Decay Constant, Lifetime and Activity

\[- \frac{dN}{dt} = \lambda N \]

\( \Rightarrow - \int_{N(0)}^{N} \frac{dN}{\lambda N} = \int_{0}^{t} dt \)

\( \Rightarrow \ln N(t) - \ln N(0) = -\lambda t \)

\( \Rightarrow N(t) = N(0)e^{-\lambda t} \)

\( T = 1/\lambda \)

\( \Rightarrow N(t) = N(0)e^{-t/\tau} \)

\( t_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \cdot \tau \)

\( \lambda = \) decay constant, probability per time unit that the decay will happen

\( T > 4.5 \times 10^9 \) years = stable nuclei
\[ A(t) = \lambda \cdot N(t) \quad \text{1 Bq} = 1 s^{-1} \]

\[ N_1 \rightarrow N_2 \rightarrow N_3 \]

\[ N_1 = -\lambda_1 \cdot N_1 \]
\[ N_2 = -\lambda_2 \cdot N_2 + \lambda_1 \cdot N_1 \]

\[ N_1 = C_{11} e^{-\lambda_1 t} \]
\[ N_2 = C_{21} e^{-\lambda_1 t} N_1 + C_{22} e^{-\lambda_2 t} \]

\[ C_{11} = N_1(0) \Rightarrow N_1(t) = N_1(0)e^{-\lambda_1 t} \]
\[ N_2(0) = 0 \Rightarrow C_{21} = -C_{22} \]

\[ \Rightarrow N_2(t) = -C_{22} e^{-\lambda_1 t} N_1 + C_{22} e^{-\lambda_2 t} \]

\[ \Rightarrow \lambda_1 C_{22} e^{-\lambda_1 t} - \lambda_2 C_{22} e^{-\lambda_2 t} \]
\[ = \lambda_2 C_{22} e^{-\lambda_1 t} - \lambda_2 C_{22} e^{-\lambda_2 t} + \lambda_1 N_1(0)e^{-\lambda_1 t} \]

\[ C_{22} = -\frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0) \]

\[ N_1(t) = N_1(0)e^{-\lambda_1 t} \]

\[ N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0) \cdot \left( e^{-\lambda_1 t} - e^{-\lambda_2 t} \right) \]

\[ A_1 = \lambda_1 N_1 \quad A_2 = \lambda_2 N_2 \]

\[ \Rightarrow A_1(t) = \lambda_1 N_1(0) \cdot e^{-\lambda_1 t} = A_1(0) \cdot e^{-\lambda_1 t} \]

\[ A_2(t) = A_1(0) \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot \left( e^{-\lambda_1 t} - e^{-\lambda_2 t} \right) \]

\[ = A_1(t) \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot \left( 1 - e^{-(\lambda_2 - \lambda_1)t} \right) \]
If a nucleus has a life time much larger than its daughter, all daughter nuclei have the same activity as the mother nucleus. *Radioactive Equilibrium*
Decay with instable daughter

Two nuclear decays:

\[ A \rightarrow B \rightarrow C \]

\[ \lambda_a > \lambda_b \]

\[ \lambda_a < \lambda_b \]
Radioactive Dating

Radioactive carbon-14 is present in all living organisms.

\[ ^{14}\text{C} \rightarrow ^{14}\text{N} + e^{-} + \nu \]

Measuring the $\beta^-$ activity of the probe one can deduce the age
\( \alpha, \beta^+ \text{ and } \beta^- \text{ decay} \)
electromagnetic ($\gamma$) decay
## Decay series of heavy elements

<table>
<thead>
<tr>
<th>Name of series</th>
<th>Type</th>
<th>Final stable nucleus</th>
<th>Longest lived nucleus</th>
<th>Half-life (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thorium</td>
<td>4n</td>
<td>$^{208}\text{Pb}$</td>
<td>$^{232}\text{Th}$</td>
<td>$1.41 \times 10^{10}$</td>
</tr>
<tr>
<td>Neptunium</td>
<td>4n+1</td>
<td>$^{209}\text{Bi}$</td>
<td>$^{237}\text{Np}$</td>
<td>$2.14 \times 10^{6}$</td>
</tr>
<tr>
<td>Uranium</td>
<td>4n+2</td>
<td>$^{206}\text{Pb}$</td>
<td>$^{238}\text{U}$</td>
<td>$4.47 \times 10^{9}$</td>
</tr>
<tr>
<td>Actinium</td>
<td>4n+3</td>
<td>$^{207}\text{Pb}$</td>
<td>$^{235}\text{U}$</td>
<td>$7.04 \times 10^{8}$</td>
</tr>
</tbody>
</table>
The system $^4_2\text{He}$ has a particular high binding energy.

**Energy Condition for the $\alpha$ decay**

$$E_\alpha = [M(A,Z) - M(A - 4,Z - 2) - M_\alpha] \cdot c^2 > 0$$
Decay chain of Uranium-238
Barrier penetration in $\alpha$-decay

$\lambda_\alpha = w(\alpha) H T$

Frequency of the Barrier collision

$\alpha$ formation Probability

Transmission through the potential barrier

Quantum-mechanically

6 MeV
Geiger-Nuttal observation in α-decay

\[ T = e^{-2G} \]

Gamov Factor G

\[
G = \frac{1}{\hbar} \int_{R}^{r_{1}} \sqrt{2m(E-V(r))} dr \approx \frac{\pi \cdot 2 \cdot (Z-2) \cdot \alpha}{\beta}
\]

\[
\beta = \frac{v}{c} = \sqrt{\frac{2E_{\alpha}}{mc^{2}}}
\]

\[
t_{\frac{1}{2}} \sim \frac{1}{\lambda} \sim \frac{1}{T} \sim e^{2G} \quad \log(t_{\frac{1}{2}}) \sim G \sim \frac{1}{\sqrt{E_{\alpha}}}
\]

Abb. 1.43: Die Geiger Nuttalsche Beobachtung [aus T. Mayer-Kuckuk [38] nach: C.J. Gallagher et al. [39]]
The fission barrier

Fig. 3.8. Potential energy during different stages of a fission reaction. A nucleus with charge $Z$ decays spontaneously into two daughter nuclei. The solid line corresponds to the shape of the potential in the parent nucleus. The height of the barrier for fission determines the probability of spontaneous fission. The fission barrier disappears for nuclei with $Z^2/A \gtrsim 48$ and the shape of the potential then corresponds to the dashed line.
Deformation of a heavy nucleus

Fig. 3.9. Deformation of a heavy nucleus. For a constant volume $V (V = \frac{4\pi R^3}{3} = \frac{4\pi ab^2}{3})$, the surface energy of the nucleus increases and its Coulomb energy decreases.