Interaction of gamma rays with matter

\[ I(x) = I_0(\lambda) \cdot e^{-\frac{\mu(\lambda,Z)}{\rho} \rho \cdot x} \]

total absorption coefficient: \( \mu/\rho \) [cm\(^2\)/g]

\[ \frac{\mu_{total}}{\rho} = \sum_{i=1}^{3} \sigma_i \]

- \( i=1 \) photoelectric effect
- \( i=2 \) Compton scattering
- \( i=3 \) pair production
Mass dependence of X-ray absorption

For X-ray radiation the **photoelectric effect** is the most important interaction.

\[
\left( \frac{\mu}{\rho} \right)_{\text{photo}} \approx \lambda^3 \cdot Z^5
\]

**Lead absorbs more than Beryllium!**

\[{}^{82}\text{Pb} \text{ serves as shielding for X-ray and } \gamma\text{-ray radiation; lead vests are used by medical staff people who are exposed to X-ray radiation.} \]

\[{}_{4}\text{Be} \text{ is often used as windows in X-ray tubes to allow for almost undisturbed transmission of X-ray radiation.} \]
Mass dependence $\mu/\rho$ of X-ray absorption

wave length dependence for Pt as absorber

$(Z=78)$

element number dependence for

$\lambda=0.1 \text{ nm or } 12.4 \text{ keV}$
X-ray image shows the effect of different absorptions

Bones absorb more radiation as tissues because of their higher $^{20}$Ca content
## Interaction of gamma rays with matter

<table>
<thead>
<tr>
<th>~100 keV</th>
<th>~1 MeV</th>
<th>~10 MeV</th>
<th>γ-ray energy</th>
</tr>
</thead>
</table>

### Photoelectric

- Probability of interaction depth
- Isolated hits

\[ E_\gamma \rightarrow e_1 \rightarrow \text{Isolated hits} \]
Interaction of gamma rays with matter

\[ E_{e,\text{kin}} = h \cdot \nu - E_{\text{Bindung}} \]

**Photo effect:**
Absorption of a photon by a bound electron and conversion of the γ-energy in potential and kinetical energy of the ejected electron. (Nucleus preserves the momentum conservation.)
Interaction of gamma rays with matter

Compton scattering:
Elastic scattering of a γ-ray on a free electron. A fraction of the γ-ray energy is transferred to the Compton electron. The wavelength of the scattered γ-ray is increased: \( \lambda' > \lambda \).

Maximum energy of the scattered electron:

\[
T(e^-)_{\text{max}} = E_\gamma \cdot \frac{2 \cdot E_\gamma}{m_e c^2 + 2 \cdot E_\gamma}
\]

Energy of the scattered γ-photon:

\[
E_\gamma' = \frac{E_\gamma \cdot m_e c^2}{m_e c^2 + E_\gamma \cdot (1 - \cos \theta)}
\]

\[
\cos \theta = 1 + \frac{m_e c^2}{E_\gamma} - \frac{m_e c^2}{E_\gamma}
\]

Special case for \( E > > m_e c^2 \):
γ-ray energy after 180° scatter is approximately

\[
E_\gamma' = \frac{m_e c^2}{2} = 256 \text{ keV}
\]

Gap between the incoming γ-ray and the maximum electron energy.

\[
E_{\text{kin}}^{\text{max}} = E_\gamma - E_\gamma' = E_\gamma \cdot \frac{2 \cdot E_\gamma / m_e c^2}{1 + 2 \cdot E_\gamma / m_e c^2}
\]
Compton scattering:
Elastic scattering of a $\gamma$-ray on a free electron. A fraction of the $\gamma$-ray energy is transferred to the Compton electron. The wavelength of the scattered $\gamma$-ray is increased: $\lambda' > \lambda$. 
Interaction of gamma rays with matter

**Compton scattering:**
Elastic scattering of a $\gamma$-ray on a free electron. The angle dependence is expressed by the **Klein-Nishina-Formula:**

$$\frac{d\sigma_C}{d\Omega} = \frac{r_0^2}{2} \left( \frac{E_{\gamma'}}{E_\gamma} \right)^2 \left\{ \frac{E_{\gamma'}}{E_\gamma} + \frac{E_{\gamma'}}{E_\gamma} - \sin^2 \theta \right\}$$

As shown in the plot, forward scattering ($\theta$ small) is dominant for $E_\gamma > 100$ keV.

$r_0 = 2.818$ fm
**Pair production:**
If γ-ray energy is \( \gg 2m_\text{e}c^2 \) (electron rest mass 511 keV), a positron-electron pair can be formed in the strong Coulomb field of a nucleus. This pair carries the γ-ray energy minus \( 2m_\text{e}c^2 \).

Pair production for \( E_\gamma > 2m_\text{e}c^2 = 1.022 \text{MeV} \)

picture of a bubble chamber
All three interaction (photo effect, Compton scattering and pair production) lead to an attenuation of the γ-ray or X-ray radiation when passing through matter. The particular contribution depends on the γ-ray energy:

- **Photo effect:** \( \sim Z^{4-5}, E_{\gamma}^{-3.5} \)
- **Compton:** \( \sim Z, E_{\gamma}^{-1} \)
- **Pair:** \( \sim Z^2, \) increases with \( E_{\gamma} \)

The absorption attenuates the intensity, but the energy and the frequency of the γ-ray and X-ray radiation is preserved!
Gamma-ray spectrum of a radioactive decay

\[ \gamma_1, 0.30 \text{ MeV} \]
\[ \gamma_2, 2.1 \text{ MeV} \]

\[ \beta^+, 0.83 \text{ MeV} \]

Stable Daughter, \(^{A(Z-1)}\)

BSc

Pb X-ray

511 keV

\[ \gamma_1 + \gamma_2 \]