Experimental Storage Ring – ESR $E_{\text{max}} = 420 \text{ MeV/u, 10} \text{Tm}$
Specification of the ESR

Particle detectors

Re-injection to SIS

Schottky pick-ups

Gas jet

Two 5 kV rf-cavities

Fast Injection

e⁻ cooler

I = 10...500 mA

Six 60° dipoles

Extraction

L = 108 m = 1/2 L_{SIS}

p = 2 \cdot 10^{-11} \text{ mbar}

E = 3...420 \text{ MeV/u}

f \approx 1...2 \text{ MHz}

β = 0.08...0.73

Q_{h,v} \approx 2.65
Bunch dimensions

- For uniform charge distributions we may use “hard edge values”
- For Gaussian charge distributions use rms values $\sigma_x, \sigma_y, \sigma_z$

We will discuss measurements of bunch size and charge distribution later
But rms values can be misleading

Gaussian beam

Beam with halo

We need to measure the particle distribution!
Each of $N_b$ particles is tracked in ordinary 3D-space

Not too helpful!
Configuration space

$6 \, N_b$-dimensional space for $N_b$ particles; coordinates $(x_i, p_i)$, $i = 1, \ldots, N_b$

The bunch is represented by a single point that moves in time

*Useful for Hamiltonian dynamics*
Configuration space example:
1 particle in an harmonic potential

\[ F_x = -kx = m\ddot{x} \]

But for many problems this description carries much more information than needed:

We don’t care about each of \(10^{10}\) individual particles

*But seeing both \(x\) & \(p_x\) looks useful*
Phase space (gas space in statistical mechanics)

6-dimensional space for $N_b$ particles
The $i^{th}$ particle has coordinates $(x_i, p_i)$, $i = x, y, z$
The bunch is represented by $N_b$ points that move in time

In most cases, the three planes are to very good approximation decoupled
$\Rightarrow$ One can study the particle evolution independently in each planes
The set of possible states for a system of \( N \) particles is referred as an \textit{ensemble} in statistical mechanics.

In the statistical approach, particles lose their individuality.

Properties of the whole system are fully represented by \textit{particle density functions} \( f_{6D} \) and \( f_{2D} \):

\[
f_{6D}(x, p_x, y, p_y, z, p_z) \, dx \, dp_x \, dy \, dp_y \, dz \, dp_z \quad f_{2D}(x_i, p_i) \, dx_i \, dp_i \quad i = 1, 2, 3
\]

where

\[
\int f_{6D} \, dx \, dp_x \, dy \, dp_y \, dz \, dp_z = N
\]
Longitudinal phase space

- In most accelerators the phase space planes are only weakly coupled.  
  → Treat the longitudinal plane independently from the transverse one  
  → Effects of weak coupling can be treated as a perturbation of the uncoupled solution

- In the longitudinal plane, electric fields accelerate the particles  
  → Use energy as longitudinal variable together with its canonical conjugated time

- Frequently, we use relative energy variation $\delta$ and relative time $\tau$ with respect to a reference particle

\[
\delta = \frac{E - E_0}{E_0}, \quad \tau = t - t_0
\]

- According to Liouville, in the presence of Hamiltonian forces, the area occupied by the beam in the longitudinal phase space is conserved
Transverse phase space

- For transverse planes \( \{x, p_x\} \) and \( \{y, p_y\} \), use a modified phase space, where the momentum components are replaced by:

\[
p_{x_i} \rightarrow x' = \frac{dx}{ds} \quad \quad p_{y_i} \rightarrow y' = \frac{dy}{ds}
\]

where \( s \) is in the direction of motion

- We can relate the old and new variables (for \( B_z \neq 0 \))

\[
p_i = \gamma \cdot m_0 \frac{dx_i}{dt} = \gamma \cdot m_0 v_s \frac{dx_i}{ds} = \gamma \cdot \beta \cdot m_0 c x'_i \quad i = x, y
\]

where \( \beta = \frac{v_s}{c} \) and \( \gamma = (1 - \beta^2)^{-1/2} \)

Note: \( x_i \) and \( p_i \) are canonical conjugate variables while \( x \) and \( x' \) are not, unless there is no acceleration (\( \gamma \) and \( \beta \) constant)
Emittance describes the area in phase space of the ensemble of beam particles.

RMS emittance

\[ \varepsilon^2 \equiv R^2 (V^2 - (R')^2) / c^2 \]
Why is emittance an important concept

1) Liouville: Under conservative forces phase space evolves like an incompressible fluid ⇒

2) Under linear forces macroscopic (such as focusing magnets) & $\gamma = \text{constant}$ emittance is an invariant of motion

3) Under acceleration $\gamma \varepsilon = \varepsilon_n$ is an adiabatic invariant

Is there any way to decrease the emittance?

This means taking away mean transverse momentum but keeping mean longitudinal momentum
What is beam cooling?

Beam cooling is synonymous for a reduction of beam temperature

Temperature is equivalent to terms as
  phase space volume, emittance and momentum spread

Beam cooling processes are not following Liouville’s Theorem:
  (which neglects interactions between beam particles)
  “In a system where the particle motion is controlled by external conservative forces the phase density is conserved”

Beam cooling techniques are non-Liouvillean processes
  e.g. interaction of beam particles with other particles (electrons, photons)

❖ Benefit of beam cooling:
  • Improved beam quality (precision experiments, luminosity increase)
Beam cooling at the ESR

What is cooling? What is temperature?

\[
\frac{3k}{2} \cdot T_{\|} = \frac{1}{2} m \cdot \langle \vec{v}_{\|}^2 \rangle
\]

\( v \) is the velocity relative to a reference particle, which moves with an average ion-velocity. The temperature is a measure of the random movement.

In an accelerator

\[
T_{\|} = M \cdot c^2 \cdot \beta^2 \cdot \langle \Delta p / p \rangle^2
\]

\[
T_{\perp} = M \cdot c^2 \cdot \beta^2 \cdot \gamma^2 \cdot \epsilon \left( \frac{1}{\langle \beta_{\|} \rangle} + \frac{1}{\langle \beta_{\perp} \rangle} \right)
\]

Why beam cooling?

Improve of the beam quality
- smaller beam size and reduction of the emittance
- broadening of the energy
- better beam intensity, accumulation
- lifetime of the beam
Beam temperature

Where does the beam temperature originate from?
The beam particles are generated in a ‘hot’ source

Thermal particle motion (temperature is conserved)

\[
\begin{align*}
\text{longitudinal} & \quad \frac{1}{2} k_B T_\parallel = \frac{1}{2} m v_\parallel^2 = \frac{1}{2} m c^2 \beta^2 \left( \frac{\delta p_\parallel}{p} \right)^2 \\
\text{transverse} & \quad \frac{1}{2} k_B T_\perp = \frac{1}{2} m v_\perp^2 = \frac{1}{2} m c^2 \beta^2 \gamma^2 \theta_\perp^2
\end{align*}
\]
Benefits of beam cooling

- Improve beam quality
  - Precision experiments
  - Luminosity increase

- Compensation of heating
  - Experiments with internal target
  - Colliding beams

- Intensity increase by accumulation
  - Weak beams from source can be increased
  - Secondary beams (antiprotons, rare isotopes)
Electron cooling

\[ V_{el} = V_{ion} \]
\[ E_e = \frac{m_e}{M_{ion}} \cdot E_{ion} \]

E.g.: 200 keV electrons cool 400 MeV/u ions

Electron temperature:
\[ k_B T_\perp \approx 0.1 \text{ eV} \]
\[ k_B T_\parallel \approx 0.1 \text{ – } 1 \text{ meV} \]

Momentum transfer by Coulomb collisions
cooling force results from energy loss in the co-moving gas of free electrons
Characteristics of electron cooling force

Analogy: energy loss in matter (electrons in the shell)

\[ F(v_{\text{ion}}) = -\frac{4\pi \cdot Q^2 e^4 \cdot n_e}{(4\pi \varepsilon_0)^2 \cdot m_e} \int L_c(v_{\text{rel}}) \cdot f(v_e) \frac{v_{\text{rel}}}{v_{\text{rel}}^3} d^3v_e \]

\[ \bar{v}_{\text{rel}} = v_{\text{ion}} - v_e \]

Cooling force \( F \)

- for small relative velocity: \( \propto v_{\text{rel}} \)
- for large relative velocity: \( \propto v_{\text{rel}}^{-2} \)

Increase with charge: \( \propto Q^2 \)

Maximum cooling force at effective electron temperature
Simple derivation of electron cooling force

Analogy: energy loss in matter (electrons in the shell)

Rutherford scattering: \[ 2 \cdot \tan \left( \frac{\theta}{2} \right) = \frac{2Z_1 Z_2 e^2}{4\pi \varepsilon_0 \Delta p \cdot v \cdot b} \]
\[ Z_1 = Q \, (ion), \quad Z_2 = -1 \, (electron) \]

Energy transfer: \[ \Delta E(b) = \frac{(\Delta p)^2}{2m_e} \approx \frac{2 \cdot Q^2 e^4}{(4\pi \varepsilon_0)^2 m_e v^2} \frac{1}{b^2} \quad \text{(for } b \gg b_{\text{min}} \text{)} \]

Minimum impact parameter: \[ b_{\text{min}} = \frac{Q e^2}{(4\pi \varepsilon_0)^2 m_e v^2} \]
from: \[ \Delta E(b_{\text{min}}) = \Delta E_{\text{max}} \approx m_e v^2 \]

Energy loss: \[ -\frac{dE}{dx} = 2\pi \int_{b_{\text{min}}}^{b_{\text{max}}} b \cdot n_e \cdot \Delta E \, db = \frac{4\pi Q^2 e^4}{(4\pi \varepsilon_0)^2 m_e v^2} n_e \cdot \ln \frac{b_{\text{max}}}{b_{\text{min}}} \]

Coulomb logarithm \[ L_C = \ln(b_{\text{max}}/b_{\text{min}}) \approx 10 \quad \text{(typical value)} \]
Example of electron cooling

transverse cooling at ESR

measured with residual gas ionization beam profile monitor

cooling of 350 MeV/u $\text{Ar}^{18+}$ ions 0.05 A, 192 keV electron beam $n_e = 0.8 \cdot 10^6$ cm$^{-3}$
Electron cooling

The ions get the sharp velocity of electrons, small size and divergence.

G.I. Budker, At. En. 22 (1967) 346
G.I. Budker, A.N. Skrinsky et al., IEEE NS-22 (1975) 2093
Ionization cooling

- Hot muon beam:
  - large transverse momentum
  - cannot fit in the beam pipe in muon accelerator

**Ionization cooling:**

Based on use of ionization energy loss of accelerated charged particles

Reduce the transfers motion and accelerate them in forward direction
Stochastic cooling is in particular efficient for hot ion beams
Principle of ‘stochastic’ cooling

**A Feedback System:** A detector or pick-up which measures the motion of the particle and a corrector, the kicker, which adjusts their angles.

Measures the deviation of the center of gravity of a sample of particles with respect to the requisite orbit and sends an error signal to the kicker.

The kicker applies an electric field to the same sample to correct the deviation measured.
Principle of ‘stochastic’ cooling

Self correction of ion trajectory

Using a pick-up probe, the position of the ion beam is measured at a fixed position via the induced signal. A deviation of the beam from the ideal orbit can be corrected by amplification of this signal.

The amplified signal is now used as a correction signal which acts on the beam at a second position (zero crossing of the betatron function) via a "kicker".

This method was invented for the cooling of hot p(bar) by van der Meer. He showed that after a cooling time of $\tau \propto N/C$ (N: particle number, C = Bandwidth of the amplifier) a momentum width of the beam of about $\Delta p/p \approx 10^{-3}$ can be achieved by stochastic cooling.

Detection of the W boson from $p \leftrightarrow p(bar)$
Stochastic cooling at GSI

fast pre-cooling of hot fragment beams

energy 400 (−550) MeV/u
bandwidth 0.8 GHz (range 0.9-1.7 GHz)

\[
\delta \frac{p}{p} = \pm 0.35\% \quad \rightarrow \quad \delta \frac{p}{p} = \pm 0.01\%
\]

\[
\varepsilon = 10 \cdot 10^{-6} m \quad \rightarrow \quad \varepsilon = 2 \cdot 10^{-6} m
\]
Comparison of Cooling Methods

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</table>
Principle of laser cooling (snowplow)

1. Absorption of photons from a laser beam:
   Energy and momentum must be conserved.

2. Absorption of photons:
   Momentum transfer in a defined direction
   (directed momentum transfer).

3. No defined direction for the spontaneous emission (isotropic re-emission):
   Momentum transfer cancels out over many absorption-emission-cycles.

*typical cooling times ~ 10 μs*
Principle of laser cooling
2-step process

\[ \vec{p}_i = m \vec{v}_i \]
\[ \vec{p} = m \vec{v}_i + \hbar k \]
\[ \vec{p} = m \vec{v}_i + \hbar k' \]

\[ \hbar k \]

Incident photons absorbed: momentum transfer = \( \hbar k \)
Spontaneous emission: total momentum transfer = 0
Net momentum transfer to atom in direction of laser

http://inms-ienm.nrc-cnrc.gc.ca/research/cesium_clock_e.html
Laser cooling at ESR

Argon ion laser (257.3 nm) frequency doubled

Fluorescence light detection
Cooling with the ESR

\[ \frac{\Delta f}{f} = -\frac{1}{\gamma^2} \frac{\Delta (m/q)}{m/q} + \frac{\Delta v}{v} \left(1 - \frac{\gamma^2}{\gamma_t^2}\right) \]
Schottky-Mass-Spectroscopy

4 particles with different m/q
Schottky mass spectroscopy

Sin(\(\omega_1\))

Sin(\(\omega_2\))

Sin(\(\omega_4\))

Fast Fourier Transform

\(\omega_4\)  \(\omega_3\)  \(\omega_2\)  \(\omega_1\)

Indian Institute of Technology Ropar  Hans-Jürgen Wollersheim  -  2017
Small-band Schottky frequency spectra

\[ \text{Intensity / arb. units} \]

\[ \begin{array}{c}
\text{Frequency / Hz} \\
33800, 33900, 34000, 34100, 34200, 34300, 34400, 34500
\end{array} \]

\[ \begin{array}{c}
\text{Intensity / arb. units} \\
0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70
\end{array} \]

\[ 143^m\text{Sm}^{62+} \quad 754 \text{ keV} \quad 143^g\text{Sm}^{62+} \]

(1 particle)

\[ m/\Delta m \approx 700000 \]