The fission barrier

Fig. 3.8. Potential energy during different stages of a fission reaction. A nucleus with charge $Z$ decays spontaneously into two daughter nuclei. The solid line corresponds to the shape of the potential in the parent nucleus. The height of the barrier for fission determines the probability of spontaneous fission. The fission barrier disappears for nuclei with $Z^2/A \geq 48$ and the shape of the potential then corresponds to the dashed line.
Deformation of a heavy nucleus

![Deformation of a heavy nucleus](image)

Fig. 3.9. Deformation of a heavy nucleus. For a constant volume \( V = 4\pi R^3 / 3 = 4\pi ab^2 / 3 \), the surface energy of the nucleus increases and its Coulomb energy decreases.

\[
B = a_V A - a_S A^3 - a_C Z(Z - 1) A^{-\frac{1}{3}} - a_{sym} \frac{(A - 2Z)^2}{A} + \delta
\]

\[
\frac{4}{3} \pi R^3 = \frac{4}{3} \pi ab^2 \quad a = R(1 + \varepsilon), \quad b = R(1 + \varepsilon)^{-1}
\]

\[
S = 4\pi R^2 (1 + \frac{2}{5} \varepsilon^2 + \ldots)
\]

\[
E_C = a_C Z^2 A^{-\frac{1}{3}} \left(1 - \frac{1}{2} \varepsilon^2 + \ldots\right)
\]

\[
E_S = a_S A^\frac{2}{3} \left(1 + \frac{2}{5} \varepsilon^2 + \ldots\right)
\]

\[
\Delta E = BE(\varepsilon) - BE(0) = \frac{\varepsilon^2}{5} \left(a_c Z^2 A^{-\frac{1}{3}} - 2a_s A^\frac{2}{3}\right)
\]

\[
\Delta E > 0 \iff \frac{Z^2}{A} > 47
\]
Fission of $^{235}\text{U}$

Energy spectrum of prompt neutrons in $^{235}\text{U}$ fission

$^{235}\text{U} + n \rightarrow ^{93}\text{Rb} + ^{141}\text{Cs} + 2n$

**Thermal Neutron Fission of U-235**

BE($^{235}\text{U}+n$) = 6.4 MeV Fission Barrier = 6.3 MeV

--> slow neutrons (E~25 meV)

BE($^{238}\text{U}+n$) = 4.8 MeV Fission Barrier = 5.8 MeV

--> fast neutrons (E>1.5 MeV)
Neutron cross sections of uranium

**Fig. 124** Übersicht über die Wirkungsquerschnitte bei Reaktionen von Neutronen mit Uran. In den Bereichen dicht liegender Resonanzen können die Strukturen in der Zeichnung nicht wiedergegeben werden. Es ist daher nur die Einhüllende der Resonanzmaxima- und -minima eingezeichnet (gestrichelt). Ein Detail ist in Figur 107 wiedergegeben.
The moderator slows down the neutron from MeV energy to thermal energies. Moreover it absorbs the kinetic energy of the emitted fission fragments.

\[ \eta = \frac{\text{fission neutrons}}{\text{absorbed neutrons}} = \nu \frac{\sigma_f}{\sigma_f + \sigma_r} > 1 \]

\[ \nu = \langle \text{number} \rangle \text{ neutrons pro fission} \]

\[ \sigma_f = \text{fission cross} - \text{section} \]

\[ \sigma_r = \text{neutron reaction cross} - \text{section} \]
$^{235}\text{U} (n, f) \rightarrow ^{239}\text{Pu}$

$fate$ of a neutron generation

Neutron must be moderated outside the core!!!
Reactor Controlling

\[ K_{\text{eff}} = \text{effective multiplication} \]

\[ K_{\text{eff}} > 1 \] to start the reactor

\[ K_{\text{eff}} = 1 \] to keep it stationary

\[
\frac{dp}{dt} = \frac{k_{\text{eff}} \rho - \rho}{t_0}
\]

\( \rho = \text{neutron number} \)

\( t_0 = \text{time between two neutron productions} \)

\[ \Rightarrow \rho(t) = \rho_0 e^{\frac{t}{\tau}} \]

\[ \tau = \frac{t_0}{k_{\text{eff}} - 1} \]

\( k_{\text{eff}} = 1.007 \) \( t_0 = 1ms \Rightarrow \tau \approx 0.1s \)

Delayed neutron emission

\[ ^{235}\text{U} + n \rightarrow ^{93}\text{Rb} + ^{141}\text{Cs} + 2n \]

These neutrons slow down the reactor period
Moderator:

$H_2O$: $p+n \rightarrow d$ ($\sigma_{\text{absorption}} = 33.3\text{ mb}$)

$D_2O$: $d+n \rightarrow t$ $\rightarrow$ ($\sigma_{\text{absorption}} = 0.5\text{ mb}$) lower neutron absorption

Graphite: difficult to stop Overheating (Cernobyl)

To compensate for the $n$ absorption in $H_2O$:

Natural Abundance $^{235}U$: 0.7%

Enriched $A.\;^{235}U \sim 4\%$
Breeding reactions

Fig. 127 Spalt-Brut-Ketten für den schnellen Brüter und den Thörium-Brüter
Weizsäcker parabola for even-odd isobars

Fig. 3.2. Mass parabola of the $A = 101$ isobars (from [Se77]). Possible $\beta$-decays are shown by arrows. The abscissa co-ordinate is the atomic number, $Z$. The zero point of the mass scale was chosen arbitrarily.
Weizsäcker parabolas for even-even and odd-odd isobars

Fig. 3.3. Mass parabolas of the $A = 106$-isobars (from [Se77]). Possible $\beta$-decays are indicated by arrows. The abscissa coordinate is the charge number $Z$. The zero point of the mass scale was chosen arbitrarily.
Beta decay of $^{40}$K

\[ t_{1/2} = 1.27 \times 10^9 \text{a} \]

- $\beta^-$ (89%)
- EC (11%)
- $\beta^+$ (0.001%)

$^{40}\text{Ar}$ → $^{40}\text{Ca}$
Modification of the beta spectrum by Coulomb interaction of $\beta^\pm$ and daughter nucleus

**Fig. 17.17.** Schematic appearance of the electron spectrum in $\beta$-decay. The phase space factor from (15.45) produces a spectrum with a parabolic fall off at both ends (dotted line). This is modified by the interaction of the electron/positron with the Coulomb field of the final state nucleus (continuous lines). These latter curves were calculated from (17.49) for $Z' = 20$ and $E_0 = 1\text{ MeV}$. 
Neutrino Detection

**Indirect method:** EC in $^{37}$Ar:

$$^{37}\text{Ar} + e^- \rightarrow ^{37}\text{Cl} + \nu_e + \Delta m (= 0.814 \text{ MeV/c}^2)$$

Monoenergetic neutrino and hence a fixed recoil momentum for $^{37}$Cl that one can measure

**Direct method:**

1) neutrino source? (Reactor) 1953: $\bar{\nu} + p \rightarrow e^+ + n$

2) how to detect? 200l water with Cadmiucloride, g detected by Scintillation Detector
SuperKamiokande

50000 Tonnen Wasser
11146 Photomultiplier
The interaction of the neutrino with the proton in the water generates either a myon or an electron and these will be detected and distinguished via their Cerenkov-Radiation.

\[ \cos \theta = \frac{1}{\beta n} \]

\[ \beta = \frac{p}{E} \]

Mass \( \mu = 105,658 \text{ MeV/c}^2 \)

Mass \( e^- = 0.511 \text{ MeV/c}^2 \)
Neutrino Oscillations

The basics idea is to find out if the atmospheric neutrinos do change their flavours on their way to the earth (about 13000 km)

\[ \nu_e + p \rightarrow e^+ + n \]

\[ \nu_\mu + p \rightarrow \mu^+ + n \]
Munich reactor