While terminal voltages of 20 MV provide sufficient beam energy for nuclear structure research, most applications nowadays require beam energies > 1 GeV

How do we attain higher beam energies?

Analogy: How to swing a child?
- Pull up to maximum height and let go: difficult and tiring (electrostatic accelerator)
- Repeatedly push in synchronism with the period of the motion
Two approaches for accelerating with time-varying fields
- Make an electric field along the direction of particle motion with Radio-Frequency (RF) cavities

**Linear Accelerators**
Use many accelerating cavities through which the particle beam passes once.

**Circular Accelerators**
Use one or a small number of RF accelerating cavities and make use of repeated passage through them: This approach leads to circular accelerators:
- Cyclotrons, synchrotrons and their variants.
Radio-Frequency Accelerators

- The electric field is no longer static but sinusoidal alternating half periods of acceleration and deceleration.

\[ V(t) = V_0 \cdot \sin \omega t \]

\[ E(t) = (V_0 / g) \cdot \sin \omega t \]

- Three important aspects of an RF linear accelerator
  - Particles must arrive bunched in time in order for efficient acceleration
  - Acceleration gaps must be spaced, so that the particle “bunches” arrive at the acceleration phase:

\[ L = v \cdot T/2 = \beta c \frac{1}{2} \frac{\lambda}{c} = \beta \frac{\lambda}{2} \]

- The acceleration field is varying while the particle is in the gap; energy gain is more complicated than in the static case.
Acceleration in the Wideroe structure

Energy gained after \( n \) acceleration gaps:

\[
E_n = n \cdot q \cdot U_0 \cdot \sin \Psi_s
\]

Kinetic energy of the particles:

\[
E_n = \frac{1}{2} m \cdot v_n^2 \quad \text{(valid for non-relativistic particles)}
\]

Velocity of the particles:

\[
v_n = \sqrt{\frac{2E_n}{m}} = \sqrt{\frac{2 \cdot n \cdot q \cdot U_0 \cdot \sin \Psi_s}{m}}
\]

Shielding of the particles during the negative half wave of the RF

Length of the \( n \)-th drift tube:

\[
l_n = v_n \cdot \frac{\tau_{RF}}{2} = v_n \cdot \frac{1}{2 \cdot v_{RF}}
\]
Linear accelerator

ions

ion beam

acceleration electrodes

acceleration only between the electrodes

\[ U \approx \]

\[ l_1 \]

\[ l_2 \]

\[ l_3 \]
Electromagnetic wave is traveling, pushing particles along with it.

Positively charged particles close to the crest of the E-M wave experience the most force forward; those closer to the centre experience less of a force. The result is that the particles tend to move together with the wave.
Principle of the acceleration

1. Positive particles just sitting there

Electromagnetic wave is traveling, pushing particles along with it

Electromagnetic Wave as seen from above (red is +, blue -)

Moving electric wave

Positively charged particles close to the crest of the E-M wave experience the most force forward; those closer to the center experience less of a force. The result is that the particles tend to move together with the wave.
Wideroe structure at GSI

27 MHz Radio frequency
● The Wideroe linac is only efficient for low-energy heavy ions
● When using 10 MHz frequency, the length of the drift tubes becomes prohibitive for high-energy protons

● **Alvarez accelerator = resonant cavity**
  Standing waves with E-field along direction of particle motion. While the electric fields point in the “wrong direction” the particles are shielded by the drift tubes.
  The accelerator consists of a long “tank” (radius determines frequency). Drift tubes are placed along the beam axis, so that the accelerating gaps satisfy synchronicity condition with drift tube length $L$ given by $L = \beta \lambda_0$ where $\lambda_0$ is the free space wavelength at the operating frequency.
Principal of an accelerated particle package

moving wave

standing wave
Suppose we couple two RF cavities together:
- Each is an electrical oscillator with the same resonant frequency
- A beampipe couples the two cavities

Remember the case of mechanical coupling of two oscillators:
Two mechanical modes are possible:
- The “zero-mode”: \( \phi_A - \phi_B = 0 \), where each oscillates at natural frequency
- The “pi-mode”: \( \phi_A - \phi_B = \pi \), where each oscillates at a higher frequency

Standing wave structures of coupled cavities are all driven so that the beam sees either the zero or \( \pi \) mode.
Standing wave cavities

The mode names correspond to the phase difference from one cell to the next.
As linacs are dominated by cavities, circular machines are dominated by magnets

- Both the accelerating field frequency and the magnetic field strength change synchronously with time to match energy and keep revolution radius constant.
- Magnetic field produced by several bending magnets increases with momentum. For high energy:
  \[ E_{\text{proton}}[\text{GeV}] \approx 0.3 \cdot B \rho [\text{T} \cdot \text{m}] \]
- Practical limitations for magnetic field → high energies only at large radius.

  example: 100 GeV protons
  - Fe-magnet \( B \sim 1.5 \text{ T} \rightarrow R = 222 \text{ m} \)
  - superconductive magnet \( B \sim 5 \text{ T} \rightarrow R = 67 \text{ m} \)
The bending field changes with particle beam energy to maintain a constant radius:

\[ \frac{1}{\rho[m]} = 0.3 \frac{B[T]}{\beta E[GeV]} = 0.3 \frac{B[T]}{cp[GeV]} \]

So \( B \) ramps in proportion to the momentum. The revolution frequency also changes with momentum.

The synchronicity condition, including now the relativistic term, is

\[ \omega = \frac{qB}{m\gamma} \]

For an electron synchrotron, the injected beam is already relativistic, so only the magnetic field changes with beam energy.

For a proton synchrotron, the injected beam is not yet relativistic, so the RF accelerating frequency and the magnetic field both ramp with energy.
**SIS - SchwerIonenSynchrotron**

- **Eff. Puls width for injection:** 47 μs, 36.2% efficiency

- **Deflecting magnets** (≤ 1.8 T)

- **Focussing magnets**

- **Acceleration:** > 100,000 turns/s

- **Period of one revolution:** 4.7 μs

- **10 turns will be accepted for injection**

**UNILAC experiment**

- **U = 216.72 m**

### Ion Number of injections | Intensity [spill⁻¹] at FRS | Ion source | Date
--- | --- | --- | ---

- **⁵⁸Ni**
  - 1
  - 6 * 10⁸
  - MEVVA
  - 3.2006

- **¹⁰⁷Ag**
  - 1
  - 3 * 10⁹
  - MEVVA
  - 2.2006

- **¹²⁴Xe**
  - 4
  - 5 * 10⁹
  - MUCIS
  - 7.2006

- **¹³⁶Xe**
  - 4
  - 5 * 10⁹
  - MEVVA
  - 7.2006

- **²⁰⁸Pb**
  - 30
  - 1.3 * 10⁹
  - PIG
  - 3.2006

- **²³⁸U**
  - 1
  - 2.0 * 10⁹
  - PIG
  - 9.2009

**Intensity [s⁻¹] = 0.5 * Intensity [spill⁻¹]**