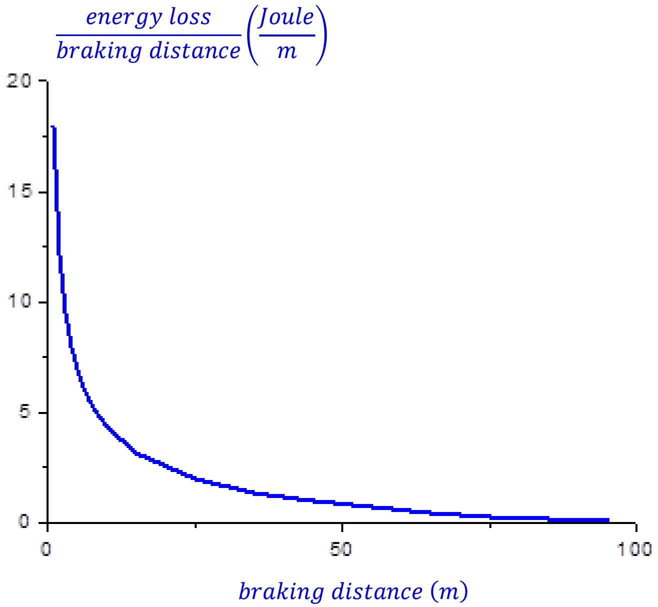
Interaction with matter

accelerated motion:

$$s = \frac{b}{2} t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2 \cdot s}{b}}$$

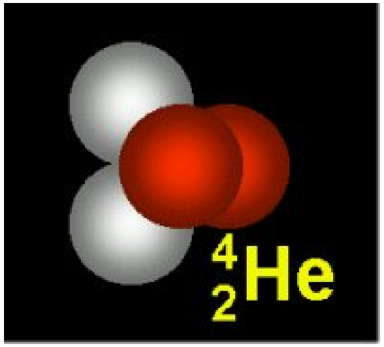
$$v = v_0 - b \cdot t = v_0 - \sqrt{2 \cdot s \cdot b}$$

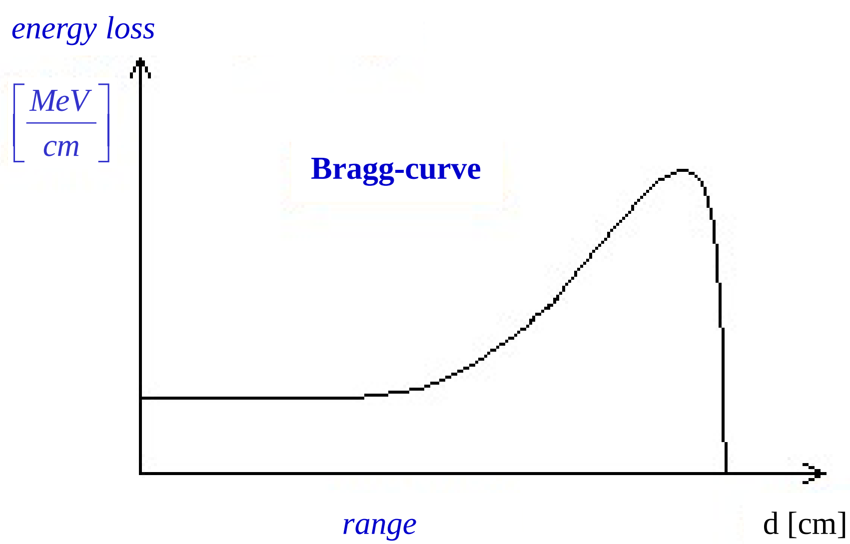
$$E = \frac{1}{2} m v^2 \quad \Rightarrow \quad \frac{dE}{ds} = m \cdot \left(\frac{v_0}{\sqrt{2 \cdot s \cdot b}} - 1 \right) \cdot b$$

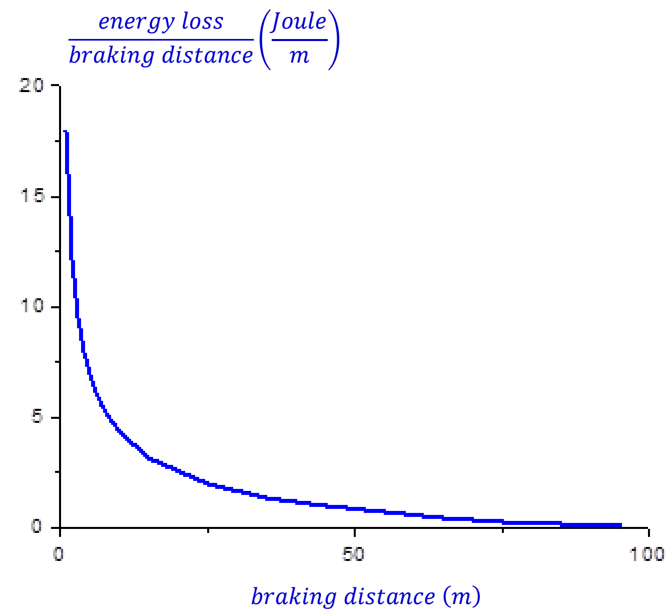


$$v_0 = 100 \frac{km}{h} = 27.78 \frac{m}{s} \qquad b = 3.86 \frac{m}{s^2}$$

Interaction with matter







Some Nuclear Units

Nuclear energies are very high compared to atomic processes, and need larger units. The most commonly used unit is the MeV.

$$1 \text{ electron Volt} = 1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ Joules}$$

$$1 \text{ MeV} = 10^6 \text{ eV; } 1 \text{ GeV} = 10^9 \text{ eV; } 1 \text{ TeV} = 10^{12} \text{ eV}$$

However, the nuclear size are quite small and need smaller units:

Atomic sizes are on the order of 0.1 nm = 1 Angstrom = 10^{-10} m. Nuclear sizes are on the order of femtometers which in the nuclear context are usually called fermis:

$$1 \text{ fermi} = 1 \text{ fm} = 10^{-15} \text{ m}$$

Atomic masses are measured in terms of atomic mass units with the carbon-12 atom defined as having a mass of exactly 12 amu. It is also common practice to quote the rest mass energy $E=m_0c^2$ as if it were the mass. The conversion to amu is:

$$1 \text{ u} = 1.66054 \cdot 10^{-27} \text{ kg} = 931.494 \text{ MeV}/c^2$$

electron mass = 0.511 MeV/c^2; proton mass = 938.27 MeV/c^2; neutron mass = 939.56 MeV/c^2

LUNDS UNIVERSITET BERKELEY LAB

Mass data: nucleardata.nuclear.lu.se/database/masses/

Relevant Formulae

The relevant formulae are calculated if A_1, Z_1 and A_2, Z_2 are the mass number (amu) and charge number of the projectile and target nucleus, respectively, and T_{lab} is the laboratory energy (MeV)

$$E = T_{lab} + m_0 \cdot c^2$$

$$m \cdot c^2 = T_{lab} + m_0 \cdot c^2$$

$$\frac{m_0 \cdot c^2}{\sqrt{1-\beta^2}} = T_{lab} + m_0 \cdot c^2$$

beam velocity:

$$\beta = \frac{\sqrt{{T_{lab}}^2 + 1863 \cdot A_1 \cdot T_{lab}}}{931.5 \cdot A_1 + T_{lab}}$$

Lorentz contraction factor:

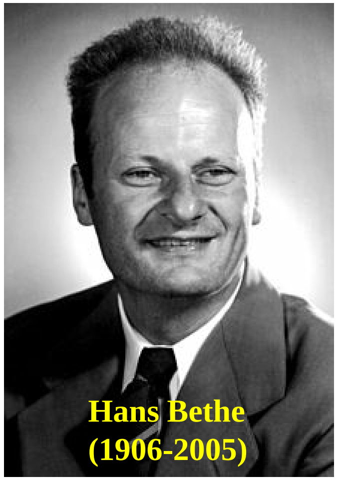
$$\gamma = (1-\beta^2)^{-1/2}$$

$$\gamma = \frac{931.5 \cdot A_1 + T_{lab}}{931.5 \cdot A_1}$$

$$\beta \cdot \gamma = \frac{\sqrt{{T_{lab}}^2 + 1863 \cdot A_1 \cdot T_{lab}}}{931.5 \cdot A_1}$$

Energetic charged particles in matter

$$E_{kin} \cong \frac{1}{2} m v^2 \qquad -\frac{dE}{dx} \propto \frac{mz^2}{E}$$



dominant in the classical limit [40 MeV/A (0.3c) - <1% deviation

$$-\frac{dE}{dx} = \overbrace{\frac{4\pi e^4 z^2}{m_0 v^2}} nZ \left[\ln \frac{2 m_0 v^2}{I} - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2} \right]$$

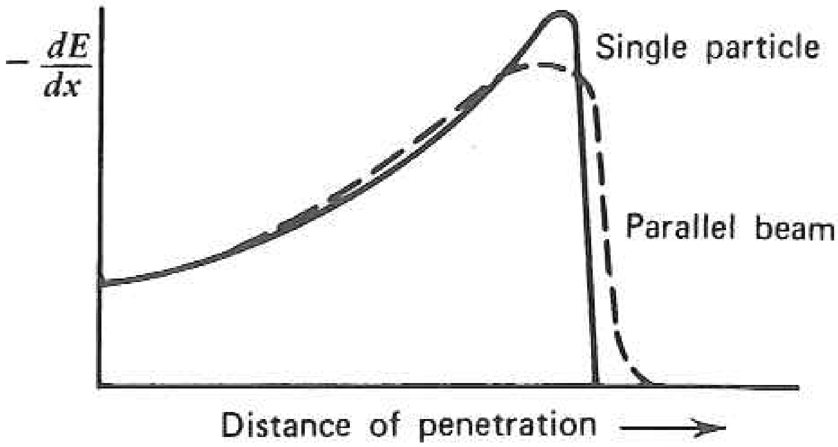
Bethe-Block formula

z – projectile atomic number
v – projectile velocity
m_0 - electron mass
e – electron charge

n – target number density
Z – target atomic number
nZ – target electron density
I – average excitation and ionization potential

Energetic charged particles in matter

Bragg curve



$$-\frac{dE}{dx} \propto \frac{mz^2}{E}$$

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_0 v^2} nZ \left[\ln \frac{2m_0 v^2}{I} - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2} \right]$$

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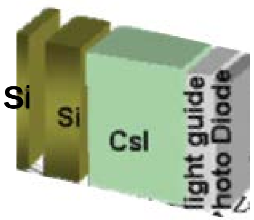
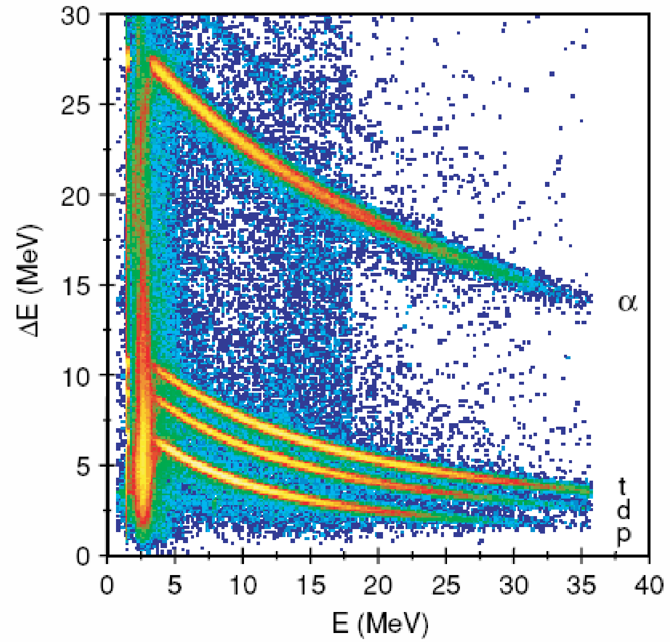
Energetic charged particles in matter

$$-\frac{dE}{dx} \propto \frac{mz^2}{E}$$

Charged particle identification with segmented or stacked detectors





$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_0 v^2} nZ \left[\ln \frac{2m_0 v^2}{I} - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2} \right]$$

Bethe-Block formula

z – projectile atomic number

v – projectile velocity

m_0 - electron mass

e – electron charge

n – target number density

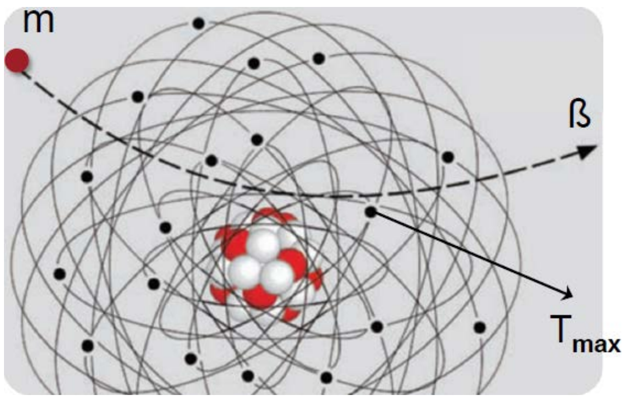
Z – target atomic number

nZ – target electron density

I – average excitation and ionization potential

Interaction of α-particles in matter

α-particles are highly ionized and lose their energy very fast by ionization and excitation when passing through matter.



➢ maximum energy transfer T_{max} of a projectile with mass **m** and velocity β on an electron $\mathbf{m_e}$ at rest

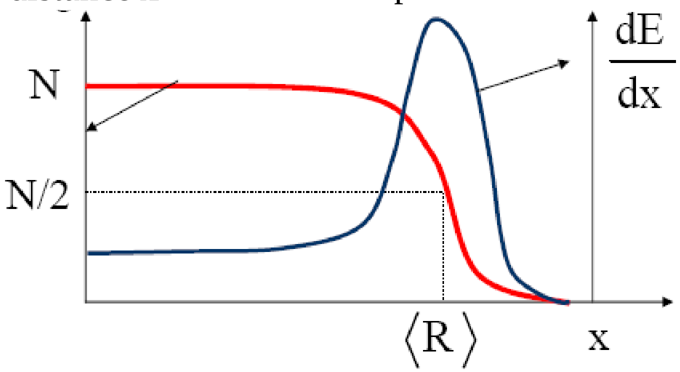
$$T_{max} = \frac{2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2 \cdot m^2}{m^2 + m_e{}^2 + 2 \cdot \gamma \cdot m \cdot m_e}$$

$$T_{max} = 2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2$$

for all heavy primary particles except electrons and positrons

number of α-particles as a function of distance **x**

energy loss of α-particles per distant unit



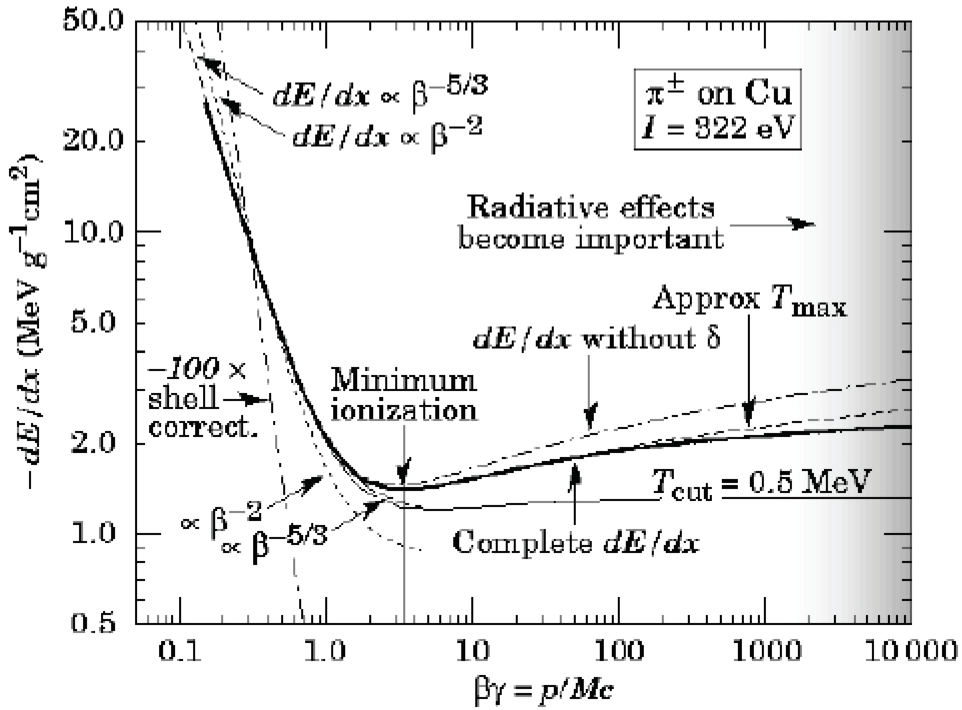
average range <R> of α-particles with 5 MeV
2,5cm in air, 2,3cm in Al, 4,3cm in tissue

Interaction of charged particles in matter

Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

$$-\frac{dE}{dx} = \underbrace{4\cdot\pi\cdot r_e^{2}\cdot N_a\cdot m_e c^{2}}_{=\,0.3071\ \mathrm{MeV\ g^{-1}cm^{2}}}\cdot\rho\cdot\frac{Z}{A}\cdot\frac{z^{2}}{\beta^{2}}\cdot\left[\frac{1}{2}ln\left(\frac{2\cdot m_e c^{2}\cdot\gamma^{2}\cdot\beta^{2}\cdot T_{max}}{I^{2}}\right)-\beta^{2}-\delta-2\cdot\frac{C}{Z}\right] \qquad \approx z^{2}\cdot\frac{Z}{A}\cdot f(\beta,I)$$

N_a : Avogadro number $6.02\cdot10^{23}$ mol^{-1}
r_e : class. electron radius $2.81\cdot10^{-13}$ cm
m_e : electron mass
ρ : density of abs. matter
Z : element number of abs. matter
A : mass of abs. matter
z : charge number of incoming particle
W_{max} : max. energy transfer in a single collision
I : average ionization potential



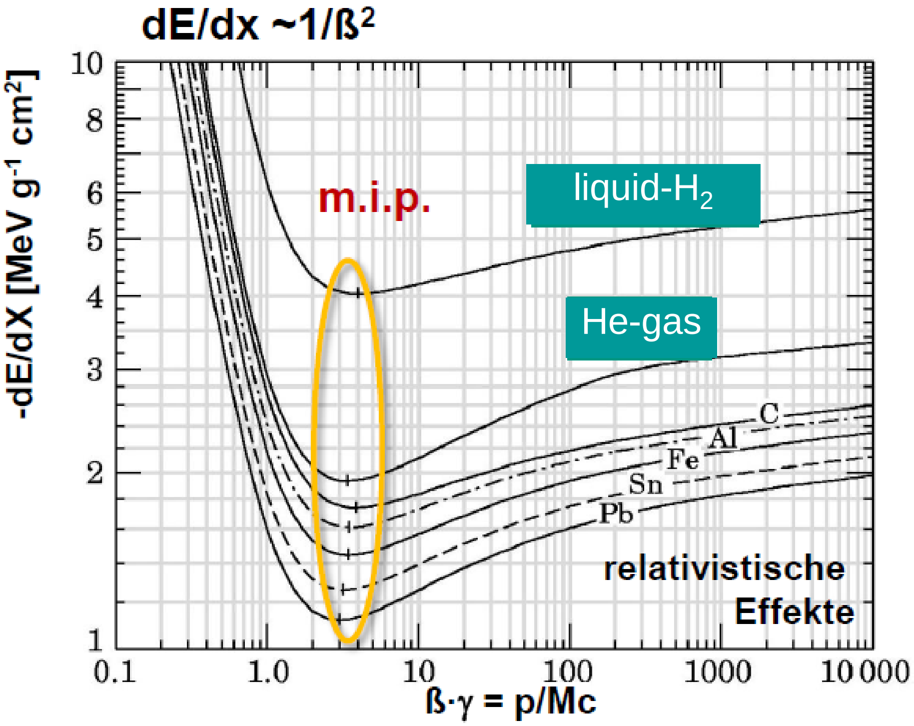

- for small β the term $1/\beta^2$ is dominant
- dE/dx has a minimum at βγ~3-4 (minimum ionizing particle)
- for high momenta dE/dx reaches a saturation

Interaction of charged particles in matter

Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

$$-\frac{dE}{dx} = \underbrace{4 \cdot \pi \cdot r_e^2 \cdot N_a \cdot m_e c^2}_{= 0.3071 \text{ MeV g}^{-1}\text{cm}^2} \cdot \rho \cdot \frac{Z}{A} \cdot \frac{z^2}{\beta^2} \cdot \left[\frac{1}{2} ln\left(\frac{2 \cdot m_e c^2 \cdot \gamma^2 \cdot \beta^2 \cdot T_{max}}{I^2}\right) - \beta^2 - \delta - 2 \cdot \frac{C}{Z}\right] \qquad \approx z^2 \cdot \frac{Z}{A} \cdot f(\beta, I)$$

- energy loss of a particle is independent of its mass!
- energy loss is an important tool for particle identification
- for minimum ionizing particles m.i.p. **dE/dx ~ 2 MeV g^{-1} cm^2**
  i.e. for a target density ρ = 1 g/cm^3
  dE/dx ~ 2 MeV/cm



- for small β the term 1/β² is dominant
- dE/dx has a minimum at βγ~3-4 (minimum ionizing particle)
- for high momenta dE/dx reaches a saturation

Energy loss and range of charged particles

$$-\frac{dE}{d\varepsilon} = -\frac{1}{\rho}\cdot\frac{dE}{dx} = z^2\cdot\frac{Z}{A}\cdot f(\beta, I)$$

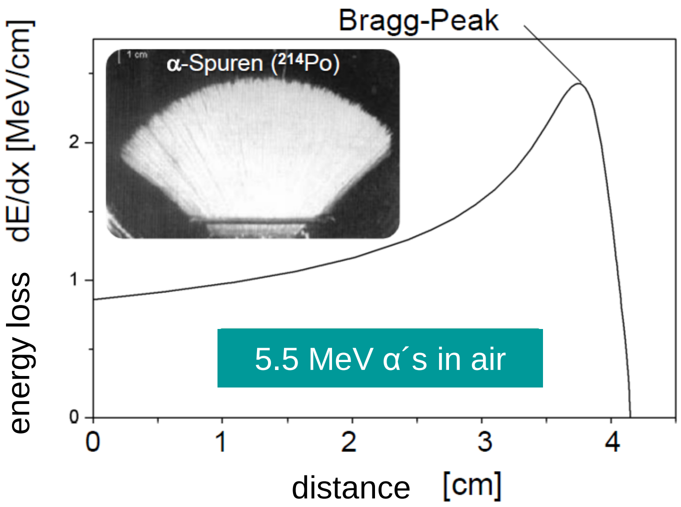
-dE/dε is independent of the material for equal particles

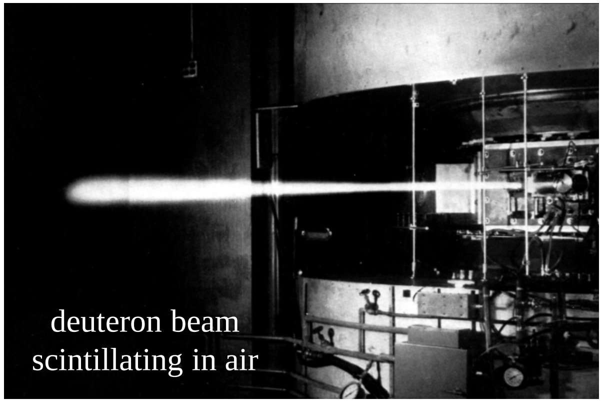
- the average range for particles with kin. energy T is obtained by integration

$$\bar{R} = \int_{E_0}^{0}\left(\frac{dE}{dx}\right)^{-1} dE$$

- 7.7 MeV α-particles in air: $\bar{R}/\rho \approx 7cm$

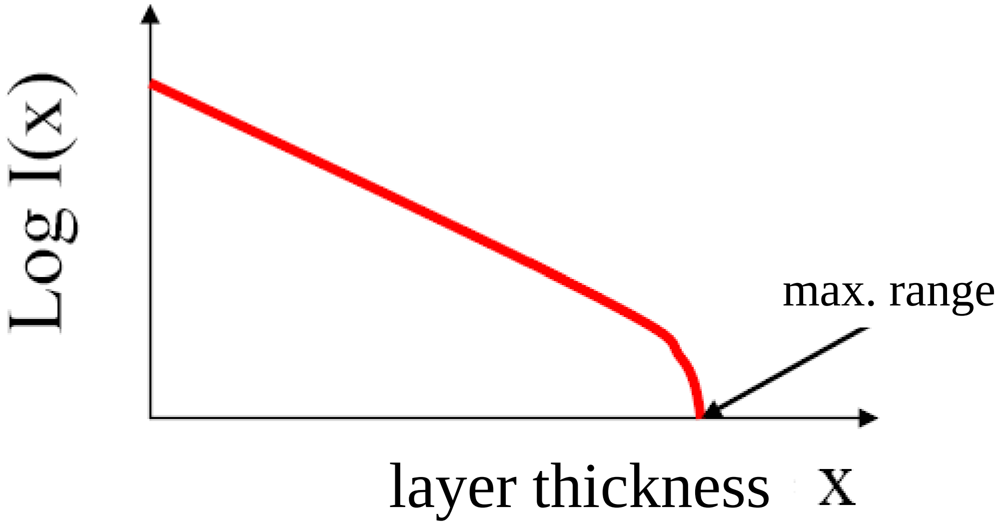
- range is not exact but there is range straggling,
  the number of interactions is a statistical process.




deuteron beam
scintillating in air

Interaction of β-particles with matter

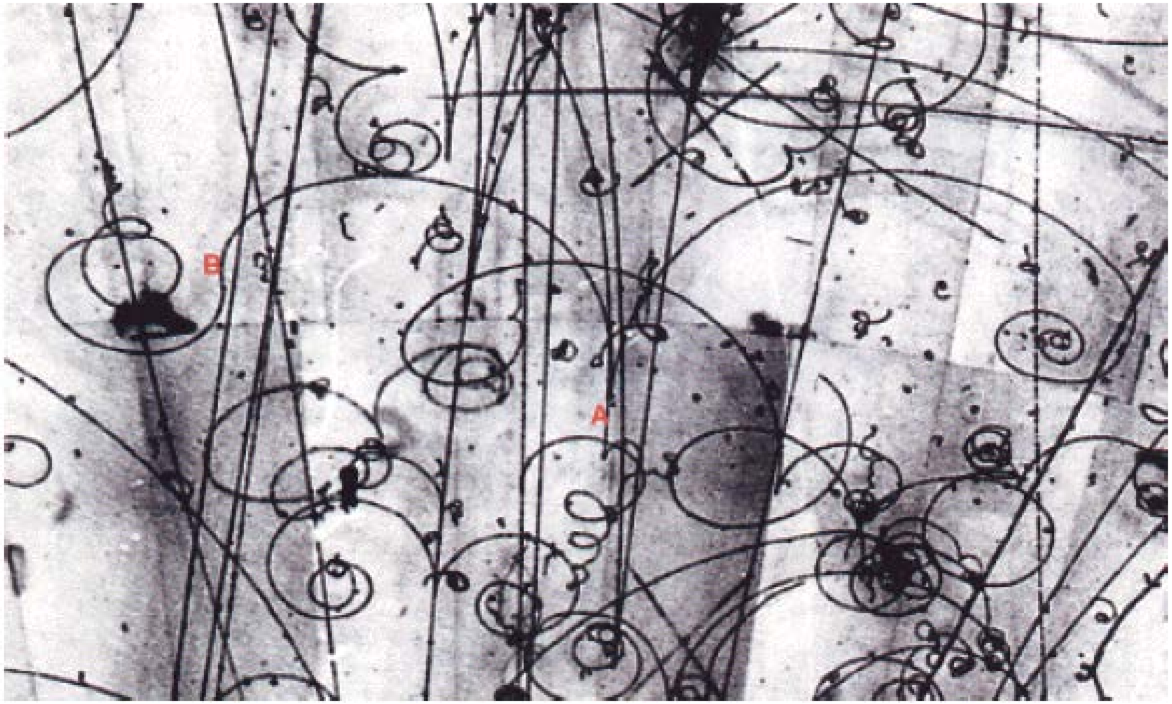
β-particles are also ionizing, similar to α-particles. Since the mass of the electrons and positrons are very small, the energy transfer per collision is small and the range large. Similar to the X-rays there is first only an attenuation, which finally leads to a maximum range for larger layer thicknesses.



Interaction of β-particles with matter

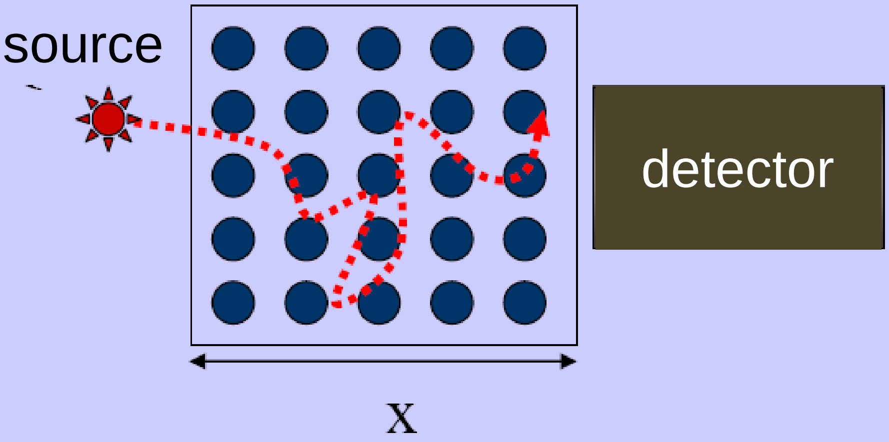
β^+ particles behave similarly as β^- particles; they are ionizing and attenuated on their way through matter.

But at the end of the their attenuation one observes a pair annihilation with an electron, which yields high energetic γ-emission. Positrons are hence more dangerous than electrons.

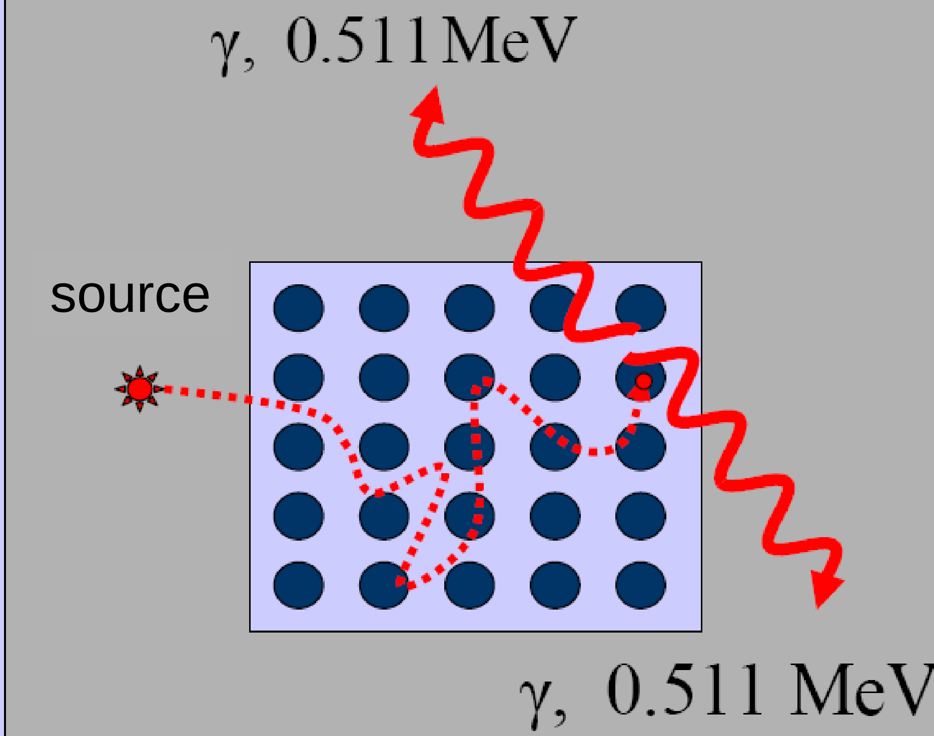


Comparison between electrons (β^-) and positrons (β^+) on their way through matter

electron



positron



Energy loss for electrons and positrons

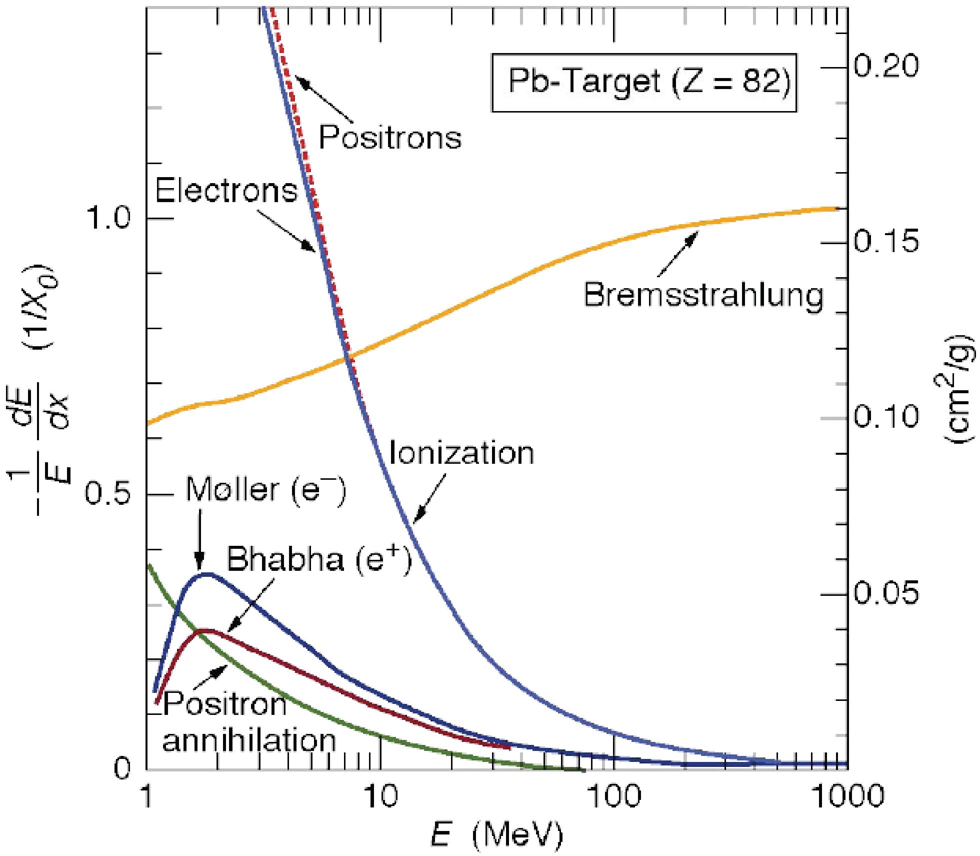
$e^{\pm}$ are exceptional cases due to their low mass. They will be deflected significantly in each collision.
In addition to the energy loss due to **ionization**, the energy loss due to **Bremsstrahlung** is of importance.

$$-\left(\frac{dE}{dx}\right)_{tot} = -\left(\frac{dE}{dx}\right)_{coll} - \left(\frac{dE}{dx}\right)_{rad}$$

For high energies the energy loss due to Bremsstrahlung is given by

$$-\left(\frac{dE}{dx}\right)_{rad} \propto E \quad \text{and} \quad -\left(\frac{dE}{dx}\right)_{rad} \propto \frac{1}{m^2}$$

Other particles like muons also radiate, especially at higher energies.

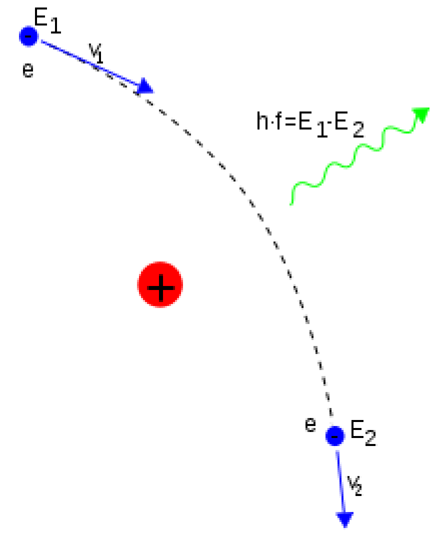


Bremsstrahlung

Bremsstrahlung ('braking radiation') is electromagnetic radiation produced by the deceleration of a charged particle when deflected by another charged particle, typically an electron by an atomic nucleus.

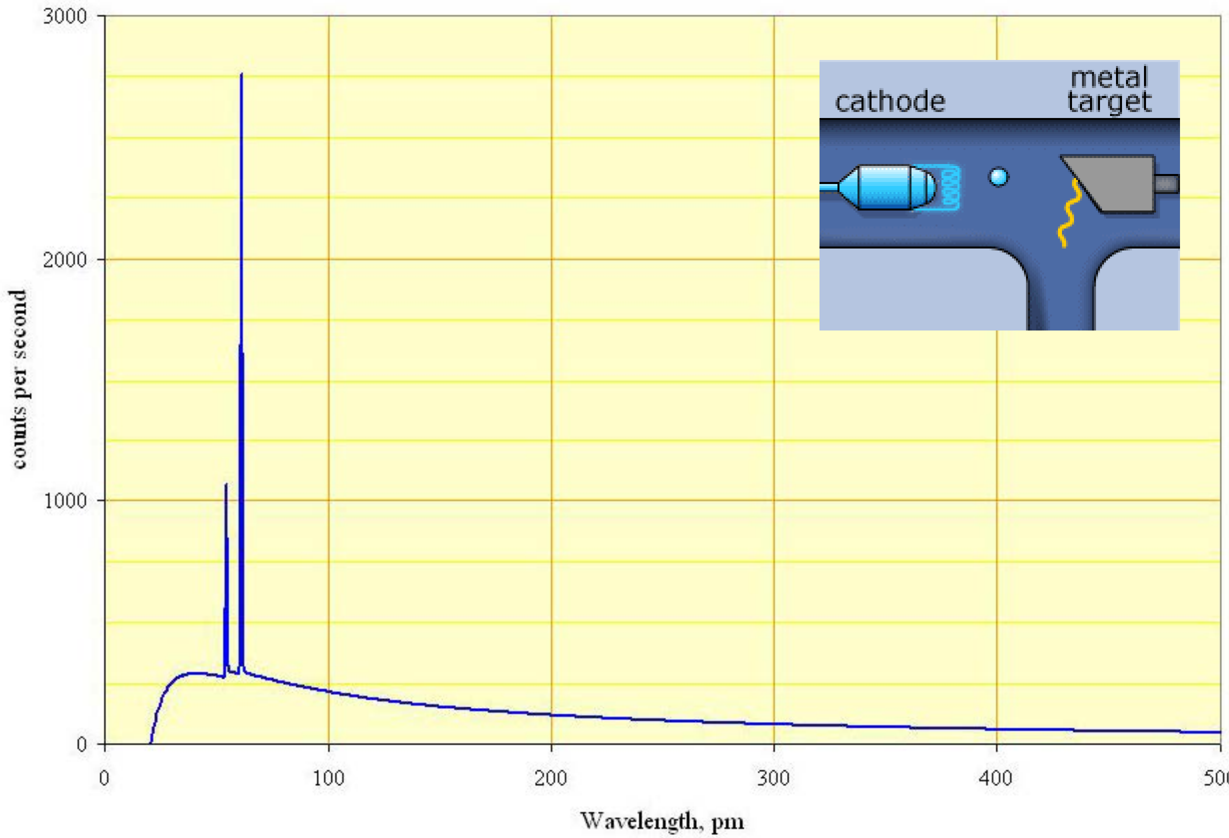
The moving particle loses kinetic energy, which is converted into a photon, thus satisfying the law of conservation of energy.

Bremsstrahlung has a continuous spectrum.



Bremsstrahlung

$$E_{photon} = h \cdot \nu = E_{kin} = e \cdot U$$



spectral distribution:

$$J(\lambda) = K \cdot I \cdot Z \cdot \left(\frac{\lambda}{\lambda_{min}} - 1\right) \cdot \frac{1}{\lambda^2}$$

K = Kramer constant
I = electron current
Z = element number of material

Spectrum of the X-rays emitted by an X-ray tube with a rhodium target, operated at 60 kV. The continuous curve is due to bremsstrahlung, and the spikes are characteristic K lines for rhodium.

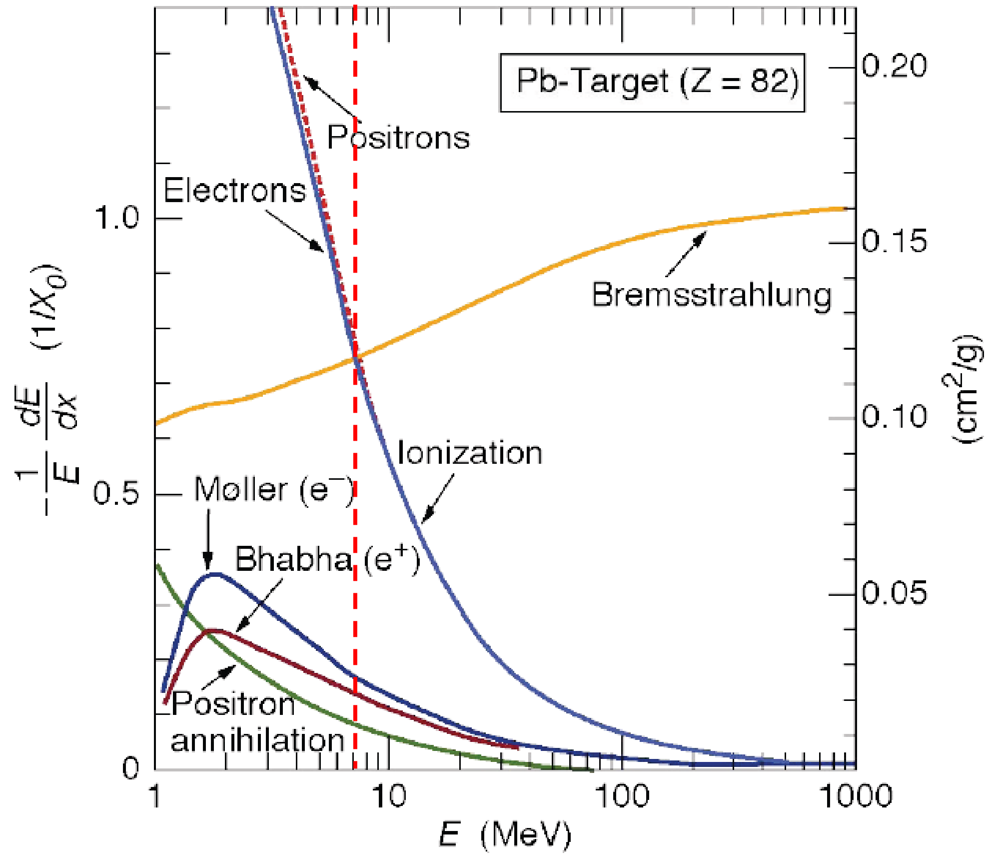
Bremsstrahlung

$$-\left(\frac{dE}{dx}\right)_{rad} = \frac{E}{X_0}$$

X_0 is the radiation length. It is the mean distance over which a high-energy electron loses all but 1/e of its energy by Bremsstrahlung

fit to data:

$$X_0 = \frac{716.4 \cdot A}{Z \cdot (Z+1) \cdot ln\left(287/\sqrt{Z}\right)}$$



Usual definition for the critical energy $\mathbf{E_c}$ (electron)

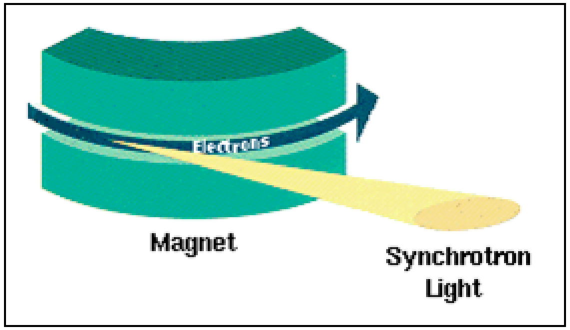
$$\left(\frac{dE}{dx}\right)_{ionization} = \left(\frac{dE}{dx}\right)_{bremsstrahlung}$$

$$E_c(e^-) = \begin{cases} \dfrac{610\ MeV}{Z+1.24} & for\ solids\ and\ liquids \\ \dfrac{710\ MeV}{Z+0.92} & for\ gases \end{cases}$$

example: Pb (Z=82, ρ = 11.34 [g/cm^3] → E_c = 7.34 MeV

Synchrotron radiation

The electromagnetic radiation emitted when charged particles are accelerated radially (**a** ⊥ **v**) is called **synchrotron radiation.** It is produced, for example, in synchrotrons using bending magnets.

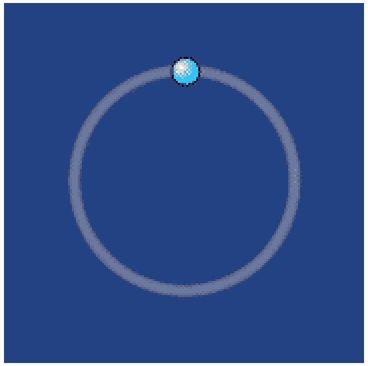


The energy loss of a charged particle (Z·e) due to radiation (during one cycle) is given by

$$\Delta E = \frac{(Ze)^2 \cdot \beta^3 \cdot \gamma^4}{\epsilon_0 \cdot 3R}$$

with Z = element number, ε_0 = electric field constant, R = radius of the storage ring, β=v/c and the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \equiv \frac{E}{m_0 c^2}$$



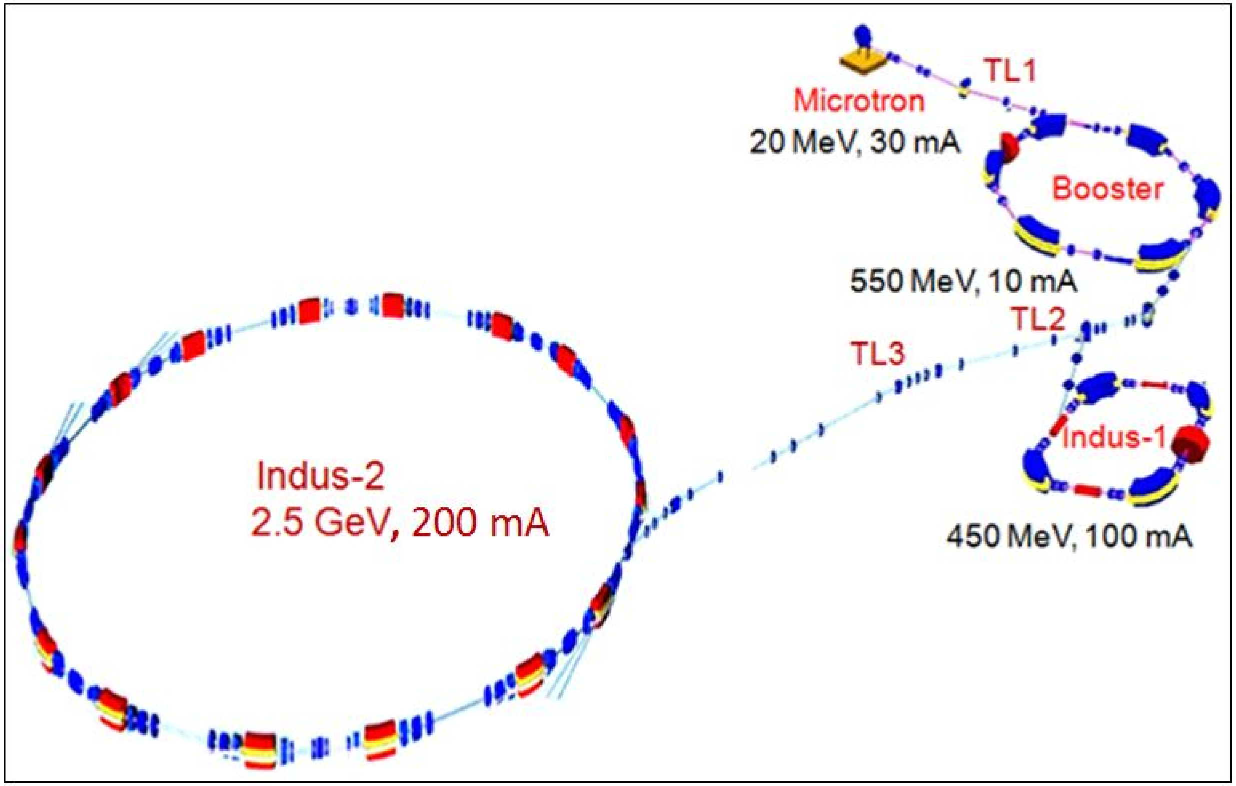
For relativistic velocities $\beta \approx 1$

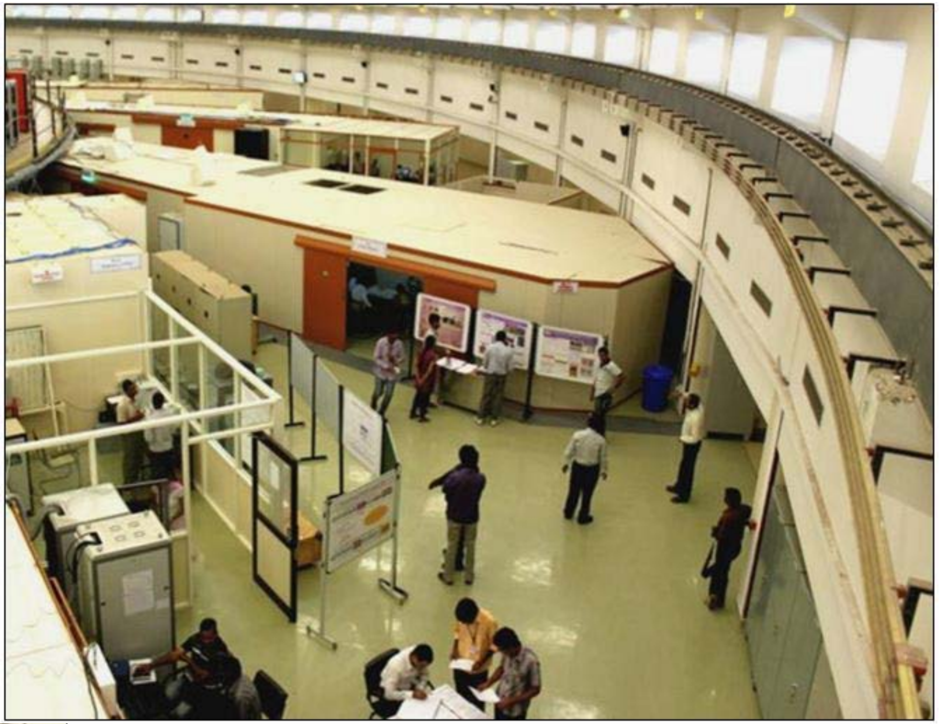
$$\Delta E = \frac{(Ze)^2 \cdot E^4}{\epsilon_0 \cdot 3R \cdot (m_0 c^2)^4}$$

It is apparent that one uses light particles to create synchrotron radiation.

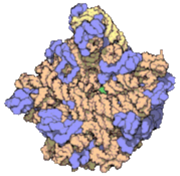
Raja Ramanna Centre for Advanced Technology

Indore





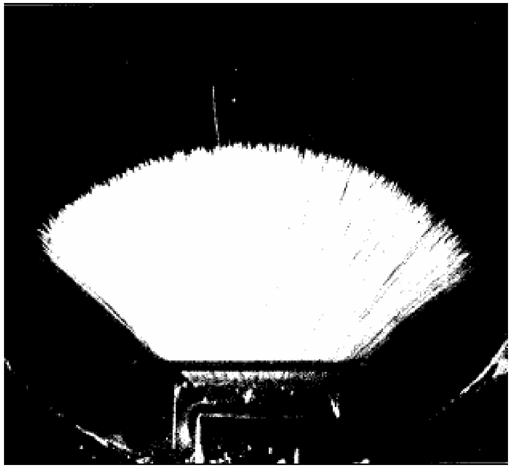
❖ Applications: condensed matter physics, material science, biology and medicine.



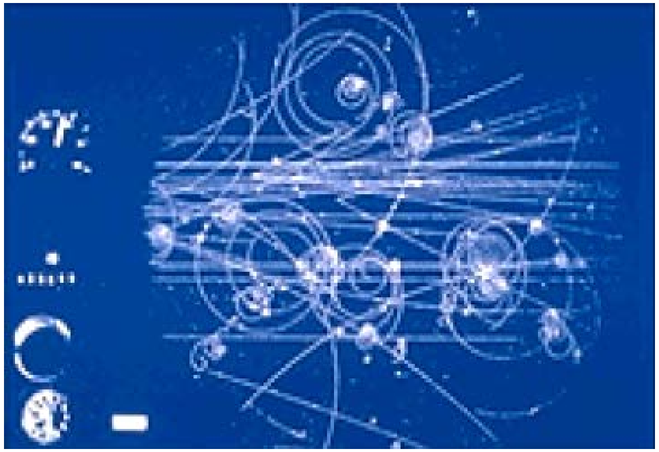
Structure of a ribosome (components of a cell)

Typical range of radioactive radiation in air

range of 5.5 MeV α-particles in air is ~ 4.2 cm



range of 1 MeV β-particles in air is ~ 4 m



range of X-rays, γ-rays and neutrons is very large.

shielding or large distances ($1/r^2$ law) are the solution