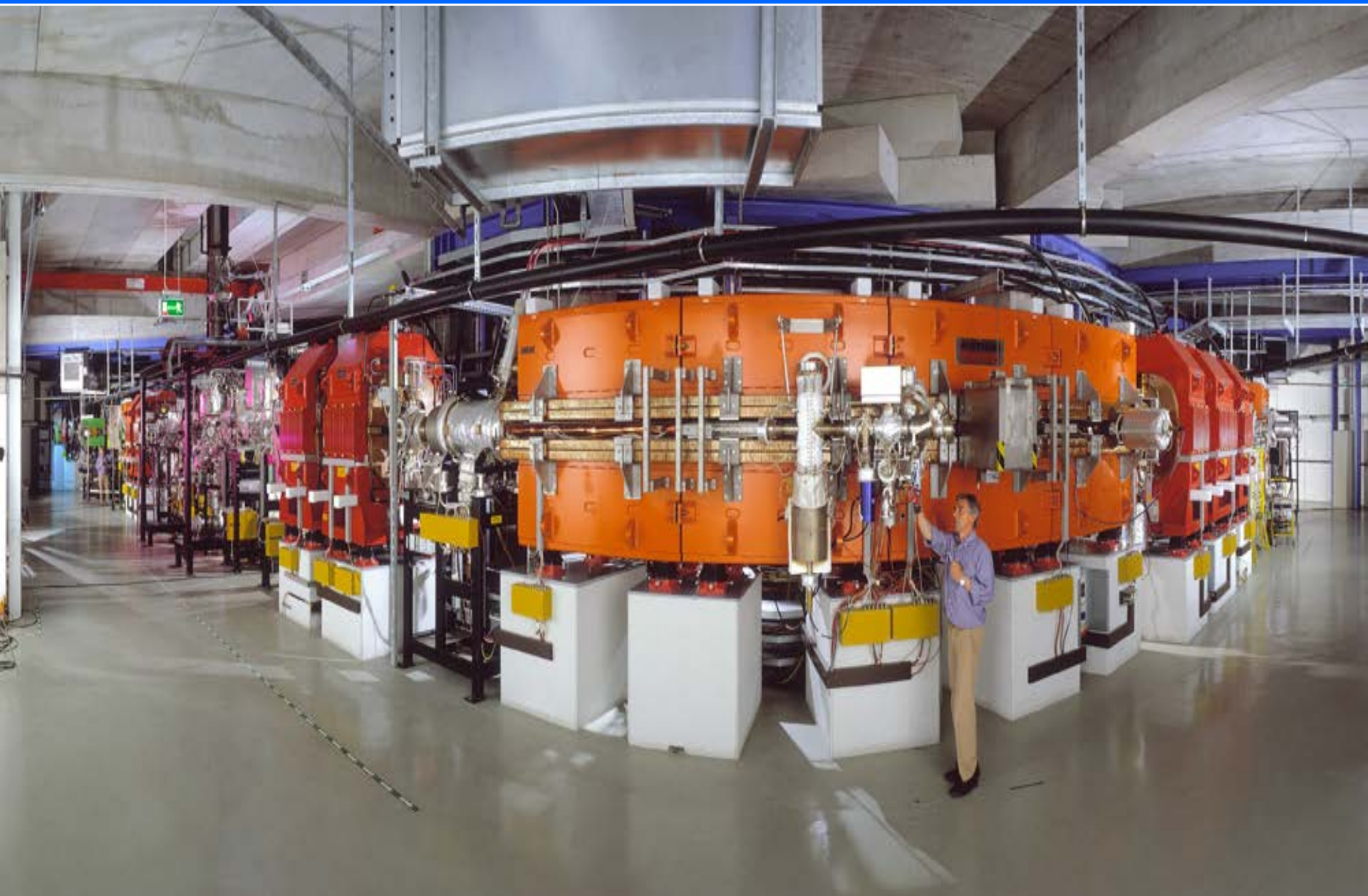
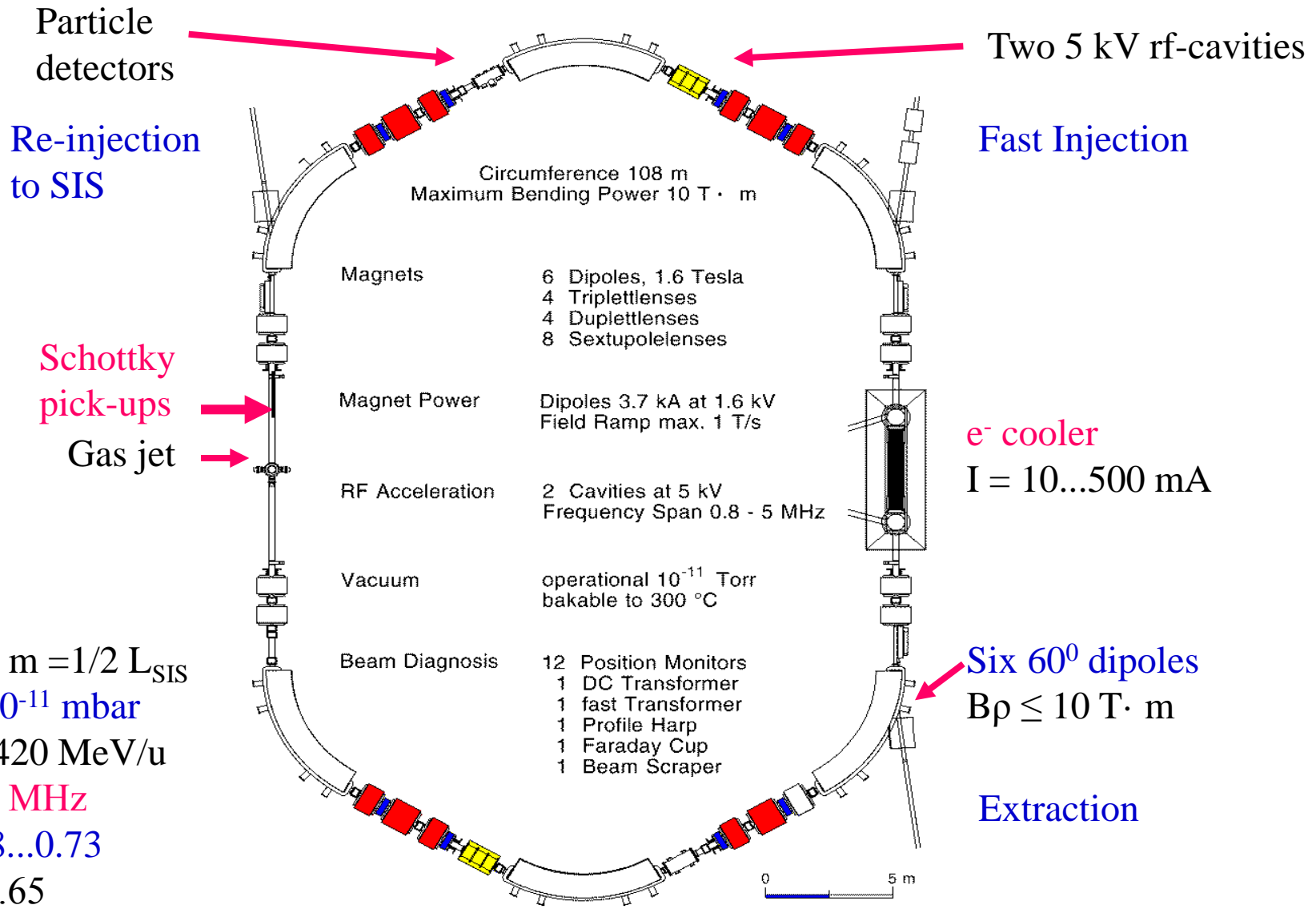


# Experimental Storage Ring – ESR $E_{\max} = 420 \text{ MeV/u}$ , 10 Tm



# Specification of the ESR



$$L = 108 \text{ m} = 1/2 L_{\text{SIS}}$$

$$p = 2 \cdot 10^{-11} \text{ mbar}$$

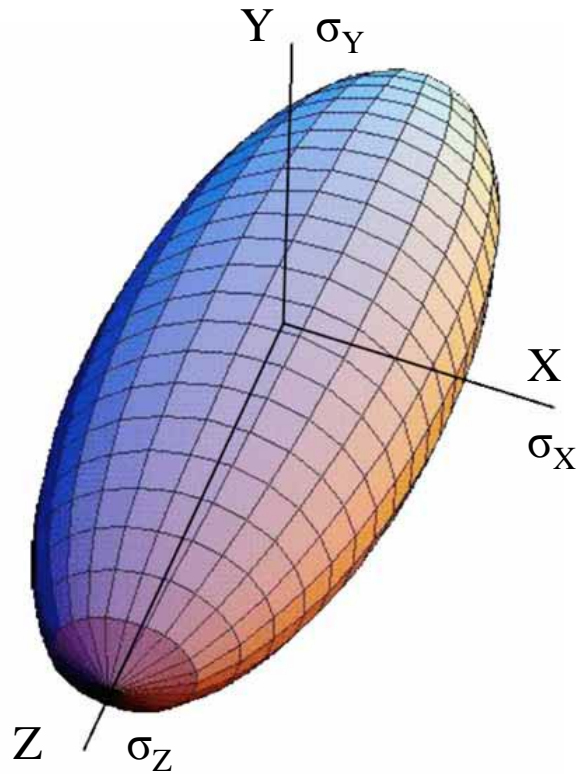
$$E = 3 \dots 420 \text{ MeV/u}$$

$$f \approx 1 \dots 2 \text{ MHz}$$

$$\beta = 0.08 \dots 0.73$$

$$Q_{h,v} \approx 2.65$$

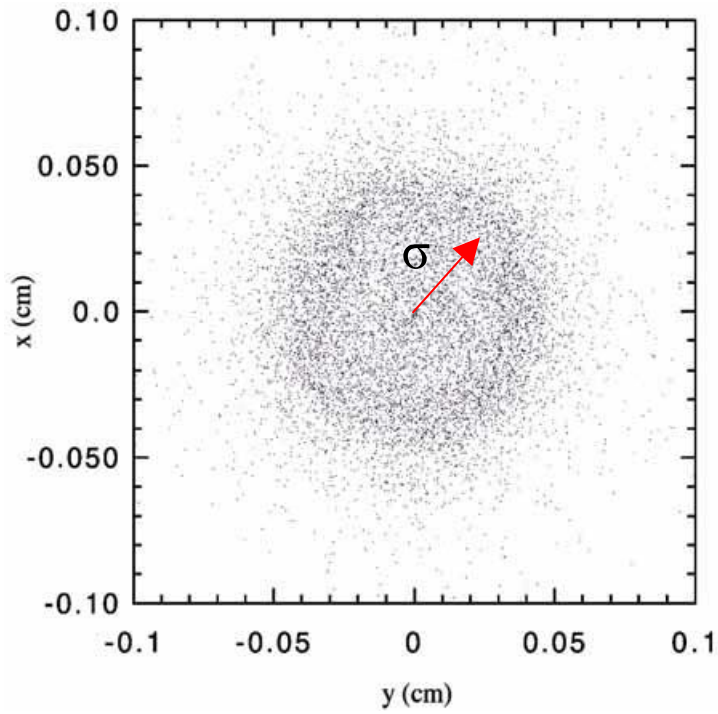
# Bunch dimensions



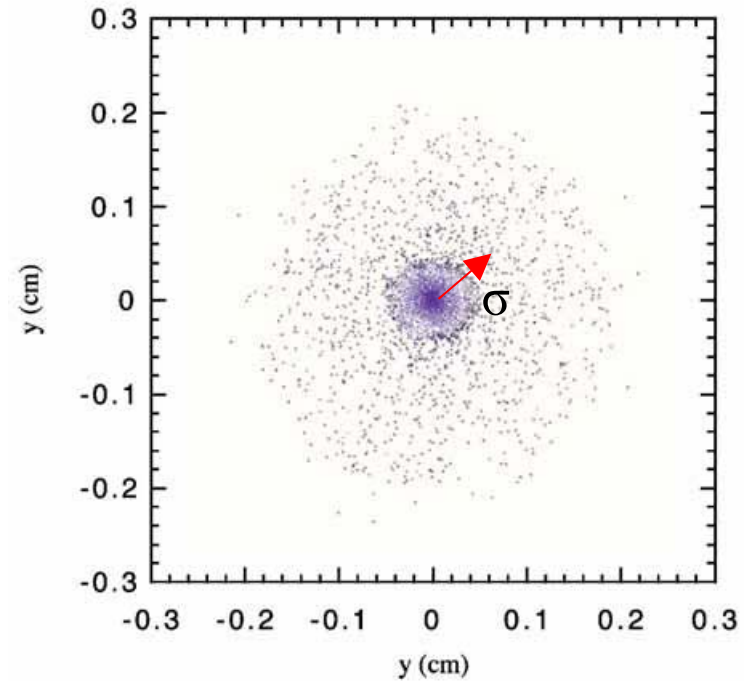
- ❖ For uniform charge distributions we may use “hard edge values”
- ❖ For Gaussian charge distributions use rms values  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$

We will discuss measurements of bunch size and charge distribution later

# But rms values can be misleading



Gaussian beam

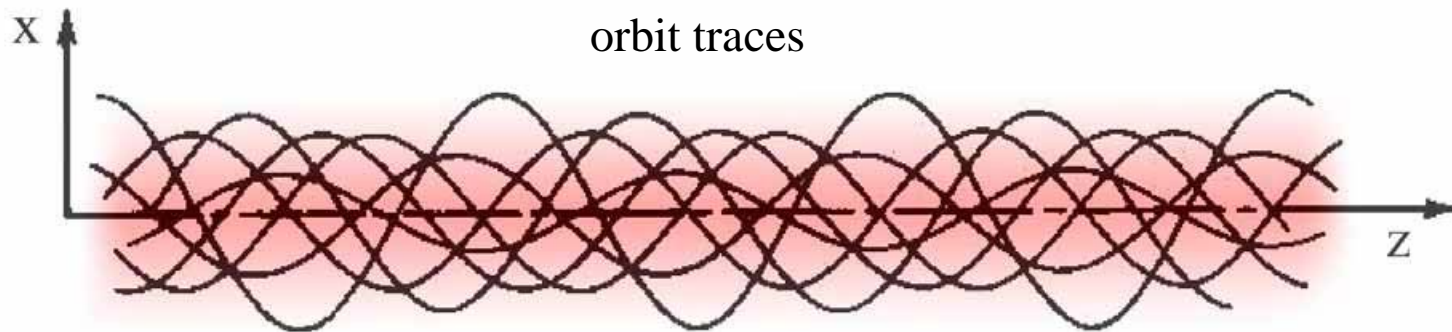


Beam with halo

*We need to measure the particle distribution!*

# Coordinate space

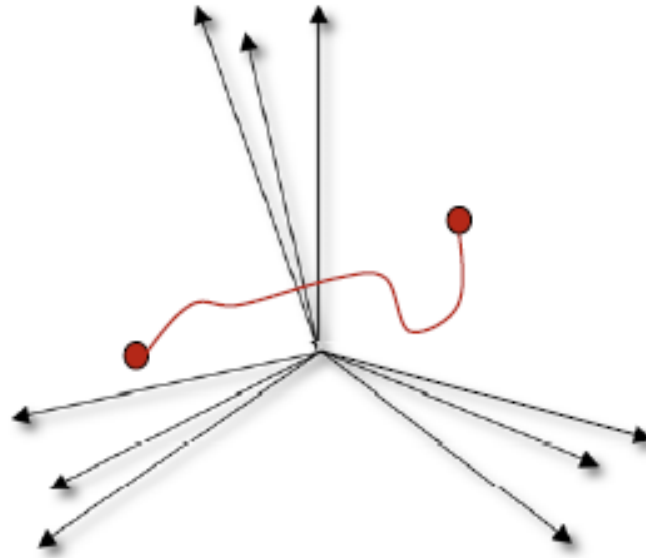
Each of  $N_b$  particles is tracked in ordinary 3D-space



*Not too helpful!*

# Configuration space

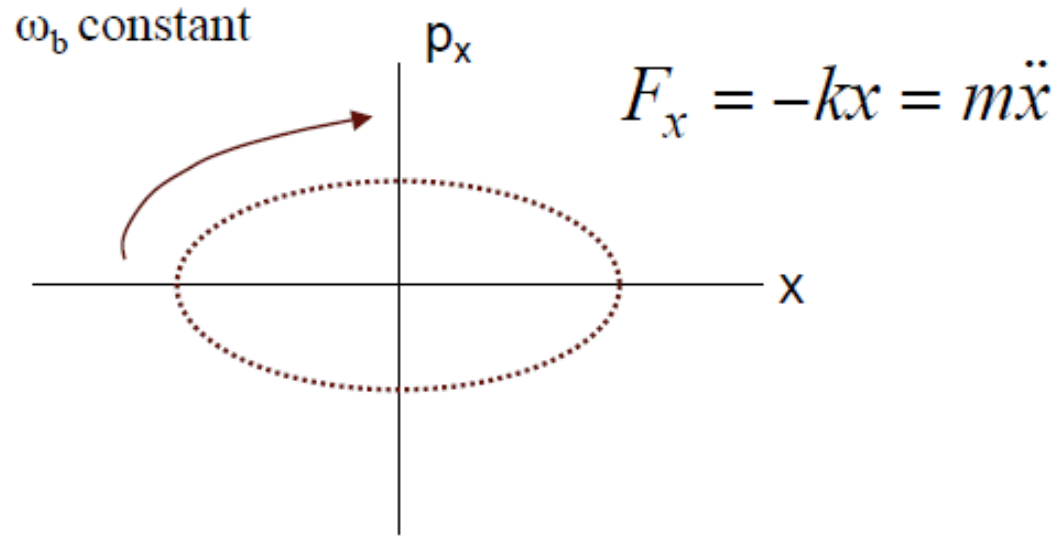
$6 N_b$ -dimensional space for  $N_b$  particles; coordinates  $(x_i, p_i)$ ,  $i = 1, \dots, N_b$   
The bunch is represented by a single point that moves in time



*Useful for Hamiltonian dynamics*

# Configuration space example:

1 particle in an harmonic potential



But for many problems this description carries much more information than needed:

We don't care about each of  $10^{10}$  individual particles

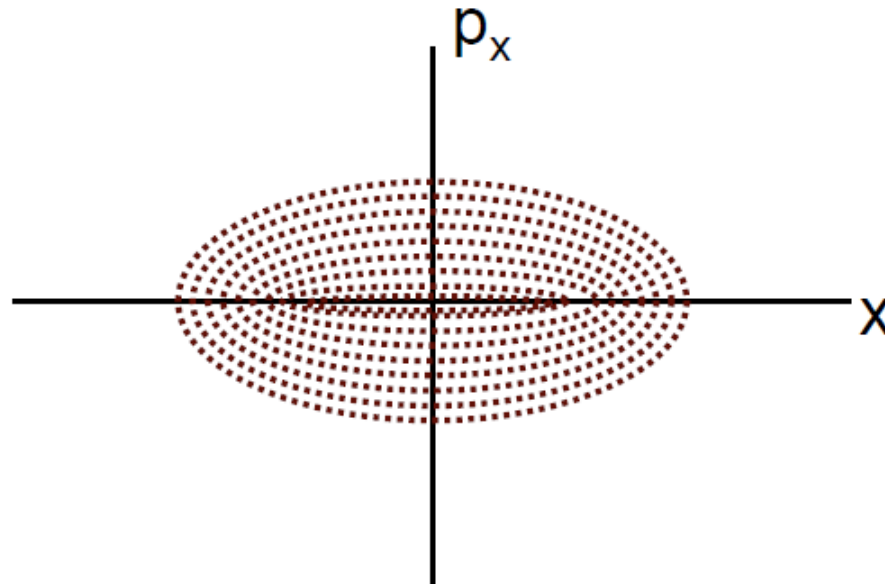
*But seeing both  $x$  &  $p_x$  looks useful*

# Phase space (gas space in statistical mechanics)

6-dimensional space for  $N_b$  particles

The  $i^{\text{th}}$  particle has coordinates  $(x_i, p_i)$ ,  $i = x, y, z$

The bunch is represented by  $N_b$  points that move in time



In most cases, the three planes are to very good approximation decoupled

$\Rightarrow$  *One can study the particle evolution independently in each planes*



# Particle Systems & Ensembles

- ❖ The set of possible states for a system of  $N$  particles is referred as an *ensemble* in statistical mechanics.
- ❖ In the statistical approach, particles lose their individuality.
- ❖ Properties of the whole system are fully represented by *particle density functions*  $f_{6D}$  and  $f_{2D}$ :

$$f_{6D}(x, p_x, y, p_y, z, p_z) dx dp_x dy dp_y dz dp_z \quad f_{2D}(x_i, p_i) dx_i dp_i \quad i = 1, 2, 3$$

where 
$$\int f_{6D} dx dp_x dy dp_y dz dp_z = N$$

# Longitudinal phase space

- ❖ In most accelerators the phase space planes are only weakly coupled.
  - Treat the longitudinal plane independently from the transverse one
  - Effects of weak coupling can be treated as a perturbation of the uncoupled solution
- ❖ In the longitudinal plane, electric fields accelerate the particles
  - Use *energy* as longitudinal variable together with its canonical conjugated *time*
- ❖ Frequently, we use relative energy variation  $\delta$  and relative time  $\tau$  with respect to a reference particle

$$\delta = \frac{E - E_0}{E_0} \qquad \tau = t - t_0$$

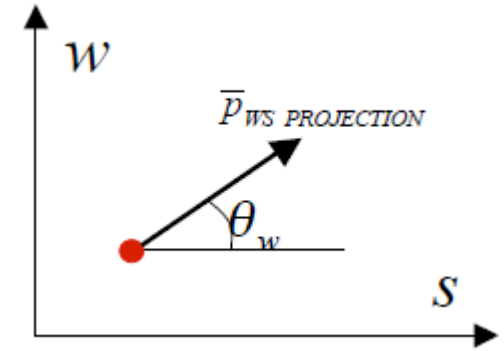
- ❖ According to Liouville, in the presence of Hamiltonian forces, the area occupied by the beam in the longitudinal phase space is conserved

# Transverse phase space

- ❖ For transverse planes  $\{x, p_x\}$  and  $\{y, p_y\}$ , use a modified phase space, where the momentum components are replaced by:

$$p_{x_i} \rightarrow x' = \frac{dx}{ds} \quad p_{y_i} \rightarrow y' = \frac{dy}{ds}$$

where  $s$  is in the direction of motion



- ❖ We can relate the old and new variables (for  $B_z \neq 0$ )

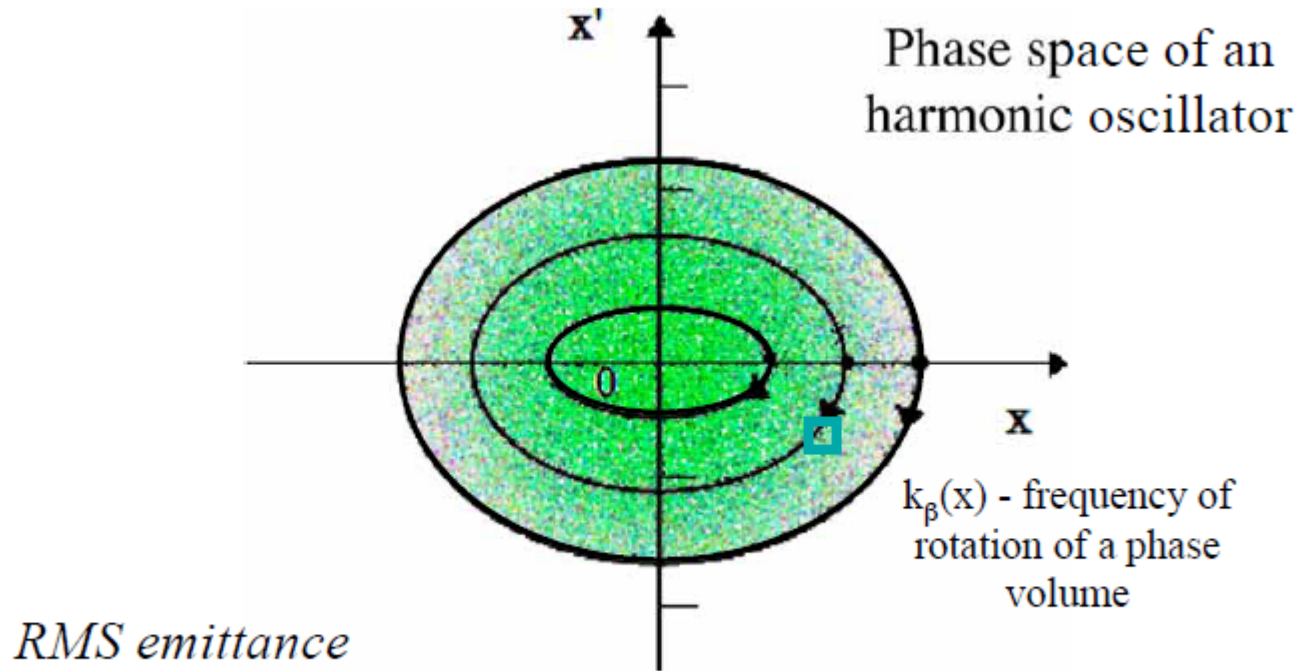
$$p_i = \gamma \cdot m_0 \frac{dx_i}{dt} = \gamma \cdot m_0 v_s \frac{dx_i}{ds} = \gamma \cdot \beta \cdot m_0 c x'_i \quad i = x, y$$

$$\text{where } \beta = \frac{v_s}{c} \text{ and } \gamma = (1 - \beta^2)^{-1/2}$$

Note:  $\mathbf{x}_i$  and  $\mathbf{p}_i$  are canonical conjugate variables while  $\mathbf{x}$  and  $\mathbf{x}'$  are not, unless there is no acceleration ( $\gamma$  and  $\beta$  constant)

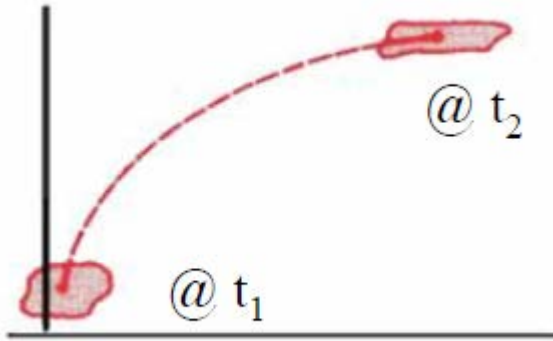
# Emittance describes the area in phase space of the ensemble of beam particles

Emittance - Phase space volume of beam



$$\varepsilon^2 \equiv R^2(V^2 - (R')^2)/c^2$$

# Why is emittance an important concept



- 1) Liouville: Under conservative forces phase space evolves like an incompressible fluid  $\Rightarrow$
- 2) Under linear forces macroscopic (such as focusing magnets) &  $\gamma = \text{constant}$  emittance is an invariant of motion
- 3) Under acceleration  $\gamma\varepsilon = \varepsilon_n$  is an adiabatic invariant

Is there any way to decrease the emittance?

This means taking away mean transverse momentum  
but  
keeping mean longitudinal momentum

# What is beam cooling?

Beam cooling is synonymous for a reduction of beam temperature

Temperature is equivalent to terms as

phase space volume, emittance and momentum spread

Beam cooling processes are not following Liouville's Theorem:

(which neglects interactions between beam particles)

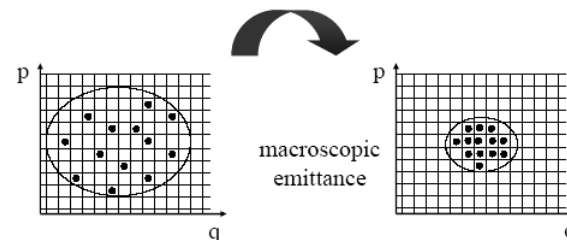
**“In a system where the particle motion is controlled by external conservative forces the phase density is conserved”**

Beam cooling techniques are non-Liouvillian processes

e.g. interaction of beam particles with other particles (electrons, photons)

## ❖ Benefit of beam cooling:

- Improved beam quality (precision experiments, luminosity increase)



# Beam cooling at the ESR

What is cooling?    What is temperature?

$$\left(\frac{3}{2} \cdot k\right) \cdot T_{\perp||} = \frac{1}{2} \cdot m \cdot \langle \tilde{v}_{\perp||}^2 \rangle$$

$\tilde{v}$  is the velocity relative to a reference particle, which moves with an average ion-velocity.  
The temperature is a measure of the random movement.

In an accelerator

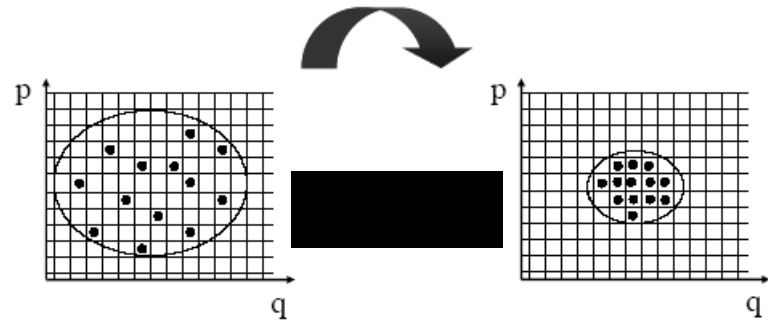
$$T_{||} = M \cdot c^2 \cdot \beta^2 \cdot \langle \Delta p / p \rangle^2$$

$$T_{\perp} = M \cdot c^2 \cdot \beta^2 \cdot \gamma^2 \cdot \varepsilon \left( \frac{1}{\langle \beta_H \rangle} + \frac{1}{\langle \beta_V \rangle} \right)$$

Why beam cooling?

Improve of the beam quality

- smaller beam size and reduction of the emittance
- broadening of the energy
- better beam intensity, accumulation
- lifetime of the beam

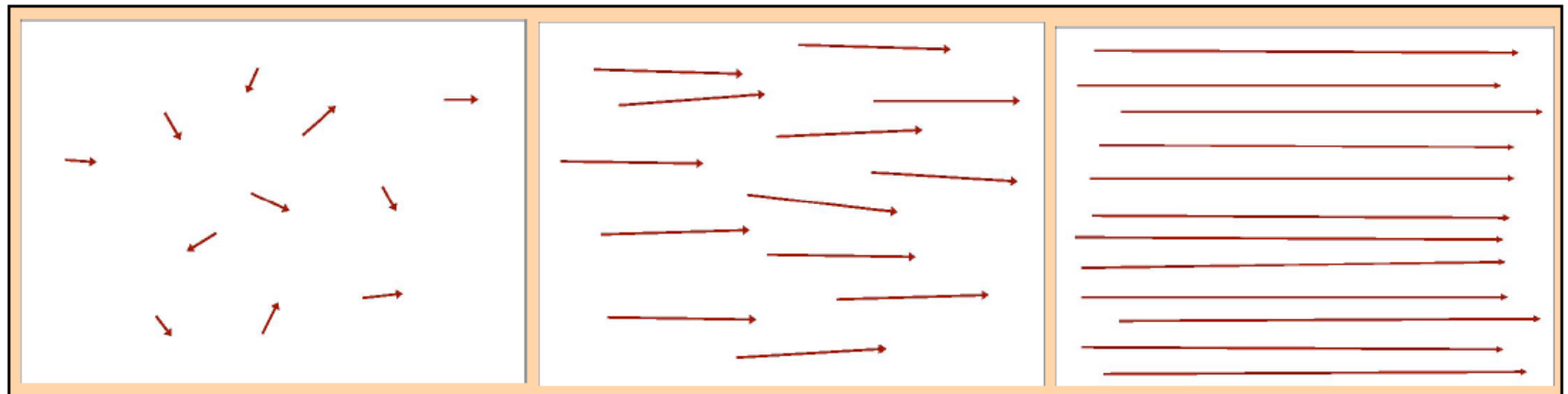


# Beam temperature

Where does the beam temperature originate from?

The beam particles are generated in a 'hot' source

Thermal particle motion (temperature is conserved)



at rest (source)

low energy

high energy

temperature

$$\text{longitudinal} \quad \frac{1}{2} k_B T_{\parallel} = \frac{1}{2} m v_{\parallel}^2 = \frac{1}{2} m c^2 \beta^2 \left( \frac{\delta p_{\parallel}}{p} \right)^2$$

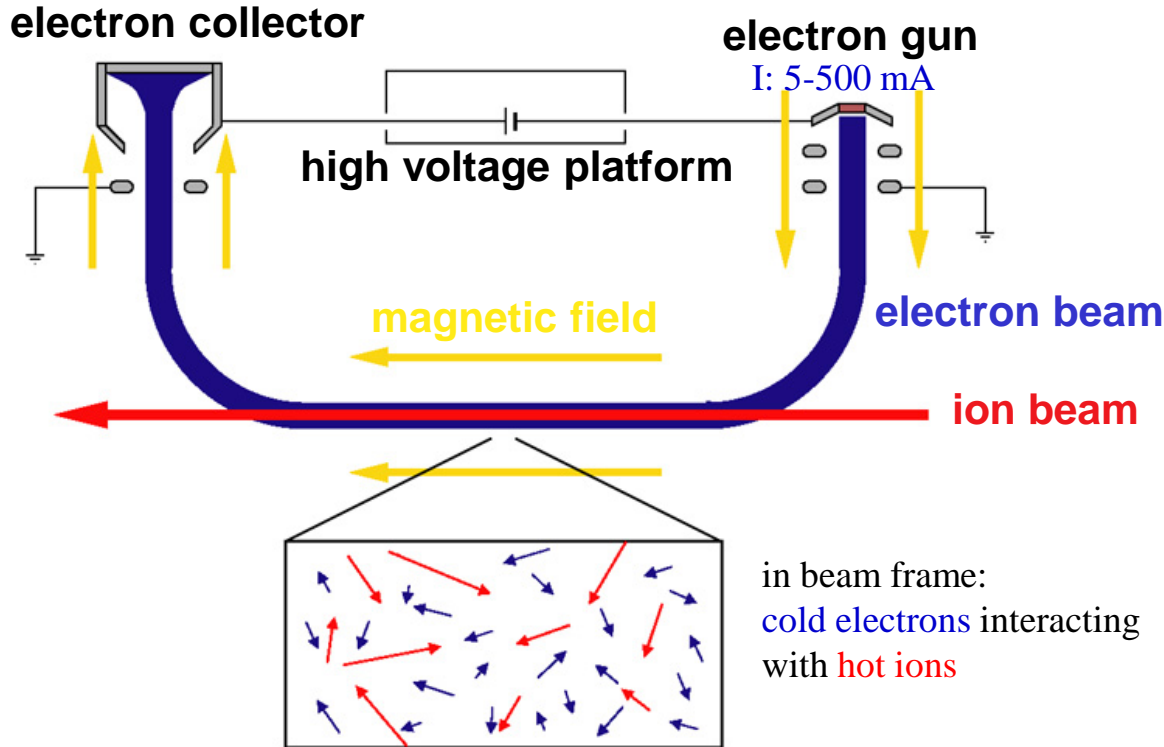
$$\text{transverse} \quad \frac{1}{2} k_B T_{\perp} = \frac{1}{2} m v_{\perp}^2 = \frac{1}{2} m c^2 \beta^2 \gamma^2 \theta_{\perp}^2$$



# Benefits of beam cooling

- ❖ Improve beam quality
  - Precision experiments
  - Luminosity increase
  
- ❖ Compensation of heating
  - Experiments with internal target
  - Colliding beams
  
- ❖ Intensity increase by accumulation
  - Weak beams from source can be increased
  - Secondary beams (antiprotons, rare isotopes)

# Electron cooling



$$v_{el} = v_{ion}$$

$$E_e = m_e/M_{ion} \cdot E_{ion}$$

e.g.: 200 keV electrons  
cool 400 MeV/u ions

electron temperature:

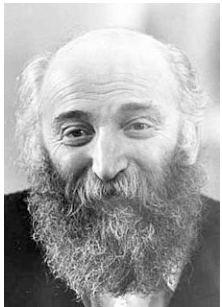
$$k_B T_{\perp} \approx 0.1 \text{ eV}$$

$$k_B T_{\parallel} \approx 0.1 - 1 \text{ meV}$$

superposition of a cold  
intense electron beam  
with the same velocity

momentum transfer by Coulomb collisions

cooling force results from energy loss  
in the co-moving gas of free electrons



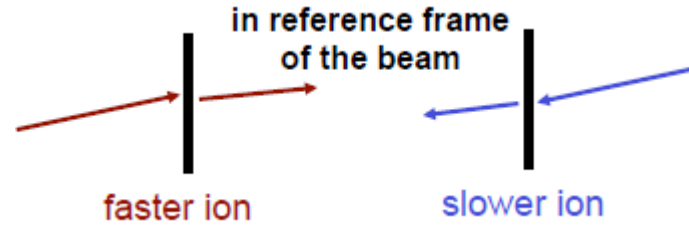
G. Budker

G.I. Budker, At. En. 22 (1967) 346

G.I. Budker, A.N. Skrinsky et al., IEEE NS-22 (1975) 2093

# Characteristics of electron cooling force

Analogy: energy loss in matter  
(electrons in the shell)



$$\vec{F}(\vec{v}_{ion}) = -\frac{4\pi \cdot Q^2 e^4 \cdot n_e}{(4\pi\epsilon_0)^2 \cdot m_e} \int L_C(\vec{v}_{rel}) \cdot f(\vec{v}_e) \frac{\vec{v}_{rel}}{v_{rel}^3} d^3\vec{v}_e$$

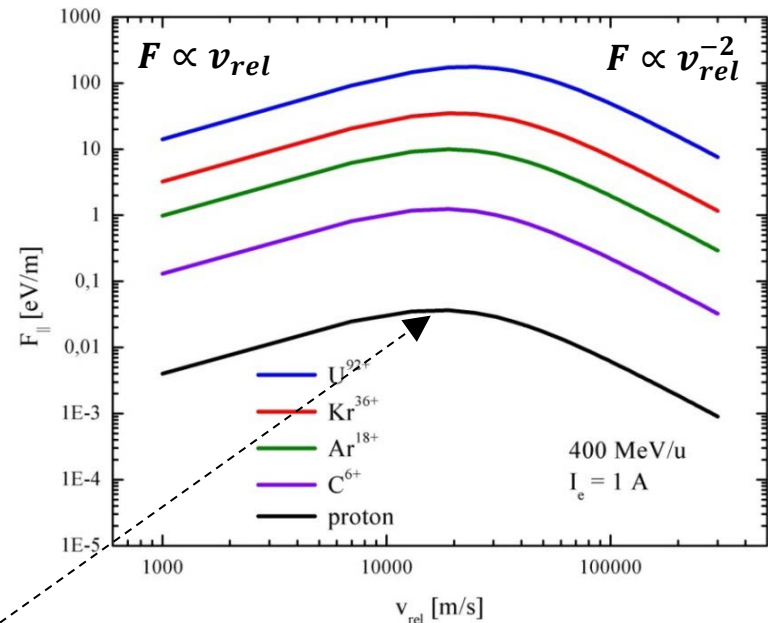
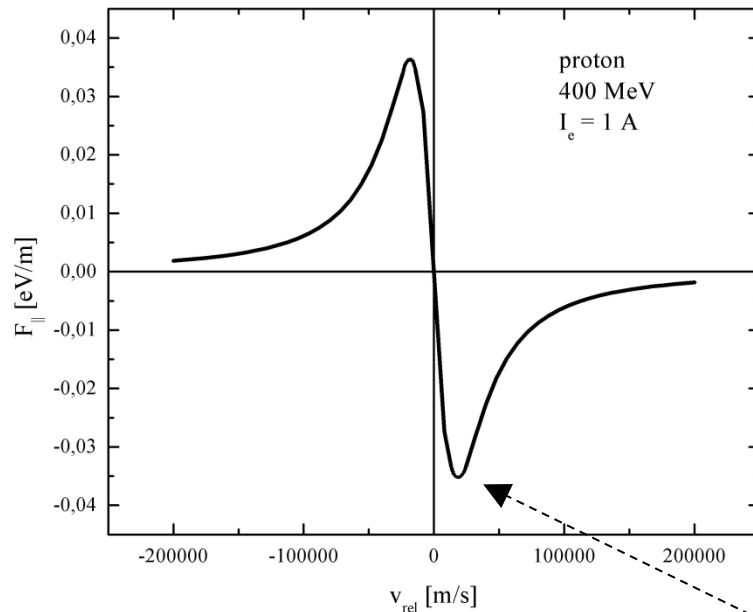
$$\vec{v}_{rel} = \vec{v}_{ion} - \vec{v}_e$$

**cooling force F**

for small relative velocity:  $\propto v_{rel}$

for large relative velocity:  $\propto v_{rel}^{-2}$

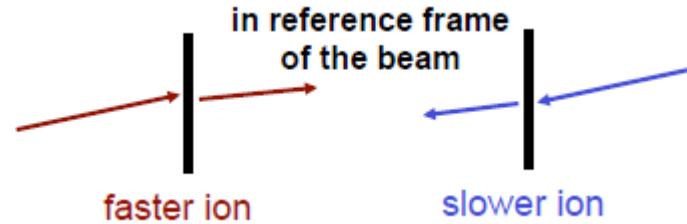
increase with charge:  $\propto Q^2$



maximum cooling force at effective electron temperature

# Simple derivation of electron cooling force

Analogy: energy loss in matter  
(electrons in the shell)



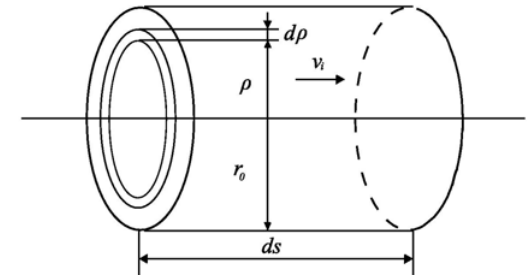
Rutherford scattering:  $2 \cdot \tan\left(\frac{\theta}{2}\right) = \frac{2Z_1 Z_2 e^2}{4\pi\epsilon_0 \Delta p \cdot v \cdot b}$   $Z_1 = Q$  (ion),  $Z_2 = -1$  (electron)

Energy transfer:  $\Delta E(b) = \frac{(\Delta p)^2}{2m_e} \cong \frac{2 \cdot Q^2 e^4}{(4\pi\epsilon_0)^2 m_e v^2} \frac{1}{b^2}$  (for  $b \gg b_{min}$ )

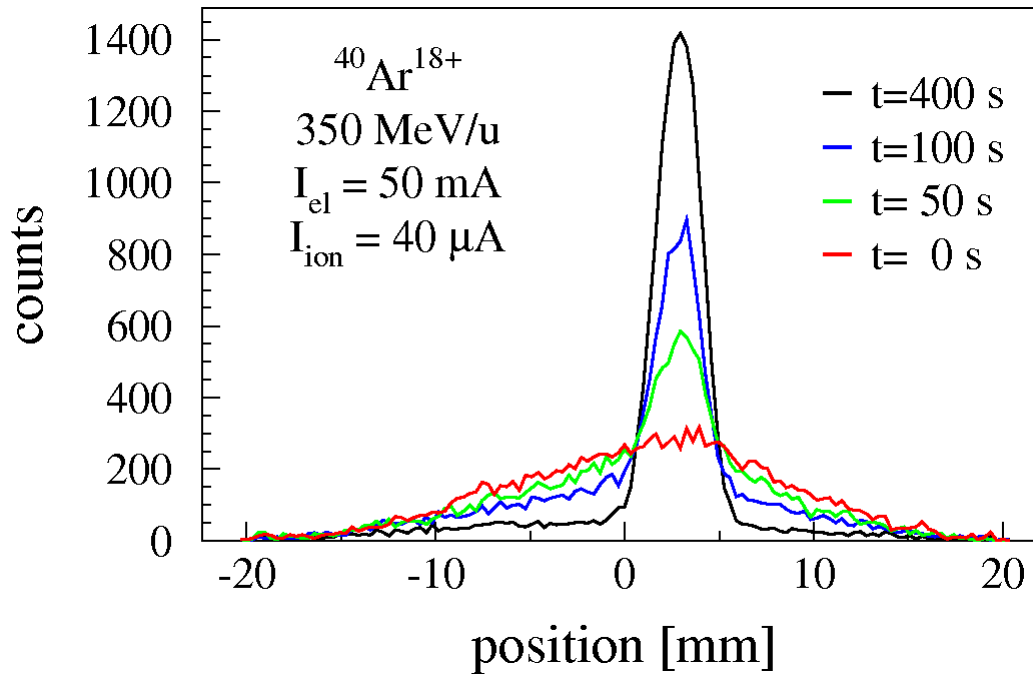
Minimum impact parameter:  $b_{min} = \frac{Qe^2}{(4\pi\epsilon_0)^2 m_e v^2}$  from:  $\Delta E(b_{min}) = \Delta E_{max} \cong m_e v^2$

Energy loss:  $-\frac{dE}{dx} = 2\pi \int_{b_{min}}^{b_{max}} b \cdot n_e \cdot \Delta E db = \frac{4\pi Q^2 e^4}{(4\pi\epsilon_0)^2 m_e v^2} n_e \cdot \ln \frac{b_{max}}{b_{min}}$

Coulomb logarithm  $L_C = \ln(b_{max}/b_{min}) \approx 10$  (typical value)



# Example of electron cooling

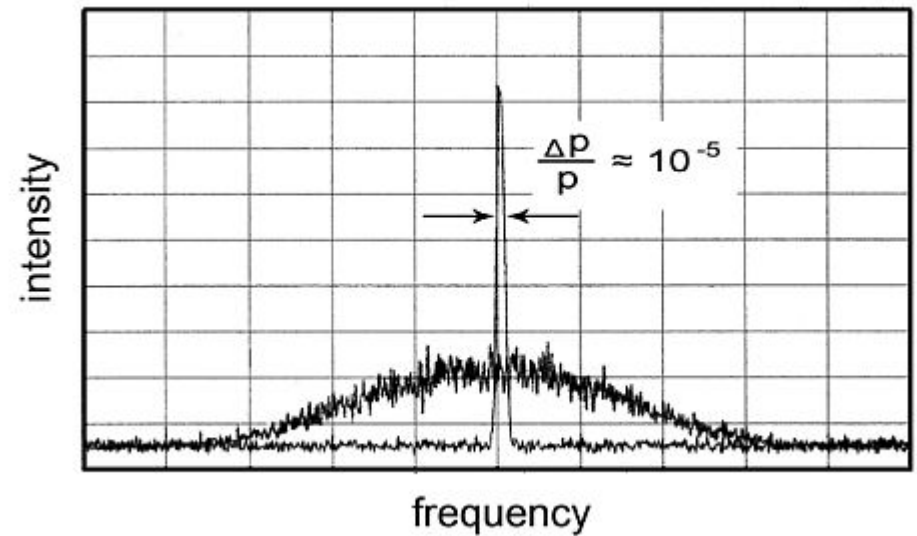


transverse cooling at ESR

measured with residual gas  
ionization beam profile monitor

cooling of  $350 \text{ MeV/u } \text{Ar}^{18+}$  ions  $0.05 \text{ A}$ ,  $192 \text{ keV}$  electron beam  $n_e = 0.8 \cdot 10^6 \text{ cm}^{-3}$

# Electron cooling



momentum spread  $\Delta p/p = 10^{-5}$   
diameter 2 mm

- The ions get the sharp velocity of electrons, small size and divergence

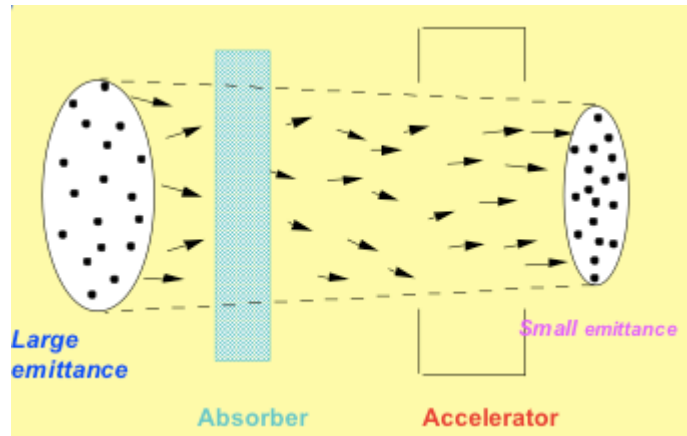
G.I. Budker, At. En. 22 (1967) 346

G.I. Budker, A.N. Skrinsky et al., IEEE NS-22 (1975) 2093

# Ionization cooling

## ❖ Hot muon beam:

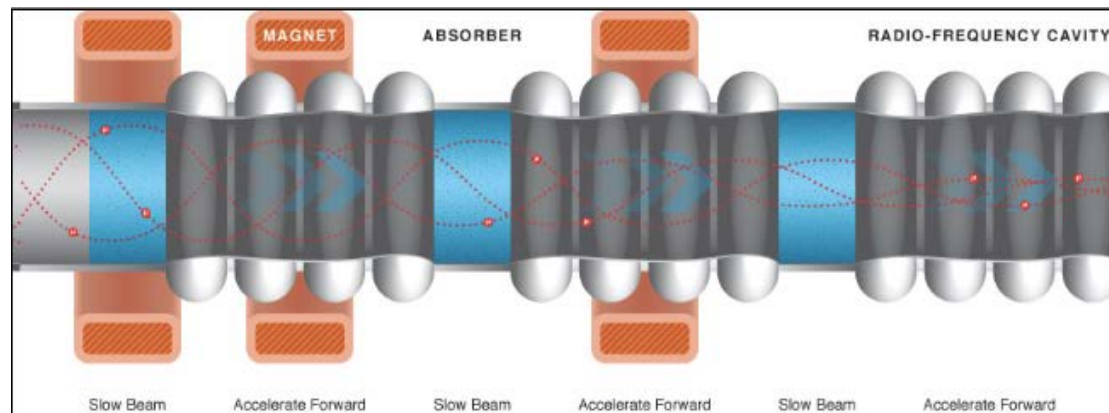
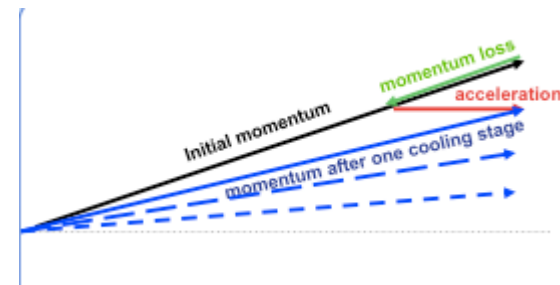
- large transverse momentum
- cannot fit in the beam pipe in muon accelerator



## *Ionization cooling:*

Based on use of ionization energy loss of accelerated charged particles

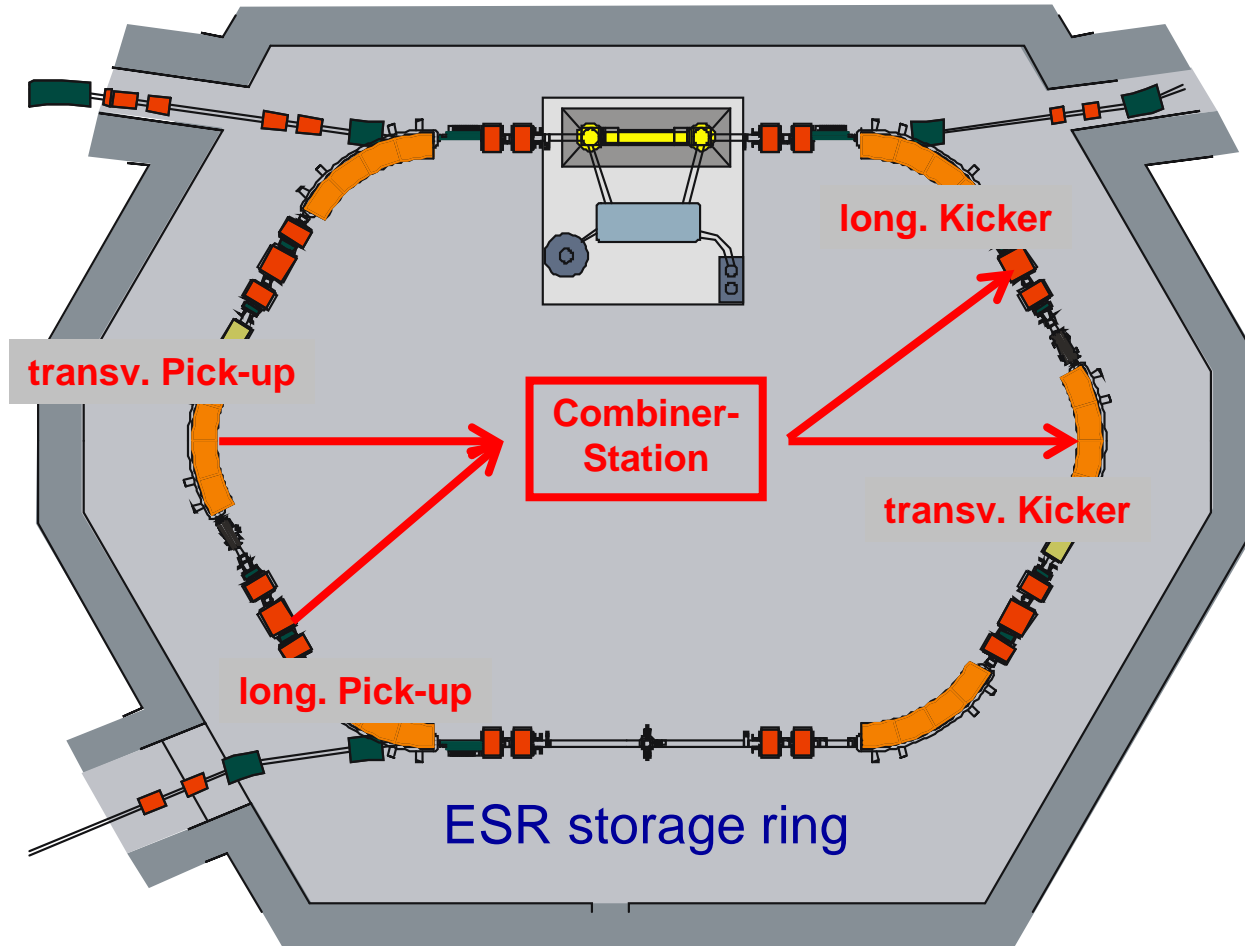
Reduce the transverse motion and accelerate them in forward direction



# Stochastic cooling: Implementation at the ESR



Simon van der Meer



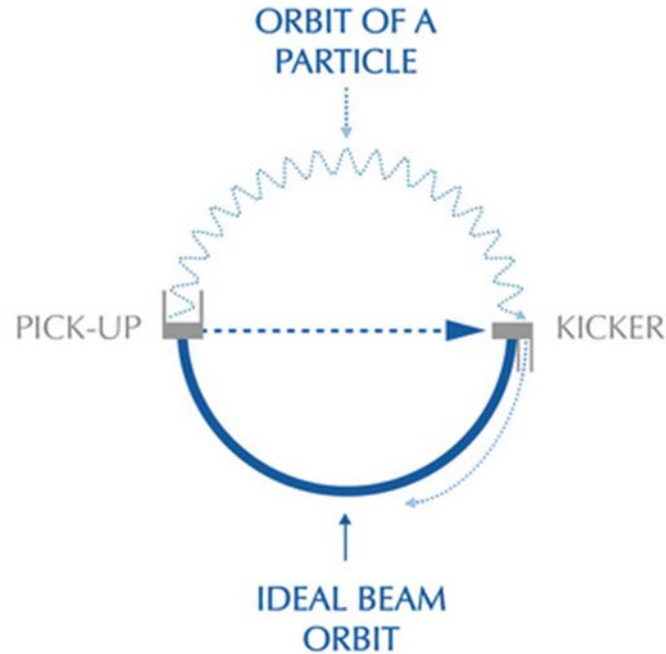
Stochastic cooling is in particular efficient for **hot** ion beams



# Principle of 'stochastic' cooling

**A Feedback System:** A detector or pick-up which measures the motion of the particle and a corrector, the kicker, which adjusts their angles.

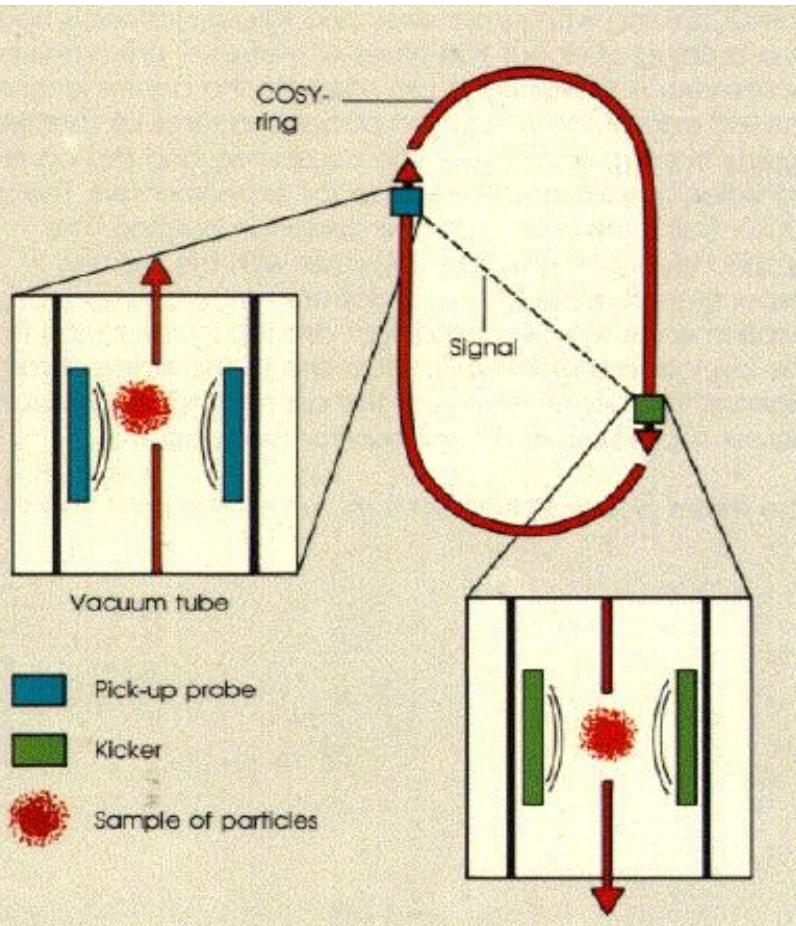
Measures the deviation of the center of gravity of a sample of particles with respect to the requisite orbit and sends an error signal to the kicker.



The kicker applies an electric field to the same sample to correct the deviation measured.

# Principle of 'stochastic' cooling

## Self correction of ion trajectory



Using a **pick-up probe**, the position of the ion beam is measured at a fixed position via the induced signal. A deviation of the beam from the ideal orbit can be corrected by **amplification of this signal**.

The amplified signal is now used as a **correction signal** which acts on the beam at a second position (zero crossing of the betatron function) via a "kicker".

This method was invented for the cooling of hot  $p(\bar{p})$  by van der Meer. He showed that after a cooling time of  $\tau \propto N/C$  ( $N$ : particle number,  $C$  = Bandwidth of the amplifier) a momentum width of the beam of about  $\Delta p/p \approx 10^{-3}$  can be achieved by stochastic cooling.

Detection of the **W boson** from  $p \leftrightarrow p(\bar{p})$

# Stochastic cooling at GSI

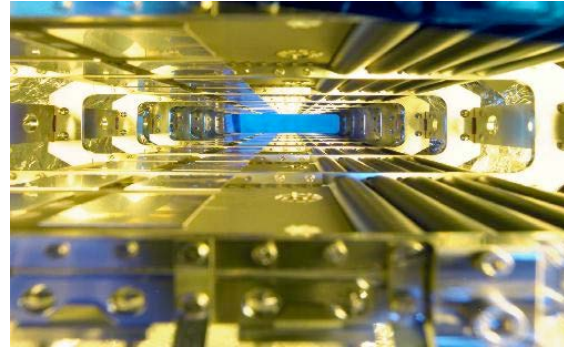
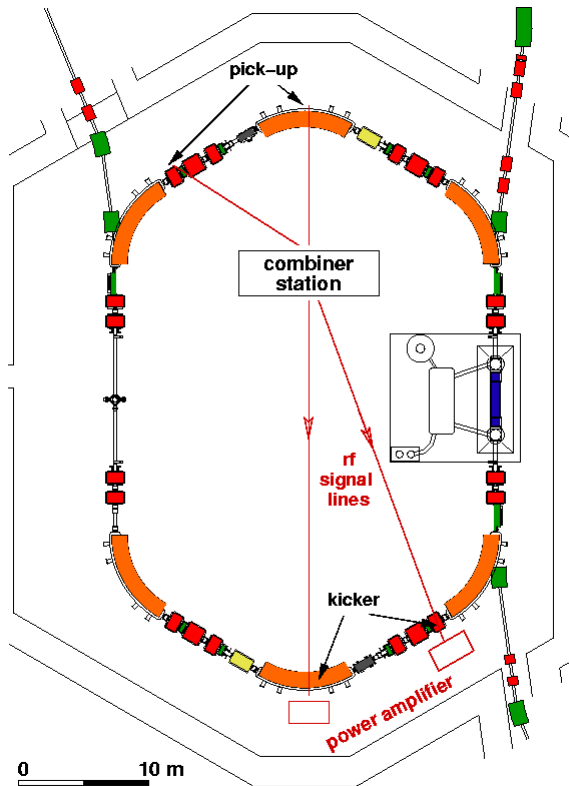
fast pre-cooling of hot fragment beams

energy 400 (-550) MeV/u

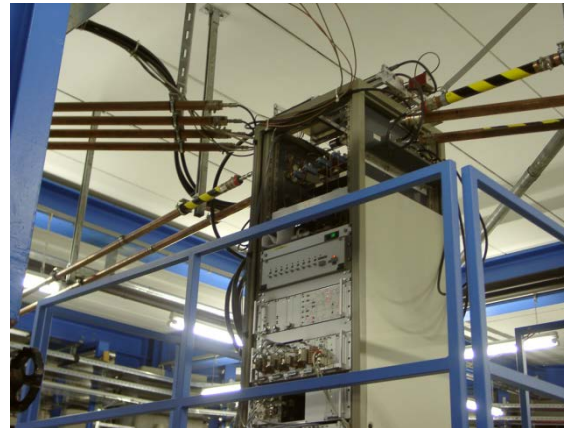
bandwidth 0.8 GHz (range 0.9-1.7 GHz)

$$\delta p/p = \pm 0.35\% \rightarrow \delta p/p = \pm 0.01\%$$

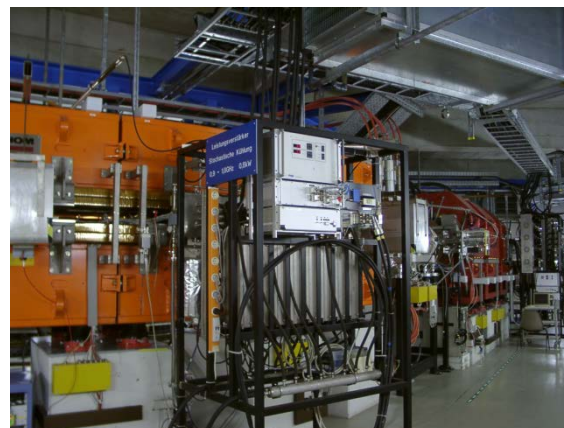
$$\varepsilon = 10 \cdot 10^{-6} m \rightarrow \varepsilon = 2 \cdot 10^{-6} m$$



electrodes installed inside magnets



combination of signals from electrodes



power amplifiers for generation of correction kicks

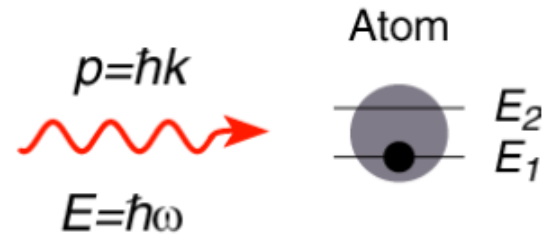
# Comparison of Cooling Methods

	<u>Stochastic Cooling</u>	<u>Electron Cooling</u>
Useful for:	low intensity beams hot (secondary) beams high charge full 3-D control	low energy all intensities warm beams (pre-cooled) high charge bunched beams
Limitations: /problems	high intensity beams beam quality limited bunched beams	space charge effects recombination losses high energy

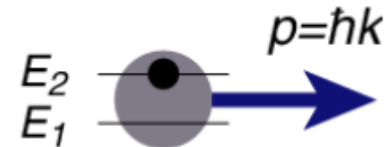
# Principle of laser cooling (snowplow)

only longitudinal cooling

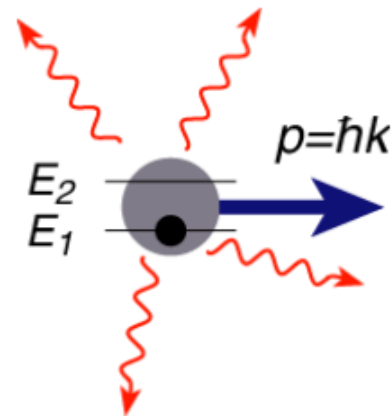
1. Absorption of photons from a laser beam:  
**Energy** and **momentum** must be conserved.



2. Absorption of photons:  
**Momentum transfer** in a defined direction  
(**directed momentum transfer**).



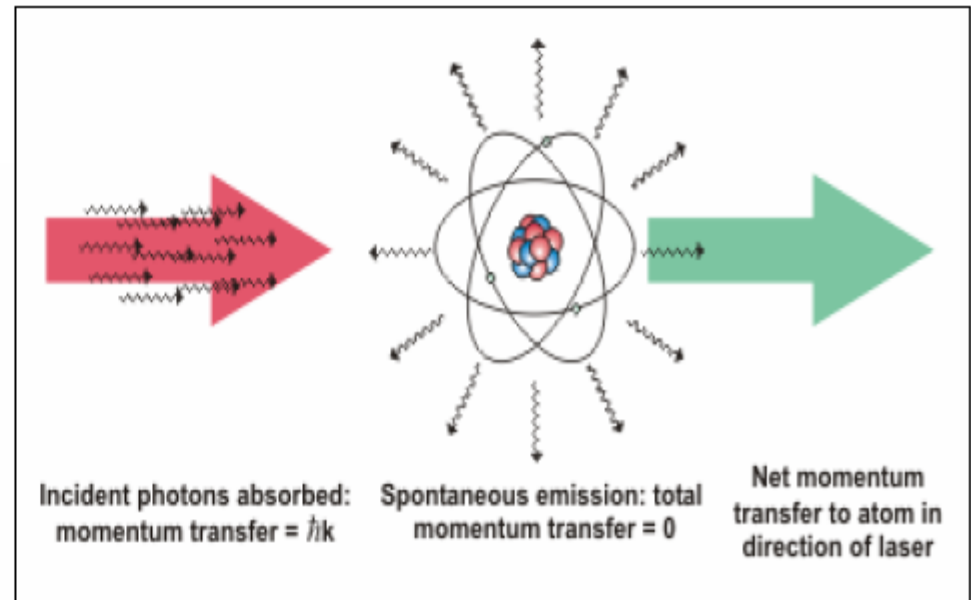
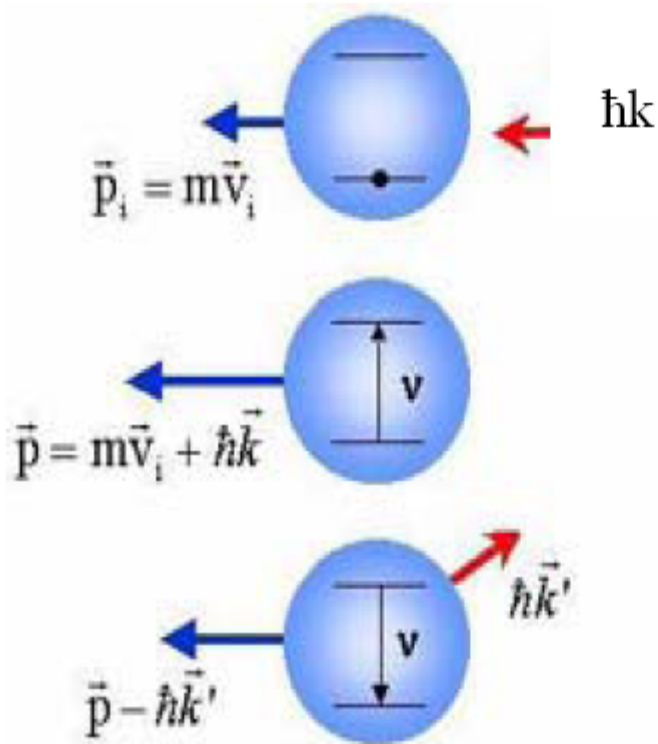
3. No defined direction for the **spontaneous emission** (**isotropic re-emission**):  
Momentum transfer cancels out over many absorption-emission-cycles.



*typical cooling times  $\sim 10 \mu\text{s}$*

# Principle of laser cooling

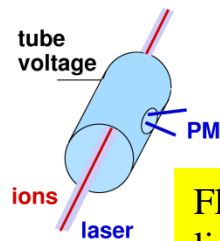
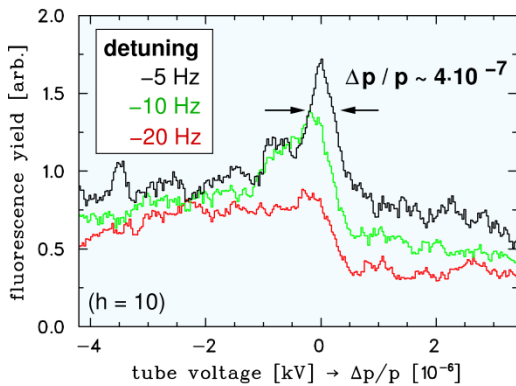
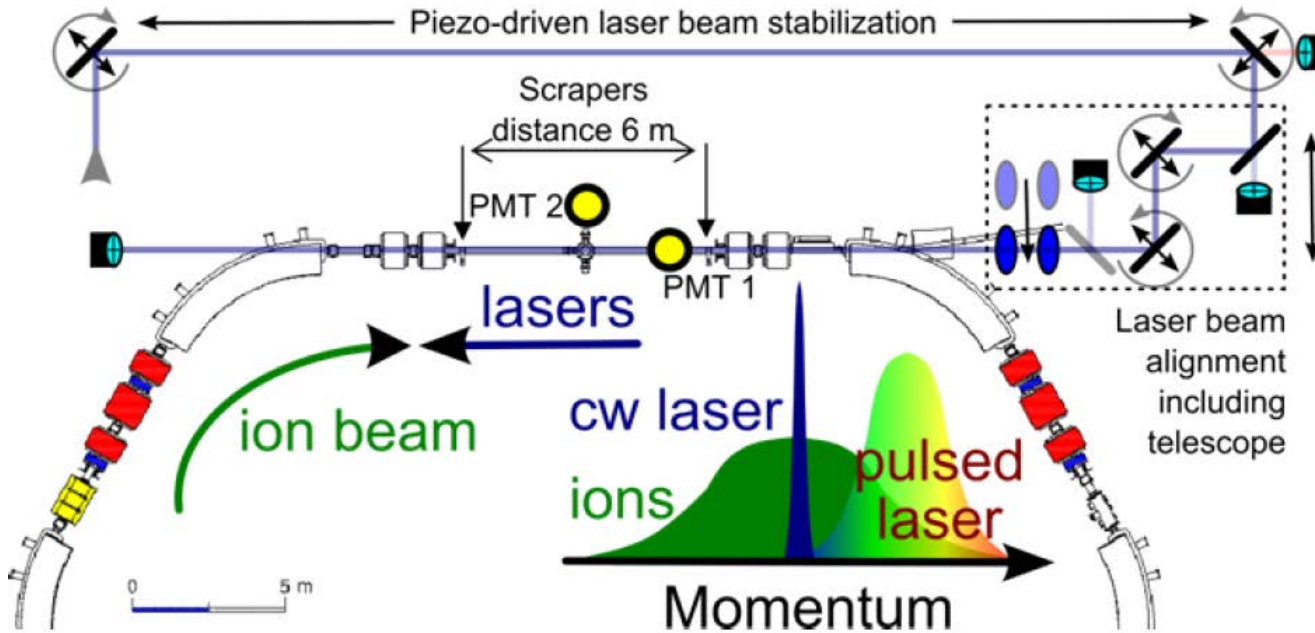
2-step process



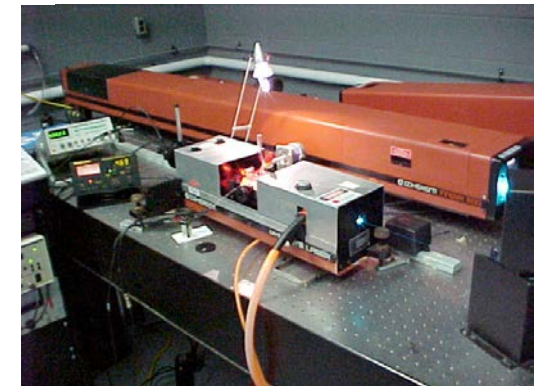
users.york.ac.uk

[http://inms-ienm.nrc-cnrc.gc.ca/research/cesium\\_clock\\_e.html](http://inms-ienm.nrc-cnrc.gc.ca/research/cesium_clock_e.html)

# Laser cooling at ESR

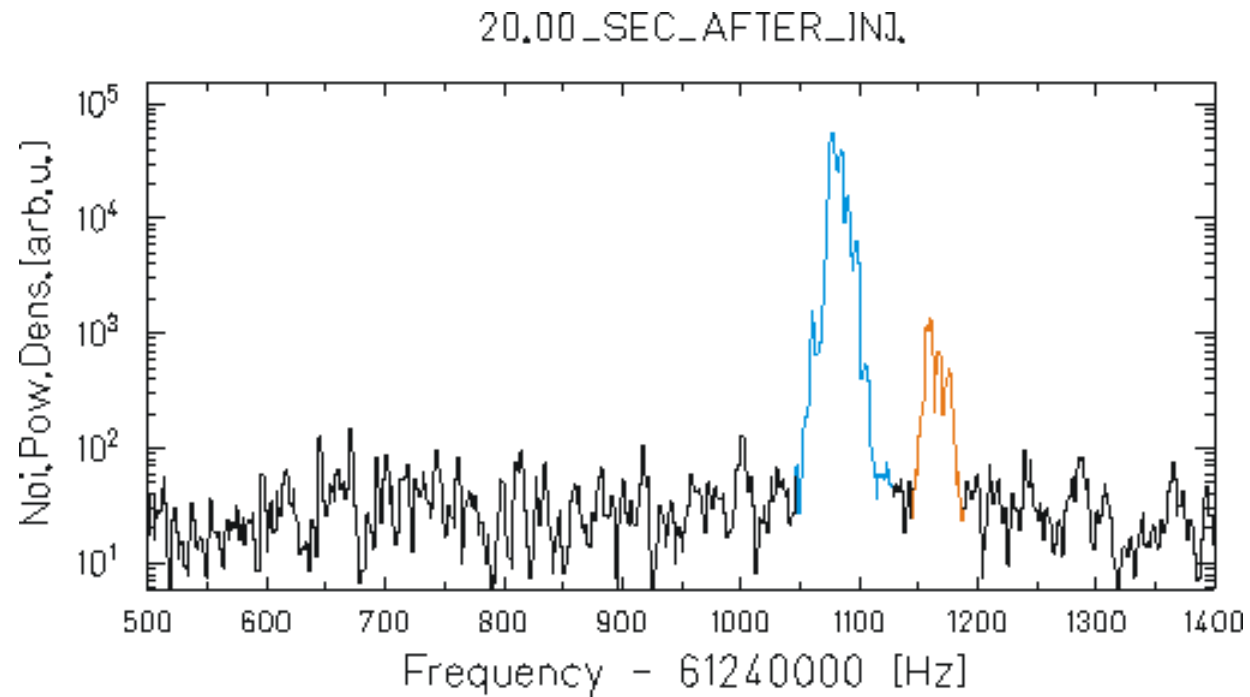


Fluorescence light detection



**Argon ion laser (257.3 nm)**  
frequency doubled

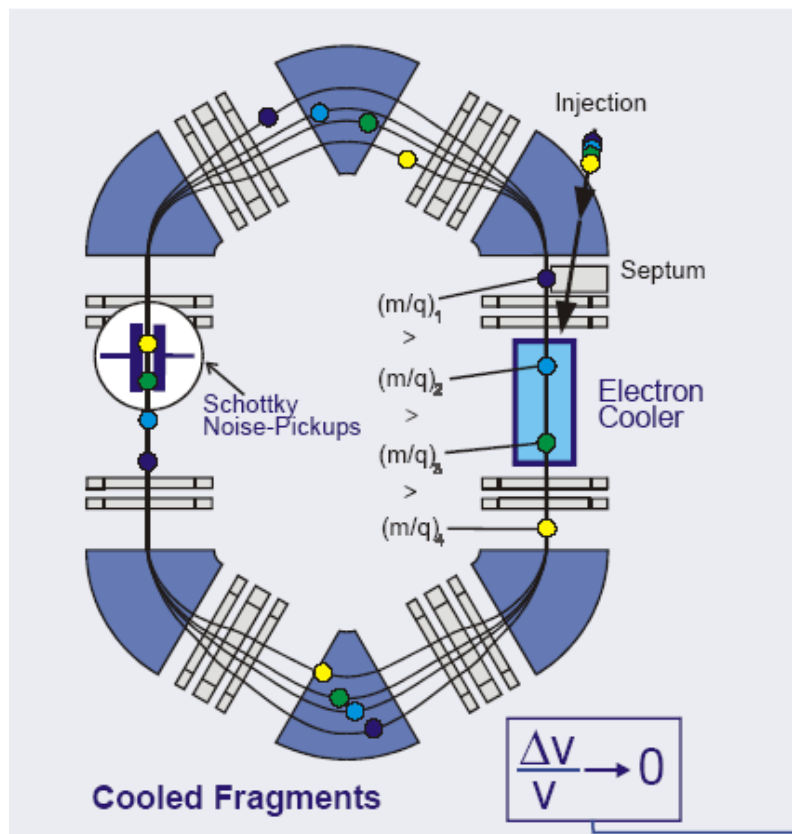
# Cooling



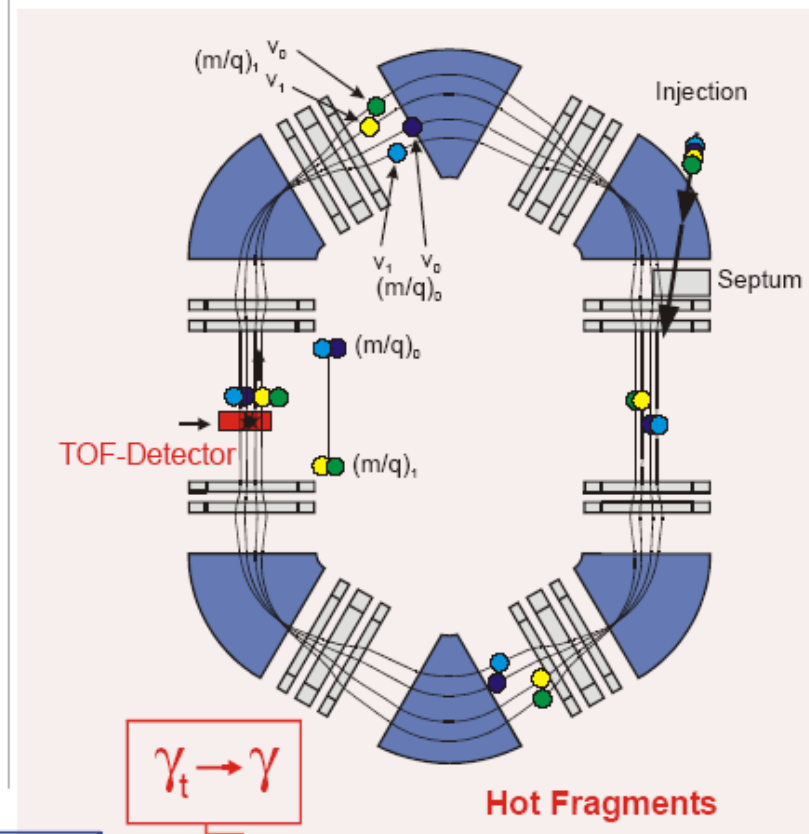


# Cooling with the ESR

## SCHOTTKY MASS SPECTROMETRY



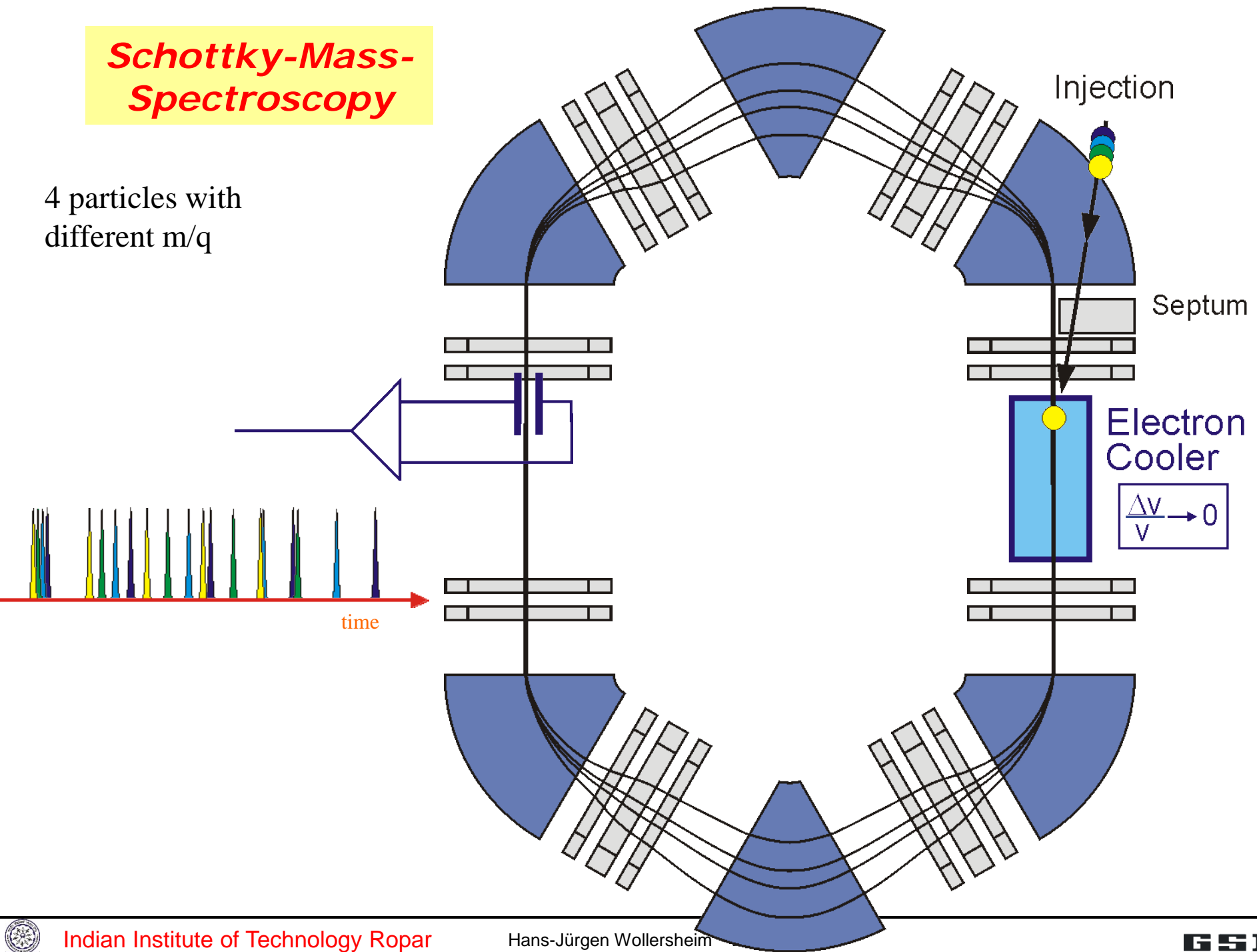
## ISOCRONOUS MASS SPECTROMETRY



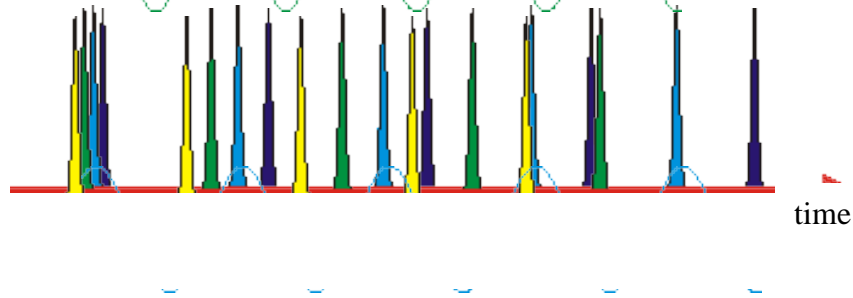
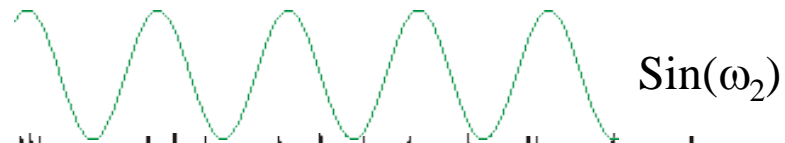
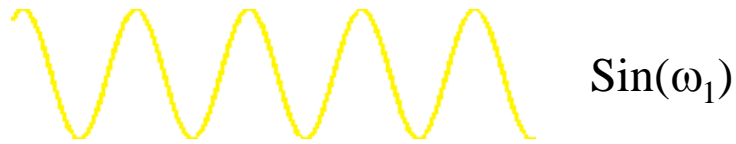
$$\frac{\Delta f}{f} = -\frac{1}{\gamma_t^2} \frac{\Delta(m/q)}{m/q} + \frac{\Delta V}{V} \left(1 - \frac{\gamma^2}{\gamma_t^2}\right)$$

# Schottky-Mass-Spectroscopy

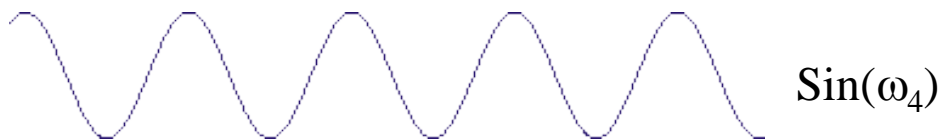
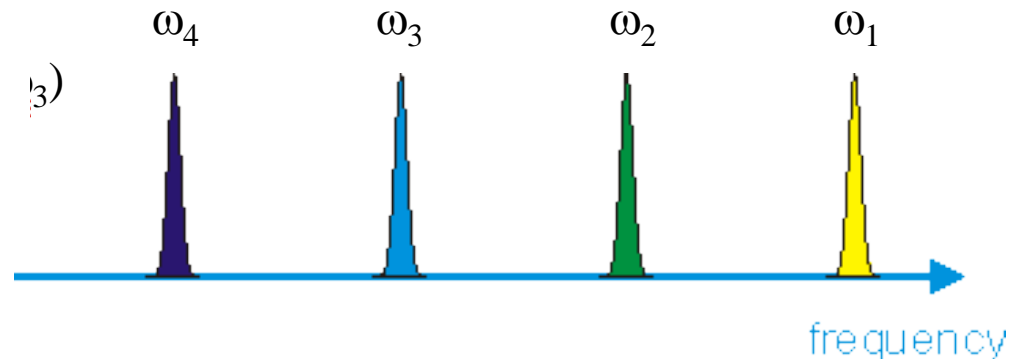
4 particles with different  $m/q$



# Schottky mass spectroscopy



## Fast Fourier Transform



# Small-band Schottky frequency spectra

