# Magnet examples


Quadrupole

Sextupole

# Dipole magnets

A dipole magnet gives a constant B-field.

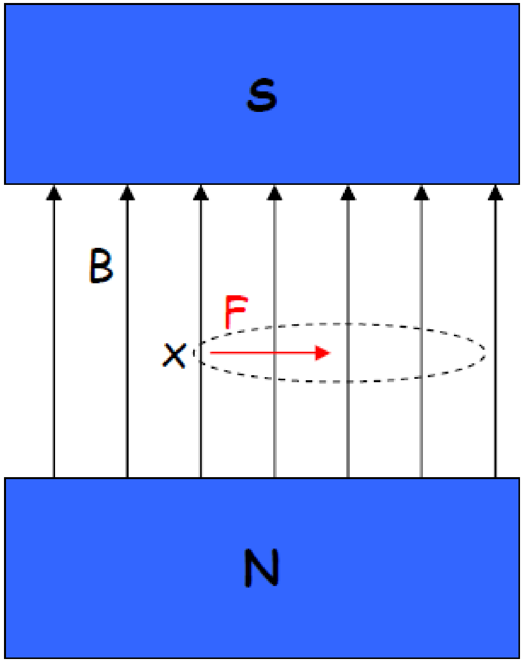
*The field lines in a magnet run from North to South.* The field shown at right is positive in the vertical direction.

Symbol convention:

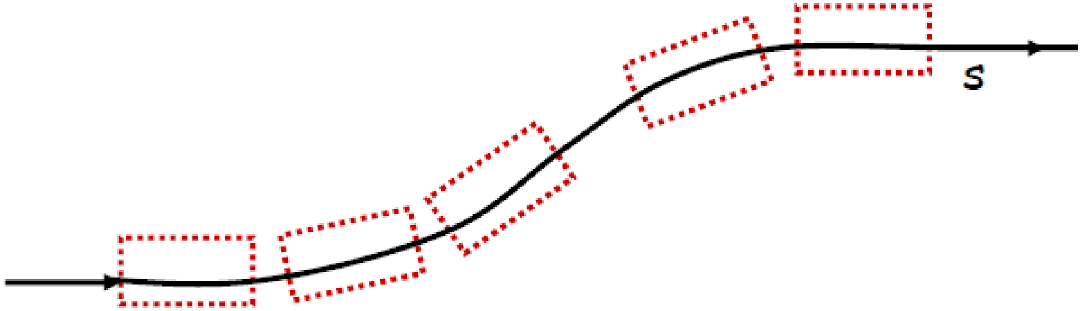
× - traveling into the page

● - traveling out of page

In the field shown, for a positively charged particle traveling into the page, the force is to the right.




In an accelerator lattice, dipoles are used to bend the beam trajectory. The set of dipoles in a lattice defines the reference trajectory:



# Field equations for a dipole

Let's consider the dipole field force in more detail. Using the Lorentz Force equation, we can derive the following useful relations:

For a particle of mass ***m***, Energy ***E*** and momentum ***p*** in a uniform ***B***-field:

1) The bending radius of the motion of the particle in the dipole field is given by:

$$\boxed{\frac{1}{\rho} = \frac{eB}{pc}}$$

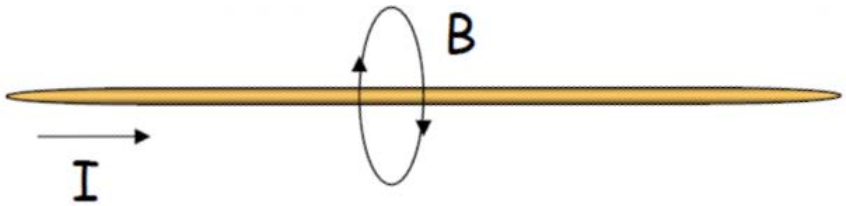
2) Re-arranging (1), we define the "magnetic rigidity" to be the required magnetic bending strength for given radius and energy:

$$B\rho = \frac{pc}{e} = \frac{\beta E}{e}$$

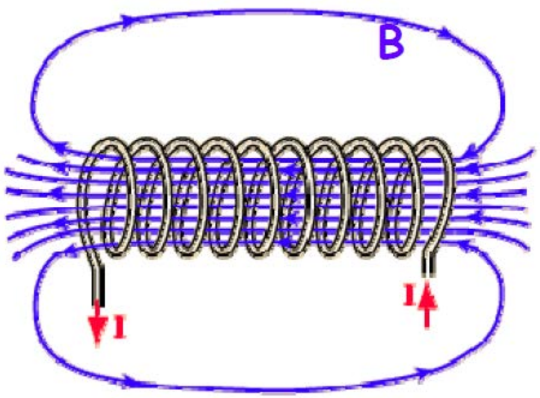
$$B\rho[T \cdot m] = \frac{10}{2.998} \cdot \beta E[GeV]$$

# Generating a B-field from a current

Recall that a current in a wire generates a magnetic B-field which curls around the wire




Or, by winding many turns on a coil we can create a strong uniform magnetic field



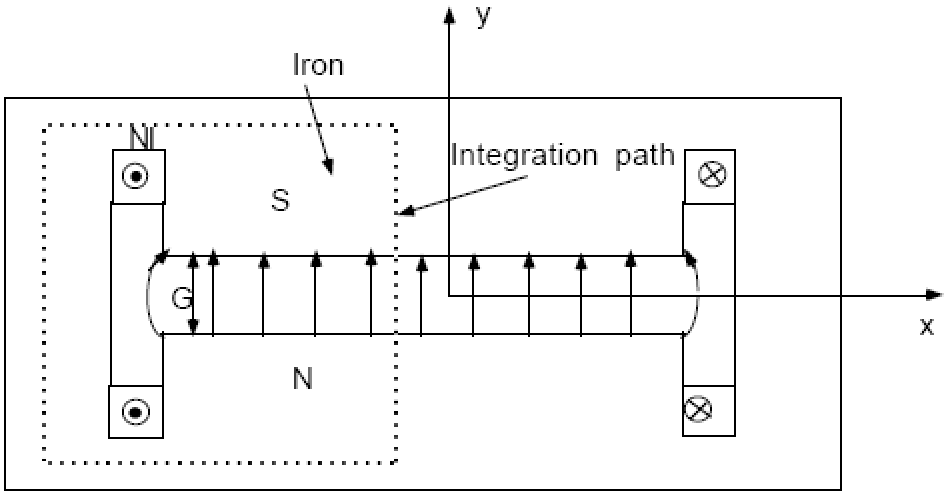

The field strength is given by one of Maxwell's equation:

$$\boxed{\Delta \times \frac{B}{\mu_r} = \frac{4\pi}{c} J}$$

$$\boxed{\mu_r = \frac{\mu_{material}}{\mu_0}}$$

# The dipole current-to-field relationship

In an accelerator dipole magnet, we use current-carrying wires and metal cores of high $\mu$ to set up a strong dipole field:




Pure dipole: NI turns/pole

***N*** turns of current ***I*** generate a small ***H = B/μ*** in the metal. Hence, the B-field across the gap, ***G***, is large.

Using Maxwell's equation for B, we can derive the relationship between ***B*** in the gap, and ***I*** in the wires:

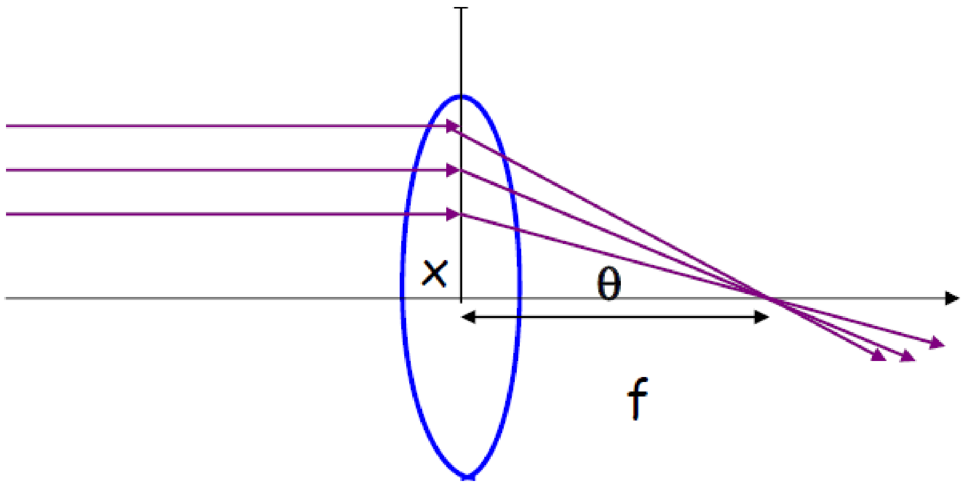
$$\boxed{I_{total} = 2 \cdot N \cdot I[A] = \frac{1}{0.4\pi} B_{\perp}[G] \cdot G[cm]}$$

# Optical analogy for focusing

We have seen that a dipole produces a constant field that can be used to bend a beam.

Now we need something that can focus a beam. Without focusing, a beam will naturally diverge.

Consider the optical analogy of focusing a ray of light through a lens:




The rays come to a focus at the focal point, f. The focusing angle depends on the distance from center, x.

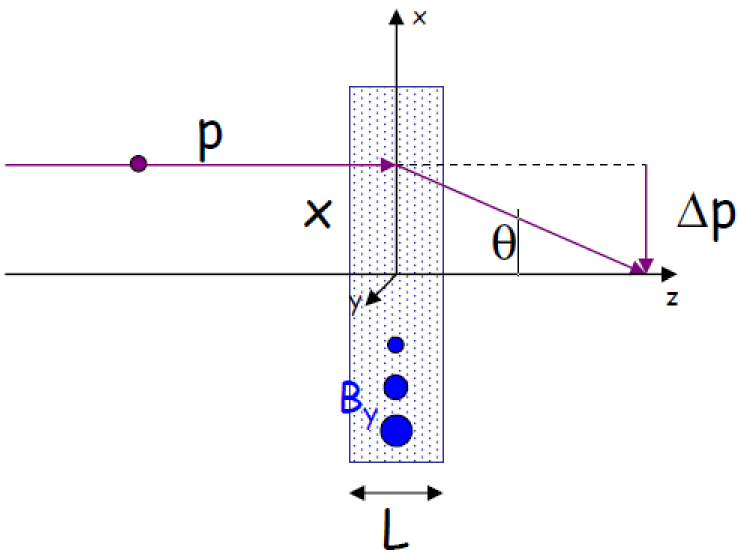
$$\tan\theta = \frac{x}{f}$$

$$\theta \approx \frac{x}{f} \qquad for\ small\ x$$

The farther off axis, the stronger the focusing effect!
The dependence is **linear** for small x.

# Focusing particles with magnets

Now consider a **magnetic lens**. This lens imparts a transverse momentum kick, ***Δp***, to the particle beam with momentum ***p***.




For a field which increases linearly with x, the resulting kick, ***Δp***, will also increase linearly with ***x***.

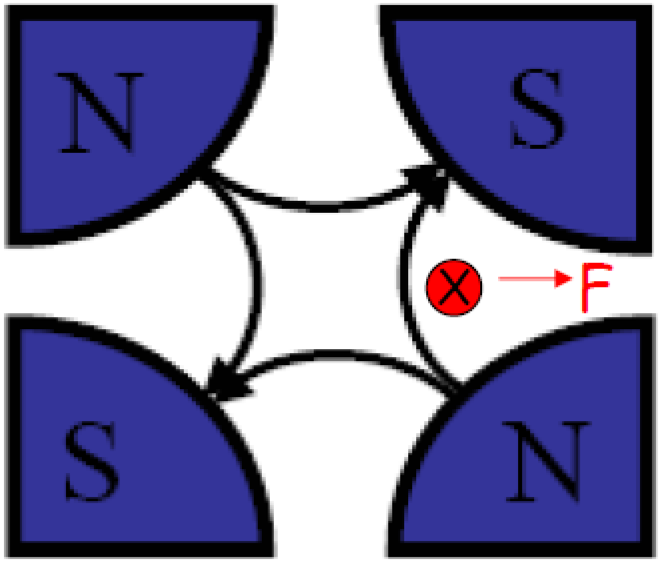
Beginning with the Lorentz force equation, we can solve for the focal length and focusing strength, ***k***:

$$\frac{1}{f} = \frac{e}{pc} \cdot gL = \frac{gL}{B\rho} \quad where \quad g = \frac{dB_y}{dx}$$

$$k[m^{-2}] = \frac{1}{fL} = \frac{e}{pc} \cdot g = \frac{0.299 \cdot g[T/m]}{\beta E[GeV]} = focusing\ strength$$

# Quadrupole magnet

A **quadrupole magnet** imparts a force proportional to distance from the center. This magnet has 4 poles:



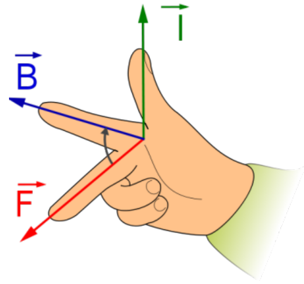

Consider a positive particle traveling into the page (into the magnet field).

According to the right hand rule, the force on a particle on the right side of the magnet is to the right, and the force on a similar particle on left side is to the left.

This magnet is horizontally defocusing. A distribution of particles in x would be defocused!

What about the vertical direction?

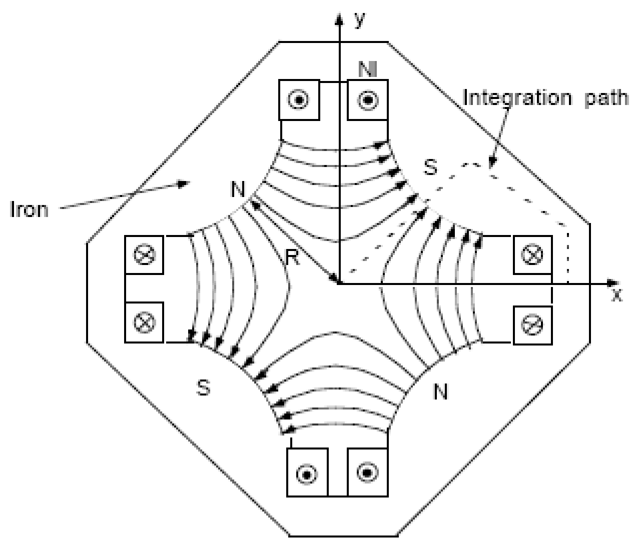
→ ***A quadrupole which defocuses in one plane focuses in the other.***




positive charge
right hand rule

# Quadrupole current-to-field equation

As with a dipole, in an accelerator we use current-carrying wires wrapped around metal cores to create a quadrupole magnet:




Pure quadrupole, NI turns/pole

The field lines are denser near the edges of the magnet, meaning the field is stronger there. The strength of $\boldsymbol{B_y}$ is a function of $\boldsymbol{x}$, and visa-versa. The field at the center is zero!

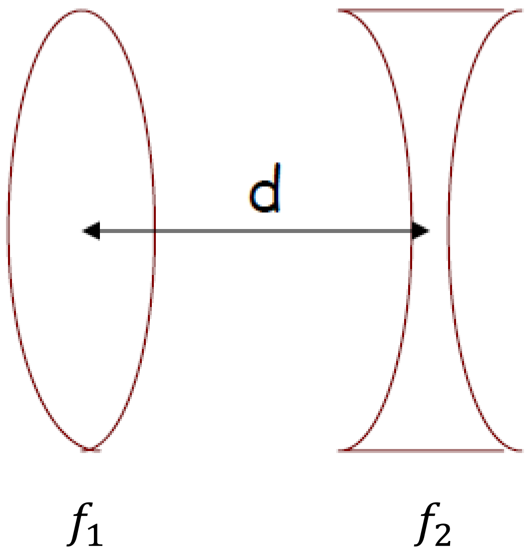
Using Maxwell's equation for B, we can derive the relationship between ***B*** in the gap, and ***I*** in the wires:

$$\boxed{\dot{B} = \frac{dB_\varphi}{dr} = \frac{8\pi I}{cR^2} \qquad \dot{B}[T/m] = \frac{2.52 \cdot I[A]}{R[mm]^2}}$$

# Focusing using arrays of quadrupoles

- Quadrupoles focus in one plane while defocusing in the other. So, how can this be used to provide net focusing in an accelerator?

Consider the optical analogy of two lenses, with focal lengths $f_1$ and $f_2$, separated by a distance d:




The combined $f$ is:

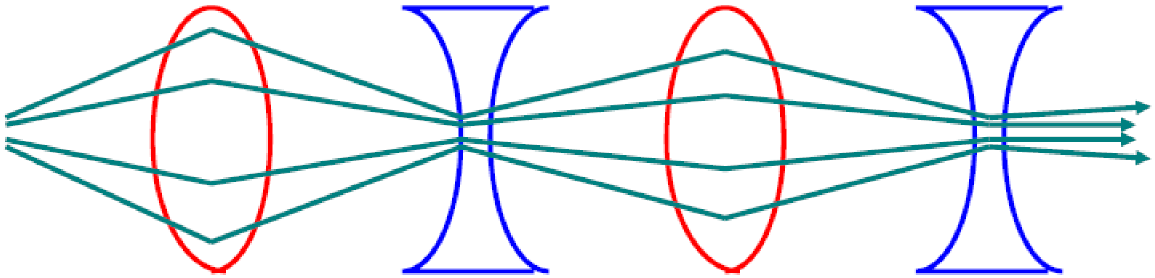
$$\frac{1}{f_{combined}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}$$

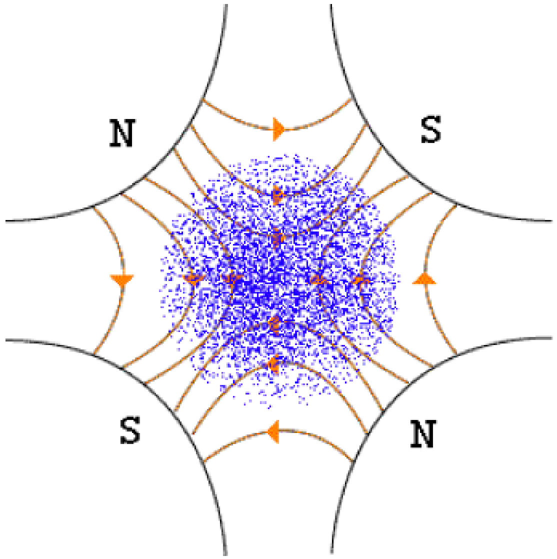
What if $f_1 = -f_2$ ?

The net effect is focusing, $\frac{1}{f_{combined}} = \frac{d}{f_1 \cdot f_2}$

# More on focusing particles …

The key is to alternate focusing and defocusing quadrupoles. This is called a FODO lattice (Focus-Drift-Defocus-Drift):

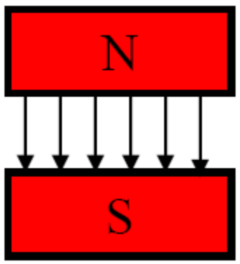




# Other n-pole magnets

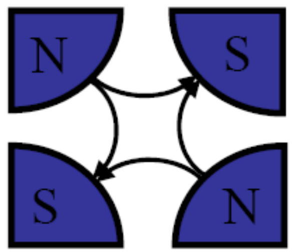
The general equation for B allows us to write the field for any n-pole magnet.
Examples of upright magnets:

**n=1: Dipole**



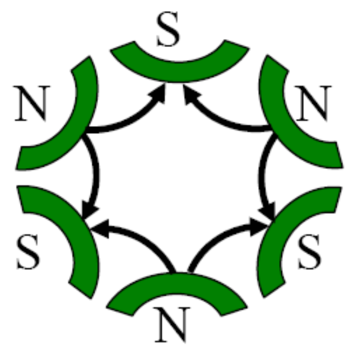

$180^0$ between poles

**n=2: Quadrupole**



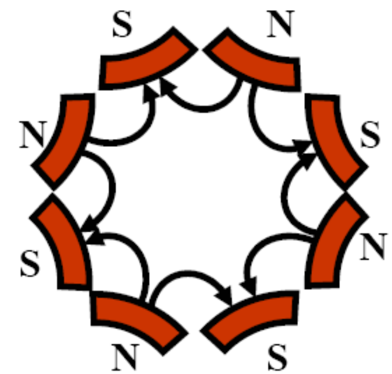

$90^0$ between poles

**n=3: Sextupole**



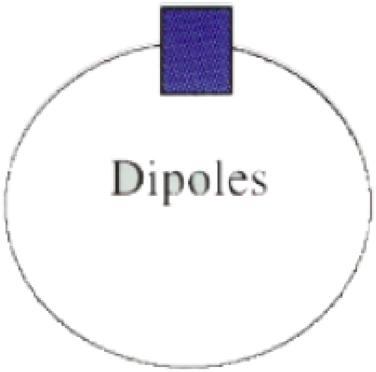

$60^0$ between poles

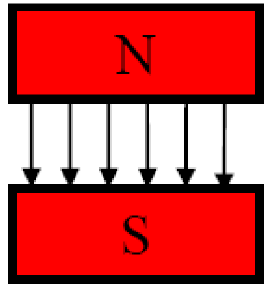
**n=4: Octupole**



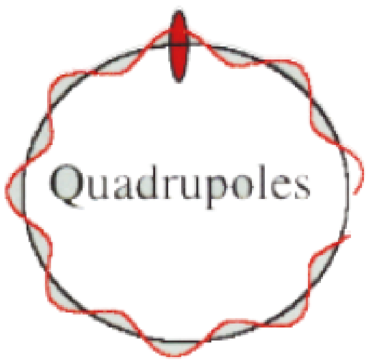

$45^0$ between poles

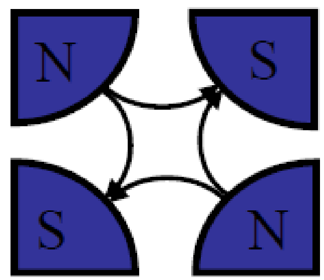
- In general, poles are 360°/2n apart.
- The skew version of the magnet is obtained by rotating the upright magnet by 180°/2n.



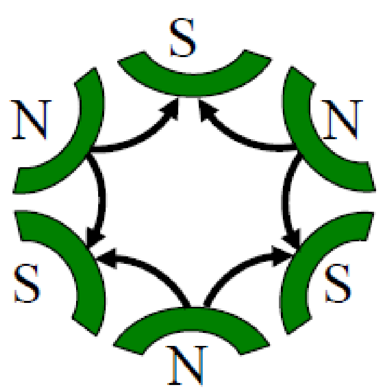




Bending (following reference trajectory)




Focusing the beam

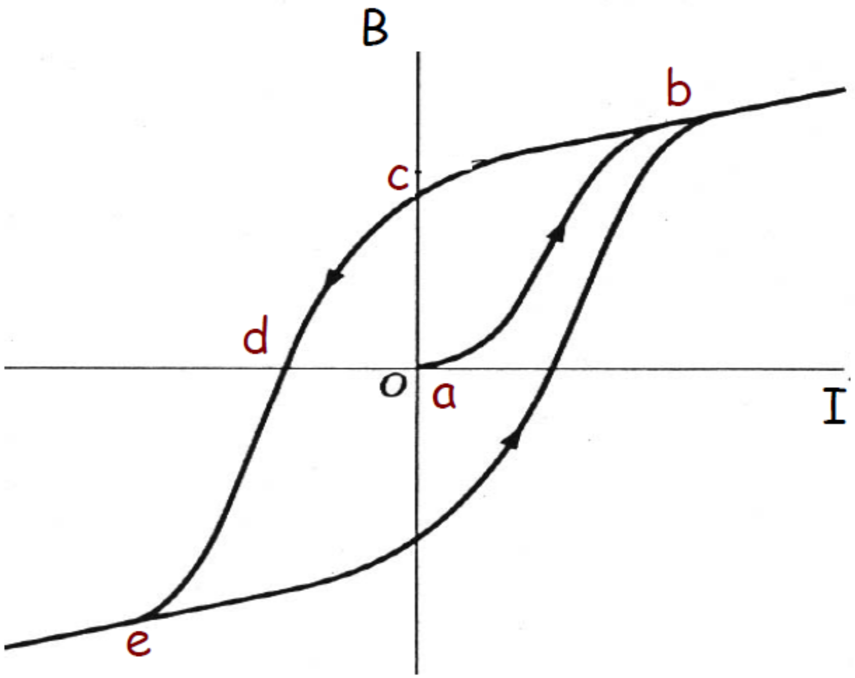




"Chromatic compensation"

# Hysteresis and magnet cycling

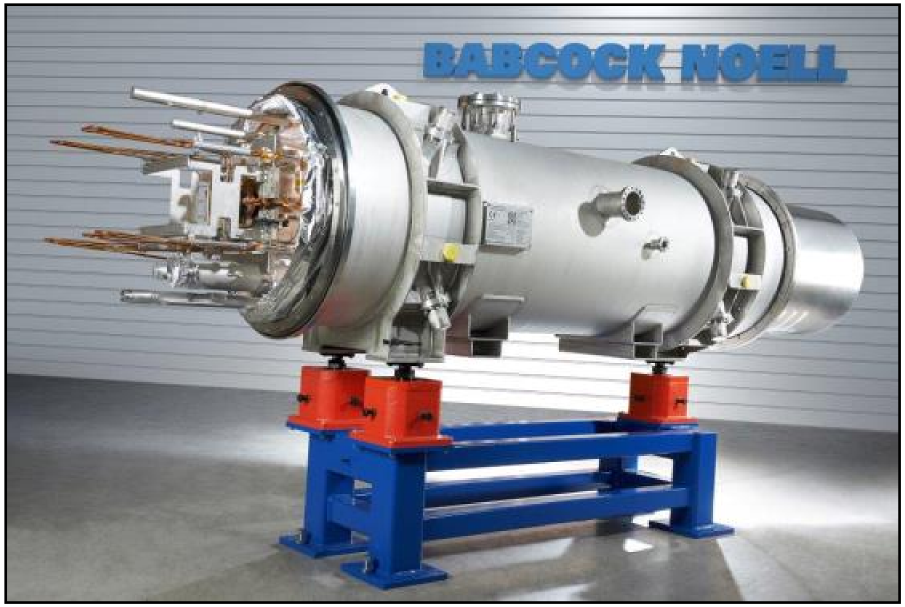
An external B-field, created by a current I, creates a B-field in iron by aligning tiny internal dipoles (electron spins) in the material.

However, if the current and external field are dropped to zero, the material remains partially magnetized. This gives rise to “hysteresis” and the need for magnet cycling.




a - start point
b - saturation
c - residual magnetization
d - B=0
e - saturation with -B

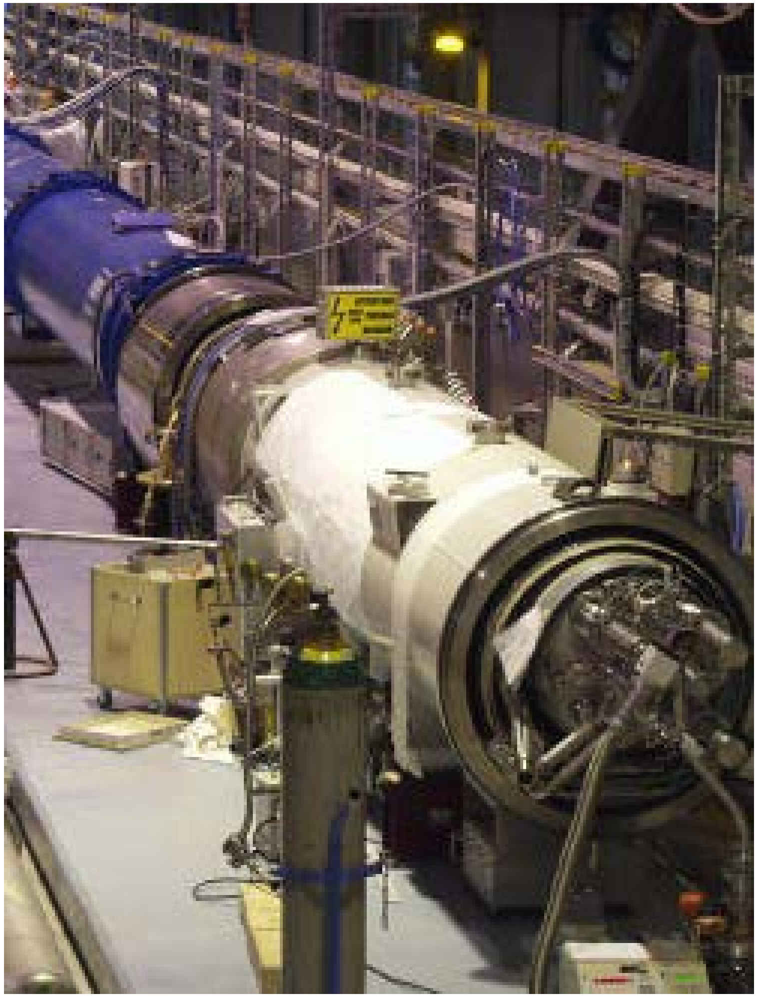
# Superconducting magnets



## *Who needs superconductivity anyway?*

### Abolish Ohm's Law

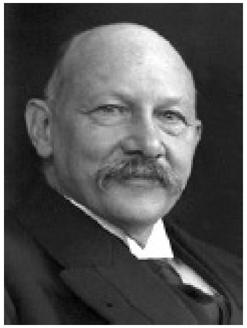
- no power consumption
  (although do need refrigeration power)
- high current density ⇒ compact windings, high gradients
- ampere turns are cheap, so we don't need iron
  (although often use it for shielding)



### Consequences

- low power bill
- higher magnetic fields mean reduced bend radius
  ⇒ smaller rings
  ⇒ reduced capital cost
  ⇒ new technical possibilities (muon collider)
- higher quadrupole gradients
  ⇒ higher luminosity
- higher rf electric fields (continuous)

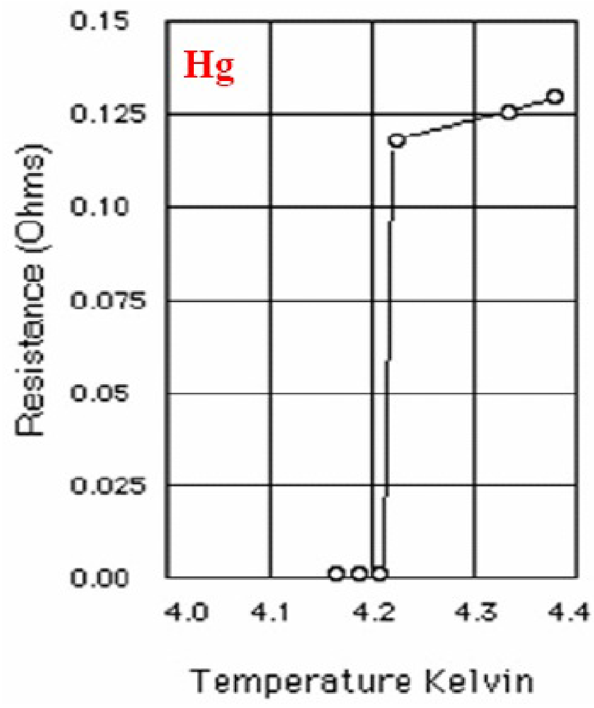
# Discovery of superconductivity



Heike Kamerlingh Onnes

Gilles Holst

- Superconductivity is a phenomenon occurring in certain materials, when their electrical resistance vanishes below a characteristic critical temperature. It is accompanied by expulsion of the magnetic field from the material.




The superconducting transition is at 4.2K. Within 0.01K, the resistance jumps from unmeasurably small ($<10^{-6}\ \Omega$ to $0.1\ \Omega$).

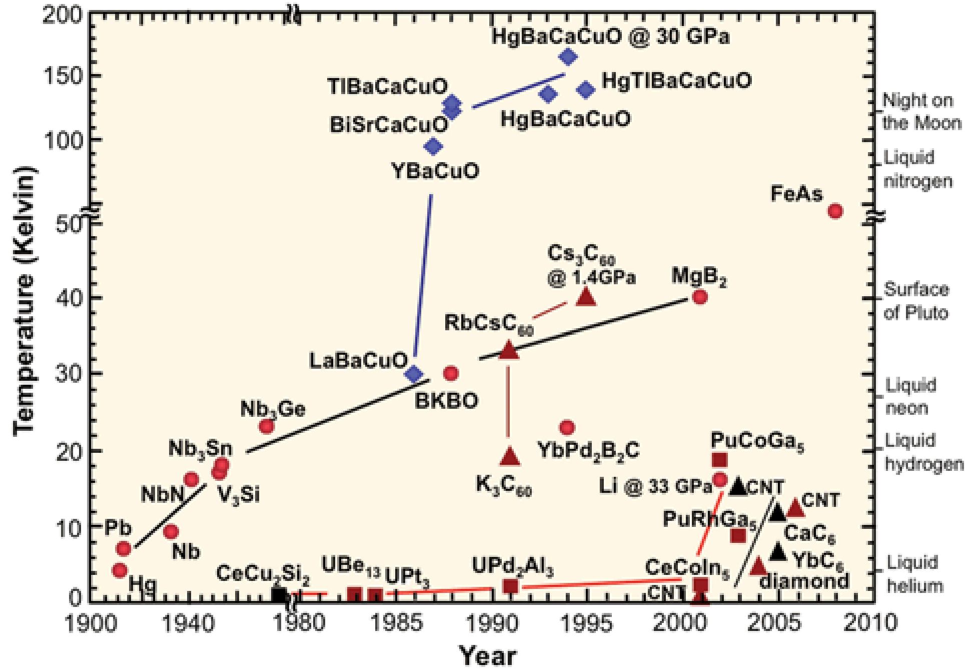
- Superconductivity was discovered by Dutch physicist Heike Kamerlingh Onnes and Gilles Holst on April 8, 1911 in Leiden.

  This discovery was made possible after Onnes was able to liquefy helium in 1908. He was awarded a Nobel prize in 1913 "*for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium*".

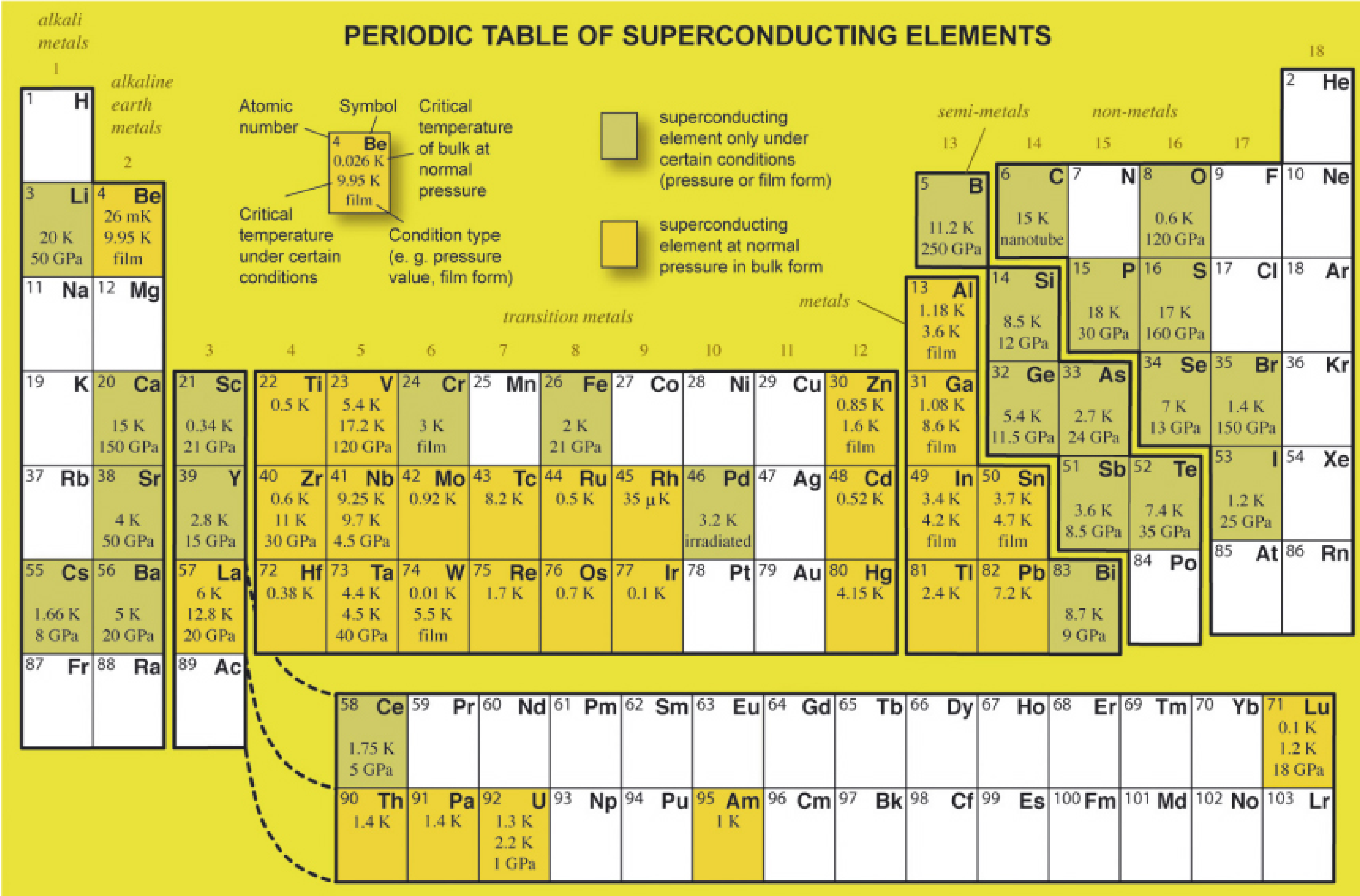
# Types of superconductors

There are many ways to classify superconductors. The most common are:

- *Response to a magnetic field: **Type I*** SC has a single critical field, above which all superconductivity is lost; ***Type II*** SC has two critical fields, between which it allows partial penetration of the magnetic field.
- *By theory of operation:* A SC is ***conventional*** if it can be explained by the BCS theory or its derivatives, or ***unconventional***, otherwise.
- *By critical temperature:* A SC is generally considered ***high temperature (HTS)*** if it reaches a SC state when cooled using liquid nitrogen ($T_c > 77K$), or ***low temperature*** otherwise.
- *By material:* SC material classes include ***chemical elements*** (e.g. Hg or Pb), ***alloys*** (such as NbTi, $Nb_3Sn$ or NbN), ***ceramics*** (YBCO and MgB2), or ***organic superconductors***

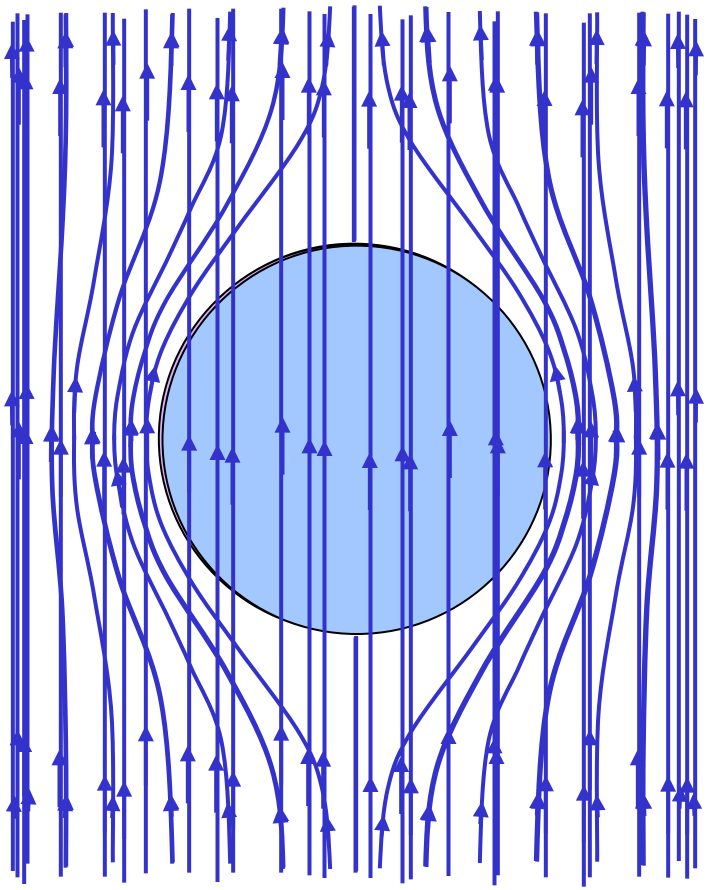


# Superconducting elements



# Two kinds of superconductor: Type 1

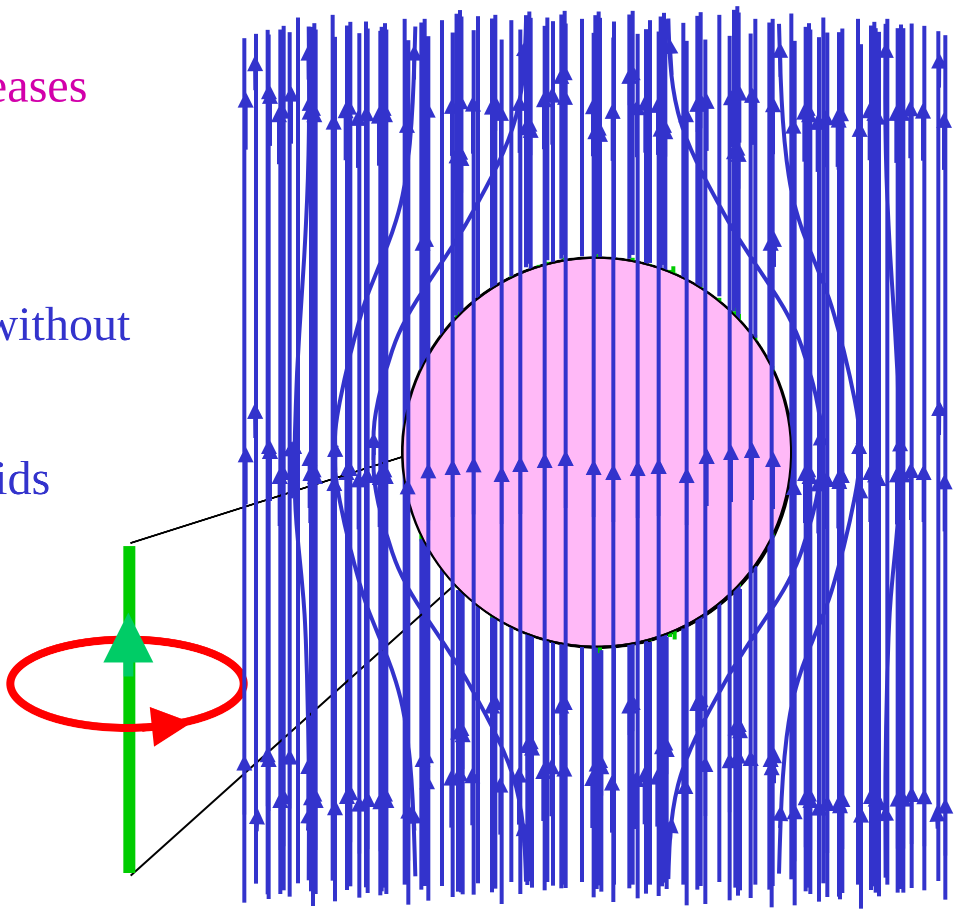
- the materials first discovered by Heike Kamerlingh Onnes in 1911 - soft metals like lead, tin mercury
- sphere of metal at room temperature
- apply magnetic field
- reduce the temperature - resistance decreases
- reduce the temperature some more
  - resistance decreases some more
- at the critical temperature $\theta_c$ the field is pushed out - the **Meissner effect** - superconductivity!
- increase the field - field is kept out
  - by Maxwell there must be surface currents
- increase the field some more - superconductivity is extinguished and the field jumps in
- thermodynamic critical field $B_c$ is trade off between **reducing** energy via condensation to superconductivity and **increasing** energy by pushing out field ~ 0.1T



*useless for magnets!*

# Two kinds of superconductor: Type 2

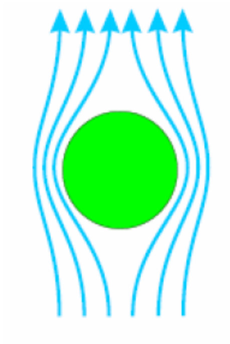
- apply magnetic field
- reduce the temperature - resistance decreases
- at the critical temperature $\theta_c$ the field is pushed out - surface currents again
- increase the field - field jumps back in without quenching superconductivity
- it does so in the form of quantized fluxoids
- lower critical field $B_{c1}$
- supercurrents encircle the resistive core of the fluxoid thereby screening field from the bulk material
- higher field $\Rightarrow$ closer vortex spacing
- superconductivity is extinguished at the (much higher) upper critical field $B_{c2}$



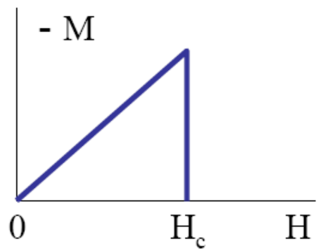
*OK for magnets!*

# Meissner effect (1933)

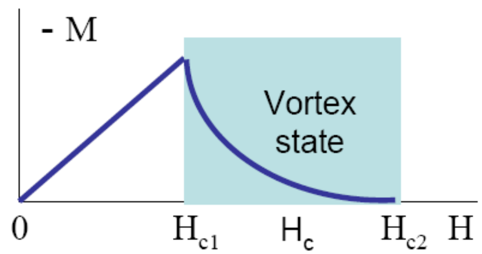
- Superconductivity is a quantum mechanical phenomenon. It is characterized by the Meissner effect (Meissner and Ochsenfeld, 1933), the complete ejection of magnetic field lines from the interior of the superconductor as it transitions into the superconducting state.
- The occurrence of the Meissner effect indicates that superconductivity cannot be understood simply as the idealization of perfect conductivity in classical physics.



Superconductor in Meissner state = ideal diamagnetic




Complete Meissner effect in type-I superconductors




High-field partial Meissner effect in type-II superconductors

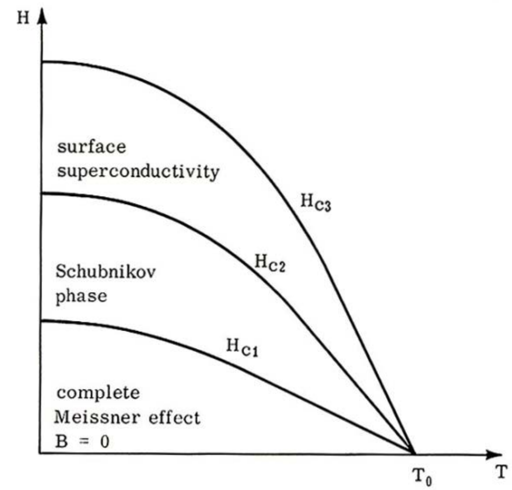



**Figure 3-1**
Phase diagram for a long cylinder of a Type II superconductor.

- Type I: Meissner state $B = H + M = 0 \; for \; H < H_c$; normal state at $H > H_c$
- Type II: Meissner state $B = H + M = 0 \; for \; H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

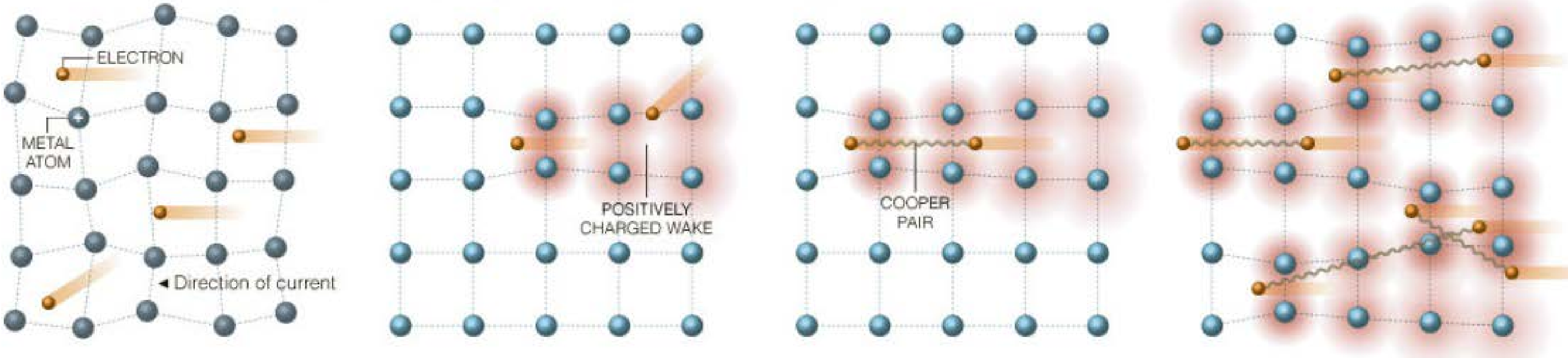
# Microscopic theory of superconductivity

Bardeen-Cooper-Schrieffer (BCS) theory (1957)
Nobel prize in 1972

The New York Times — January 7, 2008

## Low-Temperature Superconductivity

December was the 50th anniversary of the theory of superconductivity, the flow of electricity without resistance that can occur in some metals and ceramics.




**ELECTRICAL RESISTANCE**
Electrons carrying an electrical current through a metal wire typically encounter resistance, which is caused by collisions and scattering as the particles move through the vibrating lattice of metal atoms.

**CRITICAL TEMPERATURE**
As the metal is cooled to low temperatures, the lattice vibration slows. A moving electron attracts nearby metal atoms, which create a positively charged wake behind the electron. This wake can attract another nearby electron.

**COOPER PAIRS**
The two electrons form a weak bond, called a Cooper pair, which encounters less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way.

**SUPERCONDUCTIVITY**
If a pair is scattered by an impurity, it will quickly get back in step with other pairs. This allows the electrons to flow undisturbed through the lattice of metal atoms. With no resistance, the current may persist for years.

*Sources: Oak Ridge National Laboratory; Philip W. Phillips*

JONATHAN CORUM/THE NEW YORK TIMES

# BCS Theory

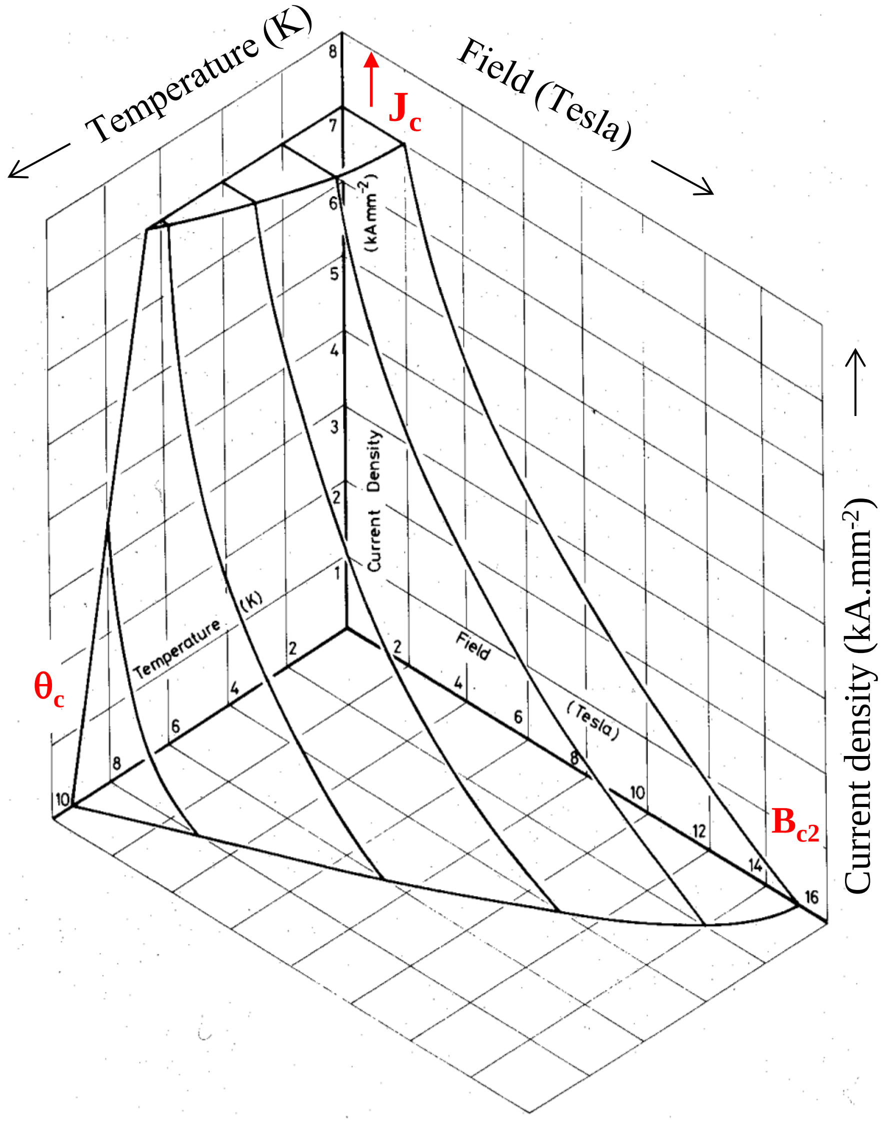
## What is the phase coherence?

Incoherent (normal) crowd:
each electron for itself

Phase-coherent (superconducting) condensate
of electrons

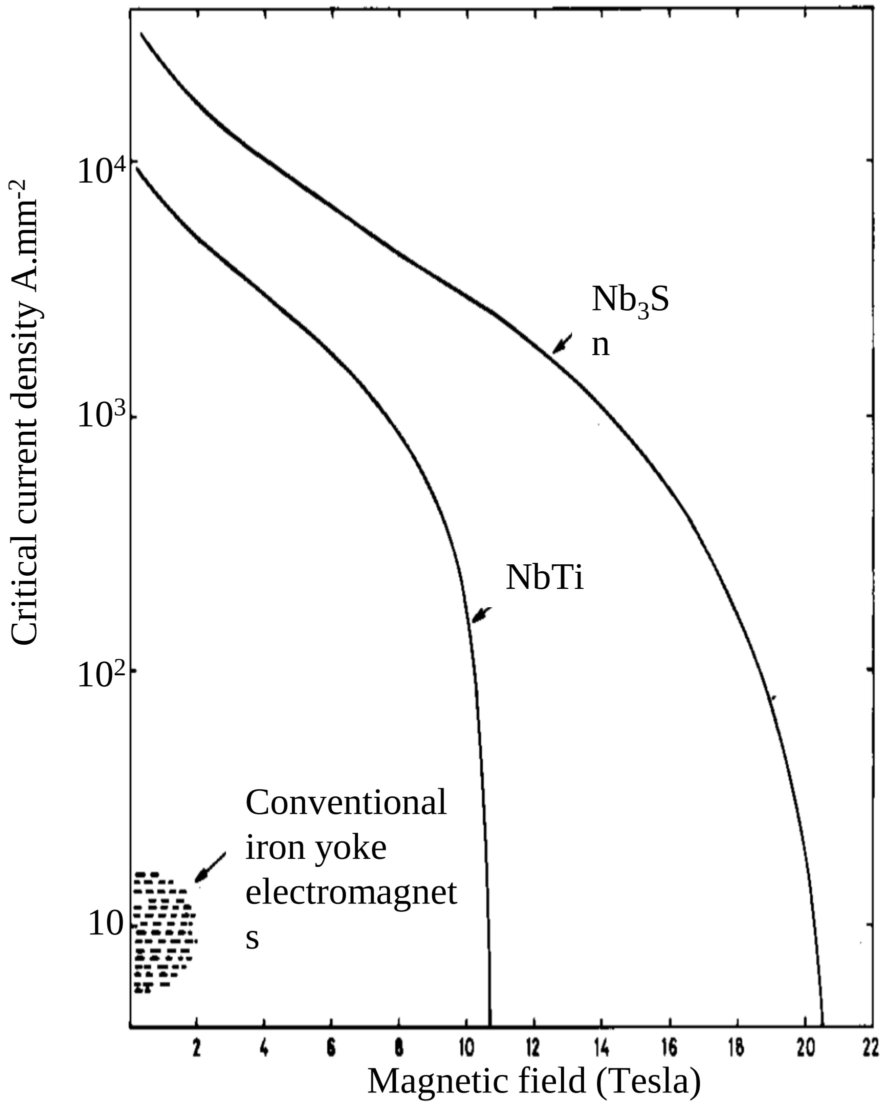
- Attraction between electrons with antiparallel momenta k and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs.
- Bose condensation of overlapping Cooper pairs in a coherent superconducting state.
- Scattering on electrons does not cause the electric resistance because it would break the Cooper pair.
- ***The strong overlap of many Cooper pairs results in the macroscopic phase coherence.***

# The critical surface for niobium titanium



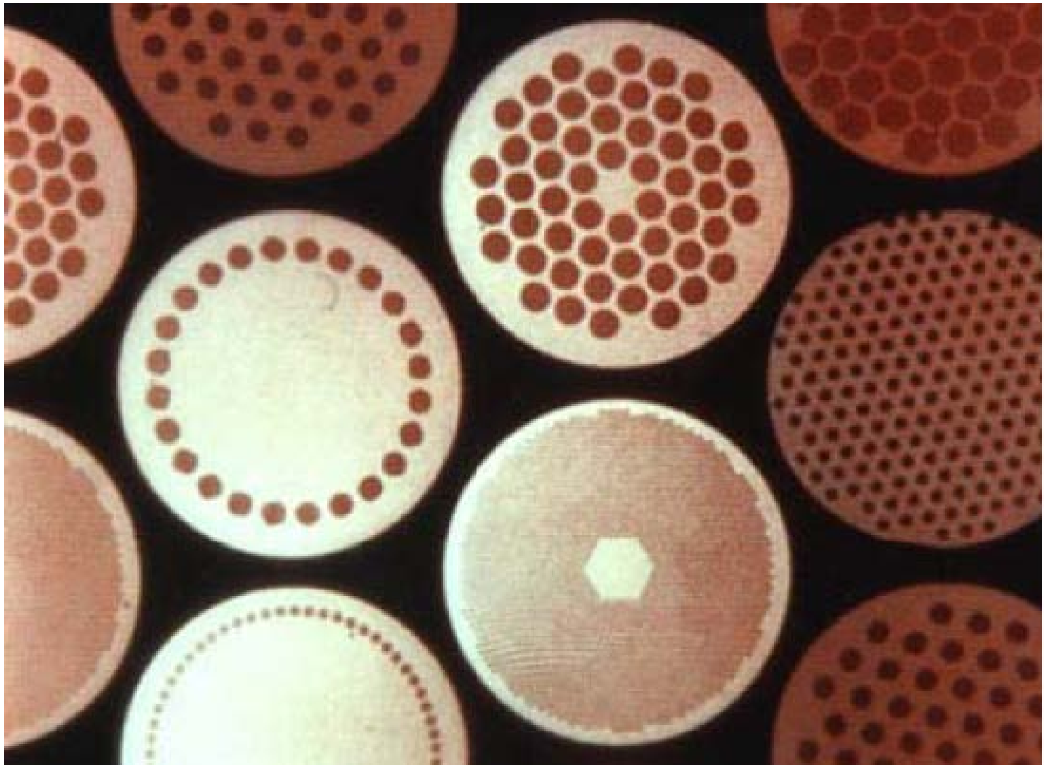

- Niobium titanium **NbTi** is the standard 'work horse' of the superconducting magnet business
- it is a ductile alloy
- picture shows the **critical surface**, which is the boundary between superconductivity and normal resistivity in 3 dimensional space
- superconductivity prevails everywhere below the surface, resistance everywhere above it
- we define an upper critical field $B_{c2}$ (at zero temperature and current) and critical temperature $\theta_c$ (at zero field and current) which are characteristic of the alloy composition
- critical current density $J_c(B,\theta)$ depends on processing

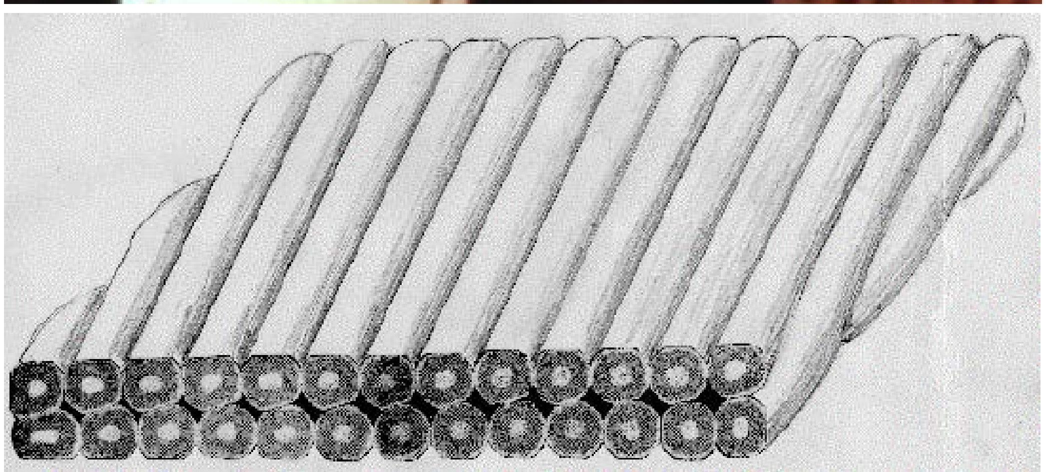
# The critical line at 4.2K




- because magnets usually work in boiling liquid helium, the critical surface is often represented by a curve of current versus field at 4.2K
- niobium tin $Nb_3Sn$ has a much higher performance in terms of critical current field and temperature than NbTi
- but it is brittle intermetallic compound with poor mechanical properties
- note that both the field and current density of both superconductors are way above the capability of conventional electromagnets

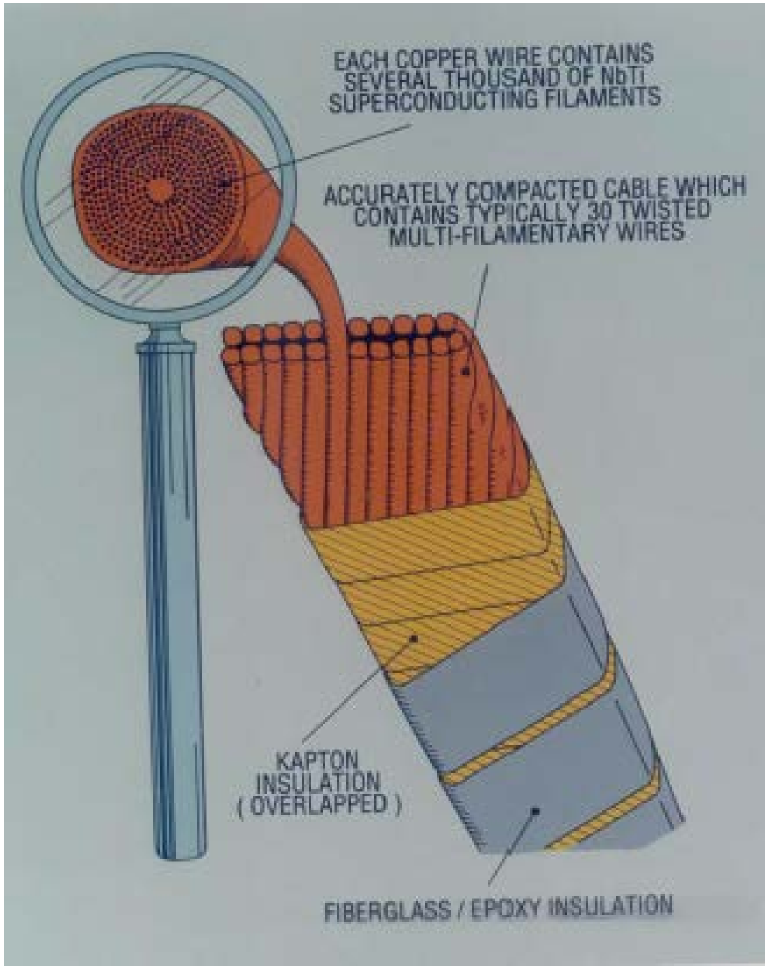
# Practical superconductors for magnets





- superconducting materials are always used in combination with a good normal conductor such as copper
- to ensure intimate mixing between the two, the superconductor is made in form of fine filaments embedded in a matrix of copper
- typical dimensions are:
  - wire diameter: 0.3 – 1.0 mm
  - filament diameter: 10 – 60 μm
- for electromagnetic reasons, the composite wires are twisted so that the filaments look like a rope
- for accelerators, many wires are combined in a cable

# A typical superconducting cable




Filament in an actual cable

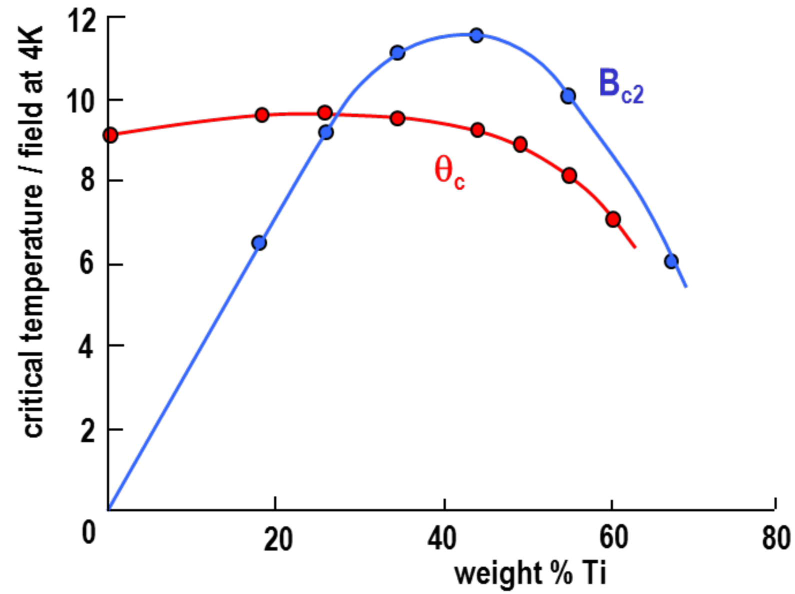
(Filament size in SSC/RHIC magnets: 6 micron

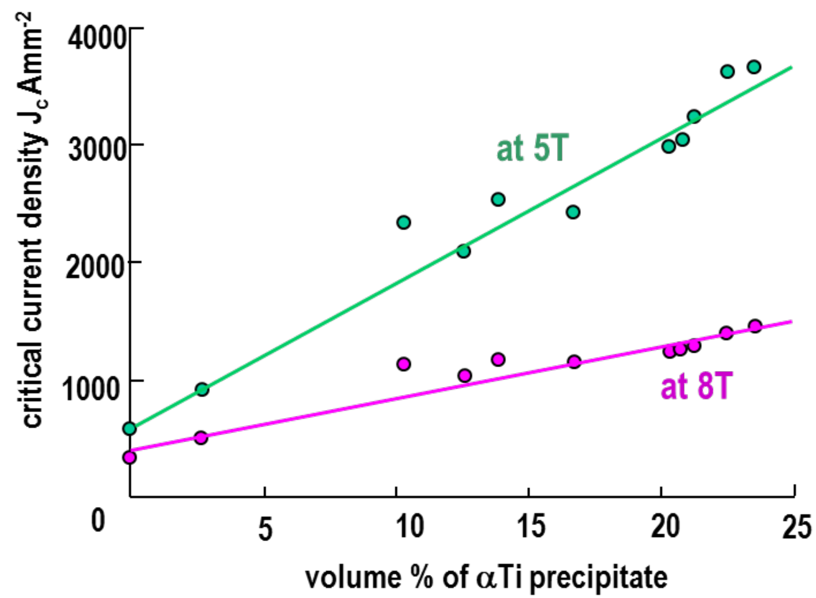
# Critical properties

- **Critical temperature** $\theta_c$: choose the right material to have a large energy gap or 'depairing energy'
  *property of the material*

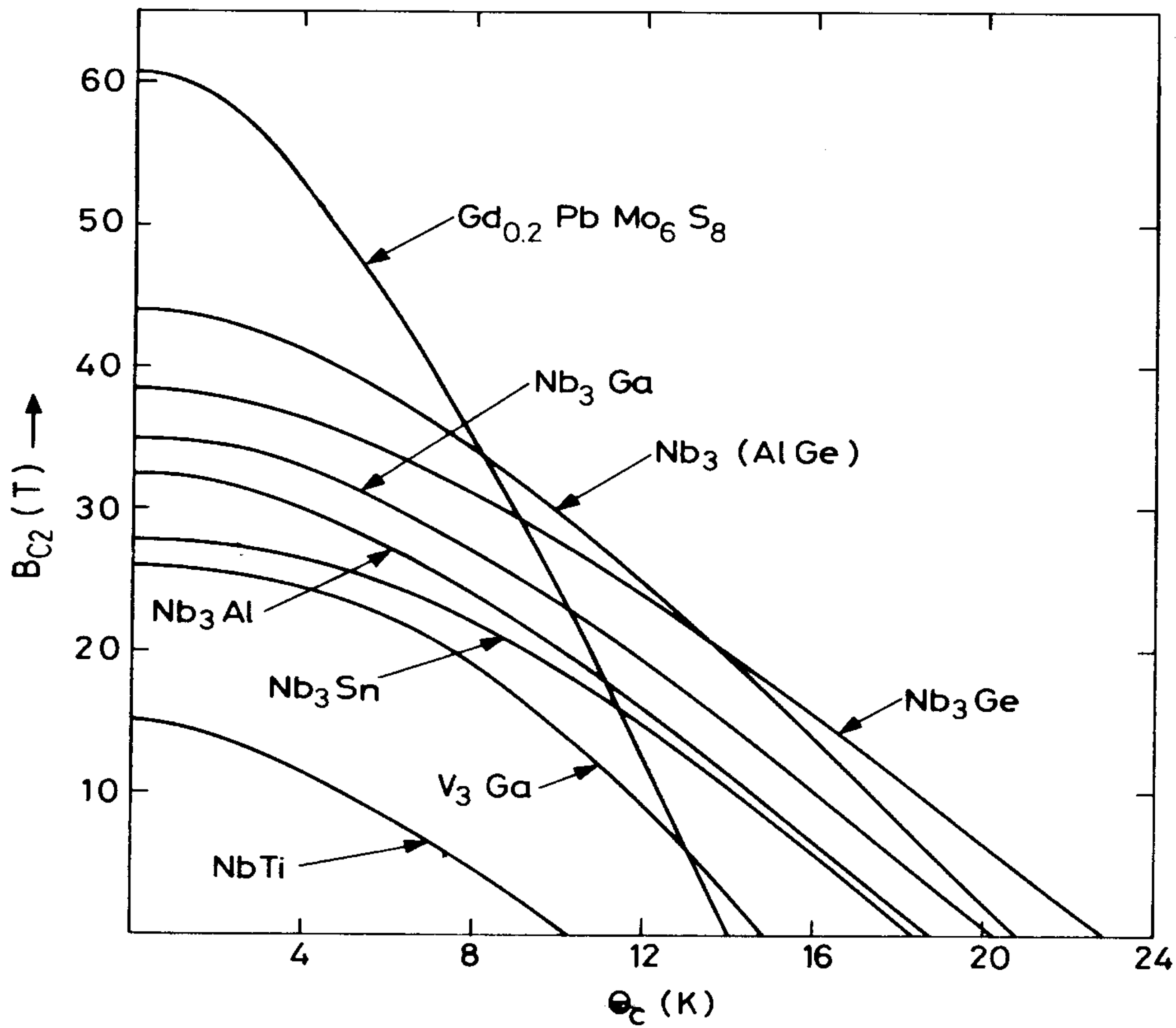
- **Upper Critical field** $B_{c2}$**:** choose a Type 2 superconductor with a high critical temperature and a high normal state resistivity
  *property of the material*

- **Critical current density** $J_c$: mess up the microstructure by cold working and precipitation heat treatments
  *hard work by the producer*



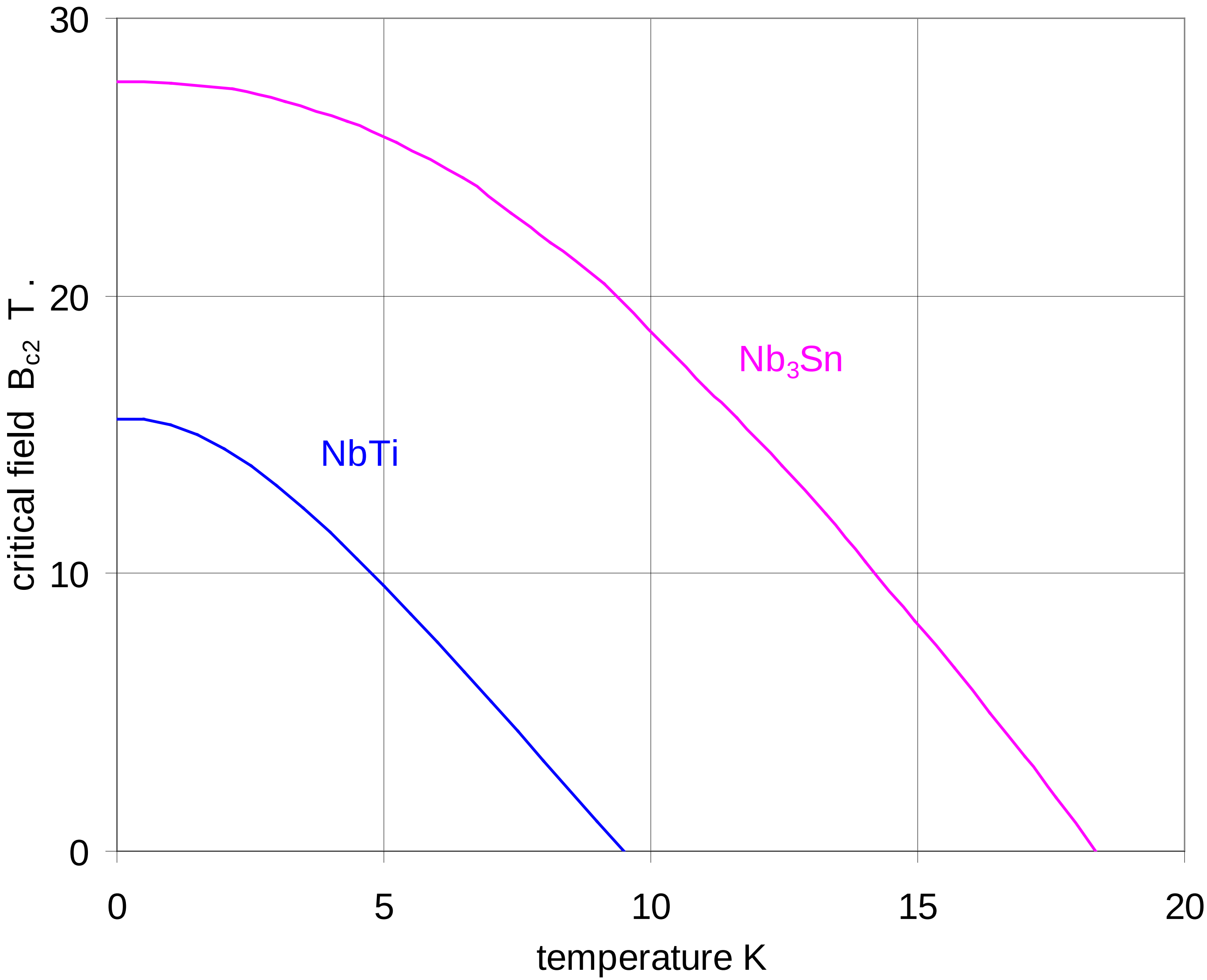
# Critical field & temperature of metallic superconductors




Note: of all the metallic superconductors, only NbTi is ductile.

All the rest are brittle intermetallic compounds

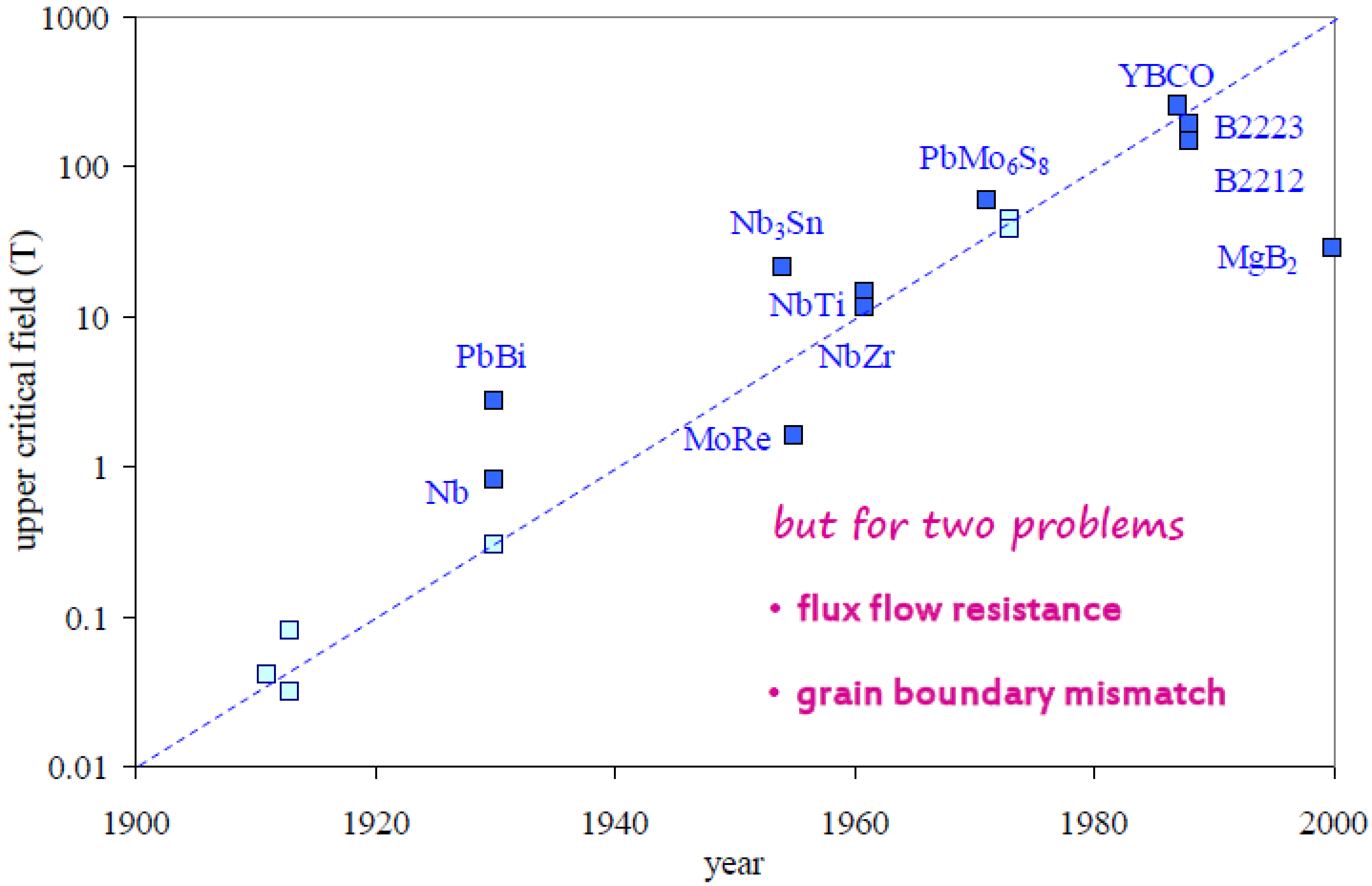
# Critical field & temperature of metallic superconductors




To date, all superconducting accelerators have used NbTi.

Of the intermetallics, only $Nb_3Sn$ has found significant use in magnets

# Wonderful materials for magnets

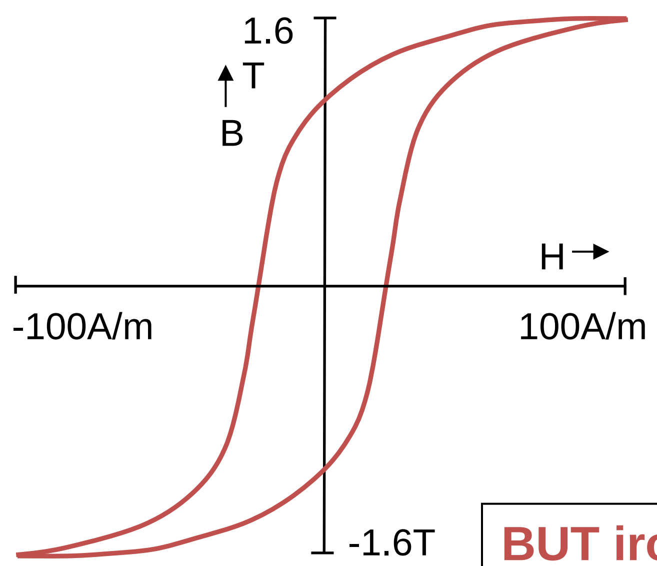


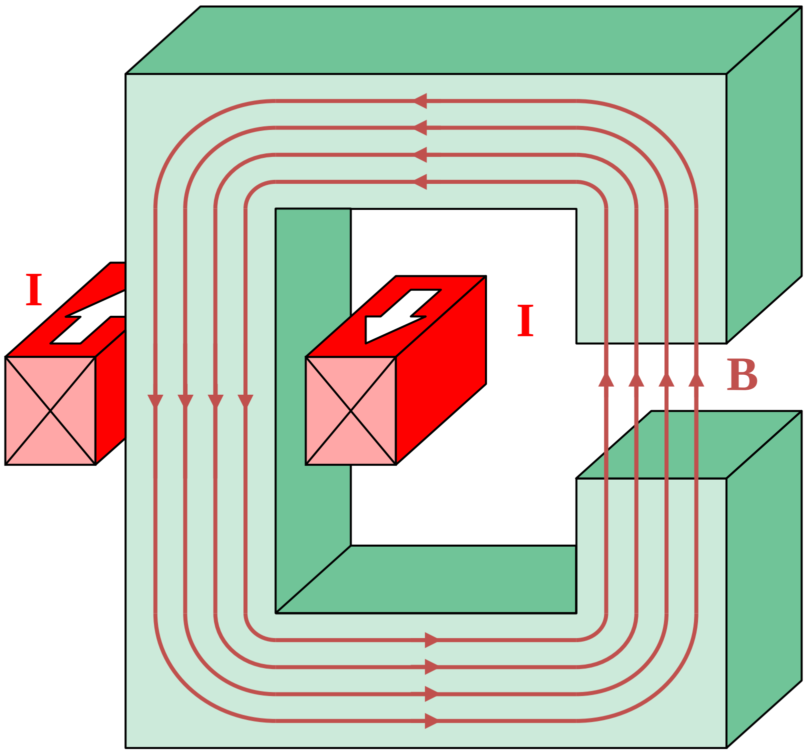

*but for two problems*

- **flux flow resistance**
- **grain boundary mismatch**

# Magnetic fields and ways to create them: (1) Iron

- Conventional electromagnets
- iron yoke reduces magnetic reluctance
  - ⇒ reduces ampere turns required
  - ⇒ reduces power consumption
- iron guides and shapes the field



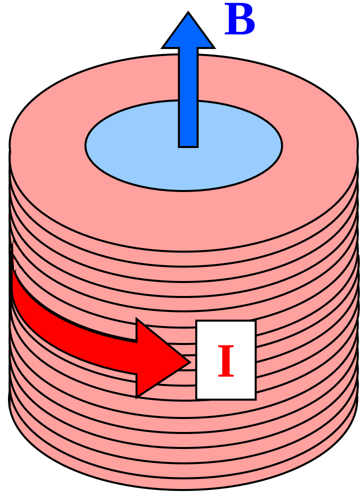

***Iron electromagnet***
***– for accelerator, HEP experiment***
***transformer, motor, generator, etc.***

**BUT iron saturates at ~ 2T**

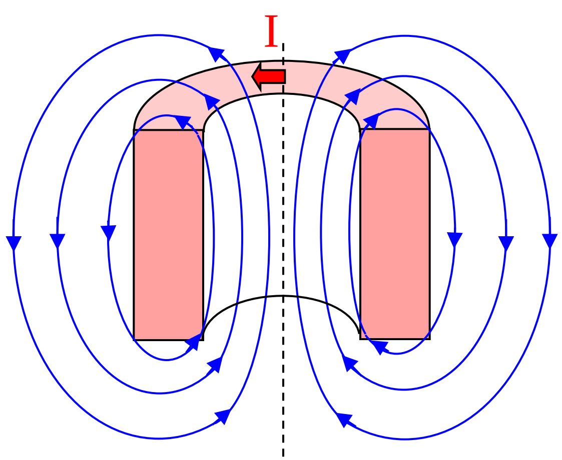
# Magnetic fields and ways to create them: (2) Solenoids

- no iron – field shape is set solely by the winding
- cylindrical winding
- azimuthal current flow

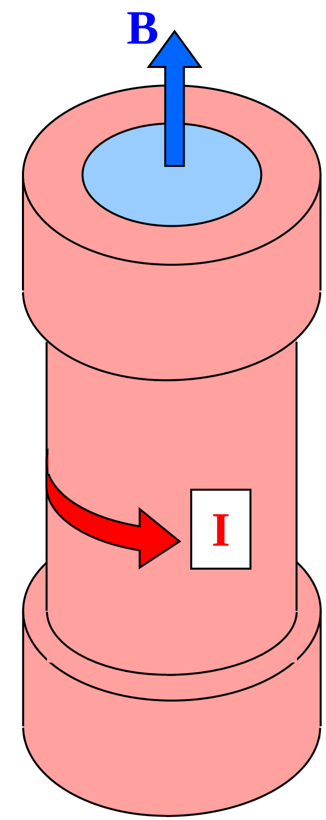
  - eg wire wound on bobbin
- axial field




- field lines curve outwards at the ends
- this curvature produces non uniformity of field
- very long solenoids have less curvature and more uniform field



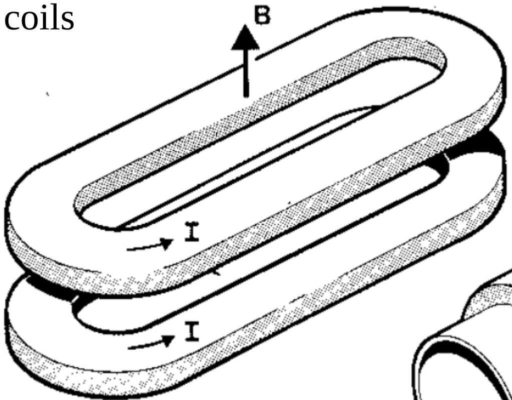

- can also reduce field curvature by making the winding thicker at the ends
- this makes the field more uniform

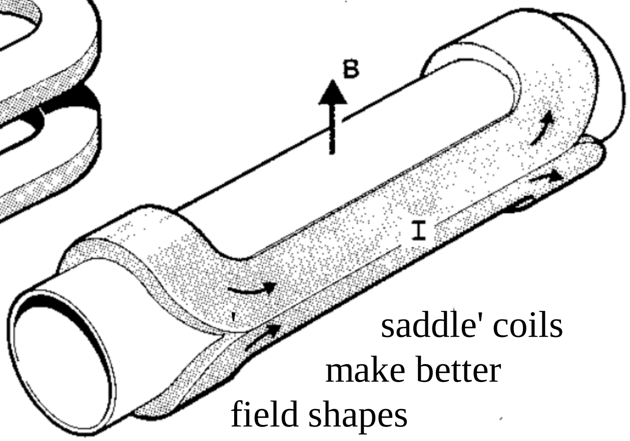



- more complicated winding shapes can be used to make very uniform fields

# Magnetic fields and ways to create them: (3) transverse uniform fields

simplest winding uses racetrack coils

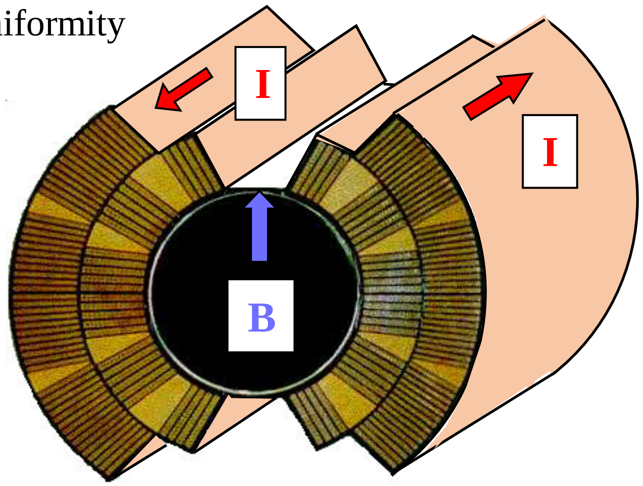


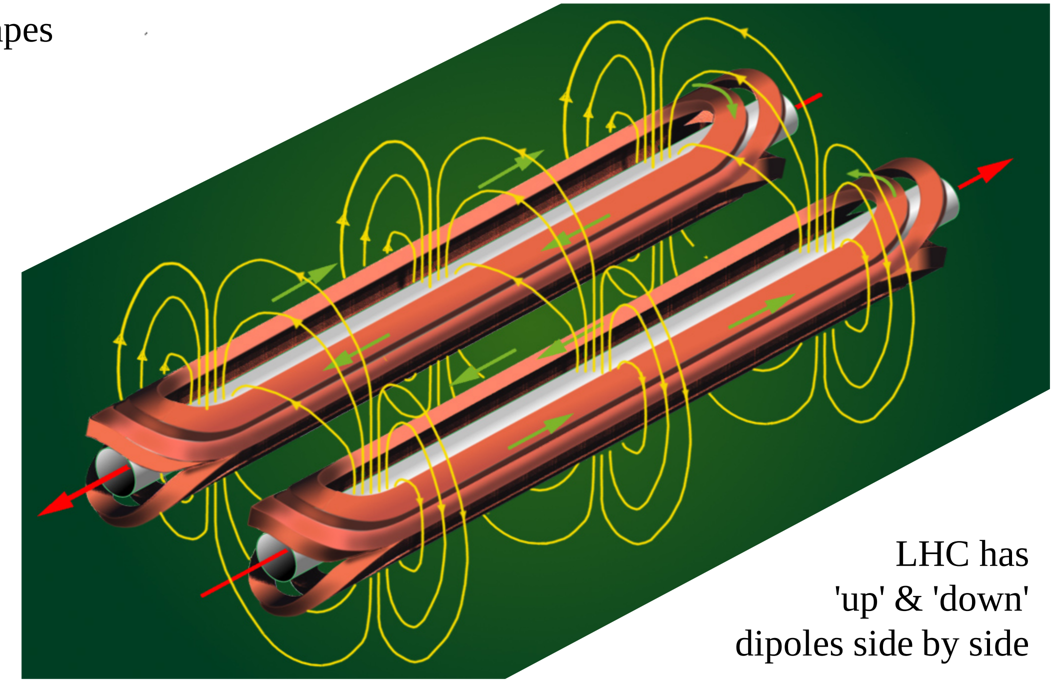




saddle' coils make better field shapes

- some iron - but field shape is set mainly by the winding
- used when the long dimension is transverse to the field, eg. accelerator magnets
- known as ***dipole*** magnets (because the iron version has 2 poles)

special winding cross sections for good uniformity



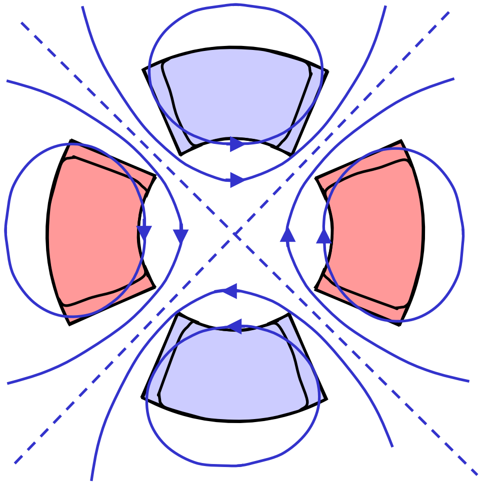
LHC has 'up' & 'down' dipoles side by side

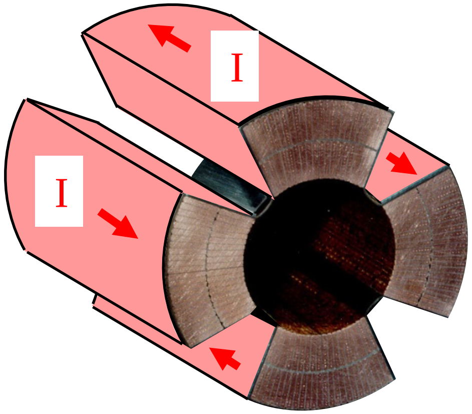
# Dipole magnets

- made from superconducting cable
- winding must have the right cross section
- also need to shape the end turns

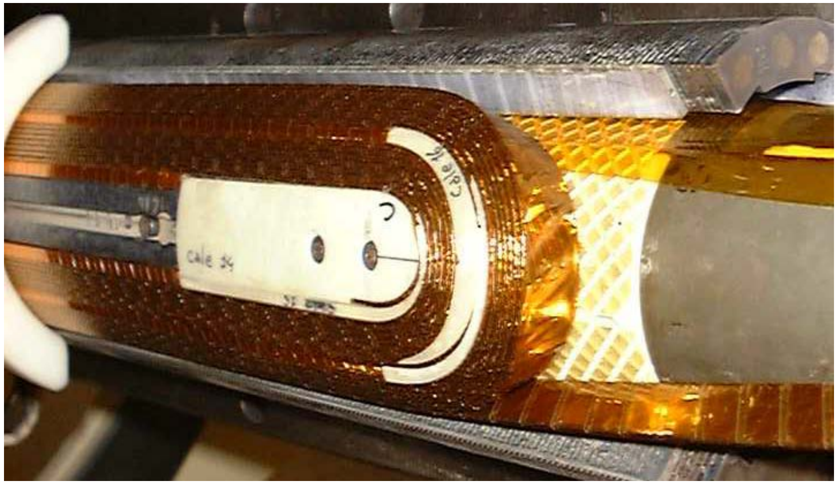
# Fields and ways to create them: (4) transverse gradient fields

- gradient fields produce focussing
- quadrupole windings






$B_x = ky$ $B_y = kx$



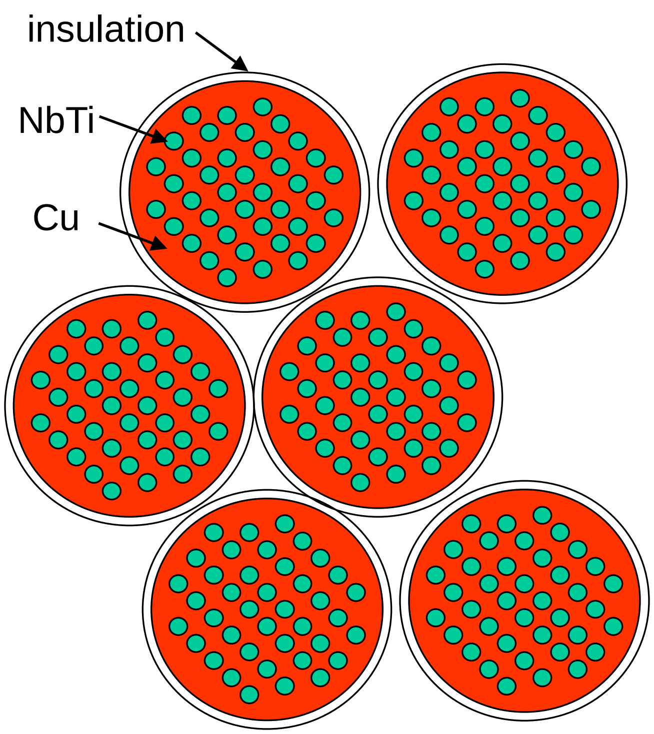
# Engineering current density

In designing a magnet, what really matters is the overall 'engineering' current density $\boldsymbol{J_{eng}}$

$$J_{eng} = \frac{current}{unit\ \ cell\ \ area} = J_{supercon} \times \lambda_{metal} \times \lambda_{winding}$$

fill factor in the wire $\lambda_{metal} = \dfrac{1}{(1+mat)}$

where *mat* = matrix : superconductor ratio




typically:

for NbTi *mat* = 1.5 to 3.0 ie $\lambda_{\mathrm{metal}}$ = 0.4 to 0.25

for $Nb_3Sn$ *mat* ~ 3.0 ie $\lambda_{\mathrm{metal}}$ ~ 0.25

for B2212 *mat* = 3.0 to 4.0 ie $\lambda_{\mathrm{metal}}$ = 0.25 to 0.2

$\lambda_{\mathrm{winding}}$ takes account of space occupied by insulation, cooling channels, mechanical reinforcement etc and is typically 0.7 to 0.8

So typically $\boldsymbol{J_{eng}}$ is only 15% to 30% of $\boldsymbol{J_{supercon}}$