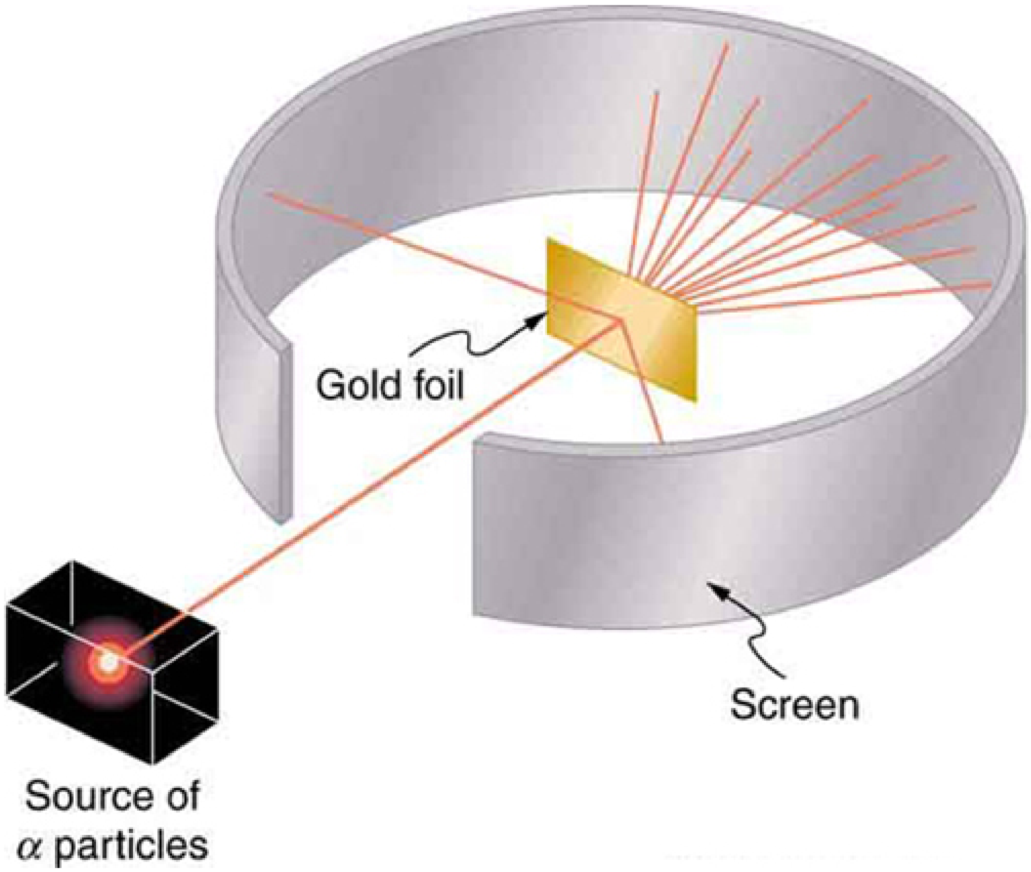
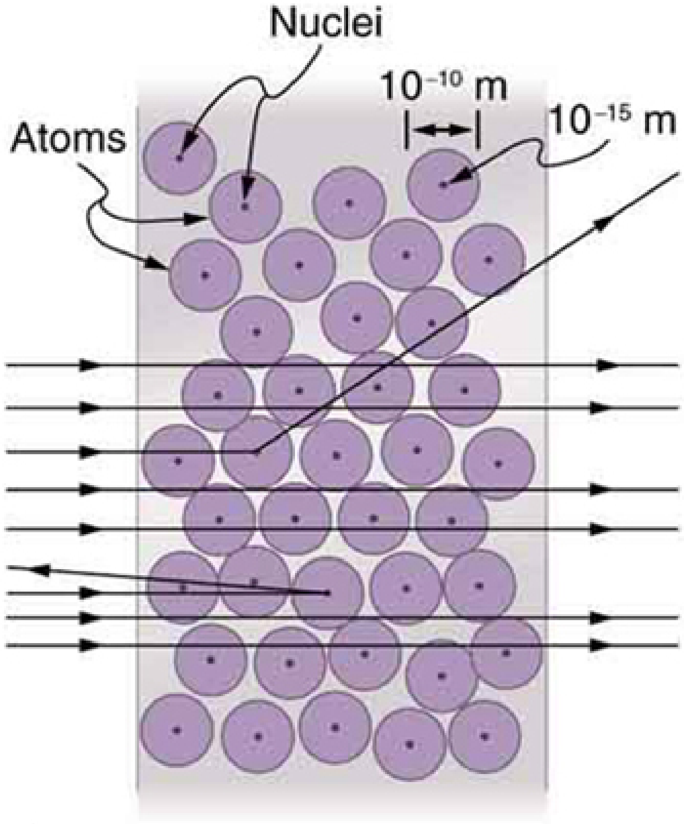
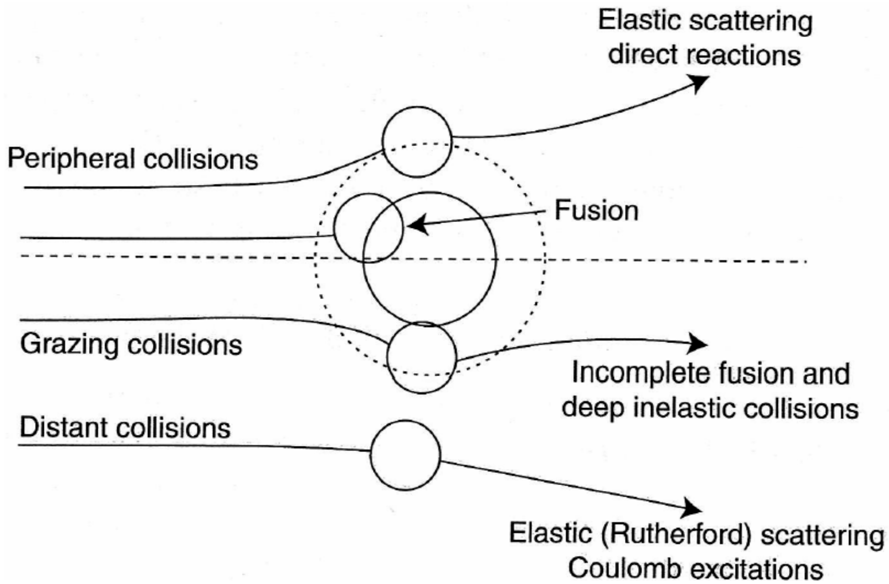
Elastic Scattering

Hans-Jürgen Wollersheim



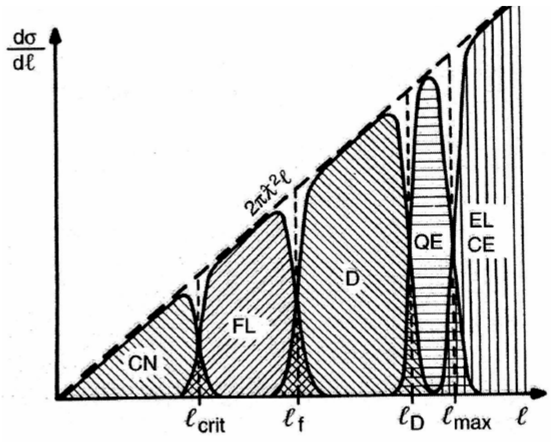


Classification of heavy ion collisions



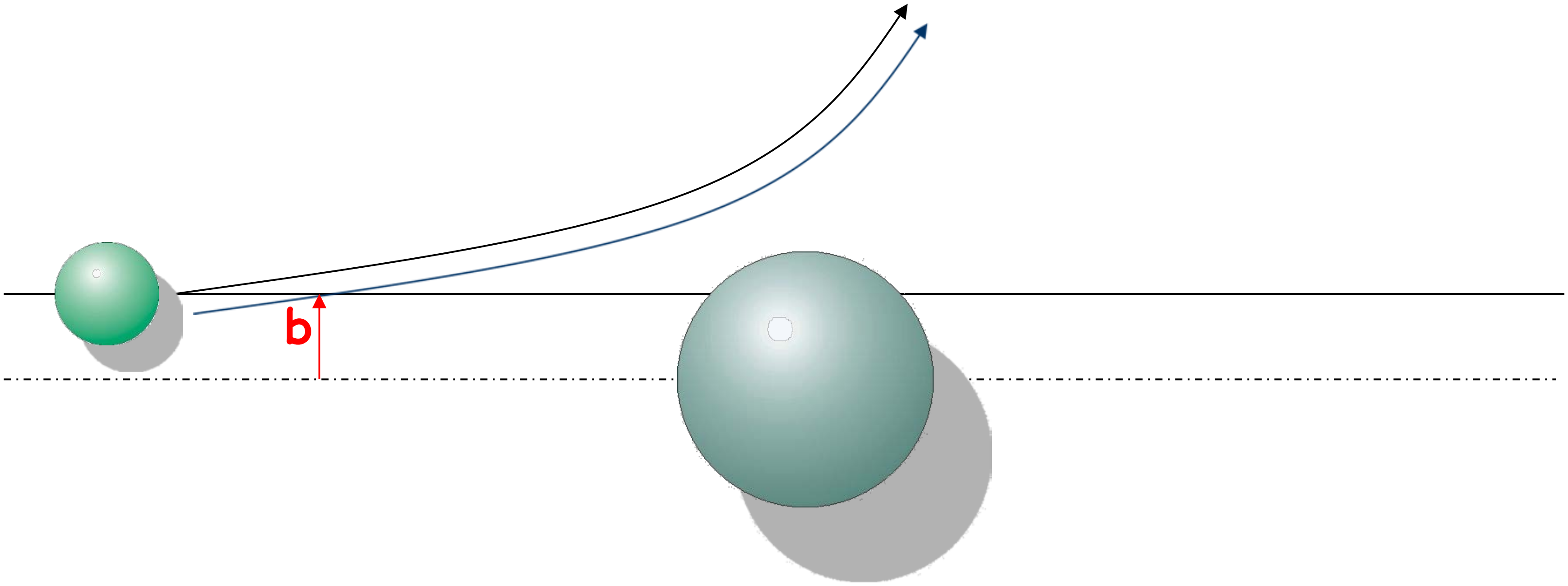
- elastic scattering
- Fresnel & Fraunhofer scattering
- scattering parameters
- differential cross section
- optical model analysis
- nuclear radius
- total reaction cross section
- cross sections at high energy
- influence of nuclear structure

partial cross section vs. angular momentum

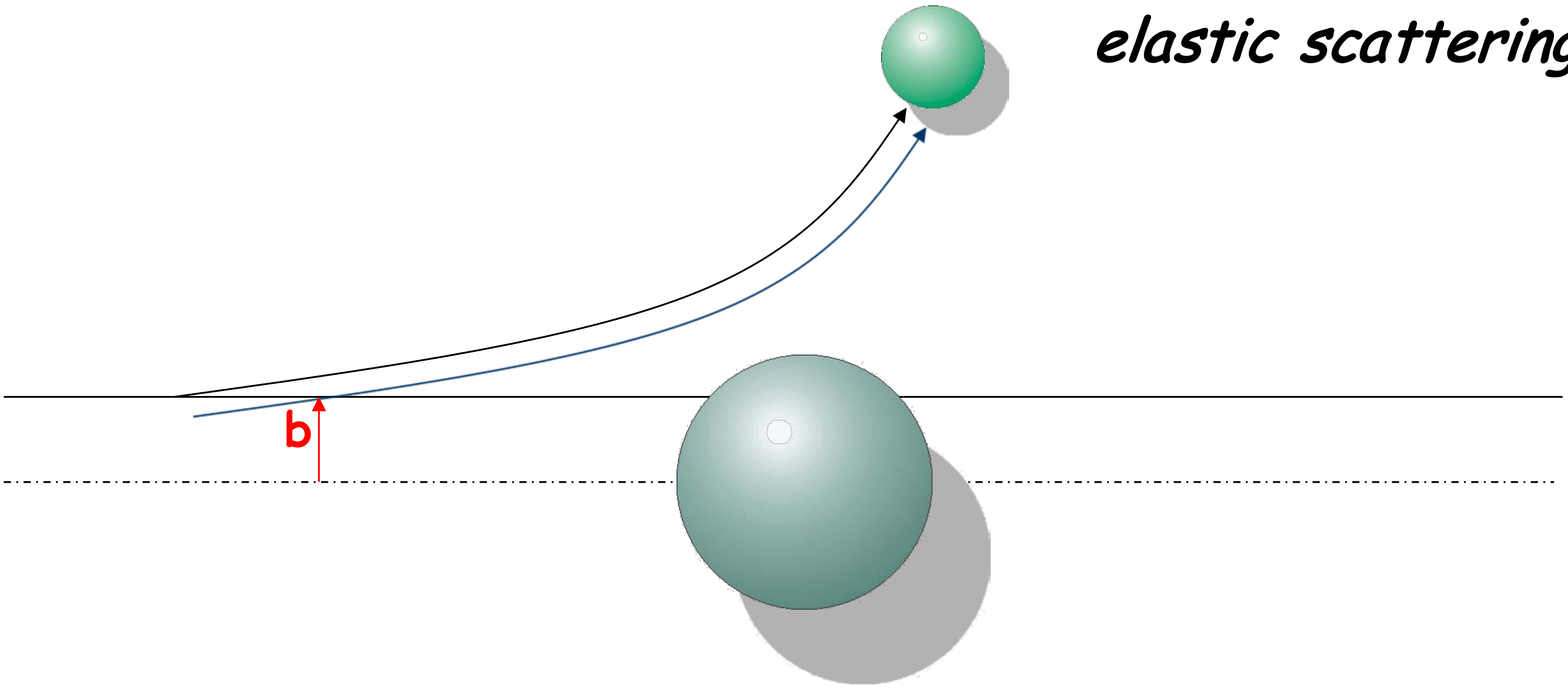


- CN: compound nucleus
- FL: fusion-like
- D: deep inelastic
- QE: quasi elastic
- CE: Coulomb excitation
- EL: elastic

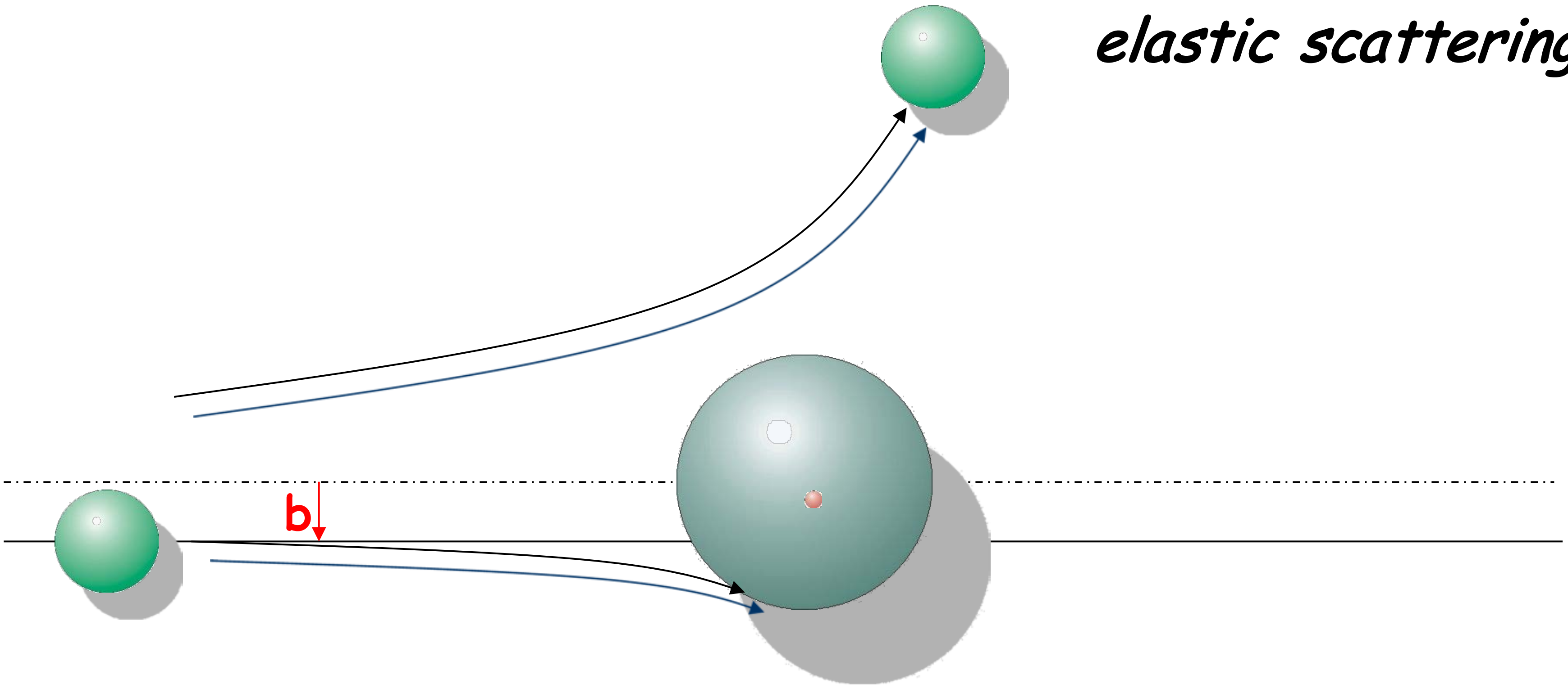
elastic scattering



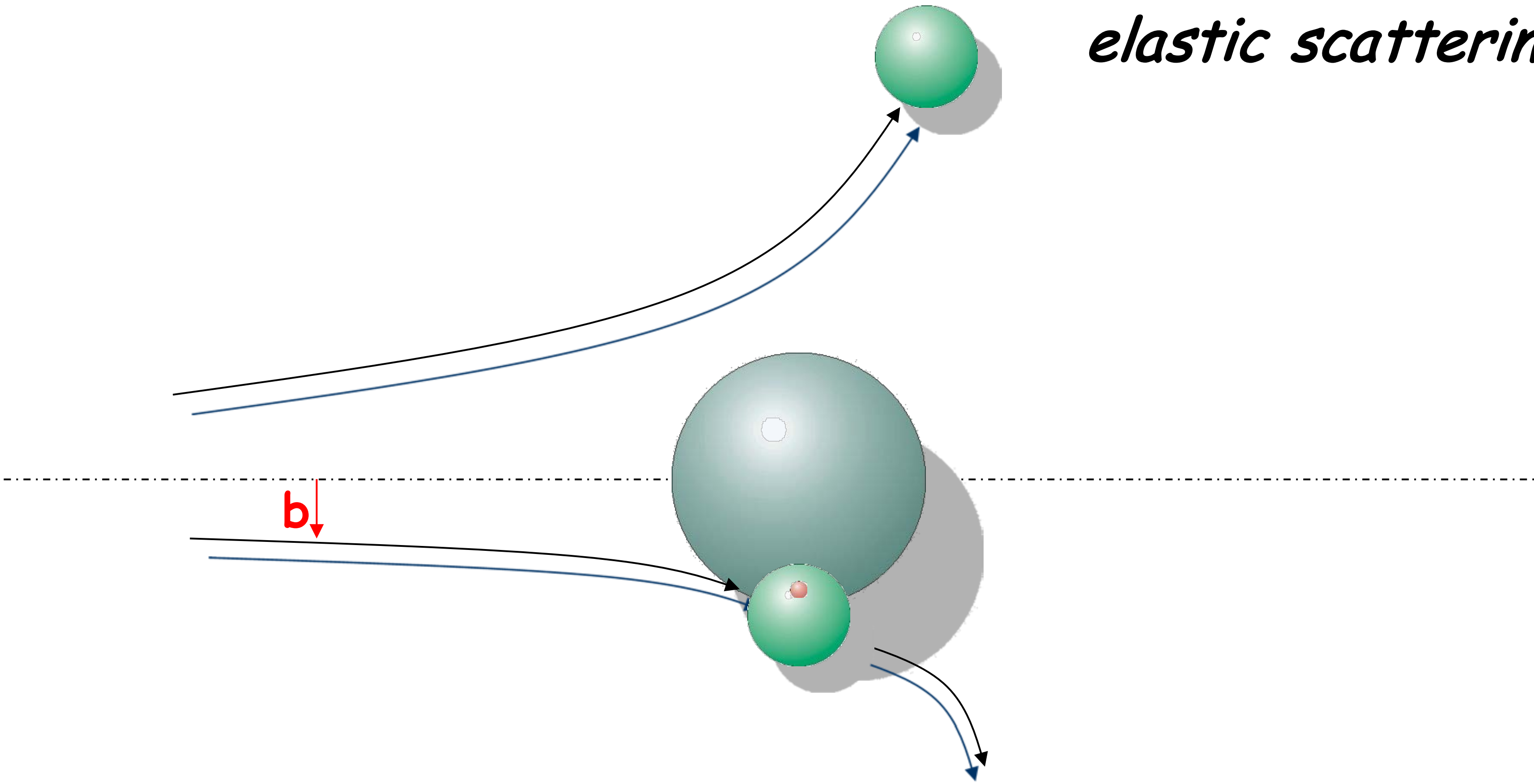
elastic scattering



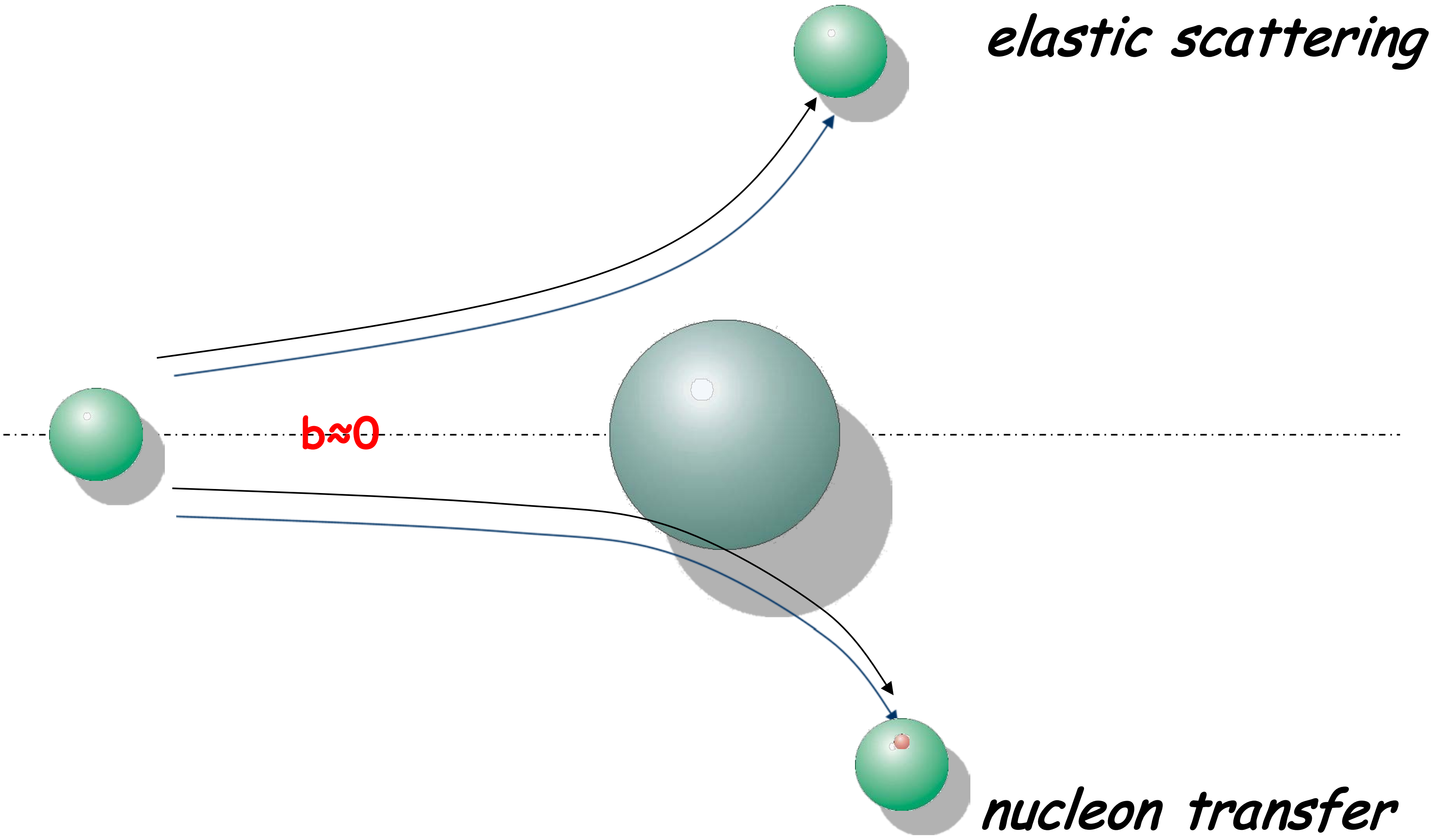
elastic scattering

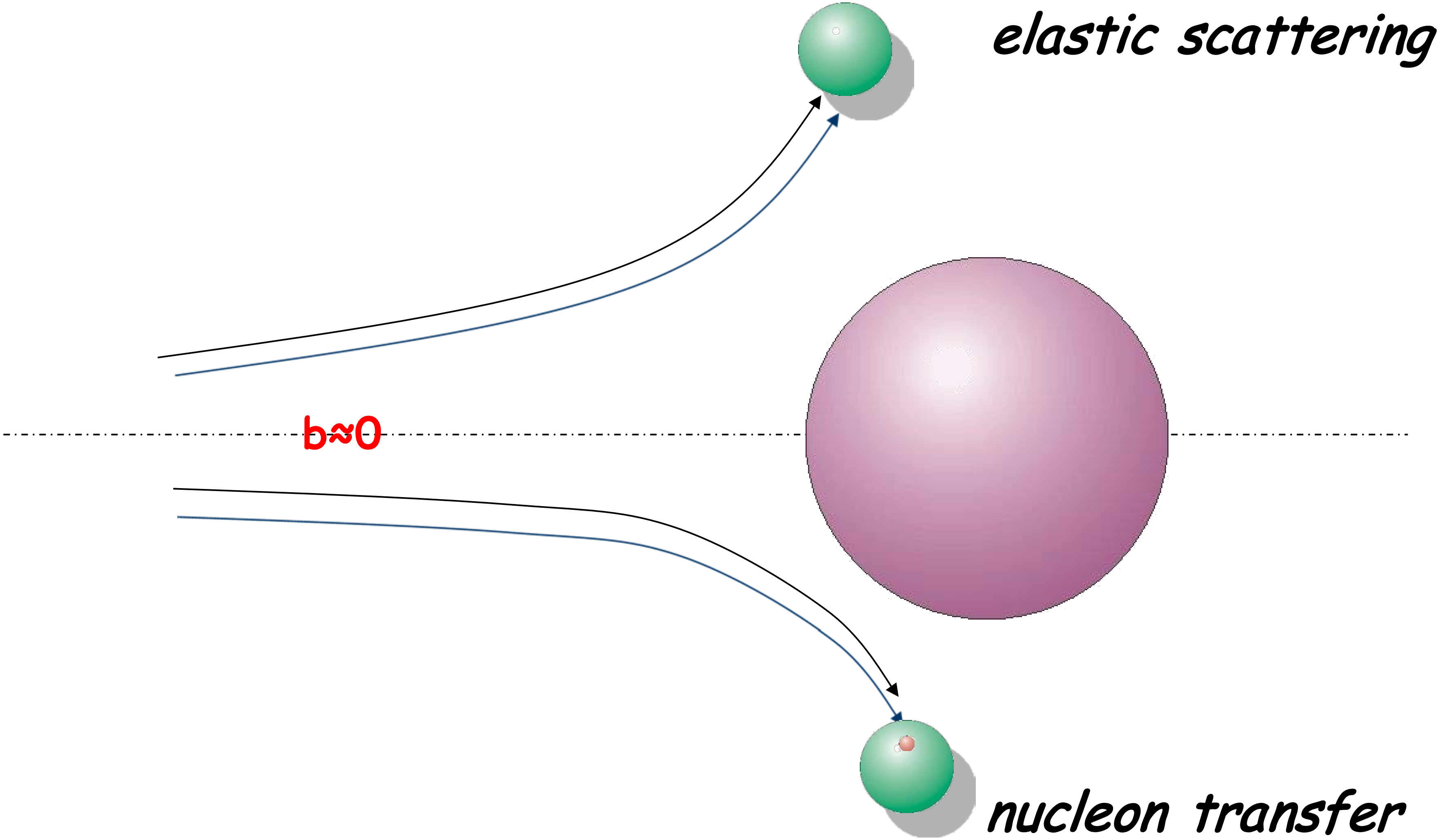


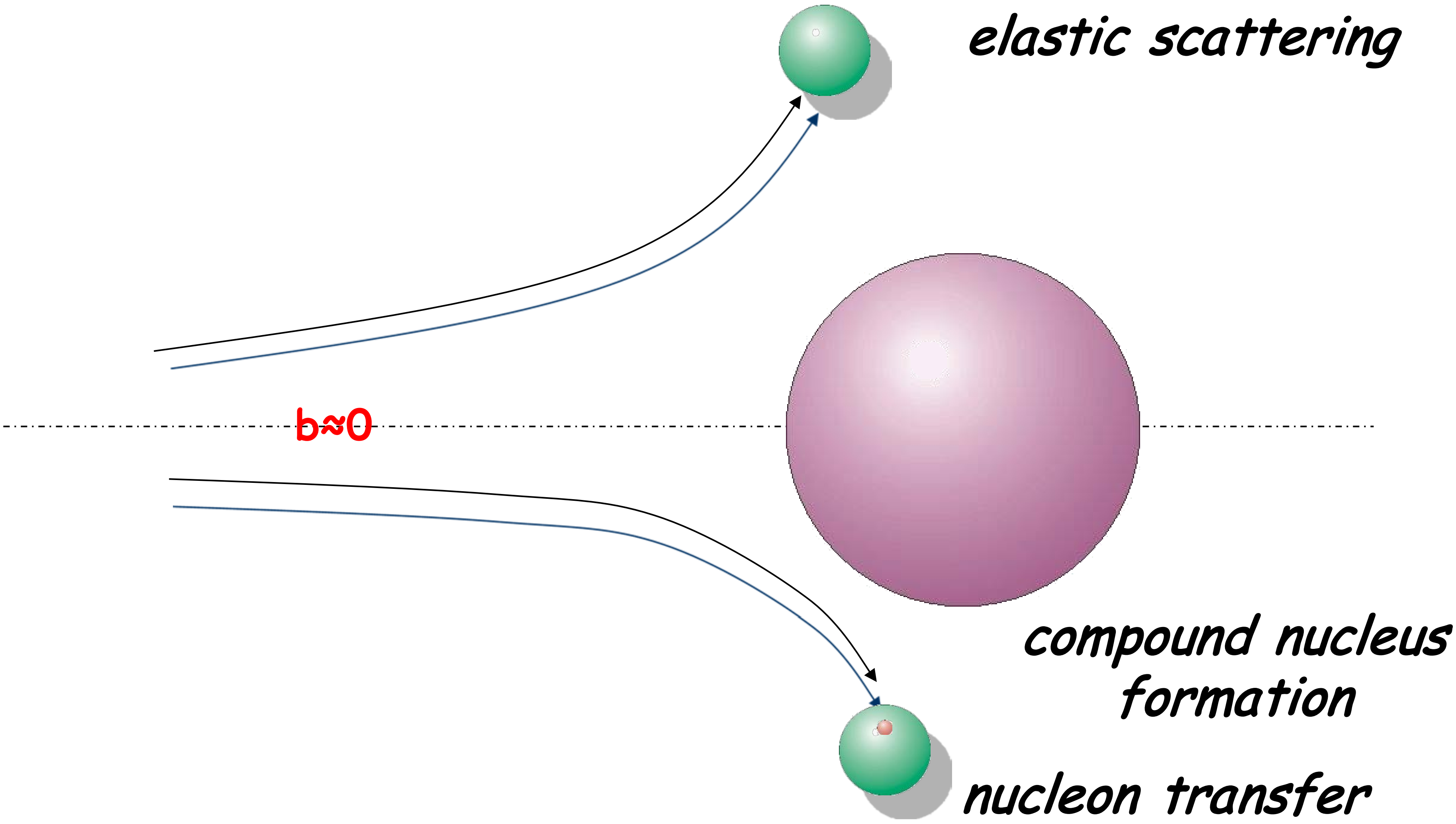
elastic scattering



nucleon transfer


elastic scattering
b≈0
nucleon transfer


elastic scattering
b≈0
nucleon transfer


elastic scattering
b≈0
compound nucleus formation
nucleon transfer

Nuclear reaction cross sections

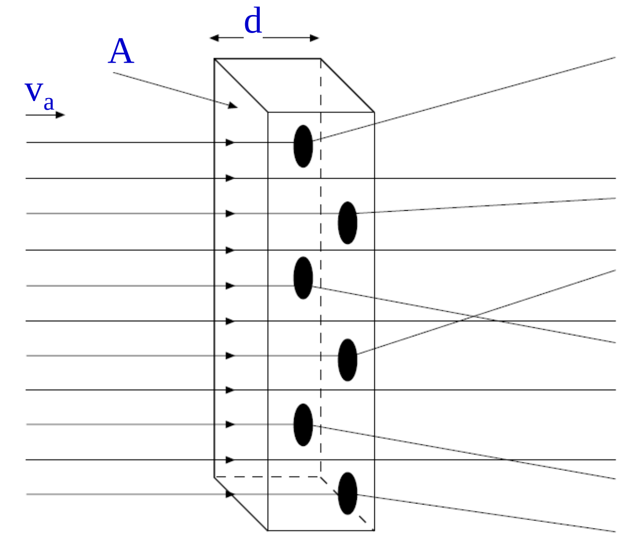
Consider a beam of projectiles of intensity Φ_a particles/sec which hits a thin foil of target nuclei with the result that the beam is attenuated by reactions in the foil such that the transmitted intensity is Φ particles/sec.
The fraction of the incident particles disappear from the beam, i.e. react, in passing through the foil is given by

$$d\Phi = -\Phi \cdot n_b \cdot \sigma \cdot dx$$

The number of reactions that are occurring is the difference between the initial and transmitted flux

$$\Phi_{initial} - \Phi_{trans} = \Phi_{initial}(1 - exp[-n_b \cdot d \cdot \sigma])$$

$$\approx \Phi_{initial} \cdot N_b \cdot \sigma \qquad \text{(for thin target)}$$



$\Phi_a = n_a \cdot v_a$ $\qquad$ $N_b = n_b \cdot A \cdot d$

Example:

A particle current of 1 pnA consists of $6 \cdot 10^9$ projectiles/s.

A ^{132}Sn target (1 mg/cm^2) consists of $5 \cdot 10^{18}$ nuclei/cm^2

$$\frac{6 \cdot 10^{23} \cdot 10^{-3}\ g/cm^2}{132g} = 4.5 \cdot 10^{18} \quad \left[\frac{target\ nuclei}{cm^2}\right]$$

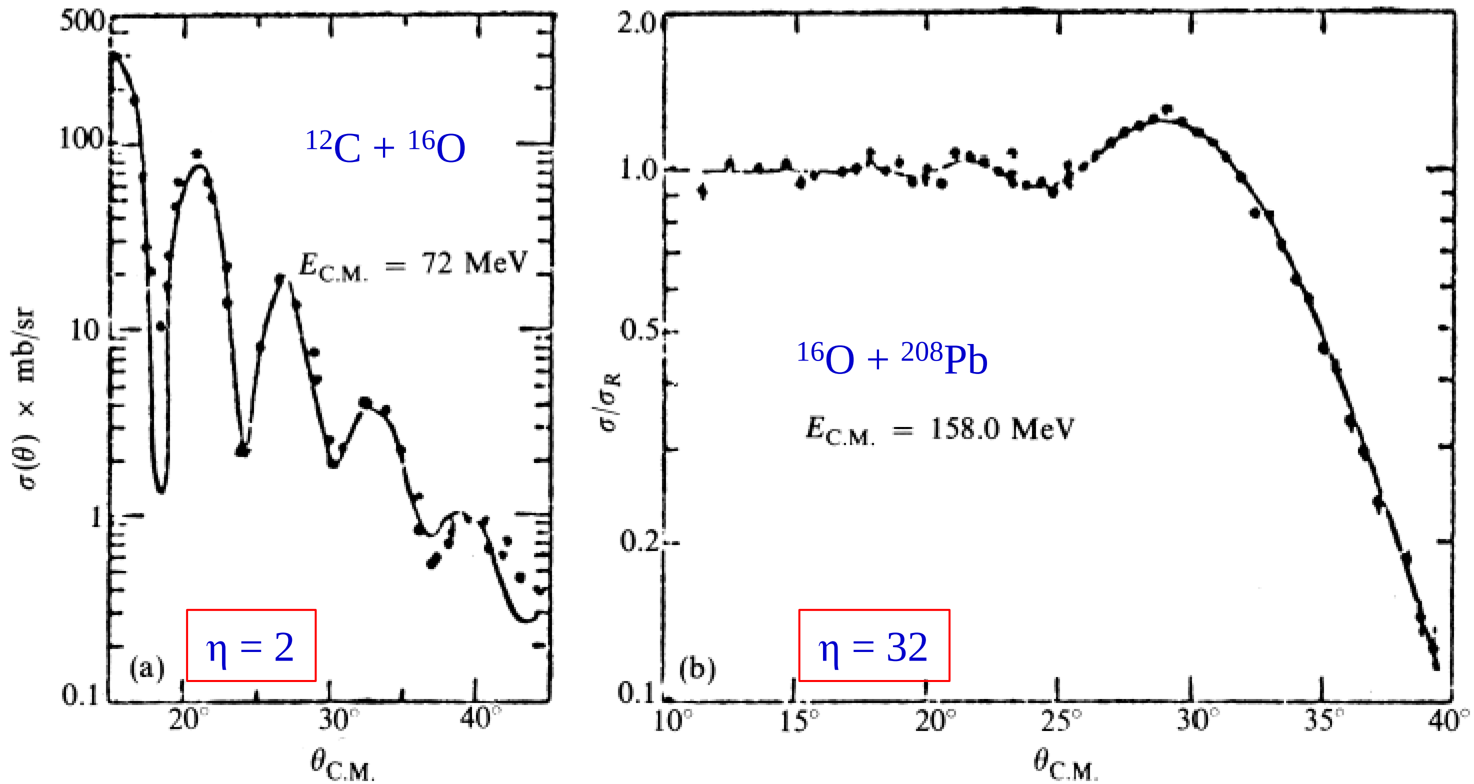
Luminosity = projectiles [s^{-1}] · target nuclei [cm^{-2}]

Luminosity (projectile → ^{132}Sn) = $3 \cdot 10^{28}$ [s^{-1}cm^{-2}]

Reaction rate [s^{-1}] = luminosity · cross section [cm^2]
= projectiles [s^{-1}] · target nuclei [cm^{-2}] · cross section [cm^2]

Elastic Scattering

Fraunhofer (left) **and Fresnel** (right) **diffraction**



Born approximation (quantum description) or ***classical description***: $\eta = \frac{a}{\lambda\!\!\!^{-}}$

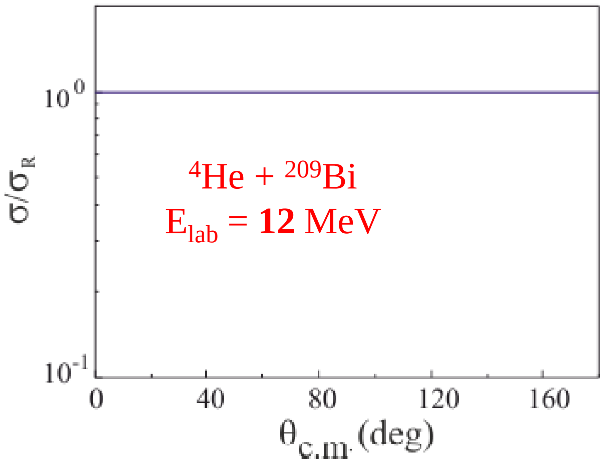
$$\eta = k_\infty \cdot a = 0.157 \cdot Z_1 Z_2 \cdot \sqrt{\frac{A_1}{T_{lab}}}$$

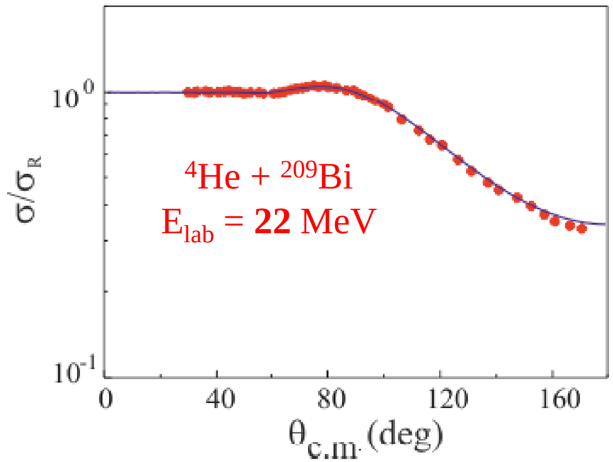
half distance of closest approach for head-on collision $a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2}$ $[fm]$

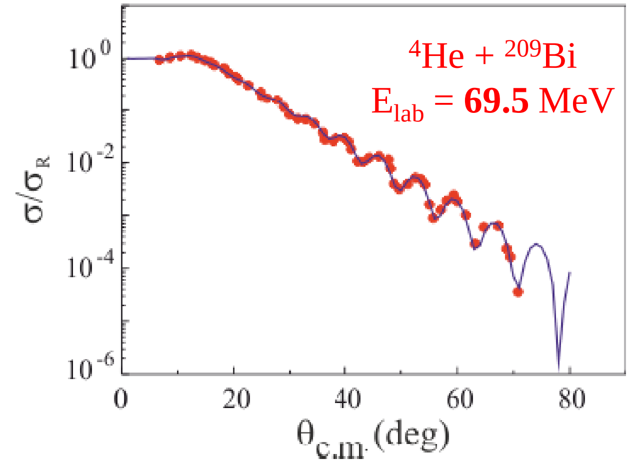
wave length of projectile $\lambda\!\!\!^{-} = (k_\infty)^{-1}$ $k_\infty = 0.219 \cdot \frac{A_2}{A_1 + A_2} \cdot \sqrt{A_1 \cdot T_{lab}}$ $[fm^{-1}]$

FAIR GSI

Elastic Scattering





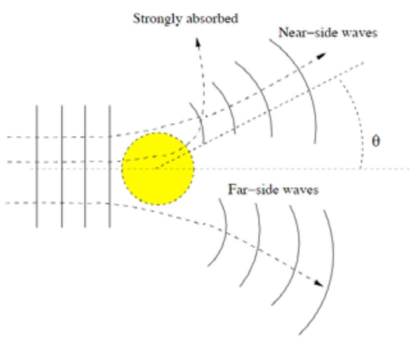


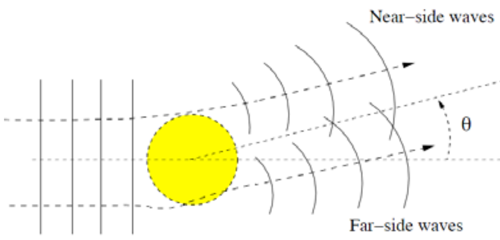
Rutherford scattering
$\eta = 15$

Fresnel scattering
$\eta = 11$

Fraunhofer scattering
$\eta = 6$

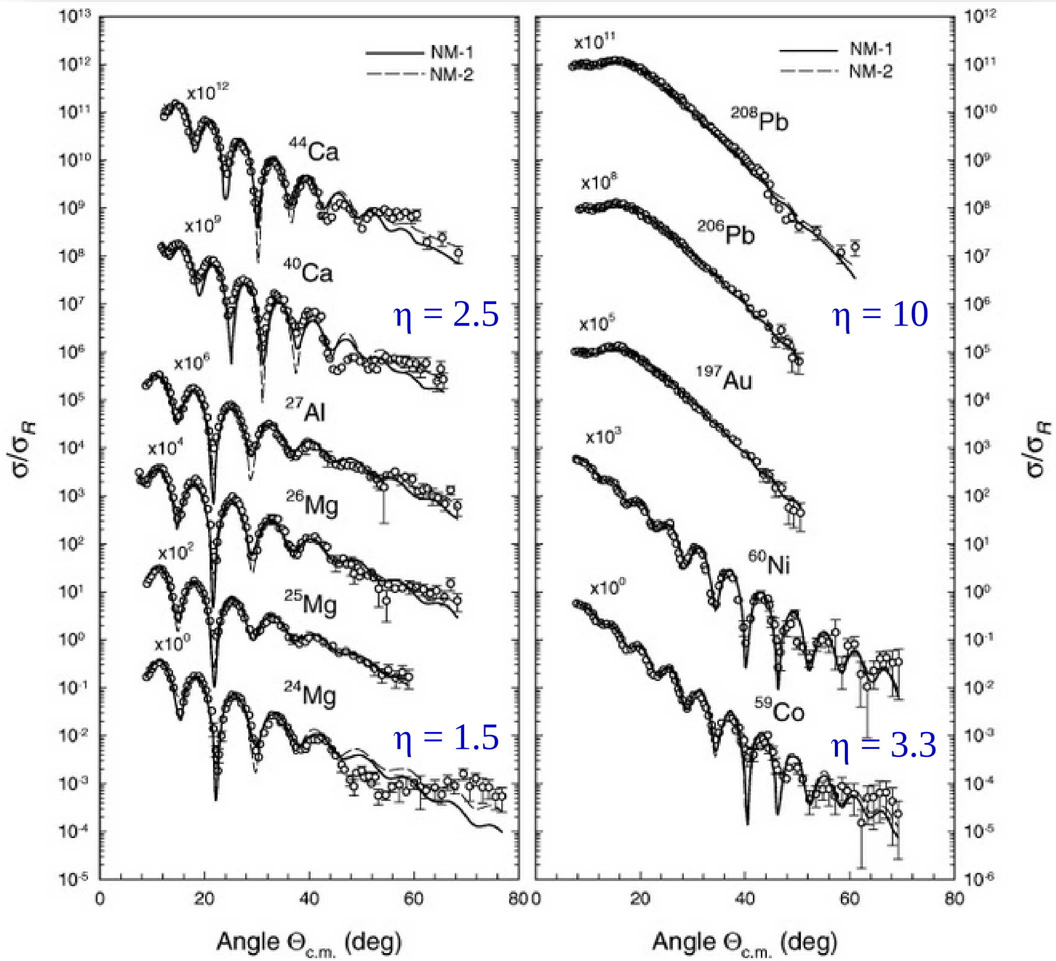
Transition from classical (optical) picture to quantum picture





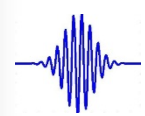
Elastic Scattering

^{6}Li elastic scattering @ 88 MeV



particle

Fresnel scattering (η≥10)



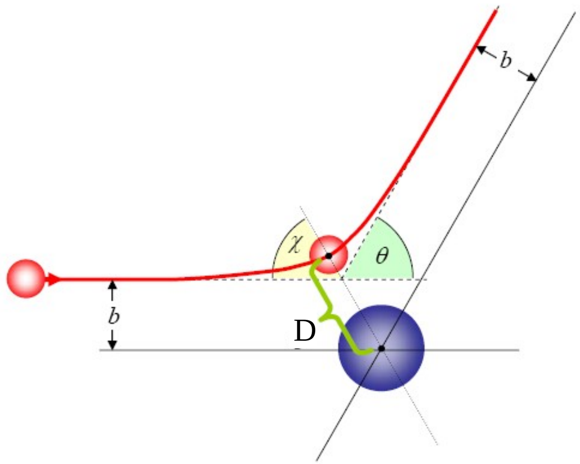
wave

Fraunhofer scattering (η<10)

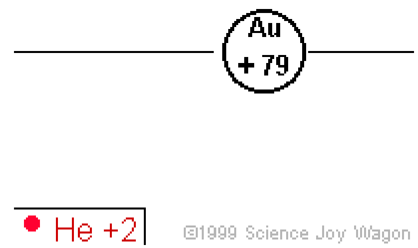
Oscillation in angular distribution → good angular resolution required

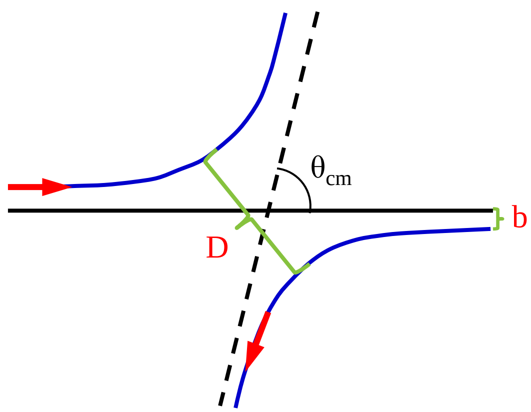
S. Hossain et al. Phys. Scr. 87 (2013) 015201

Scattering parameters



$$\theta = \pi - 2\chi$$





center of mass system

impact parameter:

$$b = a \cdot cot \frac{\theta_{cm}}{2}$$

distance of closest approach:

$$D = a \cdot \left[sin^{-1} \frac{\theta_{cm}}{2} + 1 \right]$$

orbital angular momentum:

$$\ell = k_{\infty} \cdot b = \eta \cdot cot \frac{\theta_{cm}}{2}$$

half distance of closest approach in a head-on collision ($\theta_{cm}=180^0$):

$$a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2} \quad [fm]$$

asymptotic wave number:

$$k_{\infty} = 0.219 \cdot \frac{A_2}{A_1 + A_2} \cdot \sqrt{A_1 \cdot T_{lab}} \quad [fm^{-1}]$$

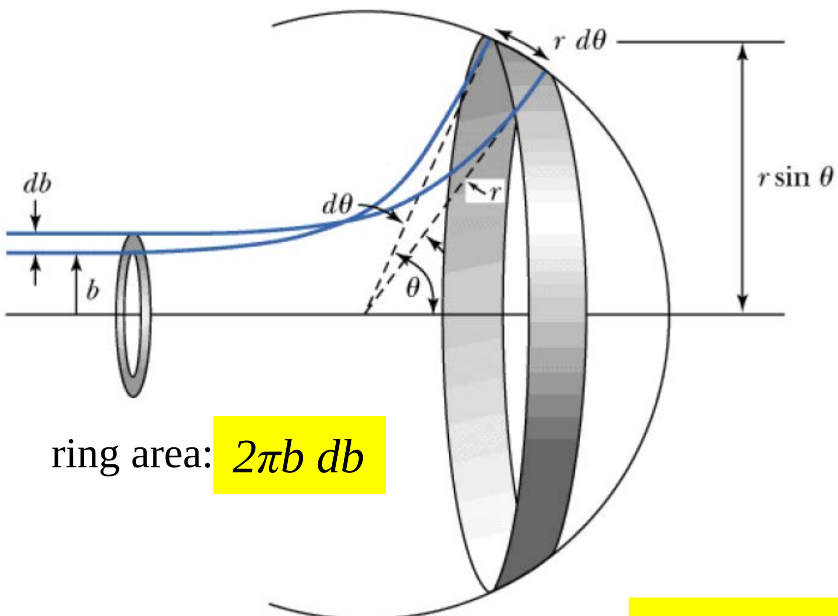
Sommerfeld parameter:

$$\eta = k_{\infty} \cdot a = 0.157 \cdot Z_1 Z_2 \cdot \sqrt{\frac{A_1}{T_{lab}}}$$

FAIR GSI

Scattering theory

- Particles from the ring defined by the impact parameter b and $b+db$ scatter between angle θ and $\theta+d\theta$



ring area: $2\pi b\, db$

solid angle: $2\pi \sin\theta\, d\theta$

$$j \cdot 2\pi \cdot b \cdot db = j \cdot 2\pi \cdot \sin\theta \cdot \frac{d\sigma}{d\Omega}$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

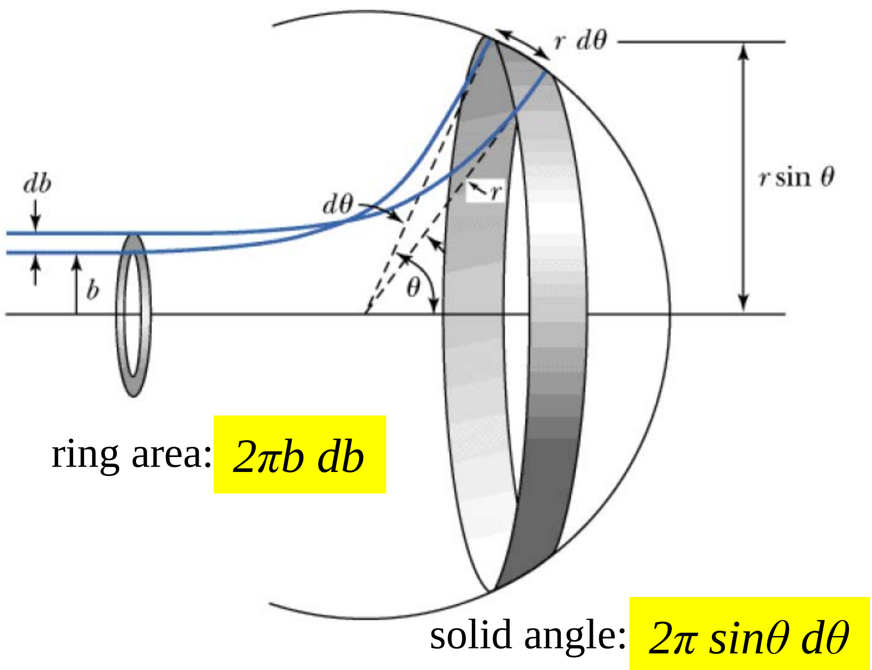
impact parameter: $b = a \cdot \cot\frac{\theta}{2}$

$$\left|\frac{db}{d\theta}\right| = \frac{a}{2} \cdot \frac{-\sin\frac{\theta}{2} \cdot \sin\frac{\theta}{2} - \cos\frac{\theta}{2} \cdot \cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} = \frac{a}{2} \cdot \frac{1}{\sin\frac{\theta}{2}}$$

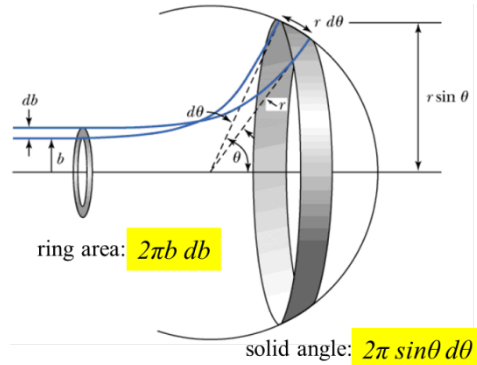
$$\frac{d\sigma}{d\Omega} = a \cdot \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \cdot \frac{1}{2 \cdot \cos\frac{\theta}{2} \cdot \sin\frac{\theta}{2}} \cdot \frac{a}{2 \cdot \sin^2\frac{\theta}{2}}$$

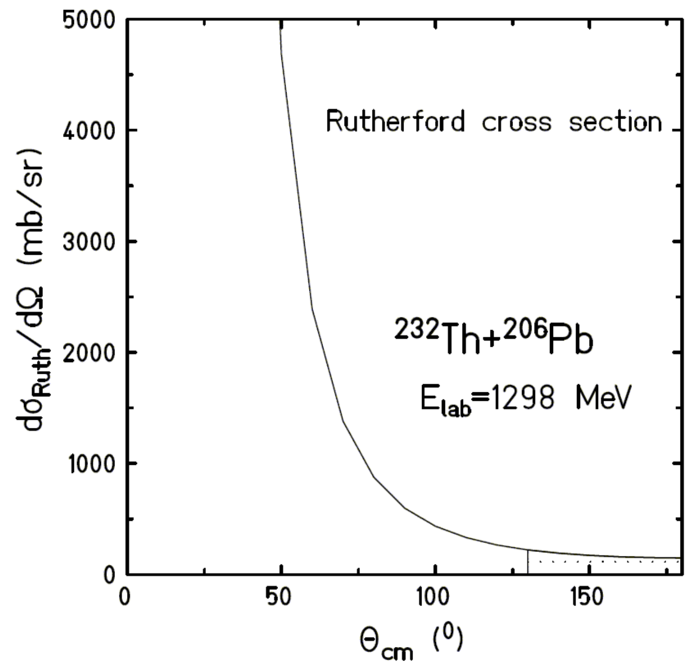
$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2}$$

Scattering theory



$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$





Scattering theory

angular momentum and scattering angle:

$$\ell = b \cdot p = b \cdot \sqrt{2 \cdot m \cdot T} \qquad k_\infty = \frac{\sqrt{2 \cdot m \cdot T}}{\hbar}$$

$$\ell = \eta \cdot cot\frac{\theta}{2} \qquad \eta = k_\infty \cdot a$$

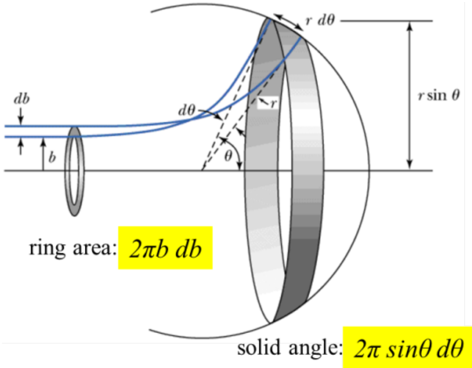
$$\frac{d\sigma}{d\ell} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{d\ell} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2} \cdot \frac{d\Omega}{d\ell}$$

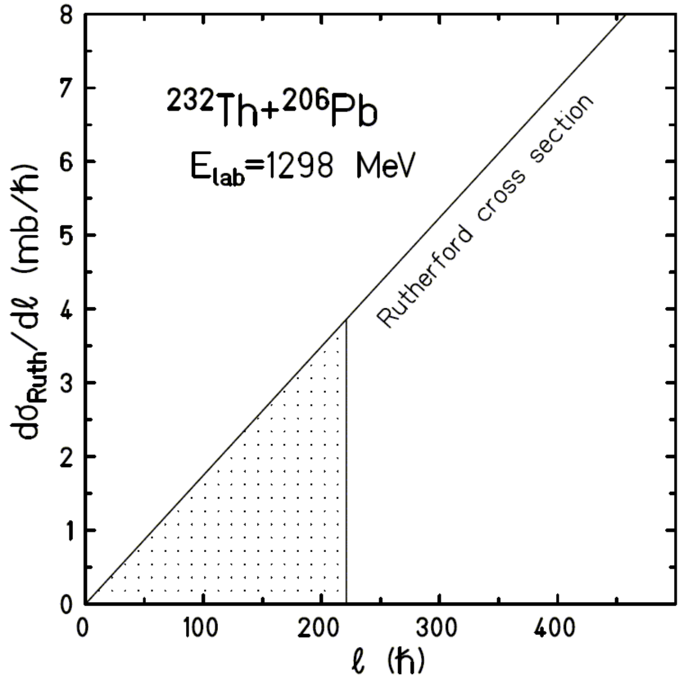
$$\frac{d\Omega}{d\ell} = 2\pi \cdot sin\theta \cdot \frac{d\theta}{d\ell} \qquad \frac{d\ell}{d\theta} = \frac{\eta}{2} \cdot sin^{-2}\frac{\theta}{2}$$

$$\frac{d\Omega}{d\ell} = 2\pi \cdot 2 \cdot cos\frac{\theta}{2} \cdot sin\frac{\theta}{2} \cdot \frac{2 \cdot sin^2\frac{\theta}{2}}{\eta} \qquad cos\frac{\theta}{2} = \frac{\ell}{\eta} \cdot sin\frac{\theta}{2}$$

$$\frac{d\sigma}{d\ell} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2} \cdot \frac{8\pi \cdot \ell}{\eta^2} \cdot sin^4\frac{\theta}{2}$$

$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$





Scattering theory

distance of closest approach and scattering angle:

$$D = a \cdot \left[\sin^{-1}\frac{\theta}{2} + 1 \right] \qquad \sin\frac{\theta}{2} = \frac{a}{D-a}$$

$$\frac{d\sigma}{dD} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{dD} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2} \cdot \frac{d\Omega}{dD}$$

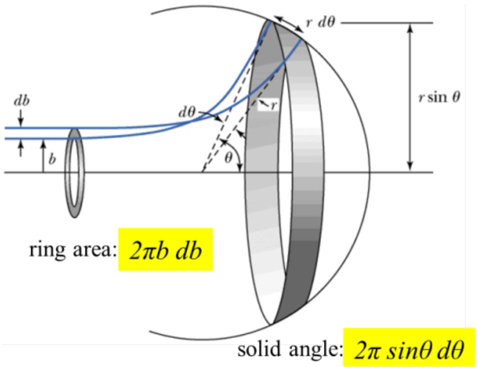
$$\frac{d\Omega}{dD} = 2\pi \cdot \sin\theta \cdot \frac{d\theta}{dD} \qquad \frac{dD}{d\theta} = \frac{a}{2} \cdot \frac{-\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}$$

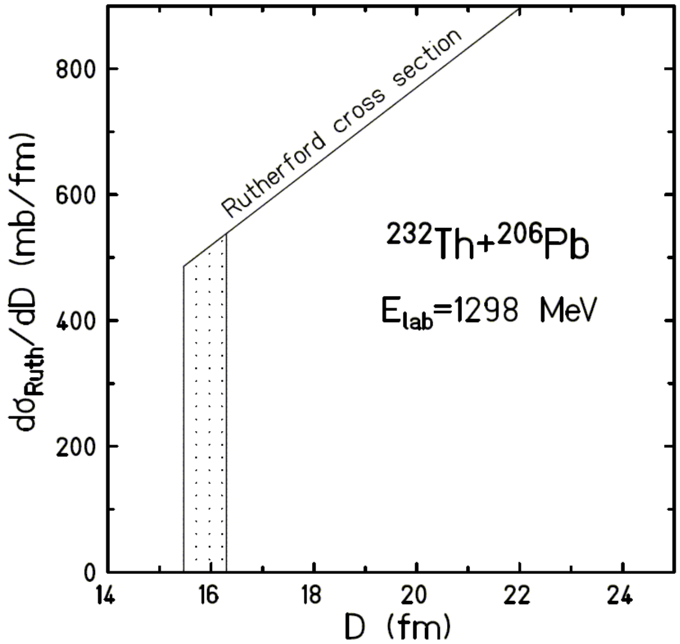
$$\frac{d\Omega}{dD} = 2\pi \cdot 2 \cdot \cos\frac{\theta}{2} \cdot \sin\frac{\theta}{2} \cdot \frac{2}{a} \cdot \frac{\sin^2\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\frac{d\Omega}{dD} = \frac{8\pi}{a} \cdot \sin^3\frac{\theta}{2}$$

$$\frac{d\sigma}{dD} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2} \cdot \frac{8\pi}{a} \cdot \sin^3\frac{\theta}{2} = \frac{2\pi \cdot a}{\sin\frac{\theta}{2}}$$

$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$



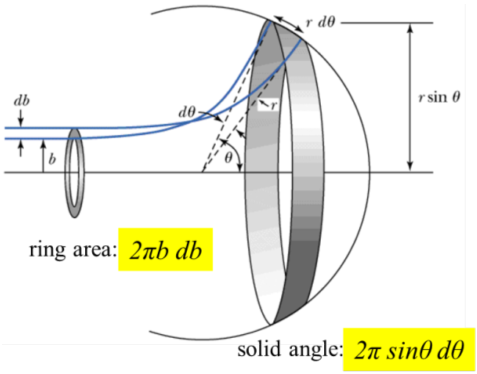


Summary

❖ impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2} \qquad a = \frac{Z_p \cdot Z_t \cdot e^2}{2 \cdot E_{cm}}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$



❖ angular momentum and scattering angle:

$$\ell = \eta \cdot cot\frac{\theta}{2} \qquad \eta = k_\infty \cdot a \qquad k_\infty = \frac{\sqrt{2 \cdot m \cdot E_{cm}}}{\hbar}$$

$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$

❖ distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$

Summary

❖ impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$

❖ angular momentum and scattering angle:

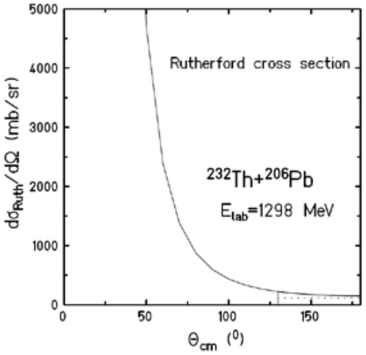
$$\ell = \eta \cdot cot\frac{\theta}{2}$$

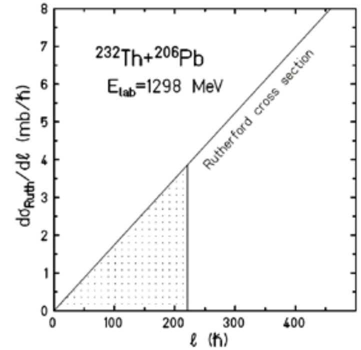
$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$

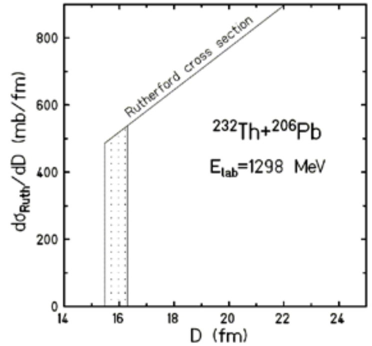
❖ distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$

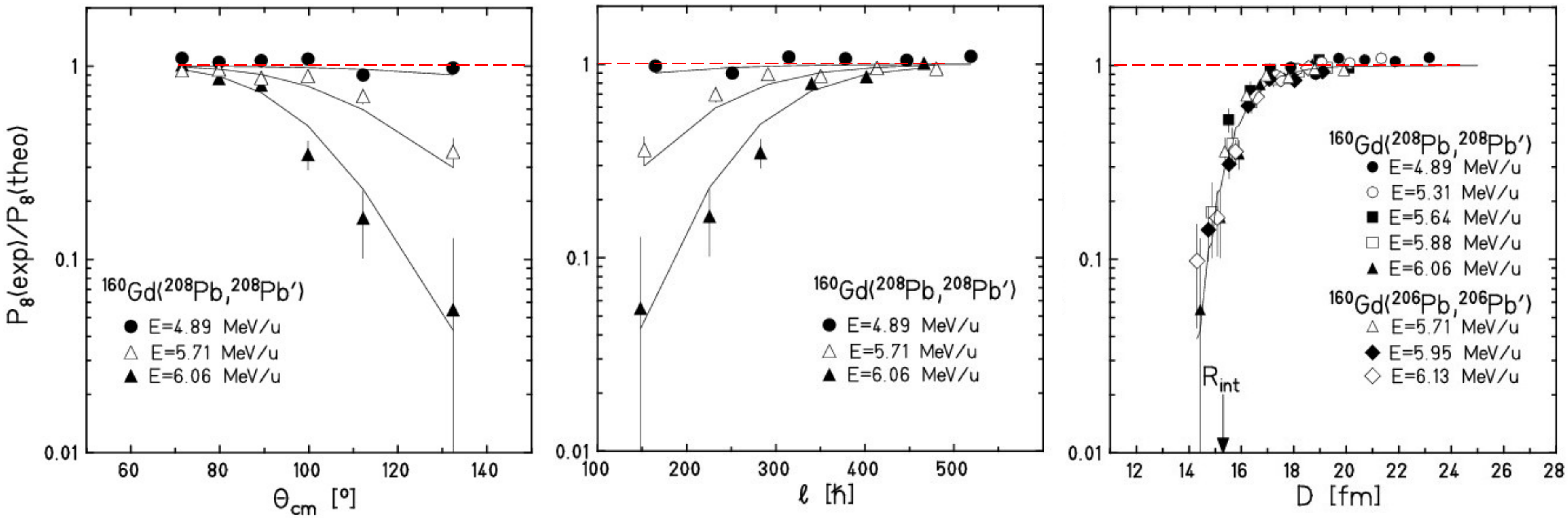






Nuclear Reactions

elastic scattering deviates from Rutherford scattering



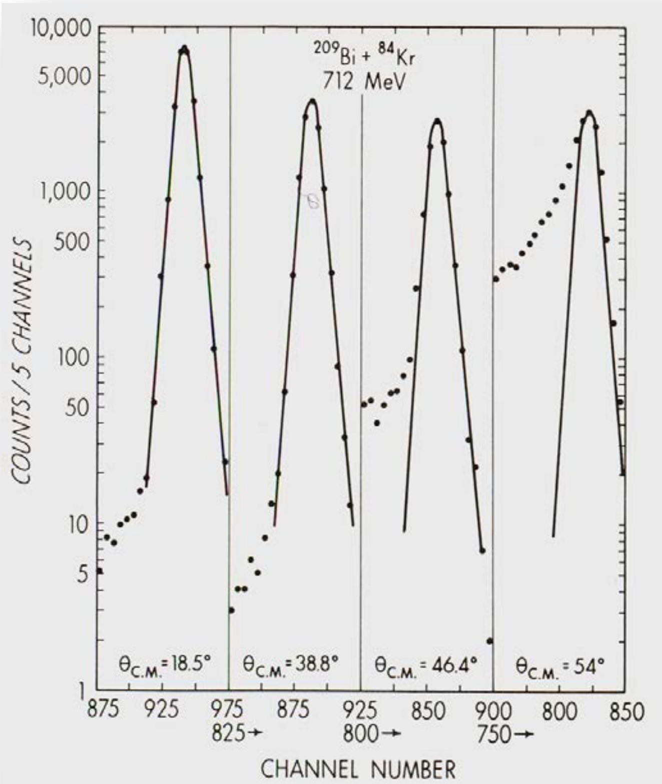
scattering angle *angular momentum* *distance of closest approach*

Ratio of the elastic scattering and Rutherford scattering (nuclear reactions) is only independent of the bombarding energy when plotted versus the distance of closest approach D.

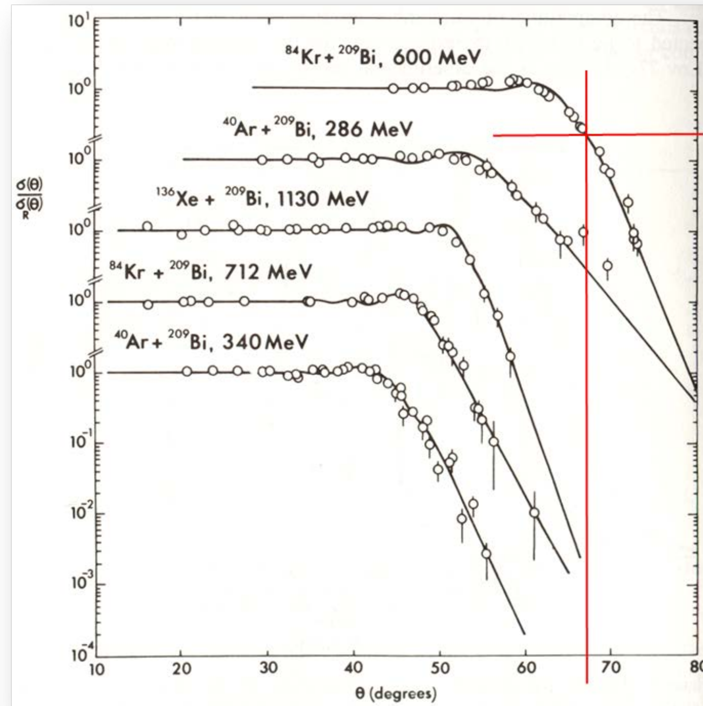
Data are from Coulomb excitation experiments: The excitation probability P_8(exp) of the low-lying rotational state $I^\pi = 8^+$ is not only excited directly but also fed from higher-lying states and is therefore a measure of the elastic scattering. When comparing with Coulomb excitation calculations P_8(theo), which corresponds to the Rutherford scattering, the observed deviations are a clear indication of nuclear interactions (nuclear reactions).

Heavy-ion elastic scattering

energy and angular distributions



determine elastic energy = f(θ), fit standard line shape,
determine elastic cross section $\sigma_{el}(\theta)$

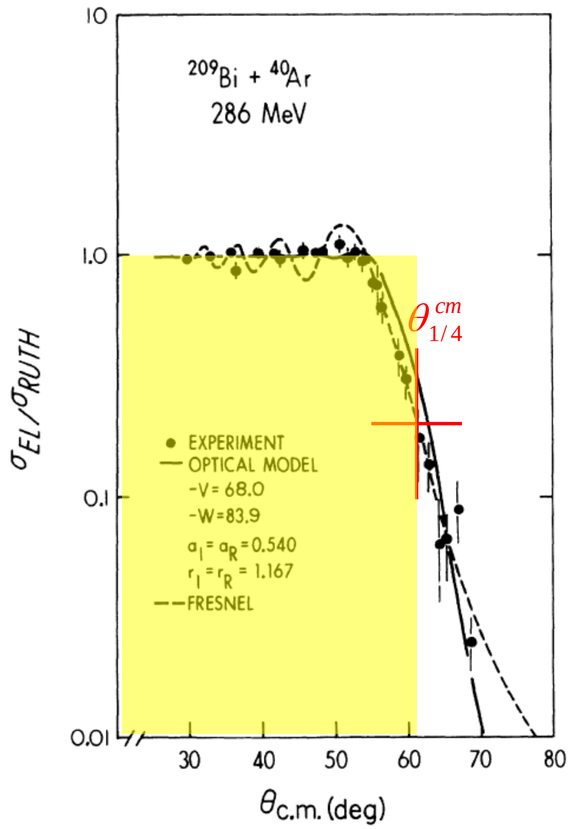


plot ratio elastic/Rutherford cross section = f(θ),
determine quarter-point $\theta_{1/4}$
→ total integrated reaction cross section σ_R

^{84}Kr (600 MeV) + ^{209}Bi: θ = 66.7^0
→ ℓ_{gr} = 268 ħ, R_{int} = 14.2 fm, σ_R = 1.9 b

J.R. Birkelund et al., Phys.Rev.C13 (1976), 133

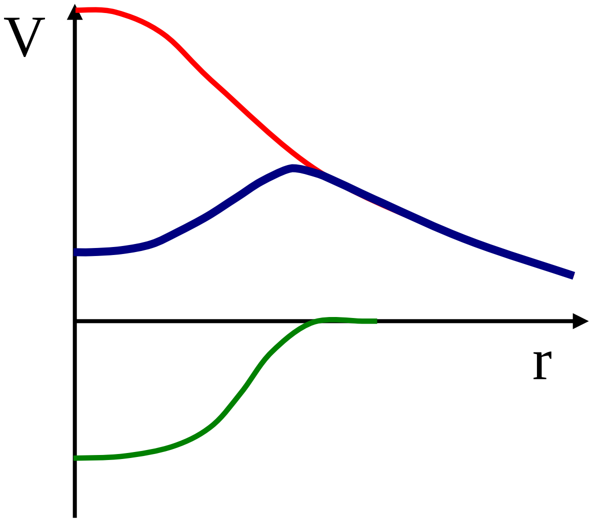
Heavy-ion elastic scattering and the optical model

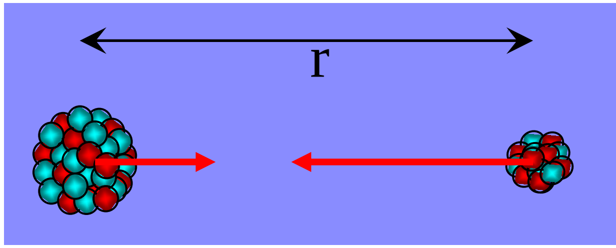


$$V_C(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{2 \cdot R_C}\left(3 - \frac{r^2}{R_C^2}\right) & r < R_C \\ \frac{Z_1 Z_2 e^2}{r} & r \geq R_C \end{cases}$$

$$V_{nucl}(r) = \frac{-V_0}{1 + exp\left[\frac{r - r_R}{a_R}\right]}$$

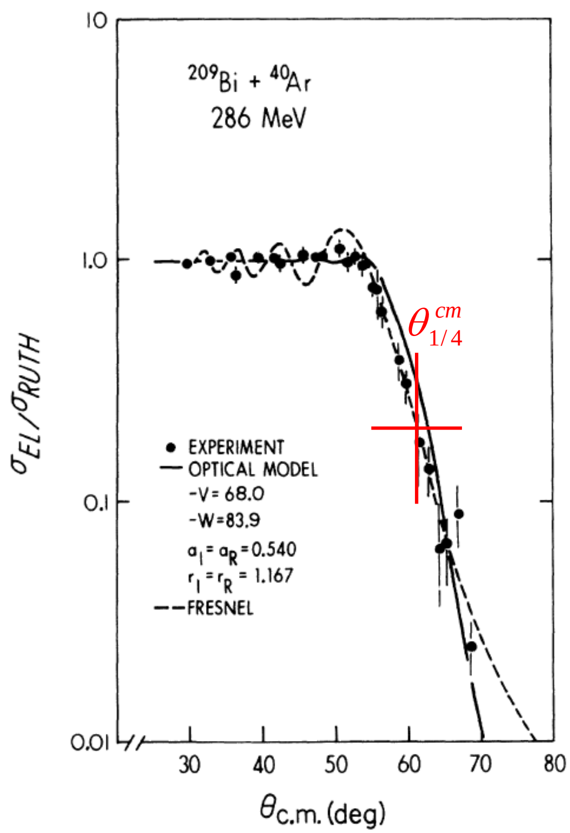
$$V_{imag}(r) = \frac{-W_0}{1 + exp\left[\frac{r - r_I}{a_I}\right]}$$





J.R. Birkelund et al., Phys.Rev.C13 (1976), 133

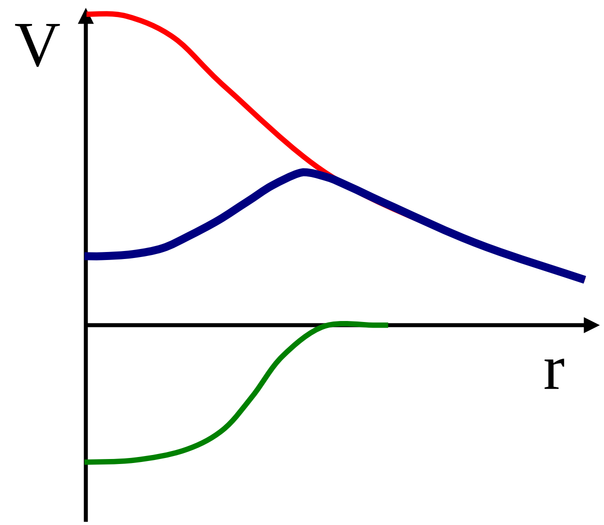
Heavy-ion elastic scattering and the optical model



$$V_C(r) = \begin{cases} \dfrac{Z_1 Z_2 e^2}{2 \cdot R_C}\left(3 - \dfrac{r^2}{R_C^2}\right) & r < R_C \\ \dfrac{Z_1 Z_2 e^2}{r} & r \geq R_C \end{cases}$$

$$V_{nucl}(r) = \frac{-V_0}{1 + exp\left[\dfrac{r - r_R}{a_R}\right]}$$

$$V_{imag}(r) = \frac{-W_0}{1 + exp\left[\dfrac{r - r_I}{a_I}\right]}$$



$\theta_{1/4} = 60^0 \rightarrow R_{int} = 13.4$ [fm]

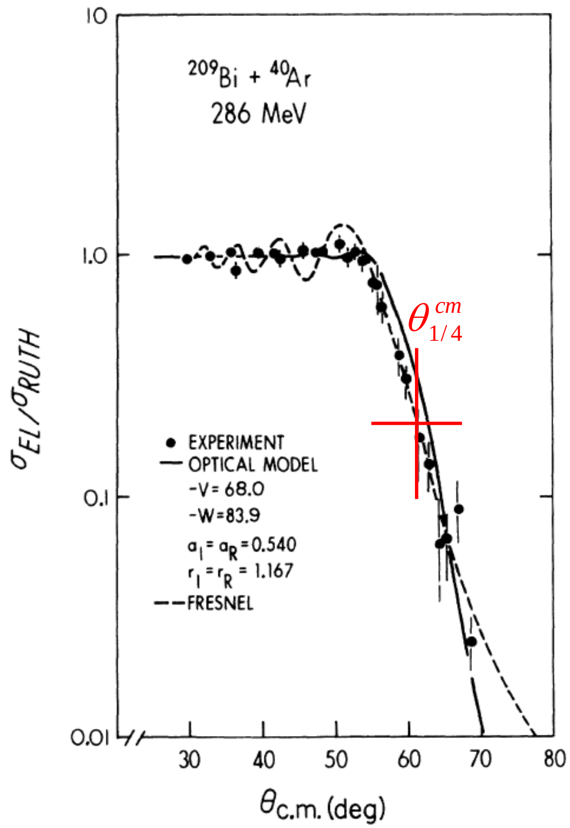
$\rightarrow \ell_{gr} = 152$ [ħ]

Parameters of the optical model fit:

V_0 (MeV)	r_R (fm)	a_R (fm)	W_0 (MeV)	r_I (fm)	a_I (fm)
68.0	1.167	0.540	**83.9**	1.167	0.540
214.5	1.104	0.536	**261.1**	1.104	0.536
43.2	1.196	0.529	**56.0**	1.196	0.529

J.R. Birkelund et al., Phys.Rev.C13 (1976), 133

Elastic scattering and nuclear radius



$\theta_{1/4} = 60^0 \rightarrow R_{int} = 13.4$ [fm]

$\rightarrow \ell_{gr} = 152$ [ħ]

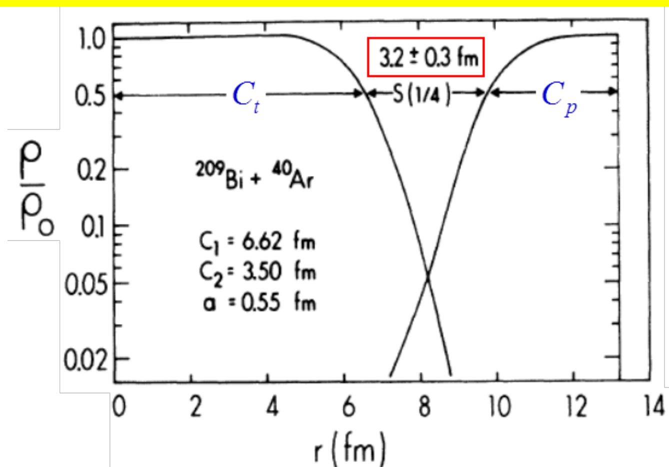
Nuclear interaction radius: (distance of closest approach)

$$R_{int} = D = a \cdot \left[sin^{-1} \frac{\theta_{1/4}}{2} + 1 \right]$$

$$R_{int} = C_p + C_t + 4.49 - \frac{C_p + C_t}{6.35} \quad [fm]$$

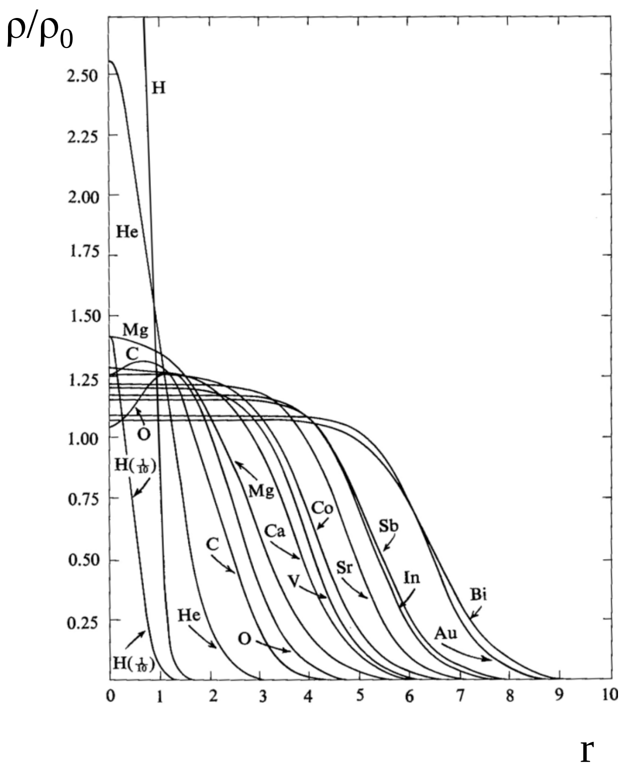
$$C_i = R_i \cdot (1 - R_i^{-2}) \quad [fm] \qquad R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$$

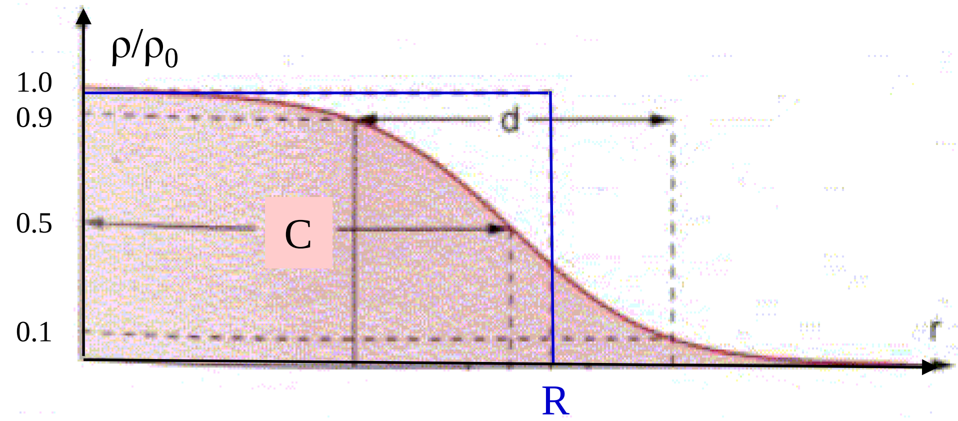
Nuclear density distributions at the nuclear interaction radius



J.R. Birkelund et al., Phys.Rev.C13 (1976), 133

Nuclear radius





nuclear radius of a homogenous charge distribution:

$$R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$$

nuclear radius of a Fermi charge distribution:

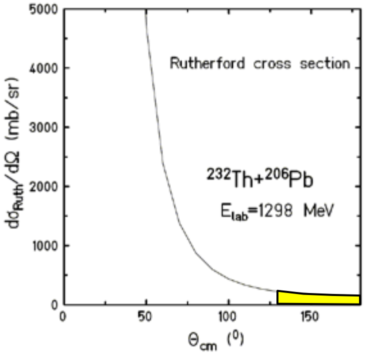
$$C_i = R_i \cdot \left(1 - R_i^{-2}\right) \quad [fm]$$

Elastic scattering – nuclear reactions

❖ impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$

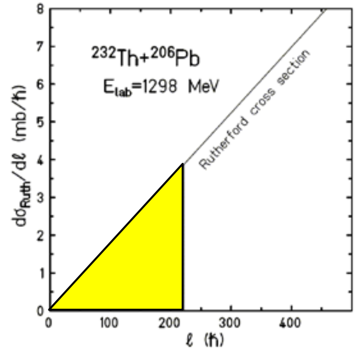


$\theta_{1/4} = 132^0$

❖ angular momentum and scattering angle:

$$\ell = \eta \cdot cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$

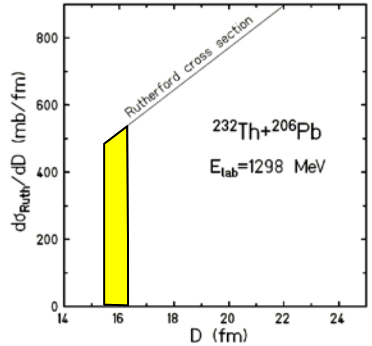


$\ell_{gr} = 206\ \hbar$

❖ distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$



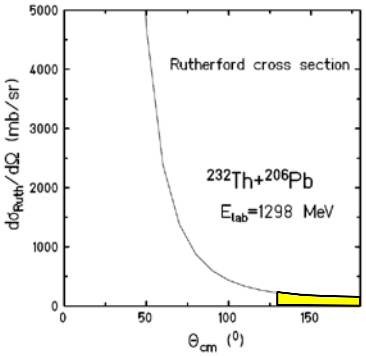
$R_{int} = 16.2$ fm

Total reaction cross section

- impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2}$$

$$\sigma_{reaction} = 2\pi a^2 \cdot \left[\left(1 - cos\ \theta_{1/4}^{cm}\right)^{-1} - 0.5\right]$$

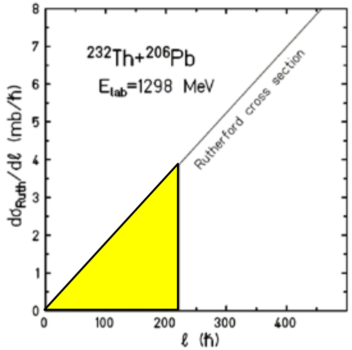


$\theta_{1/4} = 132^0$ a = 7.73 fm

- angular momentum and scattering angle:

$$\ell = \eta \cdot cot\frac{\theta}{2}$$

$$\sigma_{reaction} = \frac{\pi}{k_\infty^2} \cdot \ell_{gr}\left(\ell_{gr} + 1\right)$$

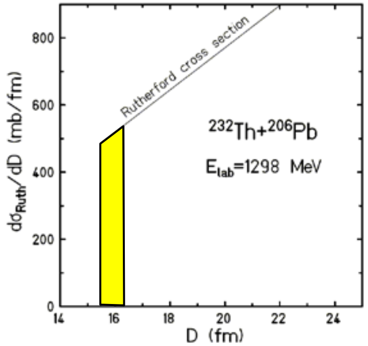


$\ell_{gr} = 206\ \hbar$ $k_\infty = 59.9\ fm^{-1}$

- distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

$$\sigma_{reaction} = \pi \cdot R_{int}^2 \cdot \left(1 - \frac{V_C(R_{int})}{E_{cm}}\right)$$



R_{int} = 16.2 fm $V_C(R_{int})$ = 656 MeV

Scattering theory – nuclear absorption

elastic cross section:

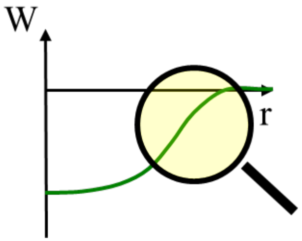
$$\frac{d\sigma_{el}}{dD} = [1 - P_{abs}(D)]\frac{d\sigma_{Ruth}}{dD}$$

attenuation coefficient:

$$[1 - P_{abs}(D)] = exp\left\{-\frac{2}{\hbar}\int_{-\infty}^{+\infty} W[r(t)]dt\right\}$$

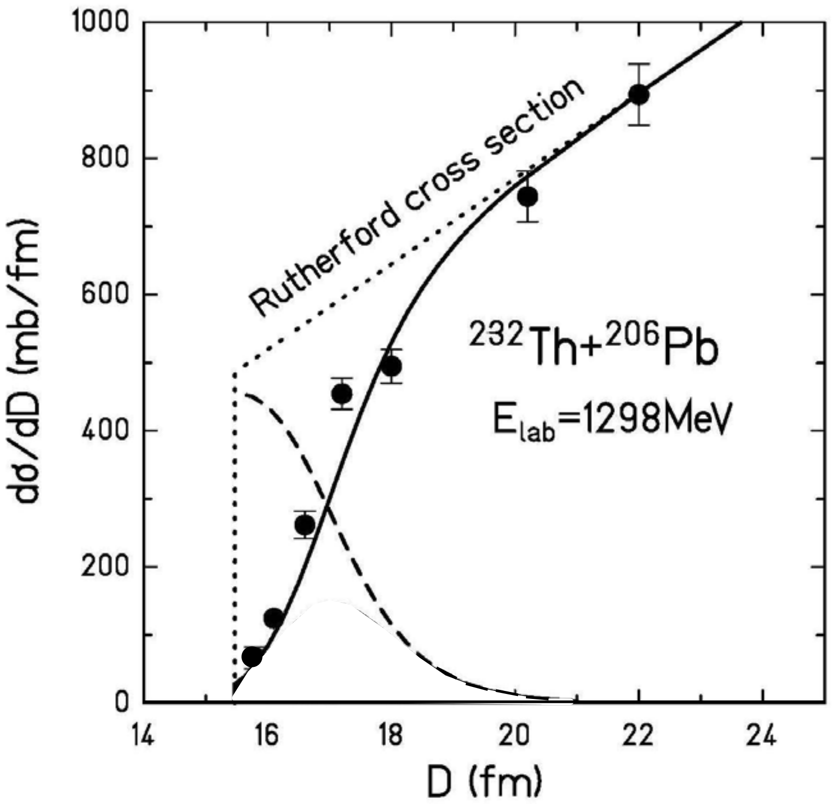
proximity potential:

$$W[r(t)] = W_0 \cdot exp\left[-\frac{r(t) - C_1 - C_2}{a_I}\right]$$



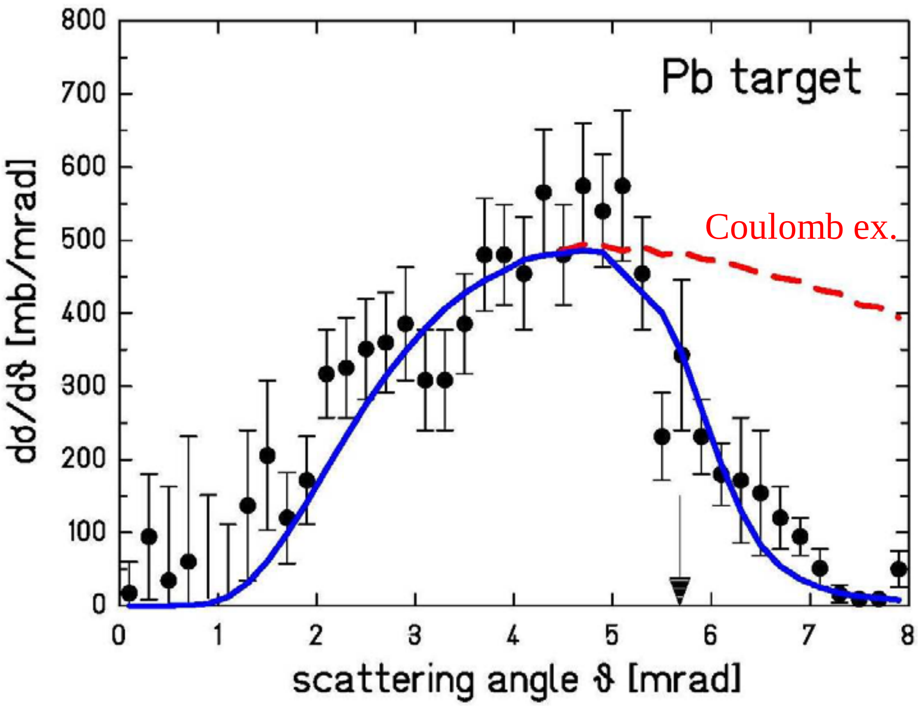
attenuation coefficient:

$$[1 - P_{abs}(D)] = exp\left\{-\frac{2}{\hbar} \cdot W_0 \cdot exp\left[-\frac{D - C_1 - C_2}{a_I}\right] \cdot \frac{D}{v}\right\}$$



High-energy Coulomb excitation

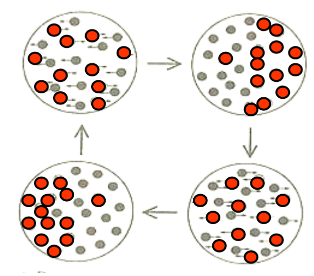
grazing angle

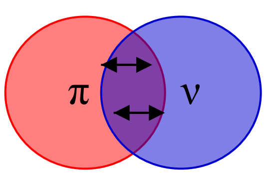


^{136}Xe on ^{208}Pb at 700 MeV/u

excitation of giant dipole resonance

$R_{\text{int}} = 15.0\ fm \ \rightarrow\ \vartheta_{1/4} = 5.7\, mrad$





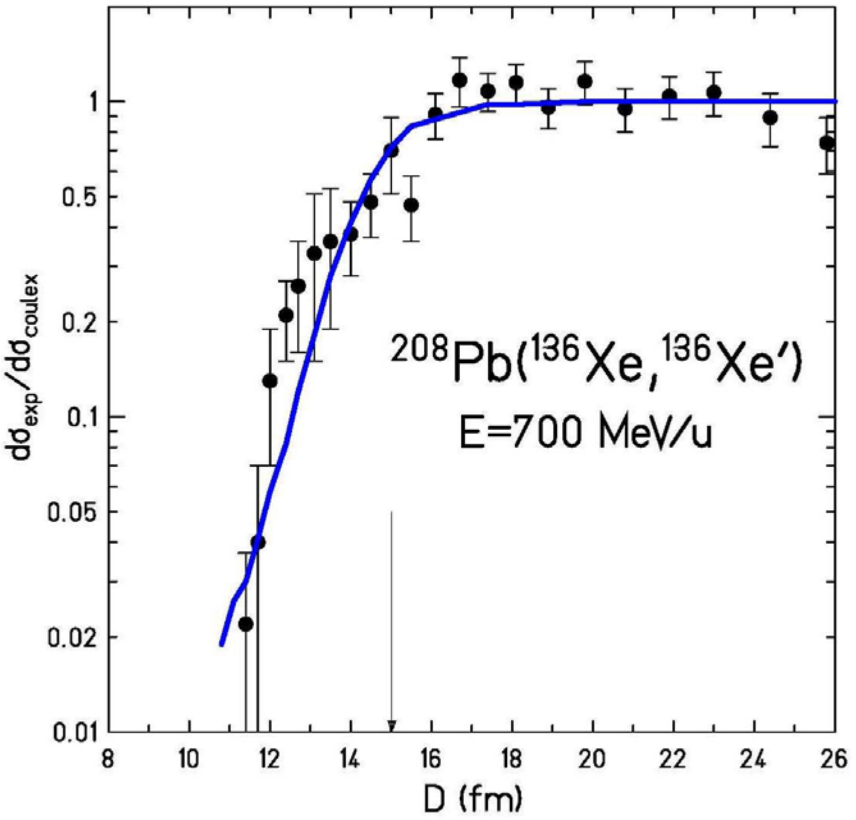
For relativistic projectiles ($\theta_{cm} \approx \vartheta_{lab}$):

$$D = \frac{2 \cdot Z_P Z_T e^2}{m_0 c^2 \beta^2 \gamma} \cdot \frac{1}{\vartheta}$$

A.Grünschloß et al., Phys. Rev. C60 051601 (1999)

High-energy Coulomb excitation

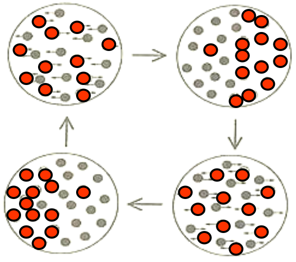
grazing angle



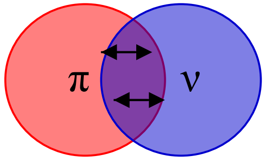
^{136}Xe on ^{208}Pb at 700 MeV/u

excitation of giant dipole resonance

$R_{int} = 15.0\ fm \rightarrow \vartheta_{1/4} = 5.7\ mrad$



Protons **Neutrons**



For relativistic projectiles ($\theta_{cm} \approx \vartheta_{lab}$):

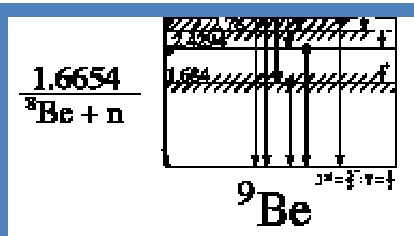
$$D = \frac{2 \cdot Z_P Z_T e^2}{m_0 c^2 \beta^2 \gamma} \cdot \frac{1}{\vartheta}$$

A.Grünschloß et al., Phys. Rev. C60 051601 (1999)

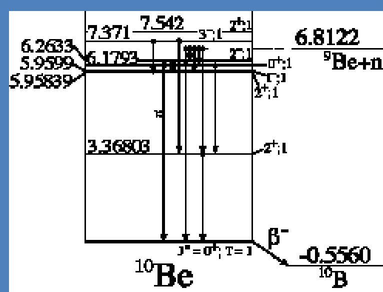
Effect of nuclear structure on elastic scattering

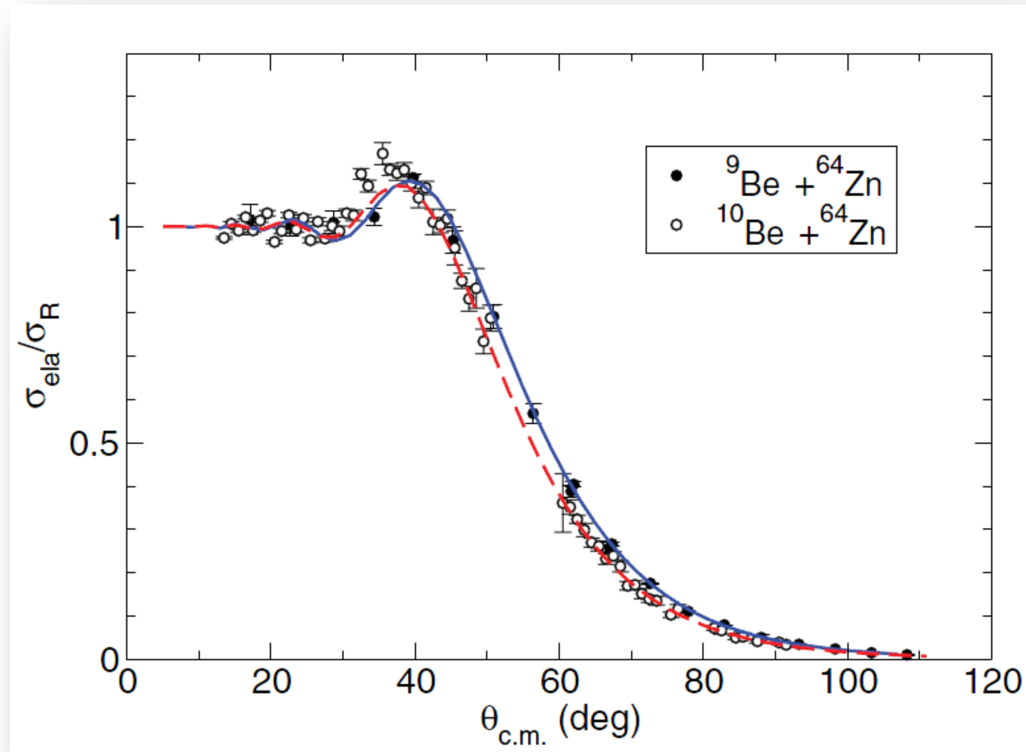
^{9,10}Be+^{64}Zn elastic scattering angular distributions @ 29MeV

^{9}Be
Sn=1.665 MeV



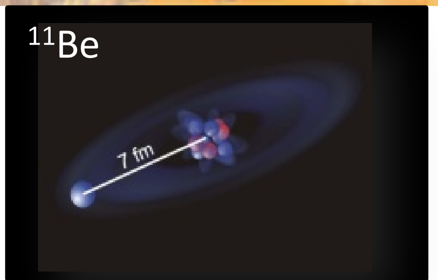
^{10}Be
Sn=6.88 MeV

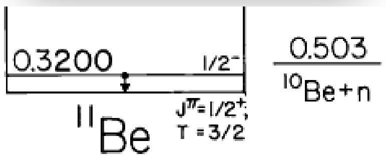




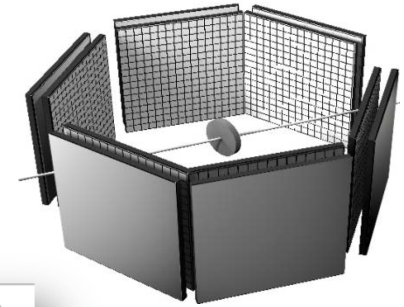
A. Di Pietro et al. Phys. Rev. Lett. 105, 022701(2010)

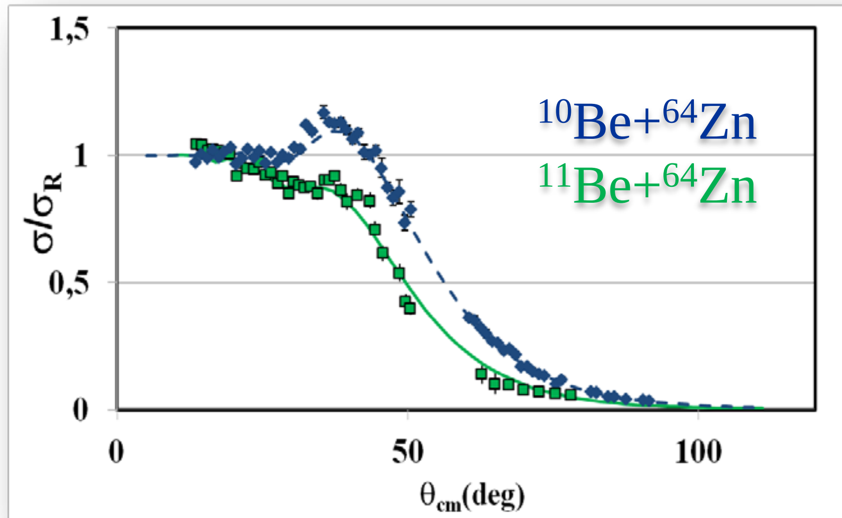
Effect of nuclear structure on elastic scattering





^{10,11}Be+^{64}Zn @ Rex-Isolde, CERN





Optical Model analysis:

- Volume potential responsible for the core-target interaction obtained from the ^{10}Be+^{64}Zn elastic scattering fit.
- plus a complex surface DPP having the shape of a W-S derivative with a very large diffuseness.
- Very large diffuseness: $a_i = 3.5$ fm similar to what found in A. Bonaccrso, NP A706 (2002), 322

Reaction cross-sections

$\sigma_R(^{9}Be) \approx 1.1b$ $\sigma_R(^{10}Be) \approx 1.2b$ $\boldsymbol{\sigma_R(^{11}Be) \approx 2.7b}$

A. Di Pietro et al. Phys. Rev. Lett. 105, 022701(2010)