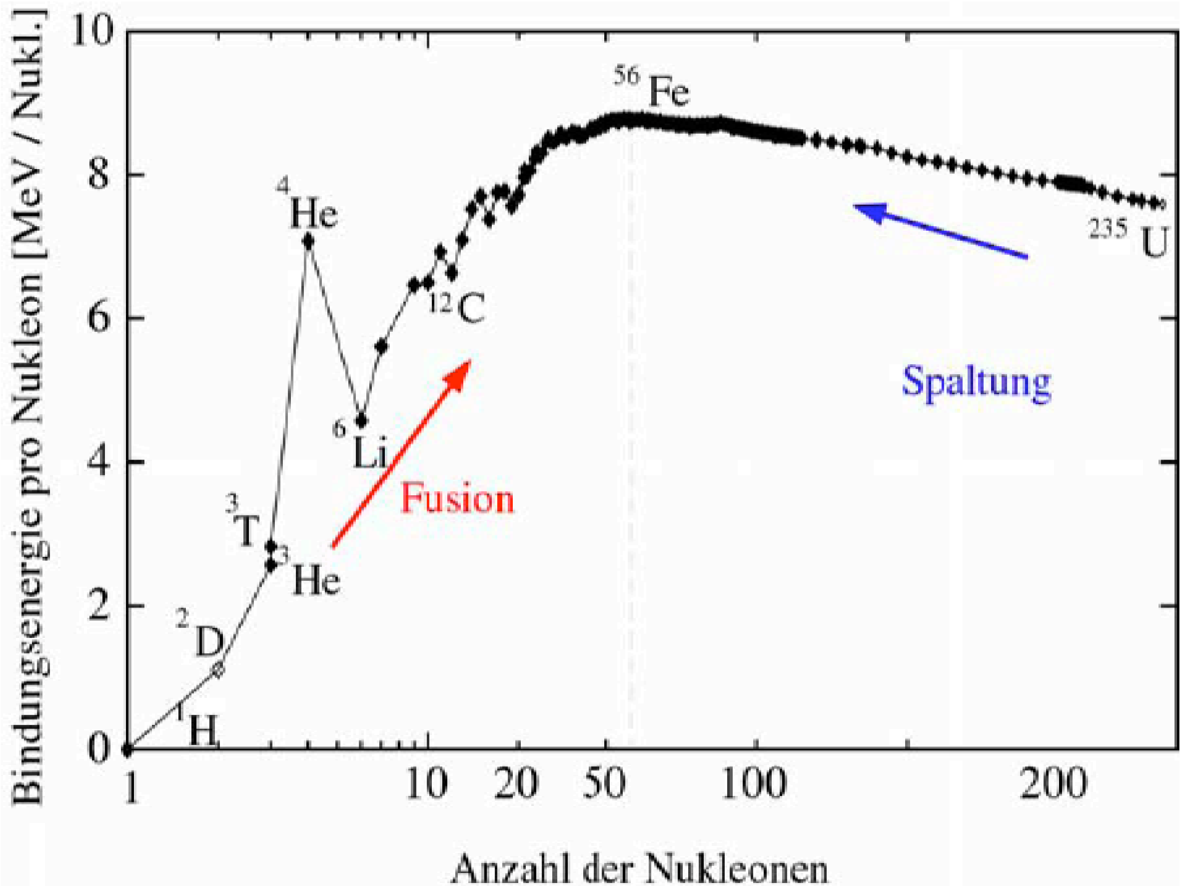
electromagnetic (γ) decay



Decay series of heavy elements

Name of series	*Type*	*Final stable nucleus*	*Longest lived nucleus*	*Half-life (a)*
Thorium	*4n*	^{208}Pb	^{232}Th	1.41×10^{10}
Neptunium	*4n+1*	^{209}Bi	^{237}Np	2.14×10^{6}
Uranium	*4n+2*	^{206}Pb	^{238}U	4.47×10^{9}
Actinium	*4n+3*	^{207}Pb	^{235}U	7.04×10^{8}

α Decay

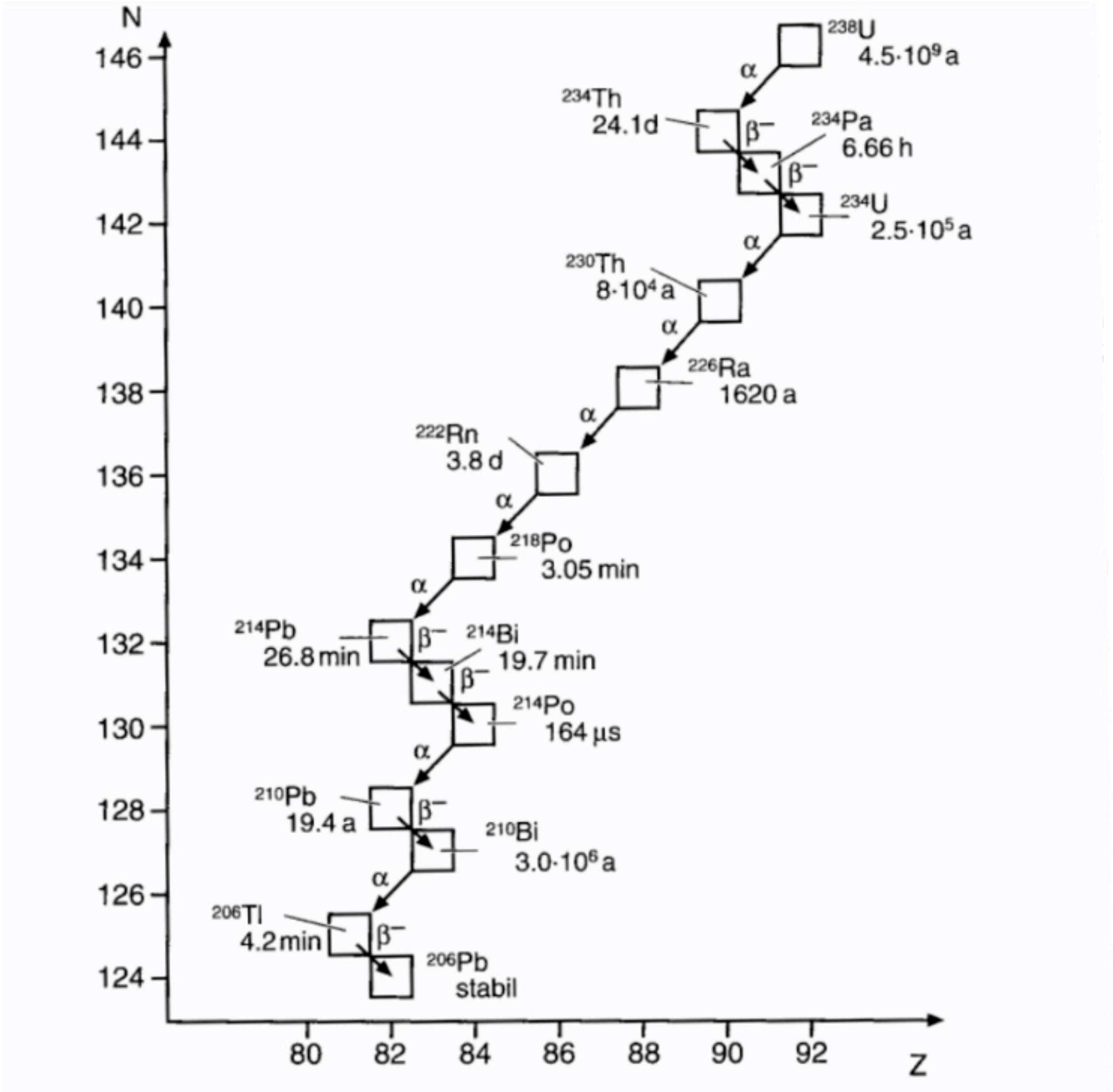


The system ^4_2He has a particular high binding energy

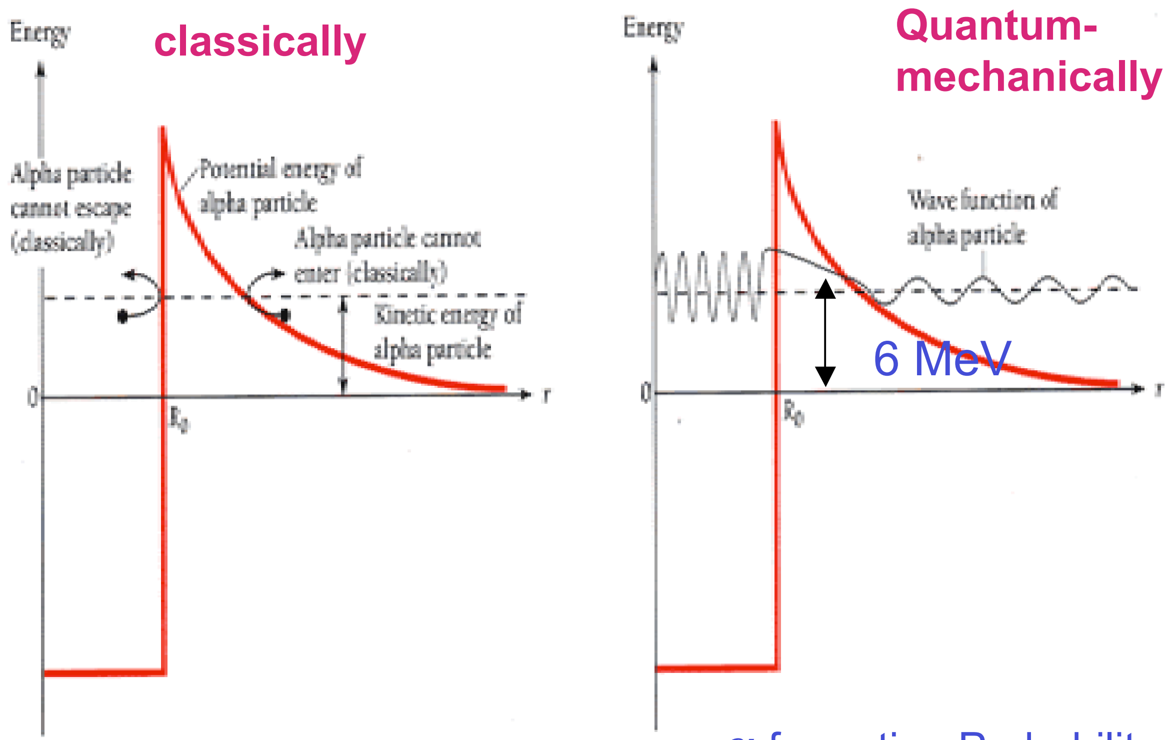
Energy Condition for the α decay

$$E_\alpha = [M(A,Z) - M(A-4,Z-2) - M_\alpha] \cdot c^2 > 0$$

Decay chain of Uranium-238



Barrier penetration in α-decay



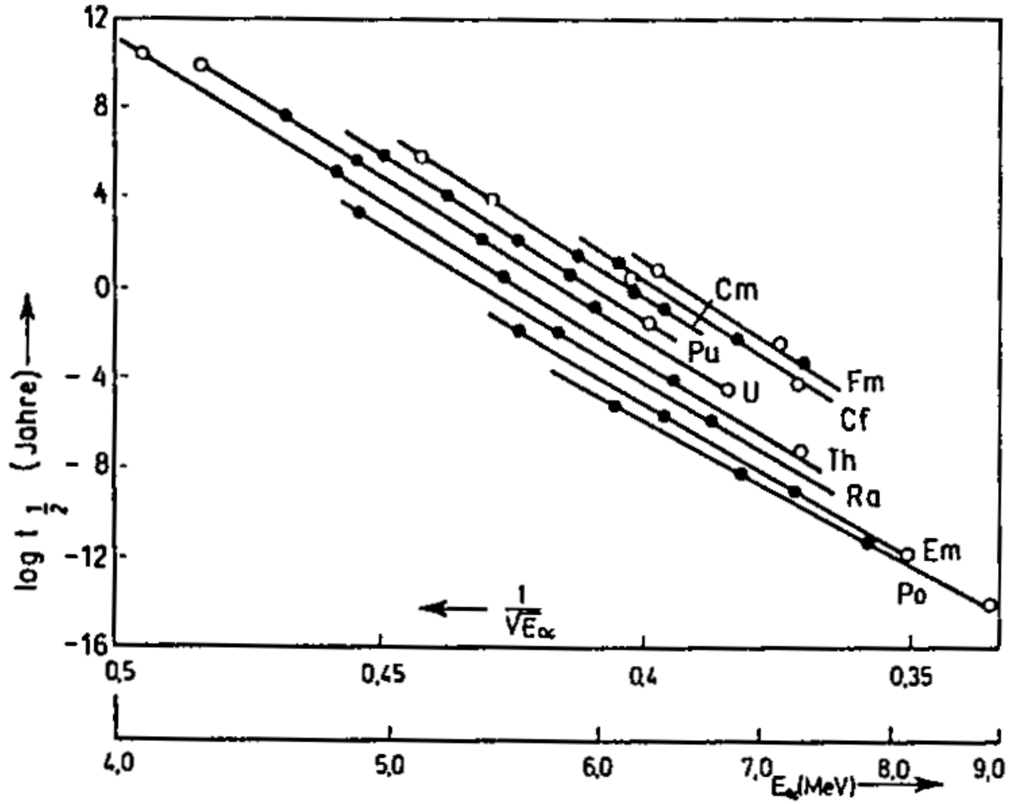
α formation Probability

$$\lambda_\alpha = w(\alpha)\ H\ T$$

Frequency of the Barrier collision

Transmission through the potential barrier

Geiger-Nuttal observation in α-decay



$$T = e^{-2G}$$

Gamov Factor G

$$G = \frac{1}{\hbar}\int_R^{r_1} \sqrt{2m|E - V(r)|}\,dr \approx \frac{\pi \cdot 2 \cdot (Z-2) \cdot \alpha}{\beta}$$

$$\beta = \frac{v}{c} = \sqrt{\frac{2E_\alpha}{m_\alpha c^2}}$$

$$t_{\frac{1}{2}} \sim \frac{1}{\lambda} \sim \frac{1}{T} \sim\sim e^{2G} \qquad \log(t_{\frac{1}{2}}) \sim G \sim \frac{1}{\sqrt{E_\alpha}}$$

Geiger-Nuttallsche

Abb. 1.43: Die Geiger Nuttalsche Beobachtung [aus T. Mayer-Kuckuk [38] nach: C.J. Gallagher et al. [39]]

The fission barrier

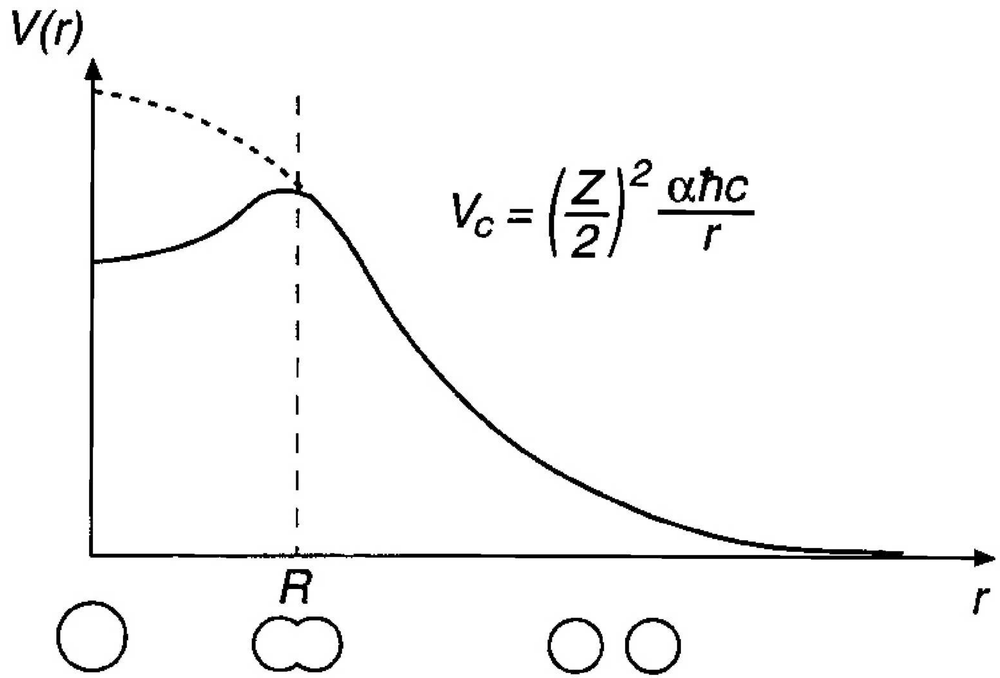


Fig. 3.8. Potential energy during different stages of a fission reaction. A nucleus with charge Z decays spontaneously into two daughter nuclei. The solid line corresponds to the shape of the potential in the parent nucleus. The height of the barrier for fission determines the probability of spontaneous fission. The fission barrier disappears for nuclei with $Z^2/A \gtrsim 48$ and the shape of the potential then corresponds to the dashed line.

Deformation of a heavy nucleus

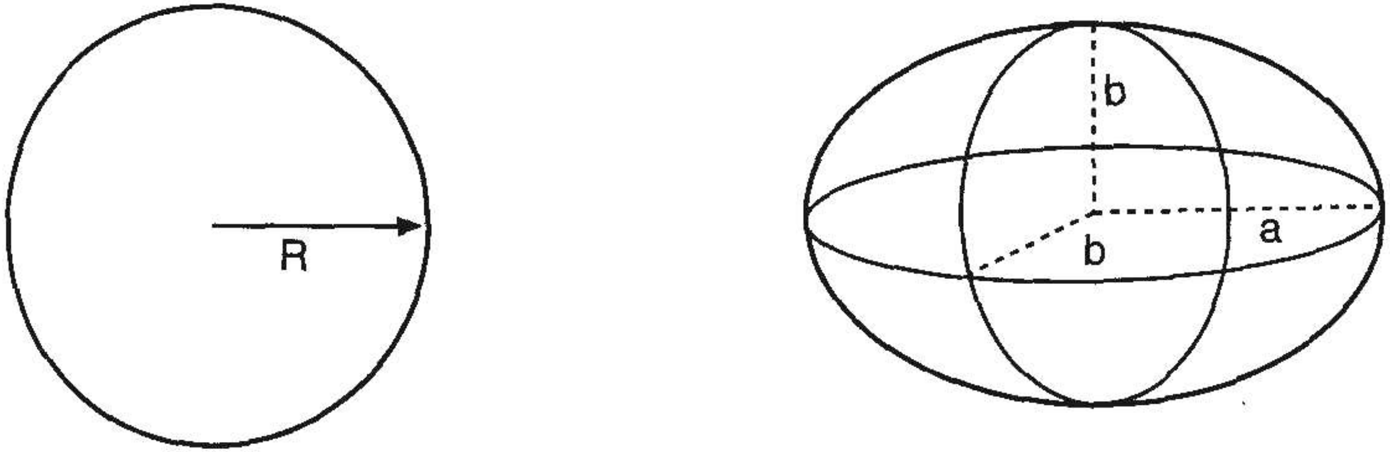


Fig. 3.9. Deformation of a heavy nucleus. For a constant volume V ($V = 4\pi R^3/3 = 4\pi ab^2/3$), the surface energy of the nucleus increases and its Coulomb energy decreases.