Radio-frequency (RF) accelerators

- Key idea: using rapidly changing high frequency voltages instead of electrostatic voltages avoids corona formation and discharge → much higher accelerating voltages possible
- But: particles must have the correct phase relation to the accelerating voltage
- But: need high power RF sources!
The Cyclotron

- 1930: Lawrence proposed the cyclotron (before he developed a workable color TV screen)
- 1931: Lawrence and Livingston built first cyclotron (80 keV)
- 1932: Lawrence and Livingston used a cyclotron for 1.25 MeV protons and mentioned longitudinal (phase) focusing

Ernest O. Lawrence (1901-1958)

M. Stanley Livingston (1905-1986)
Electromagnetic forces on charged particles

- Lorentz force equation gives the force in response to electric and magnetic fields:

\[ \vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B}) \]

- The equation of motion becomes:

\[ \frac{d\vec{p}}{dt} = \frac{d}{dt} (m_0 \gamma v) = q \cdot (\vec{E} + \vec{v} \times \vec{B}) \]

- The kinetic energy of a charged particle increases by an amount equal to the work done (Work-Energy Theorem)

\[
\Delta W = \int \vec{F} \cdot d\vec{\ell} = q \int \vec{E} \cdot d\vec{\ell} + q \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}
\]

\[
\Delta W = q \int \vec{E} \cdot d\vec{\ell} + q \int (\vec{v} \times \vec{B}) \cdot \vec{v} \, dt = q \int \vec{E} \cdot d\vec{\ell}
\]
Motion in E and B fields

- Governed by Lorentz force:
  \[
  \frac{d\vec{p}}{dt} = q \cdot [\vec{E} + \vec{v} \times \vec{B}]
  \]

  \[E^2 = \vec{p}^2 c^2 + m_0^2 c^4\]

  \[\Rightarrow E \frac{dE}{dt} = c^2 \vec{p} \cdot \frac{d\vec{p}}{dt}\]

  \[\Rightarrow \frac{dE}{dt} = \frac{c^2 \vec{p}}{E} \cdot q \cdot (\vec{E} + \vec{v} \times \vec{B}) = \frac{qc^2}{E} \cdot \vec{p} \cdot \vec{E}\]

  A magnetic field does not alter a particle’s energy. Only an electric field can do this.

- Acceleration along a uniform electric field (B = 0):

  \[x = v \cdot t\]

  \[y = \frac{1}{2} a \cdot t^2 = -\frac{1}{2} \frac{eE}{m} t^2\]

  parabolic path for \(v \ll c\)
Electromagnetic forces on charged particles

- We therefore reach the important conclusion that
  - magnetic fields cannot be used to change the kinetic energy of a particle
- We must rely on electric fields for particle acceleration
  - acceleration occurs along the direction of the electric field
  - energy gain is independent of the particle velocity
- In accelerators:
  - longitudinal electric fields (along the direction of the particle motion) are used for acceleration
  - magnetic fields are used to bend particles for guidance and focusing

- There are many possibilities, depending on existence and time-dependence of $\vec{E}$ and $\vec{B}$ fields.
  For example, if there is no magnetic field $\vec{B} = 0$ and a time-independent electric field along the z-axis, then electrostatic accelerator. If the electric field is time-dependent, then LINAC.
- If $\vec{E} = 0$ and $B_\theta = B_r = 0$ and $B_z \perp \dot{\vec{v}}$ then circular motion with $\omega_c = \dot{\theta} = \frac{q \cdot B_z}{m}$ $\omega_c =$ cyclotron frequency
- If $\rho$ radius of curvature, then $p = q \cdot B_z \cdot \rho$ or $p[MeV/c] = 300[MeV/c] \cdot B_z[T] \cdot \rho[m]$
Motion in a uniform, constant magnetic field
Constant energy with spiraling along a uniform magnetic field

\[
m_0 \cdot \gamma \cdot v^2 \rho = q \cdot v \cdot B \quad \Rightarrow
\]

\[
(a) \quad \rho = \frac{m_0 \cdot \gamma \cdot v}{q \cdot B}
\]

\[
(b) \quad \omega = \frac{v}{\rho} = \frac{q \cdot B}{m_0 \cdot \gamma} \quad \Rightarrow \quad \omega = \frac{q \cdot B \cdot c^2}{E} = \frac{v}{\rho}
\]
The Cyclotron

- This is a constant frequency orbital accelerator, but one in which the orbit radius increases. Cyclotron angular frequency given by:

\[ \omega_c = \frac{q \cdot B}{m} \quad \text{independent of particle velocity} \]

- Acceleration occurs, provided the synchronism condition, RF frequency of the source matches the cyclotron frequency \( \omega_{RF} \sim \omega_c \), is met.

- Continues gaining velocity until spirals out \( r = m \cdot \frac{v}{e} \cdot B \)

Radius increment per turn decreases with increasing energy because the revolution time must stay constant.

- Correct for low energy \((\gamma \sim 1)\) independent of \( \vec{p} \) - earlier ones were proton accelerators for a few MeV!

- As the mass increases \((m_0 \gamma)\), orbital frequency changes and resonance condition is no longer fulfilled. To overcome this, either
  - Modulate the frequency → ‘synchrocyclotron’ or
  - Allow \( B_z \) to increase with \( R \), to keep \( \omega_c = \text{constant} \). However, we will see this is unstable \((n < 0 \text{ and axial motion is unstable})\). This can be restored by abandoning cylindrical symmetry of B field, i.e. magnet is now split into segments, and using the focusing of the magnet edges → ‘sector focused cyclotron’.

**Typical parameters:** \( B = 1.5 \text{ T}, \omega = 50 \text{ MHz}, U = 200-500 \text{ kV}, I \text{ [mA]} \)

\[ E_{\text{proton}} = 20-30 \text{ MeV} \]
Basics – Cyclotron frequency and K-value

- **Cyclotron frequency (homogenous) B-field**
  \[
  \omega_c = \frac{e \cdot B}{\gamma \cdot m_0}
  \]

- **Cyclotron K-value**
  - $K$ is the relativistic kinetic energy reached for protons from bending strength:
  \[
  p^2 = m_0^2 c^2 (\gamma^2 - 1) = m_0 \cdot m_0 c^2 (\gamma - 1)(\gamma + 1) = m_0 T_{kin} (\gamma + 1) \quad \rightarrow \quad T_{kin} = \frac{p^2}{m_0 (\gamma + 1)}
  \]
  \[
  \frac{T_{kin}}{A} = \frac{p^2}{(\gamma + 1)m_u} \cdot \frac{1}{A^2} = \frac{B^2 \cdot \rho^2 \cdot q^2}{(\gamma + 1)m_u} \cdot \frac{1}{A^2}
  \]
  \[
  \frac{T_{kin}}{A} = \frac{(B \cdot \rho)^2 \cdot e^2}{(\gamma + 1)m_u} \left( \frac{q}{A} \right)^2 = K \cdot \left( \frac{q}{A} \right)^2
  \]
  - $K$ can be used to rescale the energy reach of protons to other charge-to-mass ratios ($q/A$)
  - $K$ in [MeV] is often used for naming cyclotrons
  
  example: K-130 cyclotron, Jyväskylä
During time $\Delta t$, the **velocity vector** rotates through the exact same angle $\Delta \theta$. The velocity magnitude doesn’t change. So the change in the velocity vector is $\Delta v = v \cdot \Delta \theta$.

The velocity magnitude doesn’t change. So the change in the velocity vector is $\Delta v = v \cdot \Delta \theta$.

The same statements are true about the **momentum vector**, which is parallel to the velocity vector. So the change in the momentum vector is $\Delta p = p \cdot \theta$.

Then $F = \frac{d\vec{p}}{dt} \rightarrow q \cdot v \cdot B = \frac{\Delta p}{\Delta t} = \frac{p \cdot \Delta \theta}{\Delta t} = \frac{p \cdot (v \cdot \Delta t/R)}{\Delta t} \rightarrow q \cdot B = \frac{p}{R}$

So to get high momentum, one needs a strong magnet.

It’s hard to get the field strength more than 1.5 T, because iron saturates. So one must increase the diameter.

With a 2 meter diameter one obtains $p = 450 \text{ MeV/c}$. For protons, that’s about half the speed of light, where relativity starts to become noticeable.
27-inch Cyclotron with Lawrence and Livingstone
Ideally, the “184 inch” cyclotron with a field of 2.2 Tesla and orbital radius of 2.08 meters would get to $p = 1373$ MeV/c. Since $pc = \beta \gamma mc^2$ and $mc^2 = 938.6$ MeV for protons, $\beta \gamma = \frac{1373}{938.6} = 1.463$ and $\gamma = \sqrt{1 + (\beta \gamma)^2} = 1.772$

That means the kinetic energy would be 0.772 times $mc^2$, or 724 MeV.

But, the period would be 77% longer at the maximum radius than it was at small radius. So particles would get out of phase rapidly and the acceleration stops.
In case $\gamma > 1$:

$$\omega_c = \frac{q \cdot B_z(r)}{m_0 \cdot \gamma}$$

$K = 592$ MeV
$I = 2$ mA

1.3 MW
Acceleration Scheme of Uranium

238U^{35+} 5.6 kV extraction

18GHZ ECRIS → RILAC → CSM → 18GHZ SuperECRIS

1.5 kV extraction

Charge strip with 14mg/cm² C plate
71 + 50 MeV/u → 86 + 46 MeV/u

RRC H=9 → IRC H=7 → SRC H=6

10.75 MeV/u → 114 MeV/u

Charge strip with 300μg/cm² C foil
35+ 10.75 MeV/u → 71+ 10.6 MeV/u

50 MeV/u → 345 MeV/u

Stripper