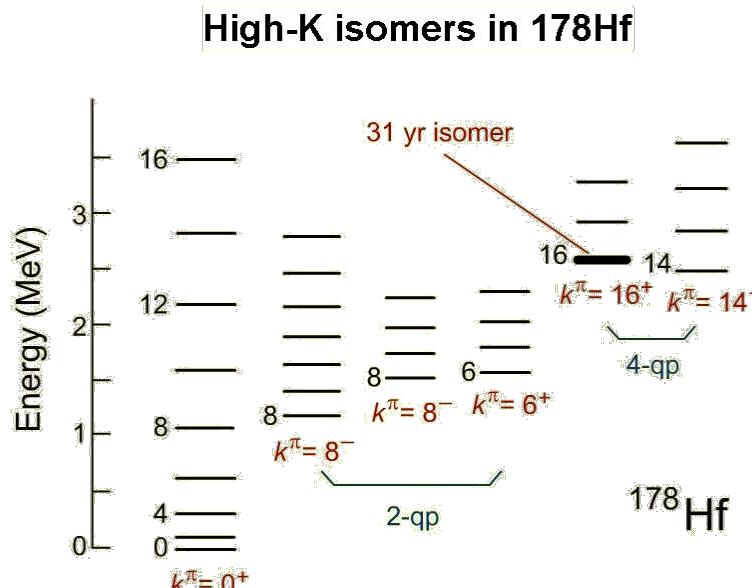
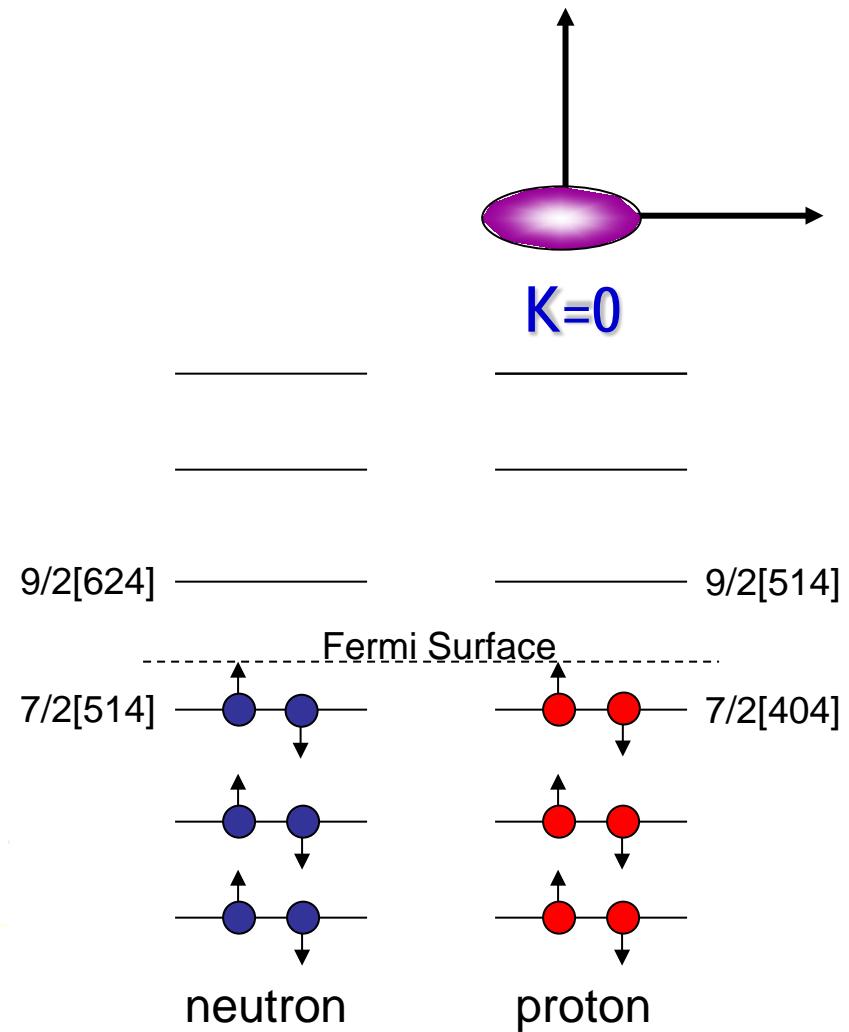


Investigation of ^{178}Hf – K-Isomers

- A well-known example:

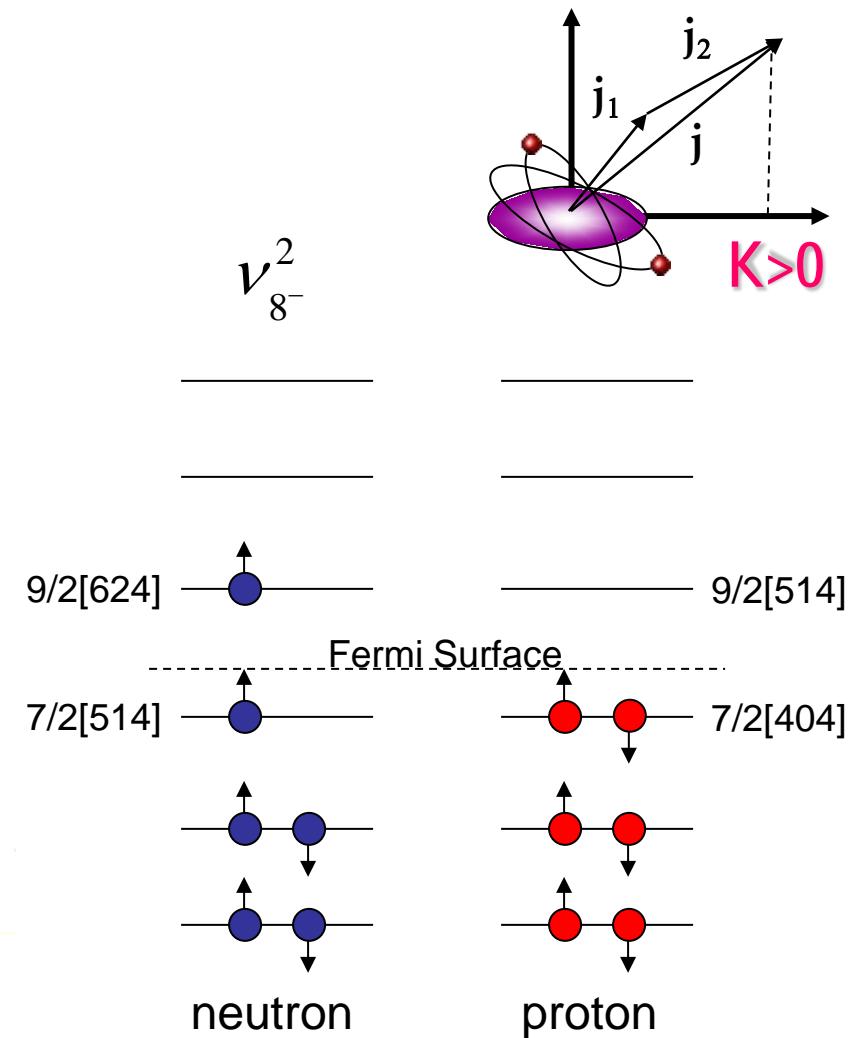
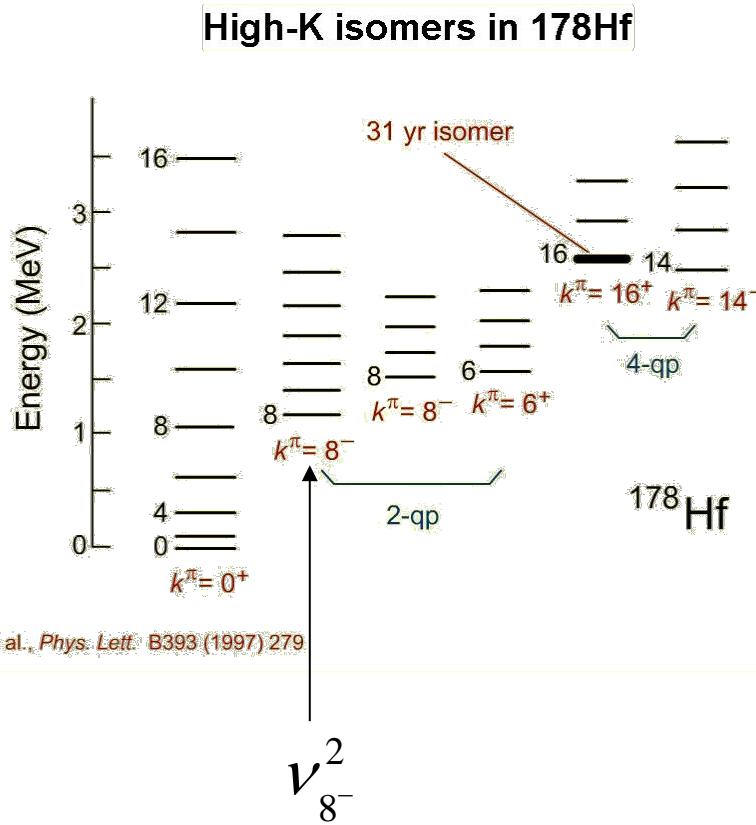


Mullins et al., Phys. Lett. B393 (1997) 279



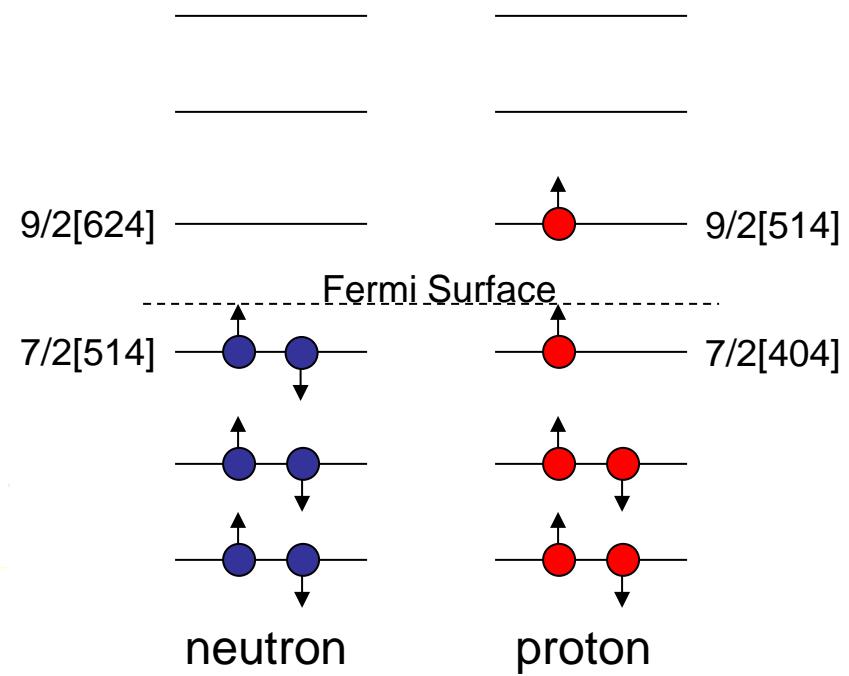
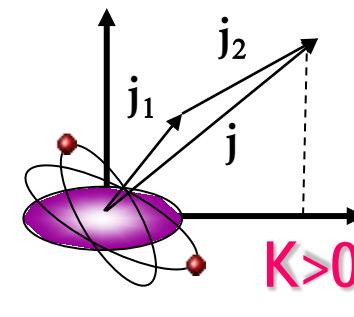
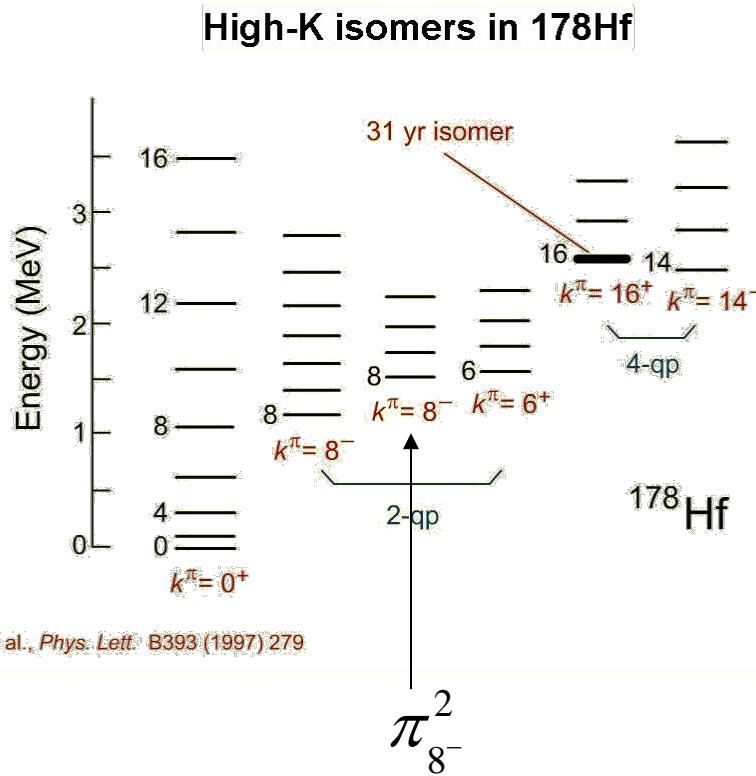
Investigation of ^{178}Hf – K-Isomers

- A well-known example:



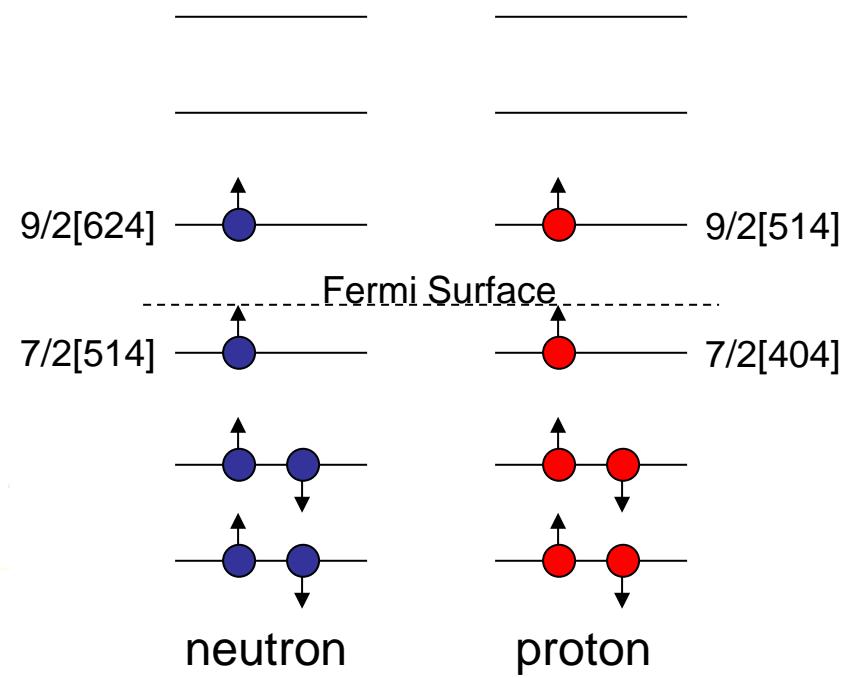
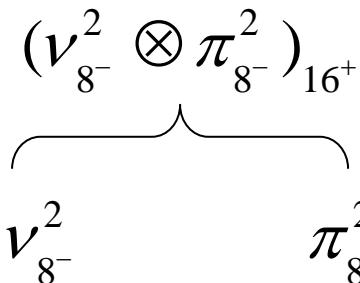
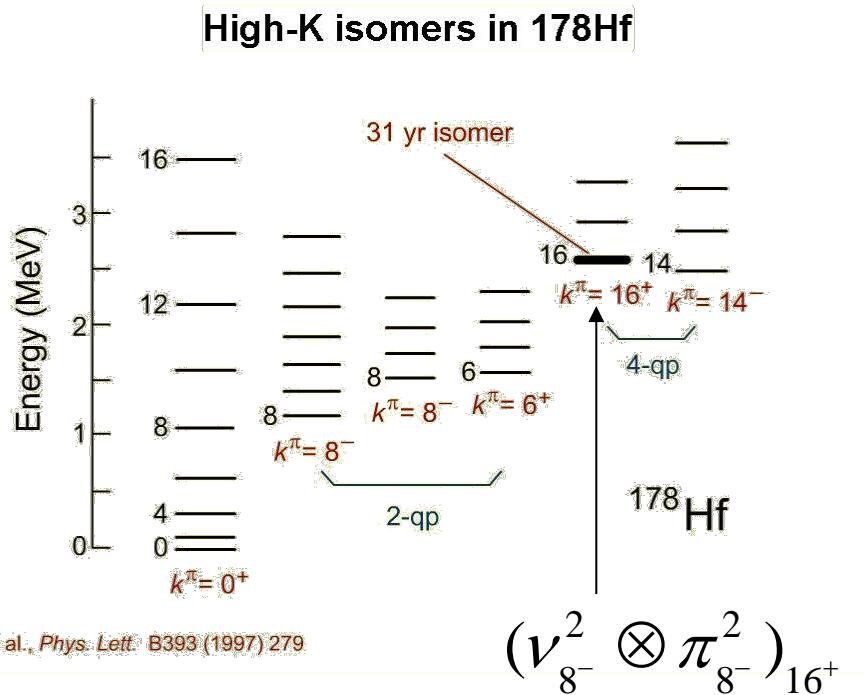
Investigation of ^{178}Hf – K-Isomers

- A well-known example:



Investigation of ^{178}Hf – K-Isomers

- ### ■ A well-known example:



Magnetic Moments in ^{178}Hf

$$g(j) = \begin{cases} \frac{2 \cdot \ell \cdot g_\ell + g_s}{2 \cdot \ell + 1} & \text{for } j = \ell + 1/2 \\ \frac{2 \cdot (\ell + 1) \cdot g_\ell - g_s}{2 \cdot \ell + 1} & \text{for } j = \ell - 1/2 \end{cases}$$

proton $g_\ell = 1$ $g_s = 5.59$
neutron $g_\ell = 0$ $g_s = -3.83$

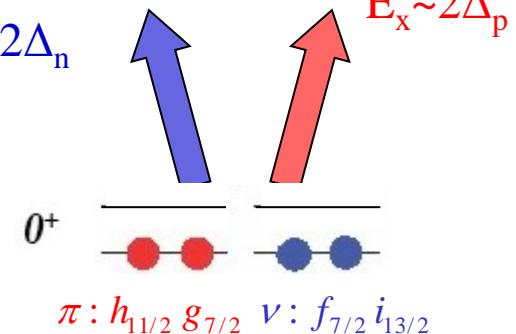
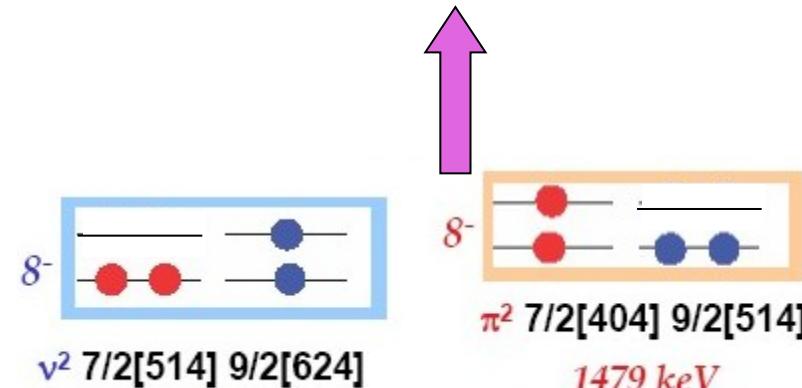
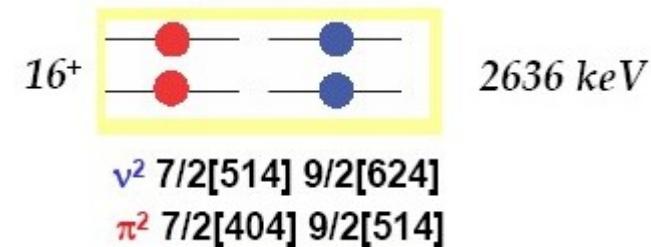
$$g(\mathbf{h}_{11/2}) = 1.42 \quad g(\mathbf{g}_{7/2}) = 0.49 \quad g(\mathbf{f}_{7/2}) = -0.55 \quad g(\mathbf{i}_{13/2}) = -0.29$$

$$g(j_1 \times j_2; J) = \frac{1}{2} \cdot (g_1 + g_2) + \frac{j_1 \cdot (j_1 + 1) - j_2 \cdot (j_2 + 1)}{2 \cdot J \cdot (J + 1)} \cdot (g_1 - g_2)$$

$$g(\mathbf{h}_{11/2} \times \mathbf{g}_{7/2}; 8^-) = 1.08 \quad g(\mathbf{f}_{7/2} \times \mathbf{i}_{13/2}) = -0.36$$

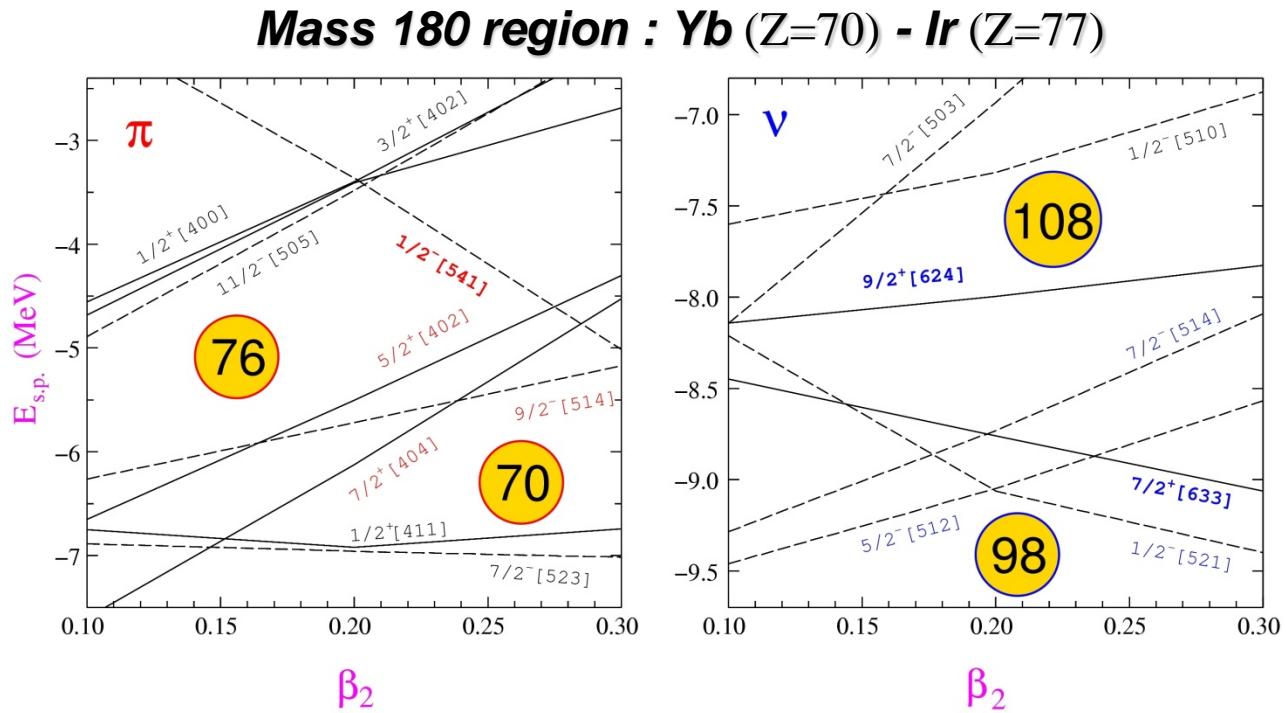
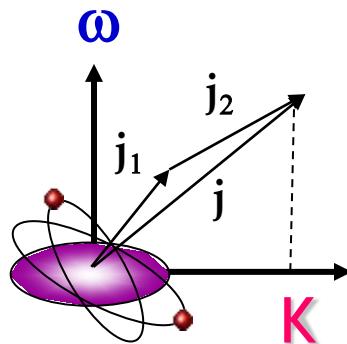
$$g(8^- \times 8^-; 16^+) = 0.36 \quad \rightarrow \quad \mu = g \cdot I = 5.76 \text{ nm}$$

$$7.26 \pm 0.16 \text{ nm}$$



K-isomers: Where to find them?

□ Deformed nuclei with axially-symmetric shape

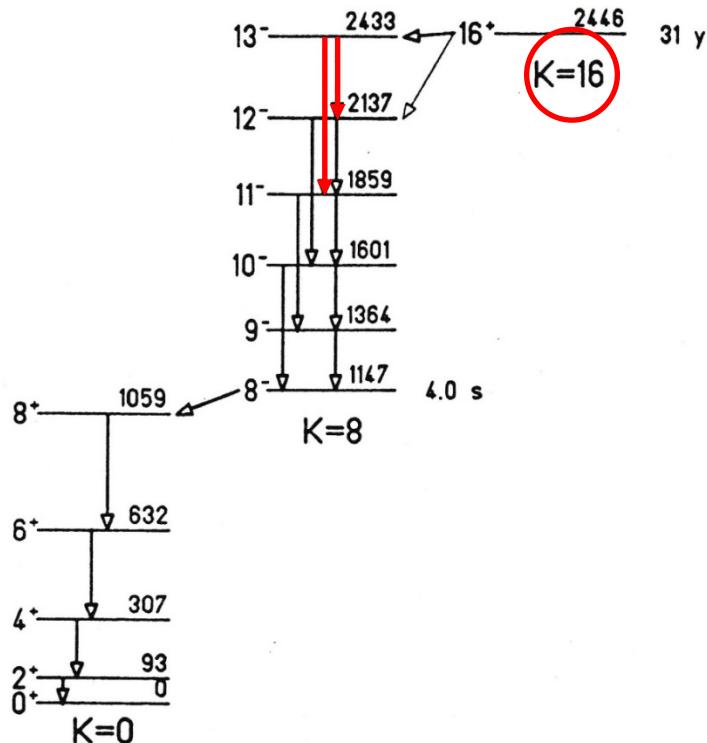


□ High-K orbitals near the Fermi surface

π : $7/2[404]$, $9/2[514]$, $5/2[402]$

ν : $7/2[514]$, $9/2[624]$, $5/2[512]$, $7/2[633]$

Decay Study of the K = 16 Isomeric State



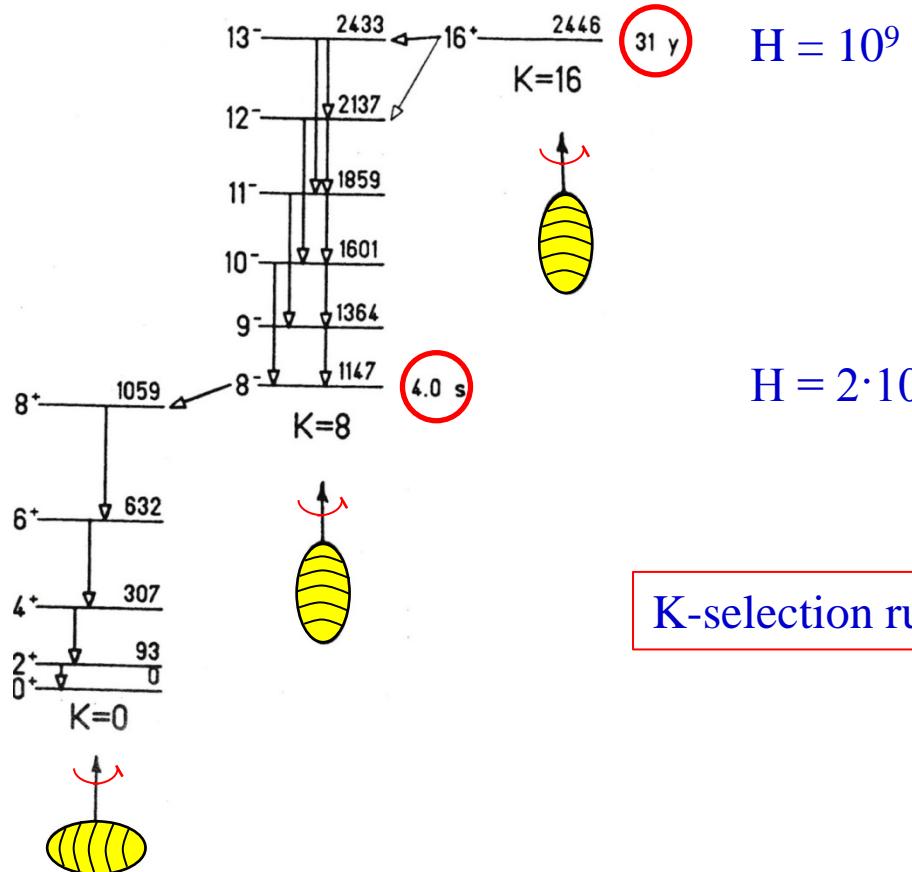
	$\frac{B(E2, I \rightarrow I - 2)}{B(E2, I \rightarrow I - 1)}$	rigid rotor K=8
10 ⁻	0.104±0.012	0.0769
11 ⁻	0.171±0.009	0.1607
12 ⁻	0.230±0.017	0.2517
13 ⁻	0.313±0.016	0.3500

E2/M1 mixing ratios are related to $\frac{g_K - g_R}{Q_{20}}$

mixed 2-quasiparticle configuration: $g_K = 0.36(2) > 0$

$\{\pi[514]9/2 + \pi[404]7/2\}8^-$ and $\{\nu[514]7/2 + \nu[624]9/2\}8^-\}$

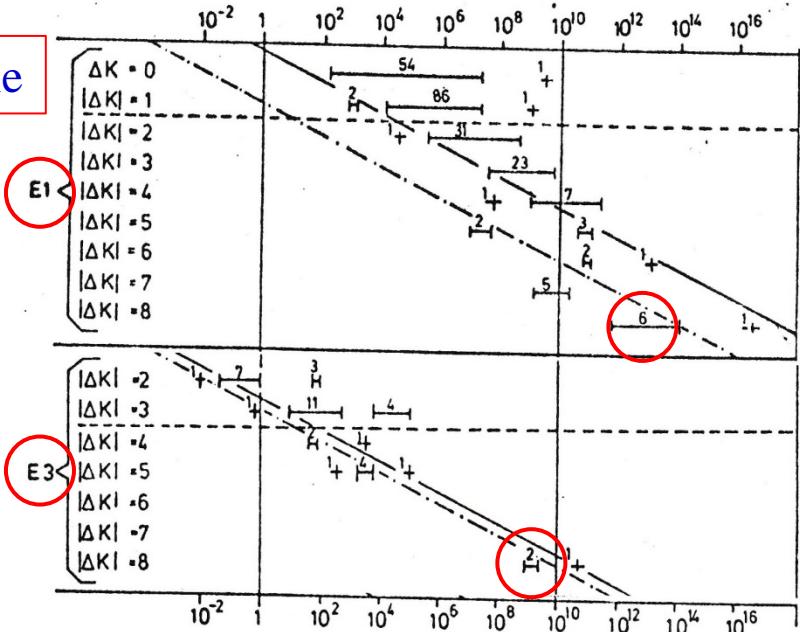
K - Isomerism



$$H = 2 \cdot 10^{13}$$

$$H = \frac{t_{1/2}(\text{experiment})}{t_{1/2}(\text{Weisskopf estimate})}$$

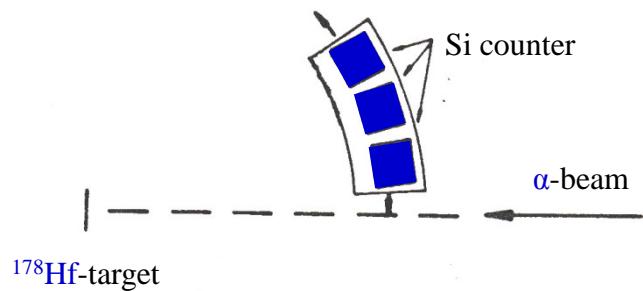
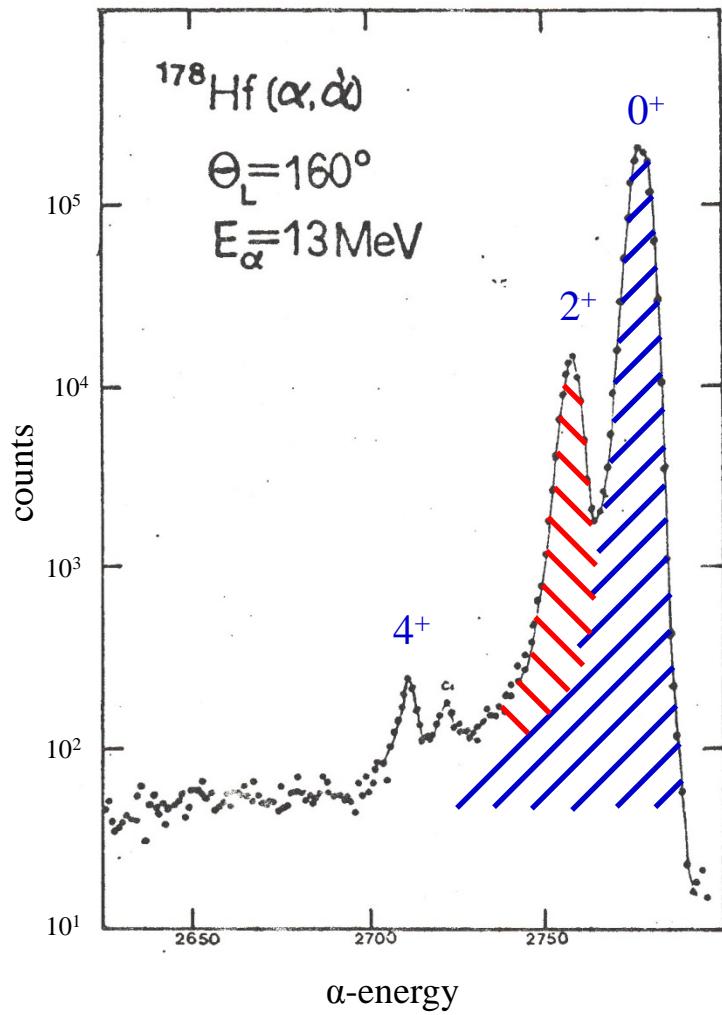
K-selection rule



Helmer & Reich; Nucl. Phys. A114 (1968), 649

K.E.G. Löbner; Phys. Lett 26B (1968), 369

^{178}Hf : Particle Spectroscopy



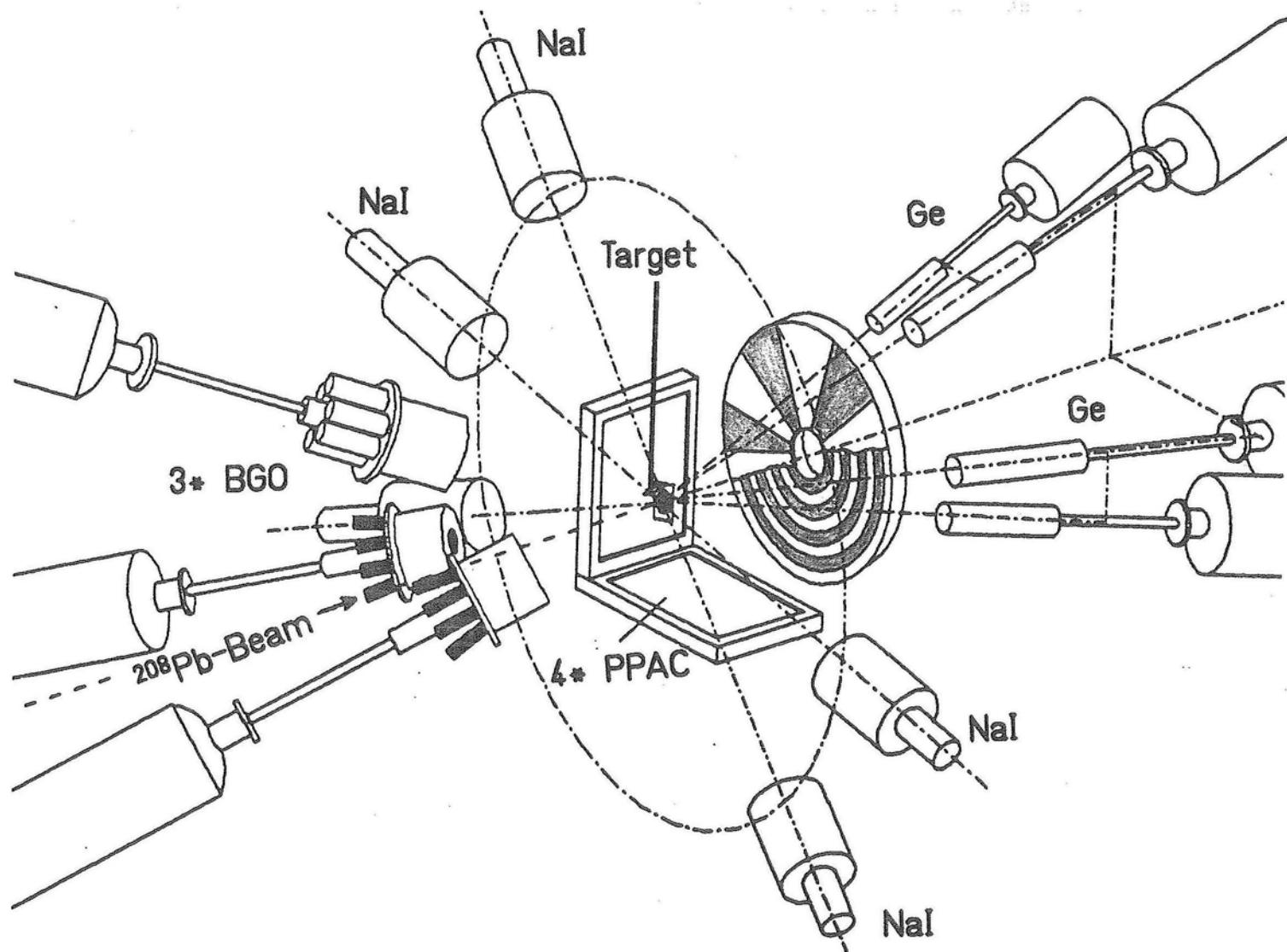
$$d\sigma_{C.E.}(E, \theta_{cm}) = P_I(E, \theta_{cm}) \cdot d\sigma_{Ruth}(E, \theta_{cm})$$

$$= 4.82 \left(1 + \frac{A_p}{A_t}\right)^{-2} \frac{A_p}{Z_t^2} (E_{lab} - \Delta E') \cdot B(E2, 0^+ \rightarrow 2^+) \cdot df_{E2}(\xi, \theta_{cm})$$

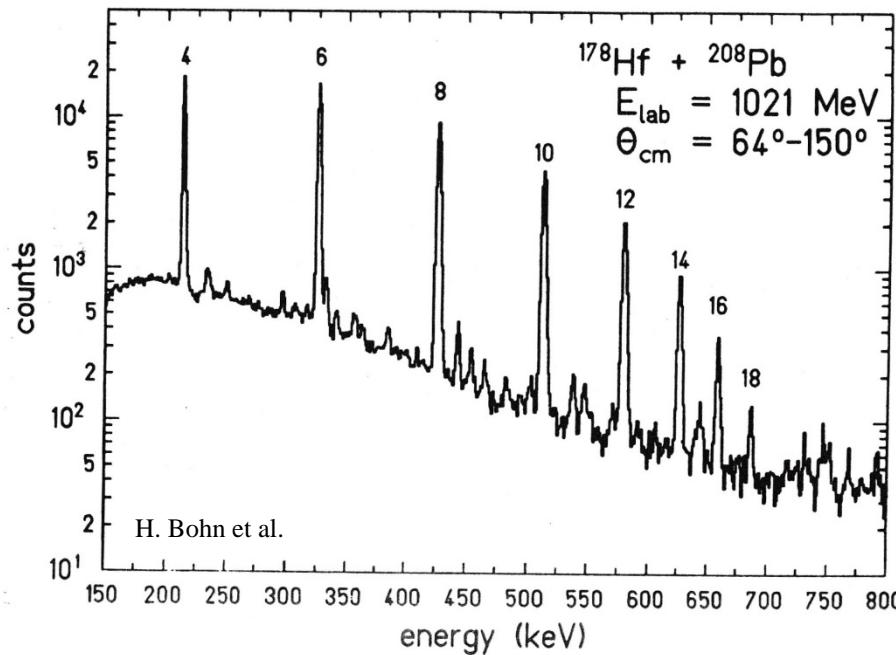
I	$P_I/P_{0^+}(160^\circ)$
2	$6.8 \cdot 10^{-2}$
4	$7.6 \cdot 10^{-4}$
17	$4.0 \cdot 10^{-4}$

$\sigma_{m2}/\sigma_{gs} = 0.05$

Coulomb Excitation of ^{178}Hf with ^{208}Pb ions



Coulomb Excitation of ^{178}Hf with ^{208}Pb ions



E2, E3 excitation of collective states:

ground state band $K = 0$

γ -vibrational band $K = 2$

determination of K-quantum number:

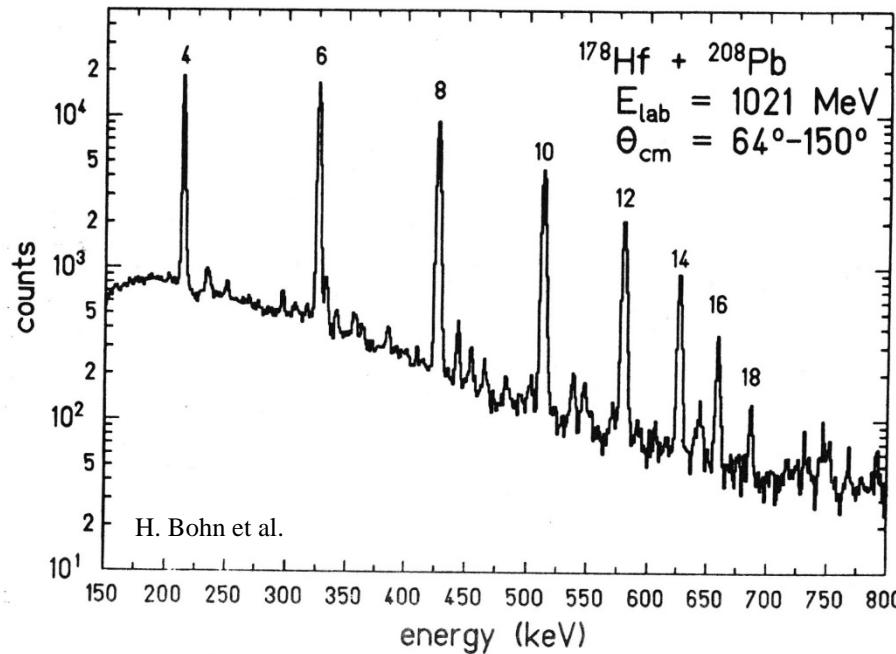
$$B(E2; I \rightarrow I-2) = \frac{[(I-1)^2 - K^2][I^2 - K^2]}{(4I^2 - 1)I(I-1)} \frac{15}{32\pi} \cdot Q_{20}^2$$

$$Q_{20} = 6.99(4) \text{ b} \quad Q_{40} = 0.54 \left(\begin{matrix} 26 \\ 47 \end{matrix}\right) \text{ b}^2 \text{ or } -1.58(48) \text{ b}^2$$

$$\beta_2 = 0.29(1) \quad \beta_4 = -0.20(4)$$

J.H. Hamilton et al.; Phys. Lett. 112B (1982), 327: [lifetime measurement](#) R.M. Ronningen et al.; Phys. Rev C16 (1977), 2218; Phys. Rev. Lett. 40 (1978), 364

Coulomb Excitation of ^{178}Hf with ^{208}Pb ions



E2, E3 excitation of collective states:

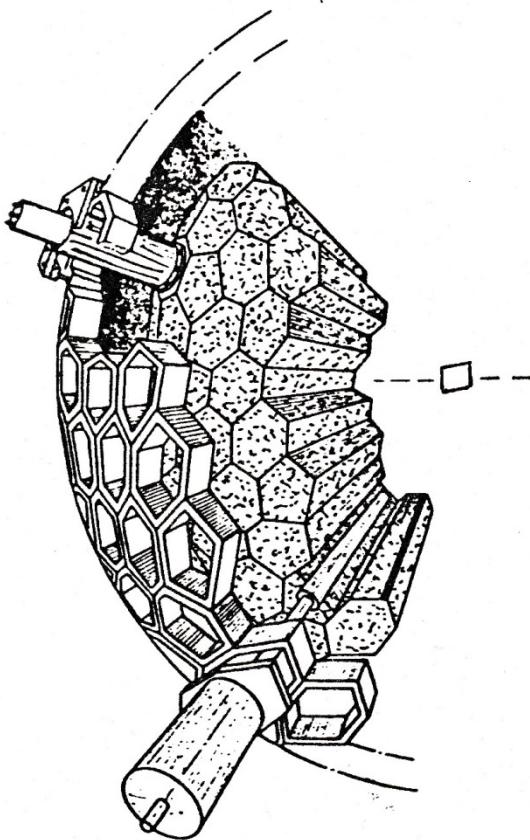
ground state band $K = 0$

γ -vibrational band $K = 2$

- excitation of the $K = 8$ isomeric state !
observation of delayed γ -rays in the $K = 0$ ground state band

- $^{178}\text{Hf} + ^{86}\text{Kr}$, $^{178}\text{Hf} + ^{136}\text{Xe}$

Coulomb Excitation of the K = 8 Isomer in ^{178}Hf



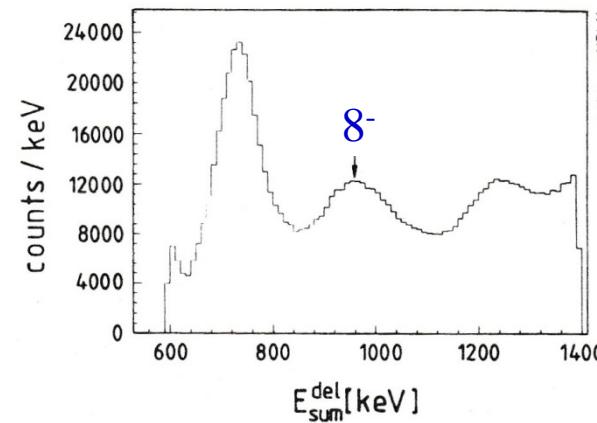
162 **NaI** detectors
6 **Ge** detector

parameters and resolutions FWHM

E_i	5.5% at 1332 keV
t_i	2.8 ns
$W(\theta)$	14^0
E_{total}	18-22% for $M_\gamma=20$
M_γ	25-30% for $M_\gamma=20$

The two basic observables which can be measured for the resulting γ -ray shower are the total energy emitted as γ -radiation and the number of γ -rays.

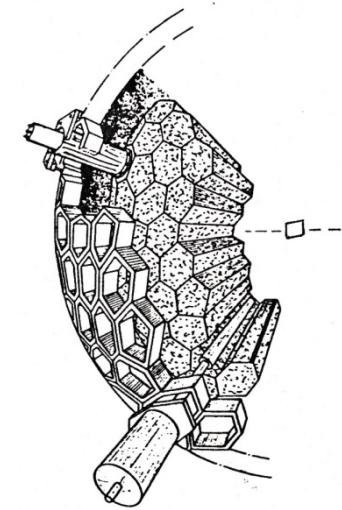
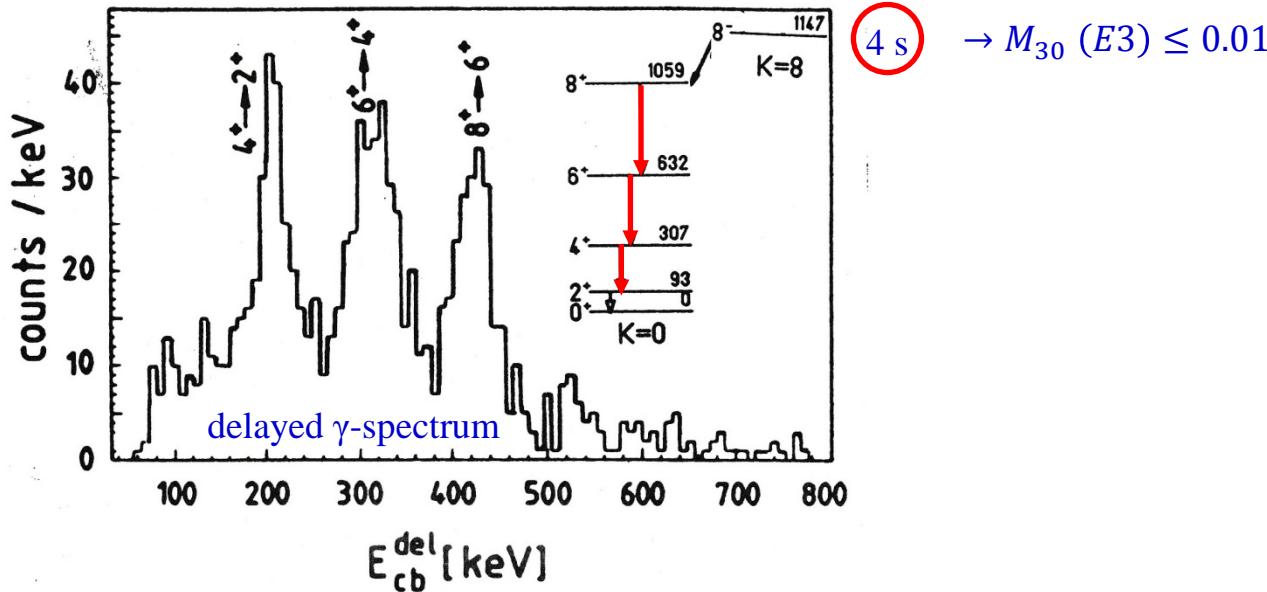
- $^{178}\text{Hf} + ^{130}\text{Te}$ at 560, 590, 620 MeV
particle detection at $\sim 180^0$, Pb catcher (0.5 mm thickness) was positioned 1cm downstream to stop the recoiling ^{178}Hf ions.



Delayed sum-energy spectrum taken at 590 MeV. (delayed time window 20-65ns with respect to beam pulse) A peak associated to the $K^\pi=8^-$ shows up.
The other peaks correspond to isomers of fusion products from target contaminants and β -decay.

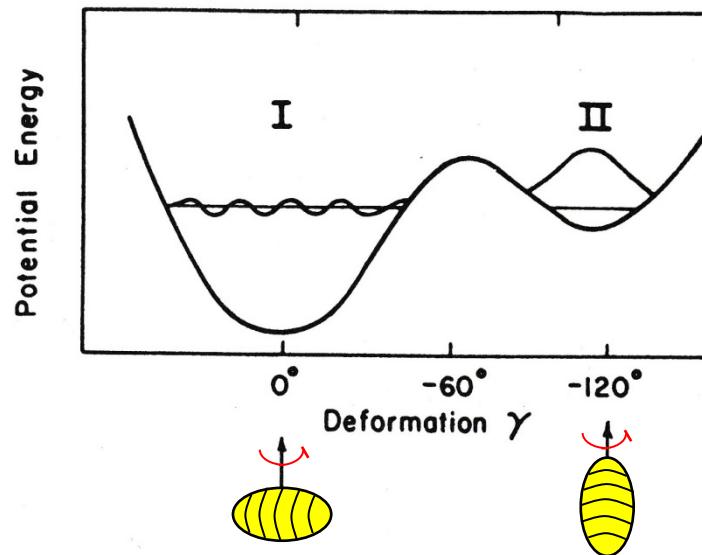
Coulomb Excitation of the K = 8 Isomer in ^{178}Hf

► $^{178}\text{Hf} + ^{130}\text{Te}$ at 560, 590, 620 MeV



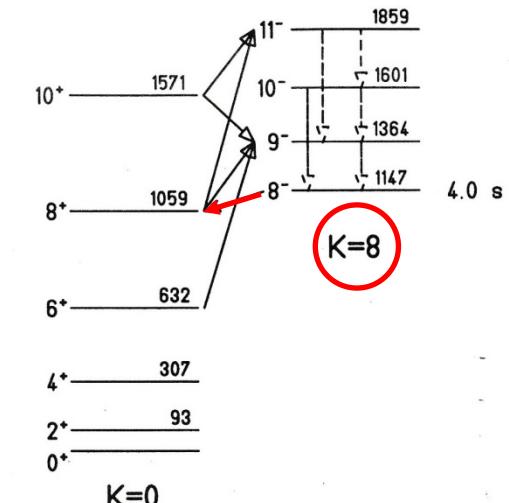
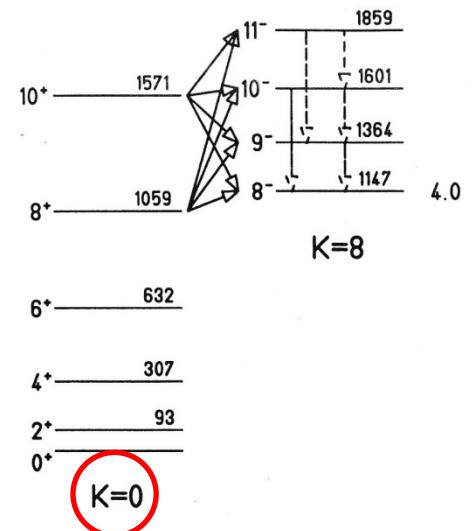
Delayed γ -ray spectrum of the Crystal Ball with $850\text{keV} \leq E_{sum}^{del} \leq 1100\text{keV}$ and $3 \leq N_{det} \leq 6$. In addition at least one of the delayed γ -rays must have been detected in one of the Ge-detectors.

Decay of the Isomer by Barrier Penetration



+ small K=8 admixture

+ small K=0 admixture



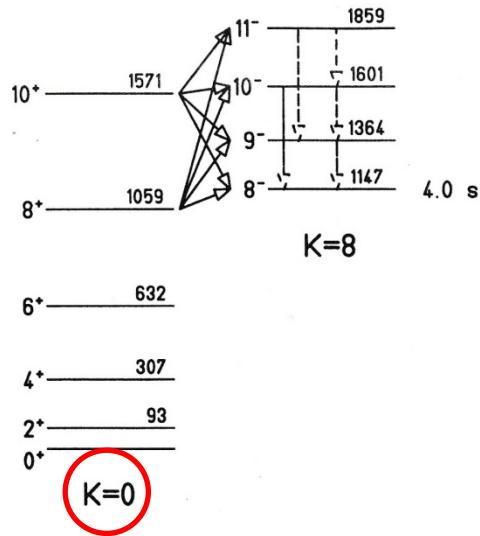
rigid rotor model:

$$\langle I_f | M(E2) | I_i \rangle = \sqrt{2I_i + 1} \cdot (I_i 3K0 | I_f K) \cdot M_{30}$$

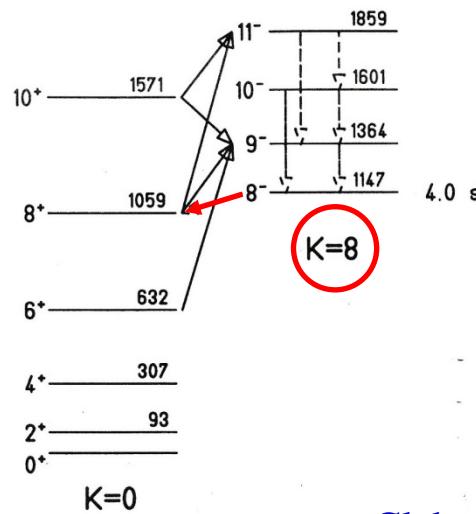
8- lifetime is independent from excitation

2-Band K-Mixing Model

+ small K=8 admixture



+ small K=0 admixture



8⁻ lifetime is independent from excitation

rigid rotor model:

$$\langle I_f | M(E3) | I_i \rangle = \sqrt{2I_i + 1} \cdot (I_i 3K0 | I_f K) \cdot M_{30}$$

Clebsch Gordan coefficient:

$$\langle I3K0 | (I-3)K \rangle = -\sqrt{\frac{5(I+K-2)(I+K-1)(I+K)(I-K-2)(I-K-1)(I-K)}{2(I-2)(I-1)I(2I-3)(2I-1)(2I+1)}} * K$$

$$\langle I3K0 | (I-2)K \rangle = \sqrt{\frac{15(I+K-1)(I+K)(I-K-1)(I-K)}{(I-2)(I-1)I(2I-1)(2I+1)(2I+2)}} * K$$

$$\langle I3K0 | (I-1)K \rangle = -\sqrt{\frac{3(I+K)(I-K)}{(I-1)I(2I-3)(2I+1)(2I+2)(2I+3)}} * (5K^2 - I^2 + 1)$$

$$\langle I3K0 | IK \rangle = \frac{5K^2 - 3I^2 - 3I + 1}{\sqrt{(I-1)I(I+1)(I+2)(2I-1)(2I+3)}} * K$$

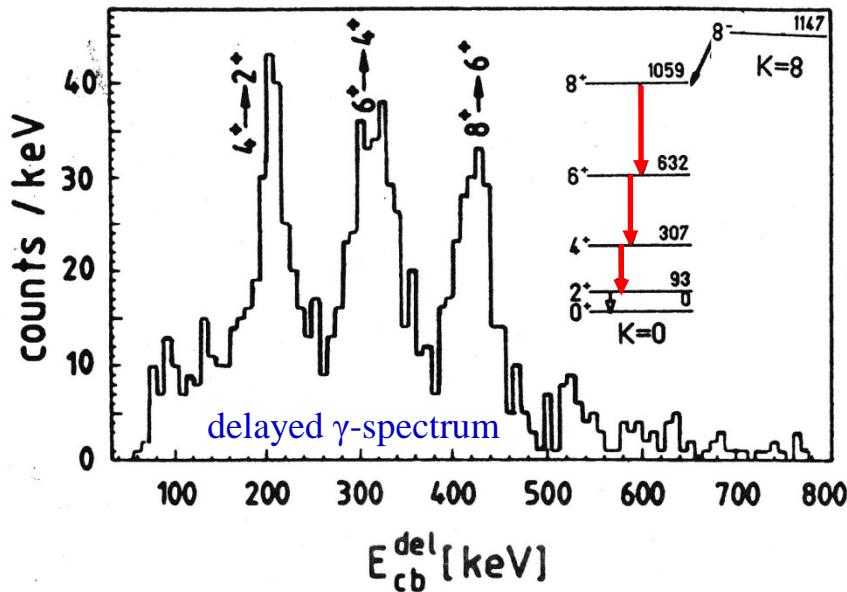
$$\langle I3K0 | (I+1)K \rangle = \sqrt{\frac{3(I+K+1)(I-K+1)}{I(I+1)(2I-1)(2I+1)(2I+4)(2I+5)}} * (5K^2 - I^2 - 2I)$$

$$\langle I3K0 | (I+2)K \rangle = \sqrt{\frac{15(I+K+1)(I+K+2)(I-K+1)(I-K+2)}{I(I+1)(I+2)(2I+1)(2I+3)(2I+6)}} * K$$

$$\langle I3K0 | (I+3)K \rangle = \sqrt{\frac{5(I+K+1)(I+K+2)(I+K+3)(I-K+1)(I-K+2)(I-K+3)}{2(I+1)(I+2)(I+3)(2I+1)(2I+3)(2I+5)}}$$

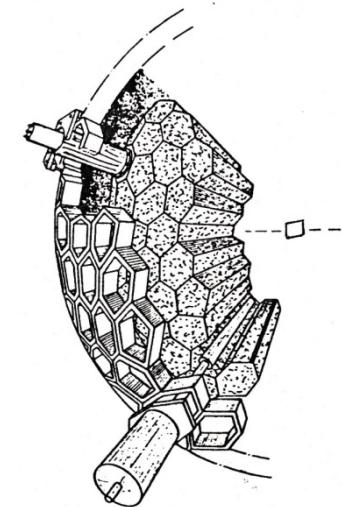
Coulomb Excitation of the K = 8 Isomer in ^{178}Hf

► $^{178}\text{Hf} + ^{130}\text{Te}$ at 560, 590, 620 MeV



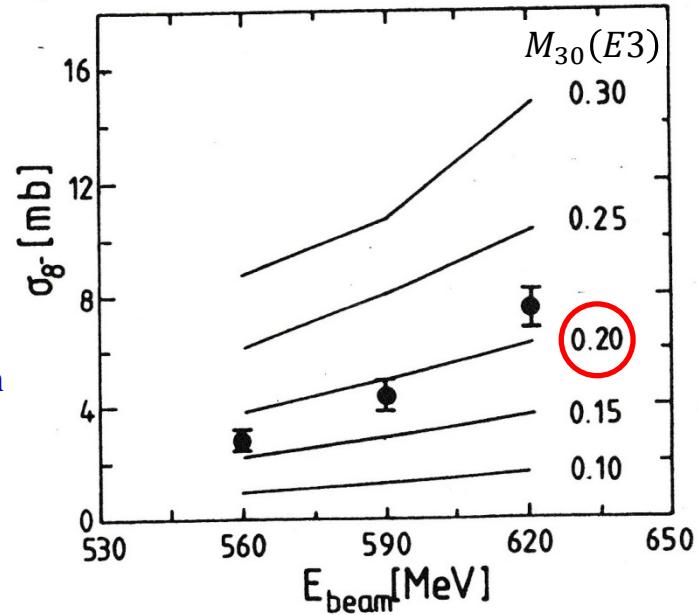
4 s

$\rightarrow M_{30}(E3) \leq 0.01$



coupling between rotational motion
and single particle excitation

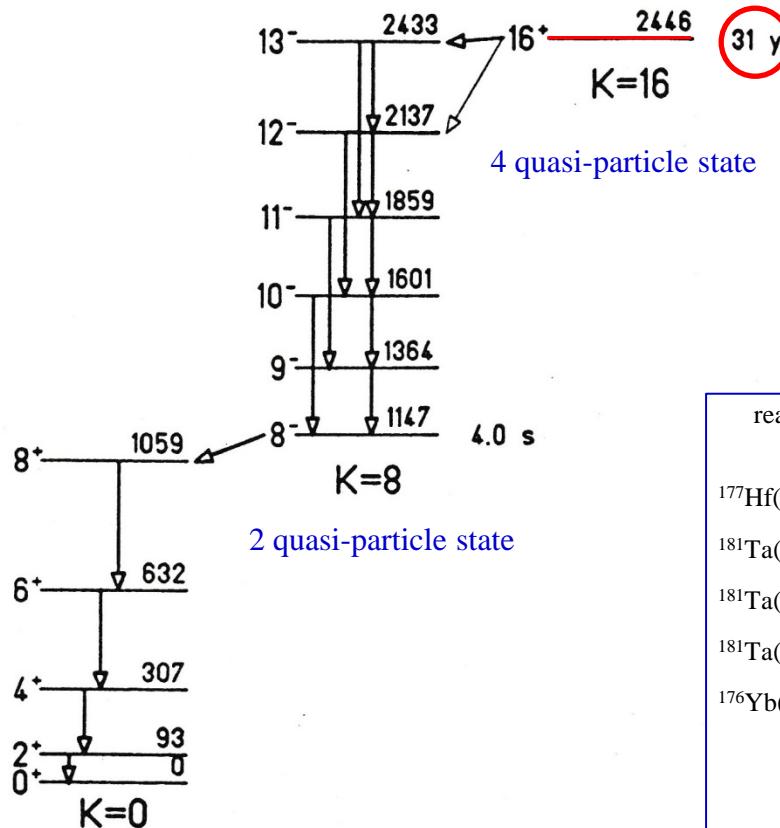
$$\psi(8^-) \cong |K=8\rangle + \alpha|K=0\rangle$$



H. Xie et al.; Phys. Rev. C48 (1993), 2517

Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$



Production of $^{178\text{m}2}\text{Hf}$ in the past

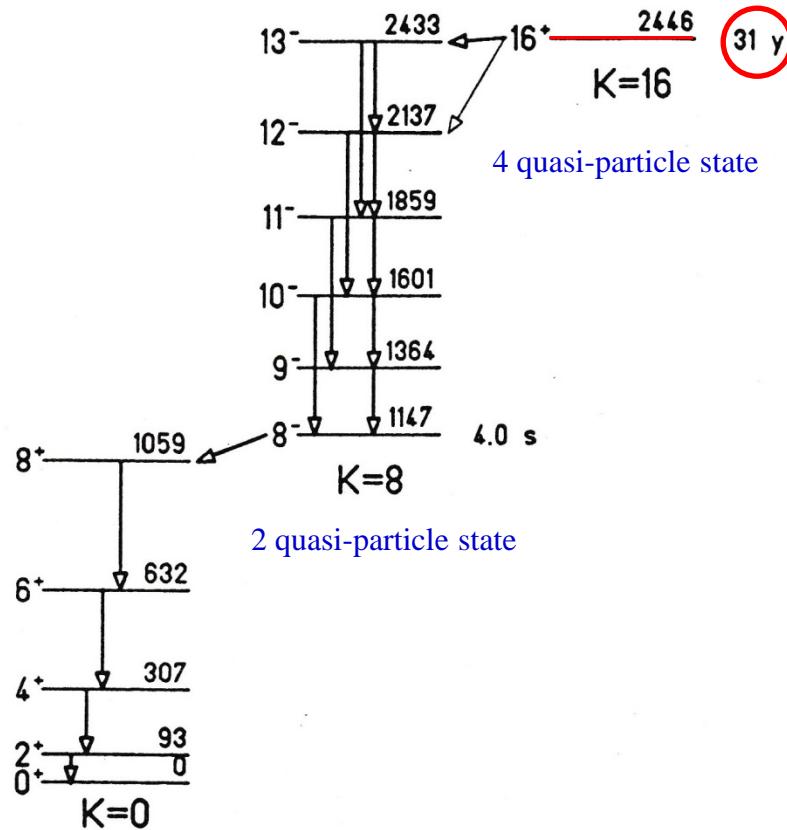
reaction	projectile energy (MeV)	σ_m/σ_{gs}	reference
$^{177}\text{Hf}(n,\gamma)$	thermal	10^{-9}	R.G. Helmer et al.; Nucl. Phys. A211, 1 (1973)
$^{181}\text{Ta}(\gamma,p2n)$	≤ 85	low	F.W.N. de Boer et al.; Nucl. Phys. A263, 1 (1976)
$^{181}\text{Ta}(\alpha,\alpha p2n)$	120	-	J. Van Klinken et al.; Nucl. Phys. A339, 189 (1980)
$^{181}\text{Ta}(p,\alpha)$	92.5	10^{-3}	W. Kutschera et al.; Radio. Beam Conf., 345 (1989)
$^{176}\text{Yb}(\alpha,2n)$	26	0.01	T. Khoo et al.; Phys. Lett. 67B, 271 (1977)
	27	0.005	A. Kugler et al.; priv. communication
	32	0.05	NUPECC News 1991
	40	0.08	

Helmer & Reich; Nucl. Phys. A114 (1968), 649

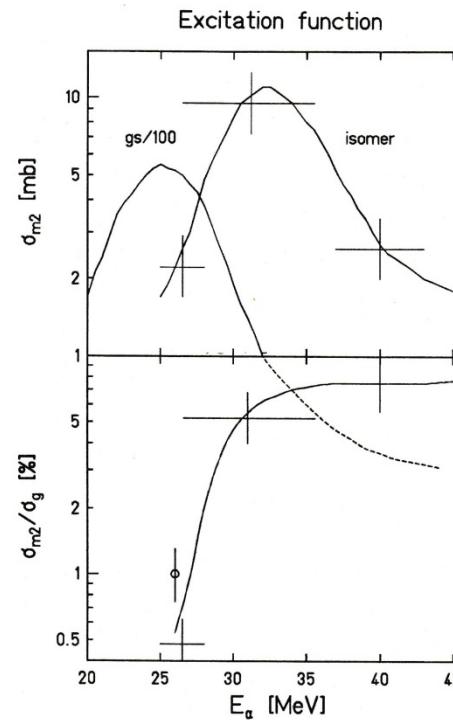


Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$



Production of $^{178\text{m}2}\text{Hf}$ using $^{176}\text{Yb}(\alpha, 2n)$

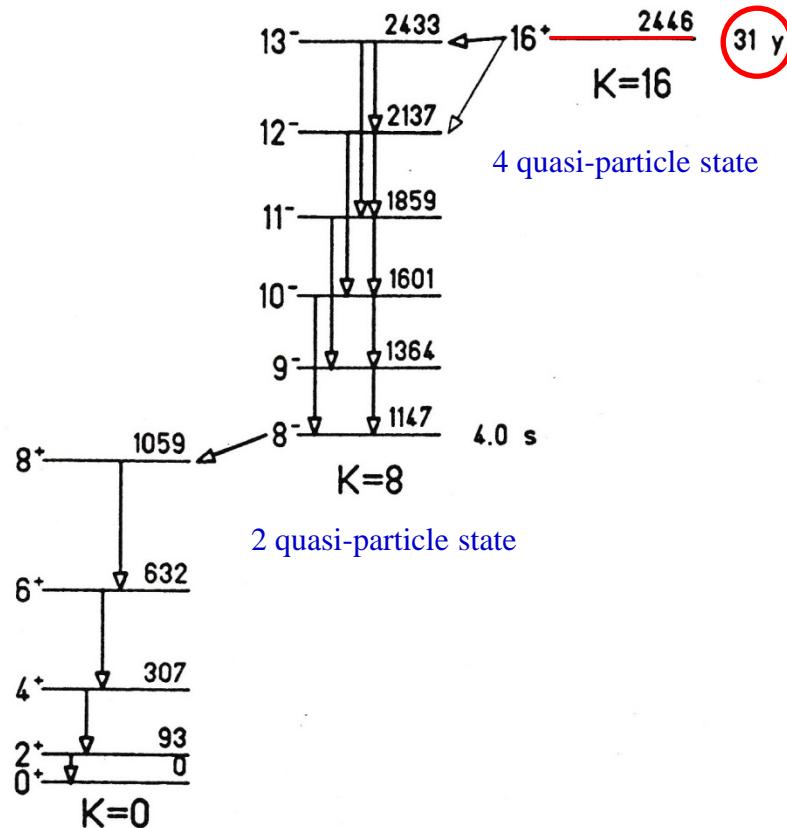


Helmer & Reich; Nucl. Phys. A114 (1968), 649



Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$

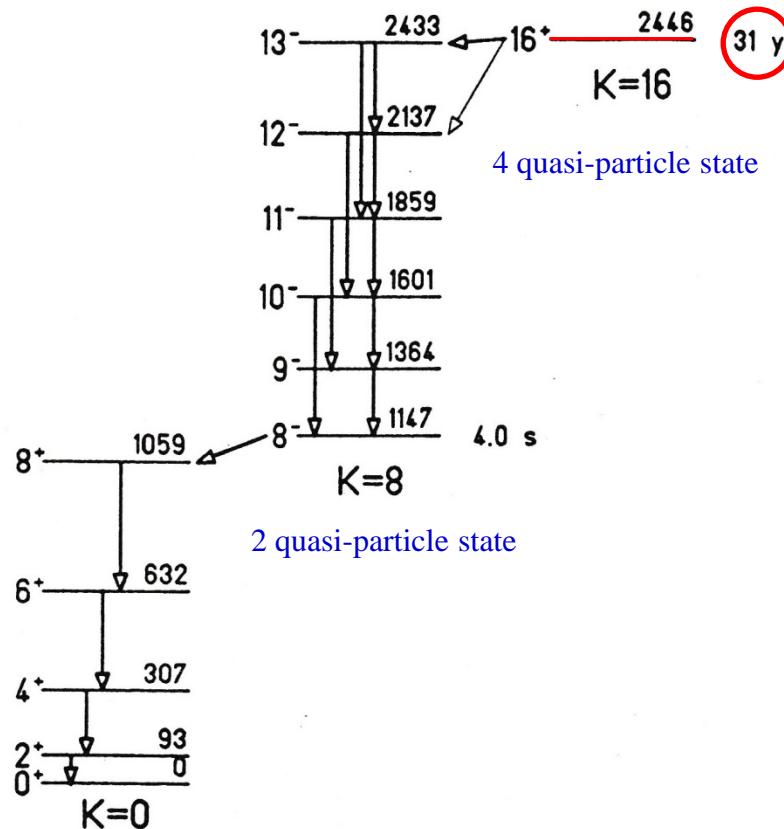


The isotopic distribution of radioactive Hf nuclei produced in the irradiation $^4\text{He} + ^{176}\text{Yb}$

Mass	170	171	172	173	175	$^{178\text{m}}$	$^{178\text{g}}$	$^{179\text{m}}$
$T_{1/2}$	15.9h	12.2h	1.9y	24h	70d	31y	stable	24.8d
Experiment	$5 \cdot 10^{10}$	$1.5 \cdot 10^{11}$	$4.5 \cdot 10^{12}$	$1.3 \cdot 10^{13}$	$2.2 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	--	$7.6 \cdot 10^{11}$
Calculation	$8 \cdot 10^{10}$	$1.5 \cdot 10^{12}$	$4.4 \cdot 10^{12}$	$0.8 \cdot 10^{13}$	$2.2 \cdot 10^{13}$	--	$2.9 \cdot 10^{14}$	--

Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$



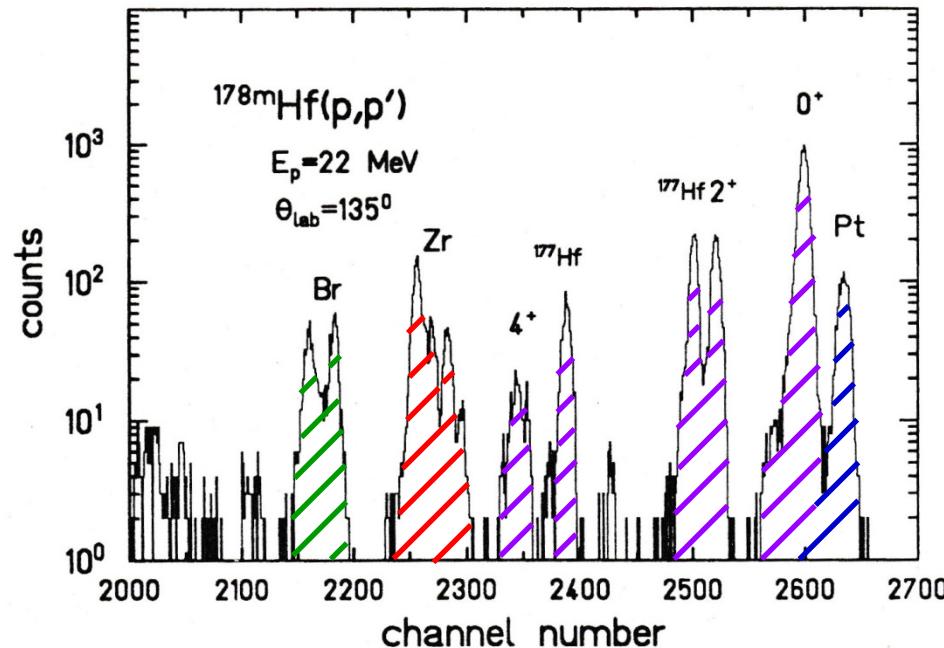
- magnet moment $\mu(16^+) = 7.3 \pm 0.2 \text{ nm}$
- 4-qp state: $\pi[514]9/2 + \pi[404]7/2 + \nu[514]7/2 + \nu[624]9/2$
- production: $^{176}\text{Yb}(\alpha, 2n)^{178}\text{Hf}$ Dubna
- chemical separation Orsay, Dubna
- mass separation Orsay
- target preparation Orsay

chemistry by M. Hussonnois (IN2P3-CNRS Orsay, France)

Yu. Ts. Oganessian; J. Phys. G: Nucl. Part. Phys. 18 (1992), 393

Target Analysis

- inelastic p-scattering experiment

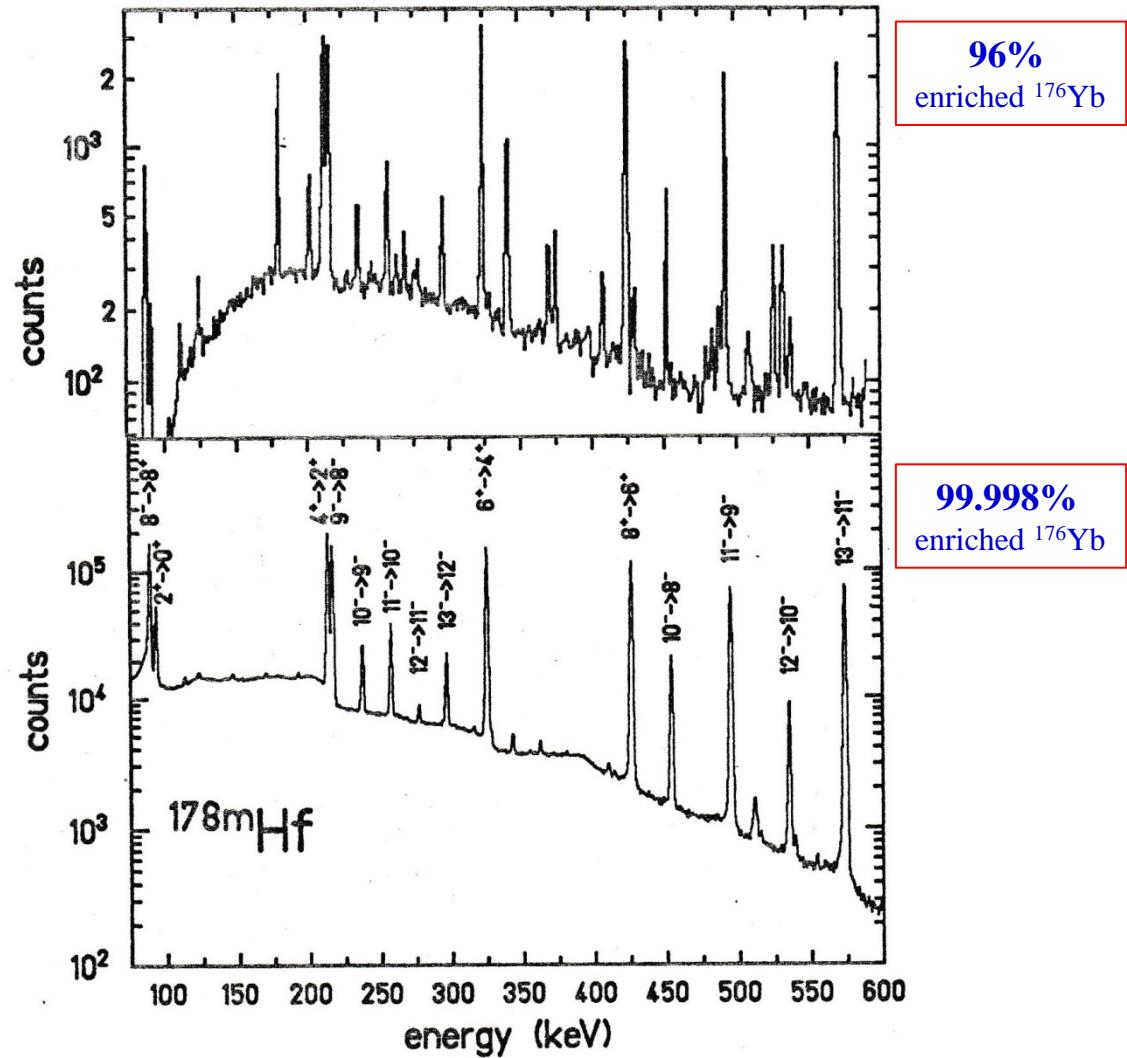
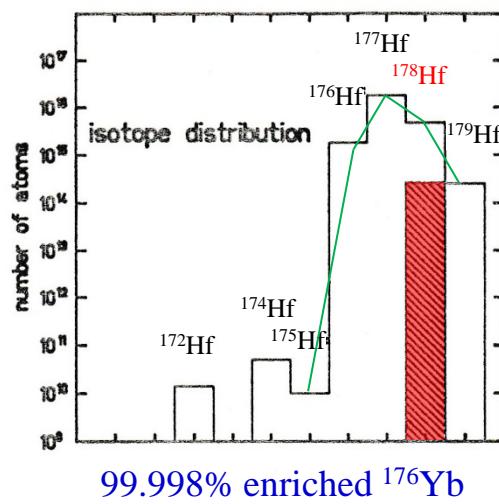
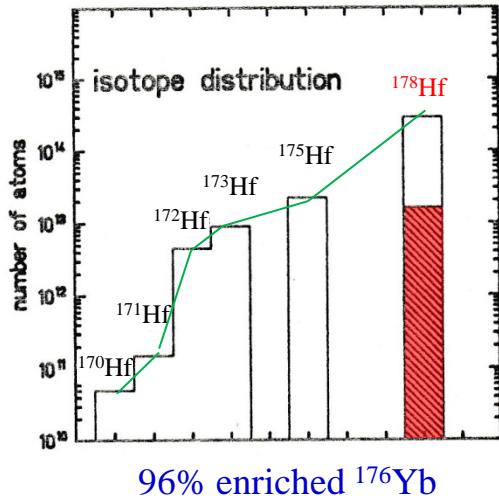


- X-ray analysis

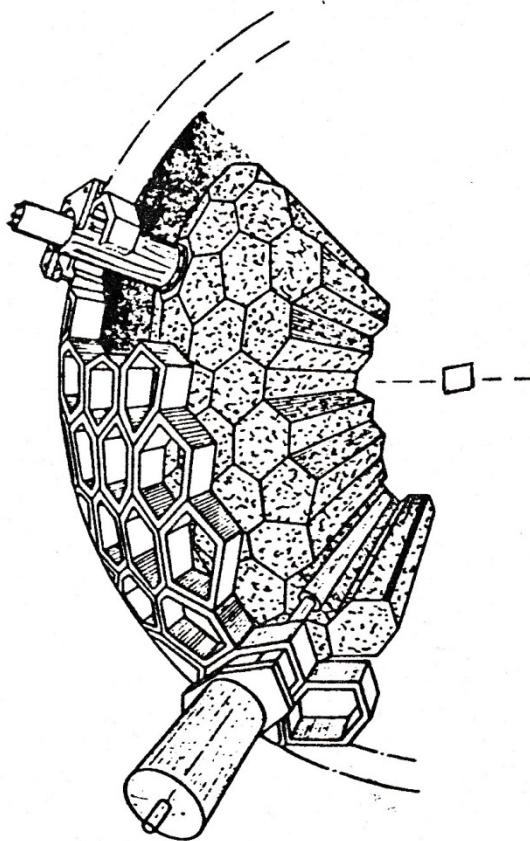
		Br	Zr	Hf	Pt	
$^{178m^2}\text{Hf}$	15 μg	107	320	1278	33	counts/s
^{nat}Hf	40 μg	-	223	3645	-	counts/s

- isomer-to ground state ratio: $\sigma_{m^2}/\sigma_{gs} = 2\%$

The $^{176}\text{Yb}(\alpha, 2\text{n})$ Reaction



Decay Study with the GSI-MPI Crystal Ball



162 **NaI** detectors
1 **Ge** detector

parameters and resolutions FWHM

E_i 5.5% at 1332 keV

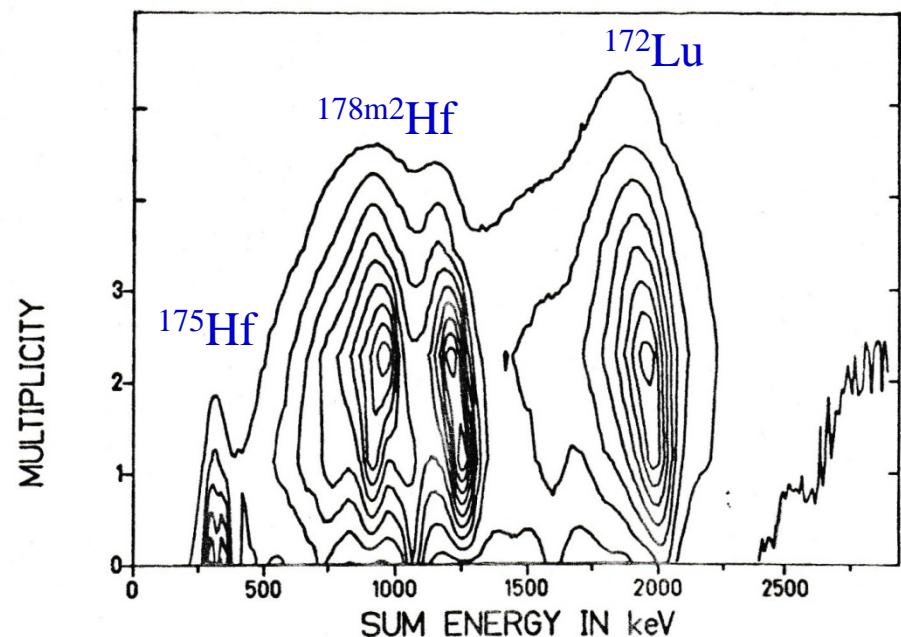
t_i 2.8 ns

$W(\theta)$ 14^0

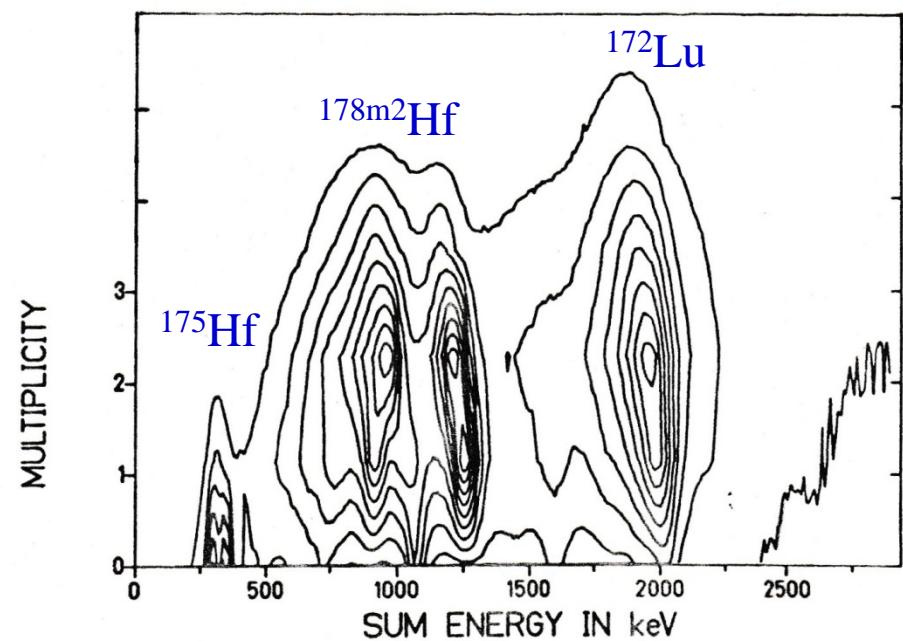
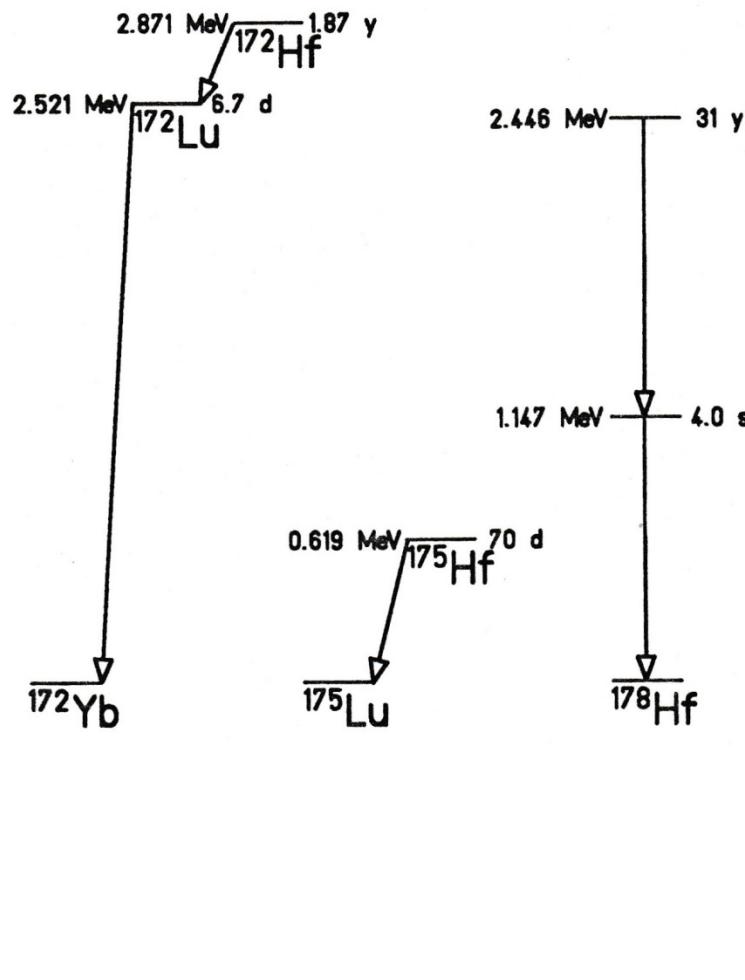
E_{total} 18-22% for $M_\gamma=20$

M_γ 25-30% for $M_\gamma=20$

The two basic observables which can be measured for the resulting γ -ray shower are the total energy emitted as γ -radiation and the number of γ -rays.



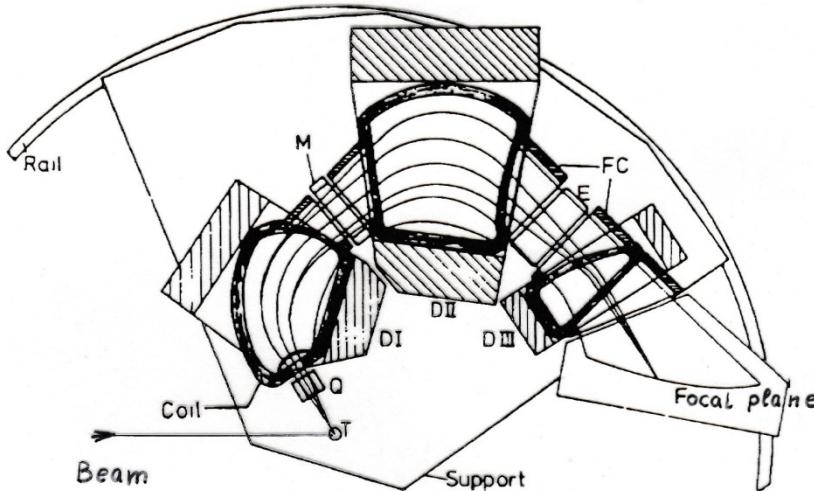
Decay Scheme of $^{175,178m^2}\text{Hf}$, ^{172}Lu



H. Xie et al.; Phys. Rev. C48 (1993), 2517

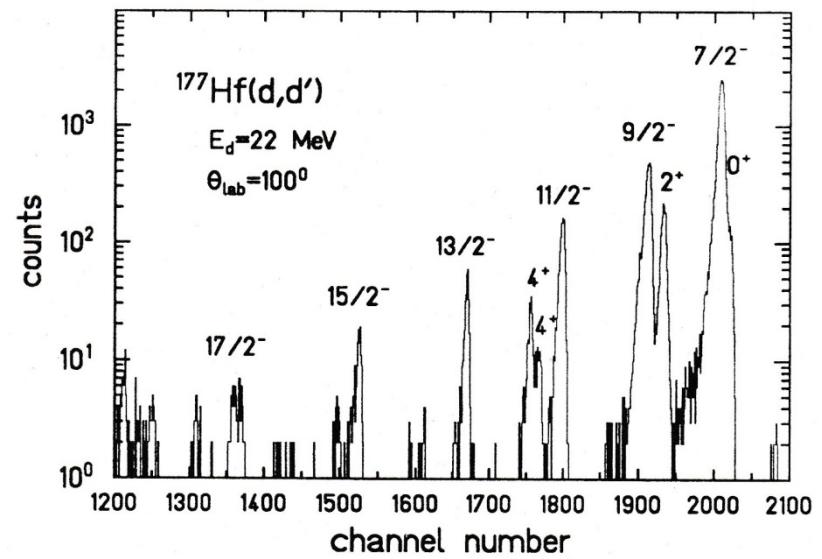


Inelastic d-Scattering Experiment



Q3D measurement at the Munich tandem

energy resolution: 12 keV



S. Deylitz et al. Phys. Rev. C53 (1996), 1266

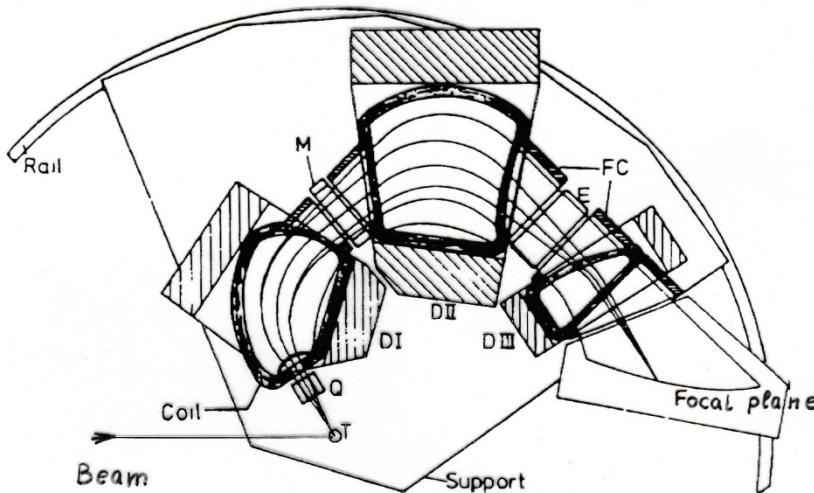


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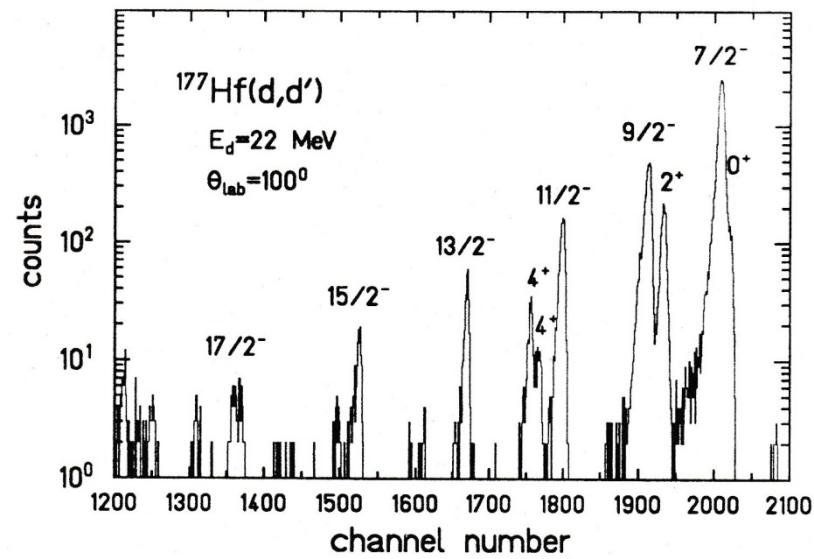
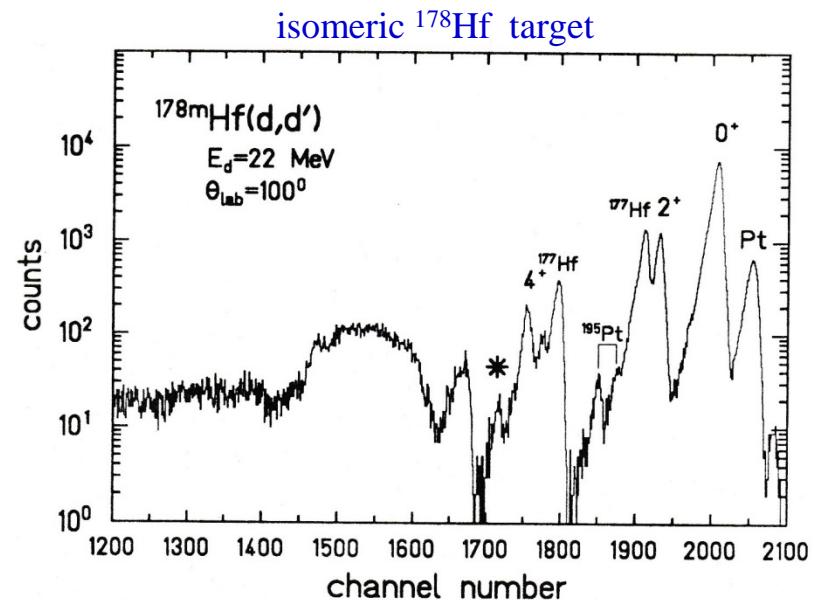


Inelastic d-Scattering Experiment

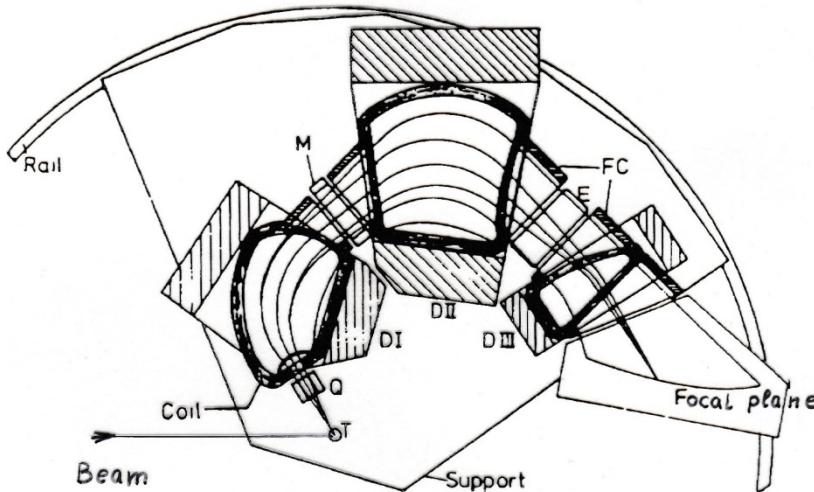


Q3D measurement at the Munich tandem

energy resolution: 12 keV

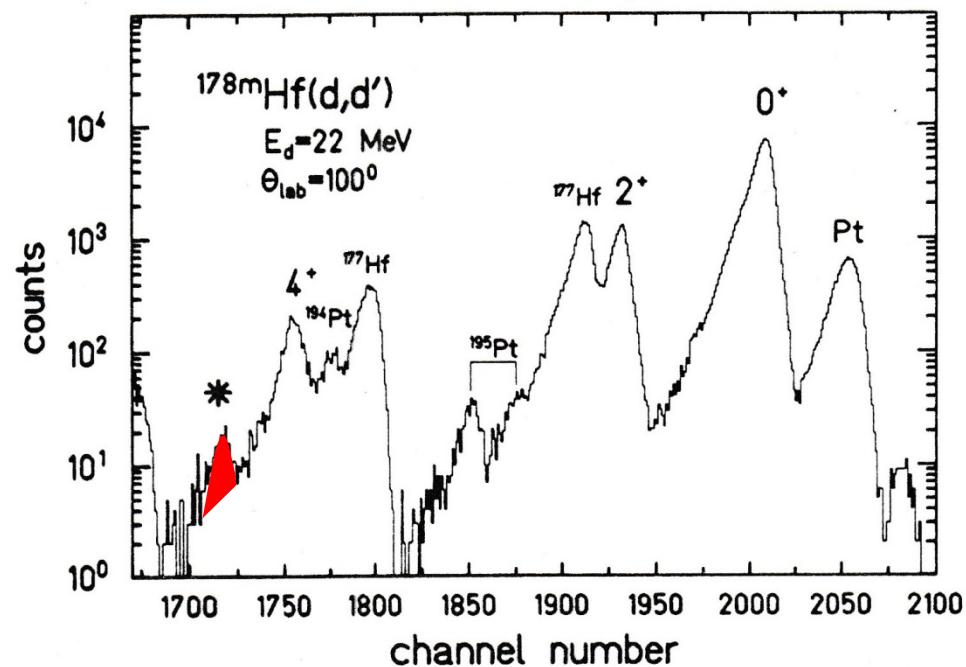


Inelastic d-Scattering Experiment



Q3D measurement at the Munich tandem

energy resolution: 12 keV



S. Deylitz et al. Phys. Rev. C53 (1996), 1266

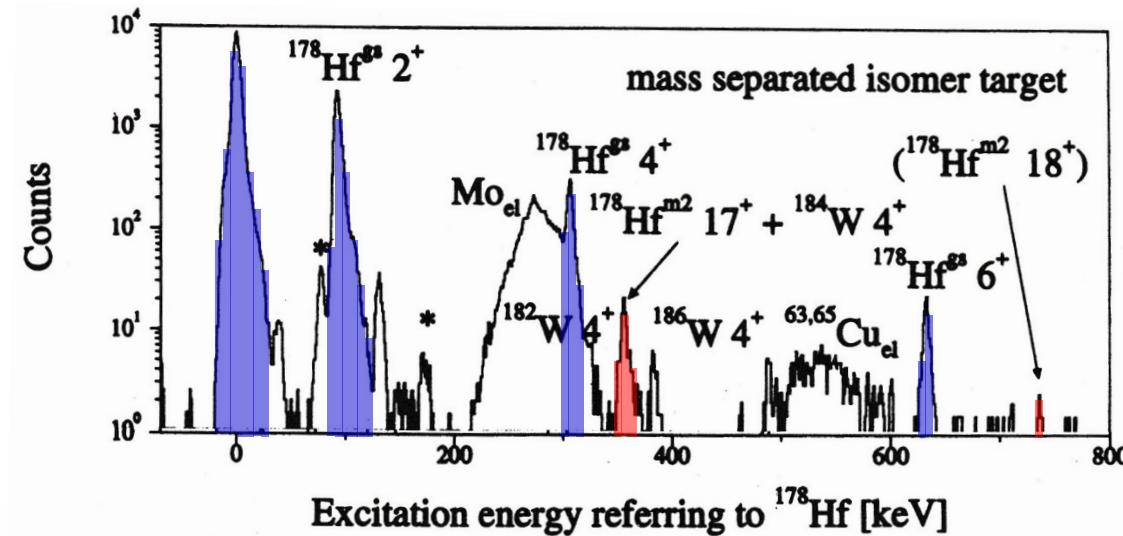


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Inelastic Deuteron Scattering from K=16 Isomer



$$\frac{^{178m2}Hf}{^{178}Hf} \cong 3\%$$

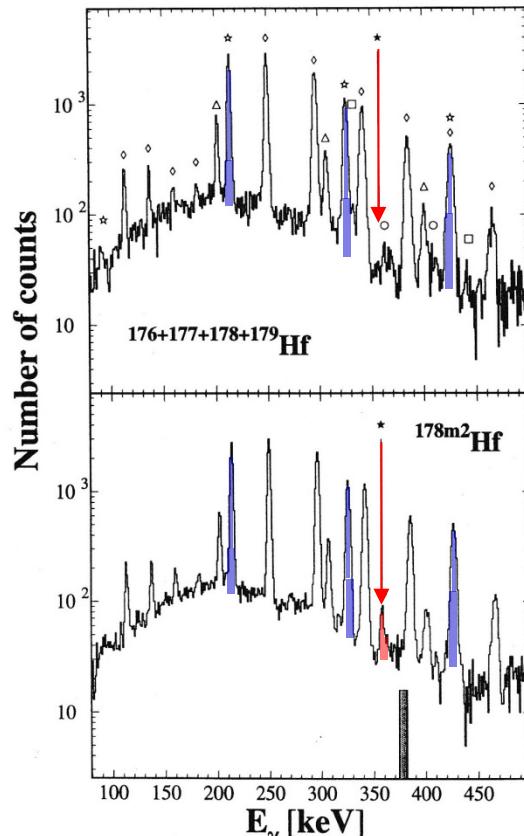
experiment performed at Munich tandem accelerator with Q3D spectrograph ($45^0 - 100^0$)

W, Ta, Mo, Cu result from sputtering and migration process in the mass-separator facility

- ❖ $E_x = 356.5 \pm 0.4$ keV 17⁺ member of rotational band

$$\frac{\Im(^{178m2}Hf)}{\Im(^{178}Hf)} = 1.48$$

Coulomb Excitation of the K=16 Isomer



$^{208}\text{Pb} \rightarrow \text{Hf}; 4.8 \text{ MeV/u}$

“artificial” γ -spectrum

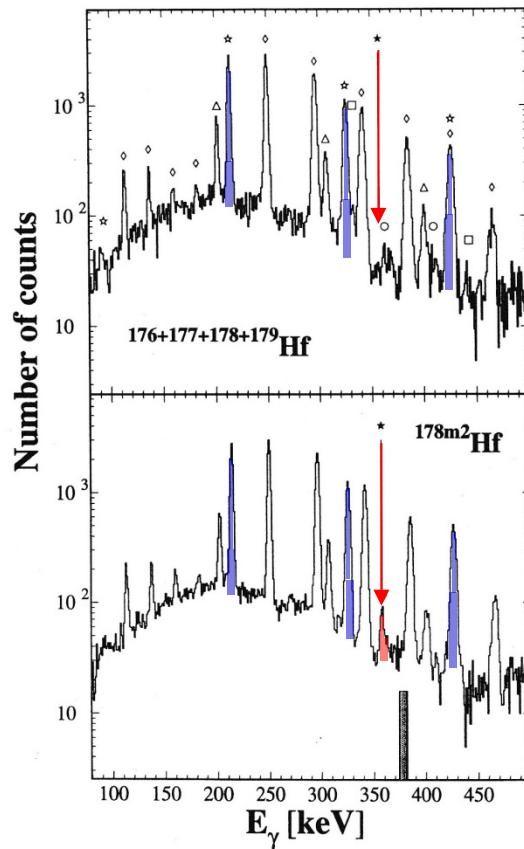
γ -spectrum with isomeric target

$$\frac{^{178m^2}\text{Hf}}{\text{Hf}} = 0.6\% \quad \mathbf{10^{14} \, ^{178m^2}\text{Hf atoms}}$$

experiment performed at UNILAC accelerator

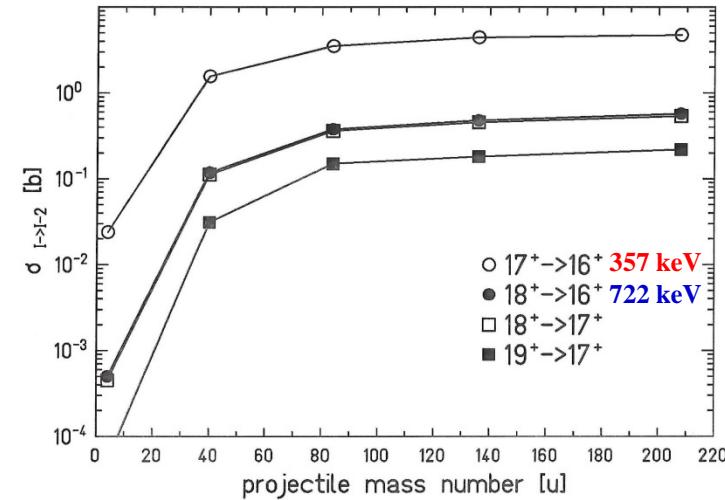
- ❖ $E_\gamma = 357.4 \pm 0.3 \text{ keV}$ $17^+ \rightarrow 16^+$ transition
- ❖ $Q_0 (\text{K}=16) = 8.2 \pm 1.1 \text{ b}$ rigid rotor model

Coulomb Excitation of the K=16 Isomer



$^{208}\text{Pb} \rightarrow \text{Hf}$; 4.8 MeV/u

“artificial” γ -spectrum



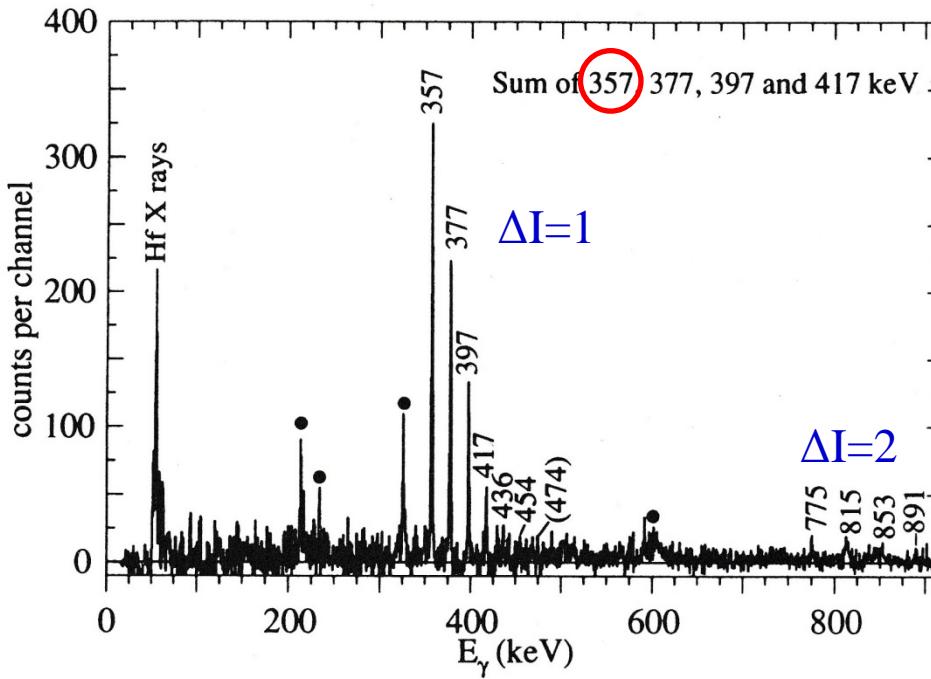
γ -spectrum with isomeric target

$$\frac{178m^2 Hf}{Hf} = 0.6\% \quad \boxed{10^{14} \text{ } 178m^2 Hf \text{ atoms}}$$

experiment performed at UNILAC accelerator

- ❖ $E_\gamma = 357.4 \pm 0.3$ keV $17^+ \rightarrow 16^+$ transition
- ❖ Q_0 (K=16) = 8.2 ± 1.1 b rigid rotor model

Rotational Band on the K=16 Isomer in ^{178}Hf



Summed coincidence spectrum from projections made on transitions in the rotational band assigned to $^{178\text{m}2}\text{Hf}$.

$^{176}\text{Yb}(\alpha, 3n)$ ^{178}Hf 55, 60 MeV

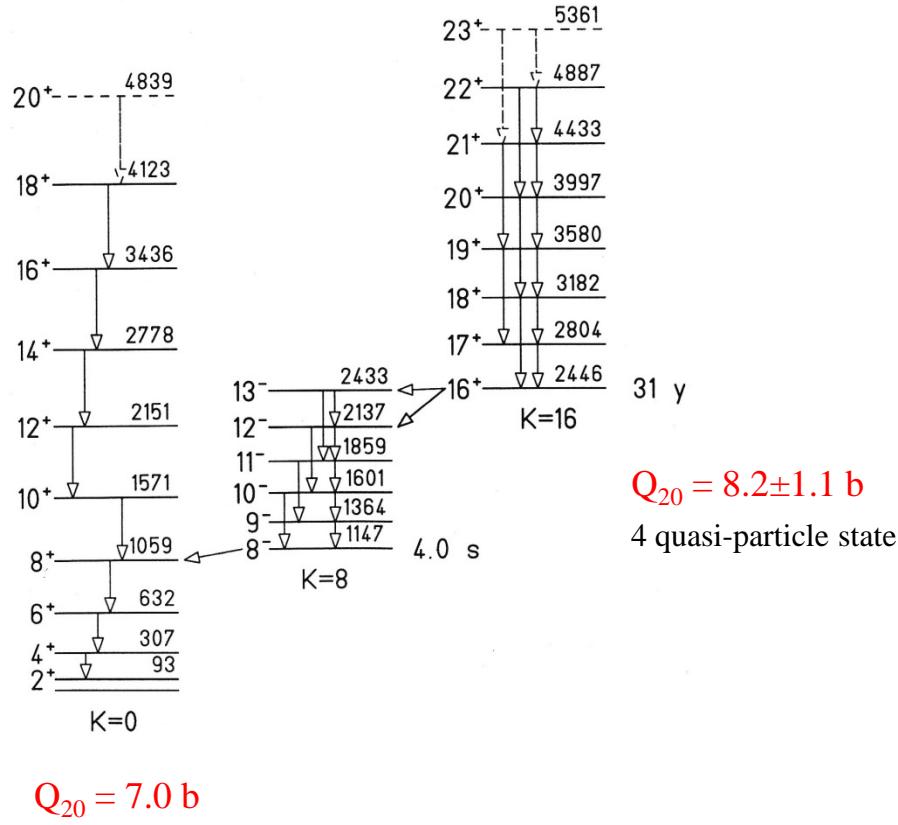
incomplete fusion reaction

14 charged-particle detectors cover 85% of 4π

E2/M1 mixing ratios $g_K = 0.52(6)$ $v^2\pi^2 - \text{qp band}$

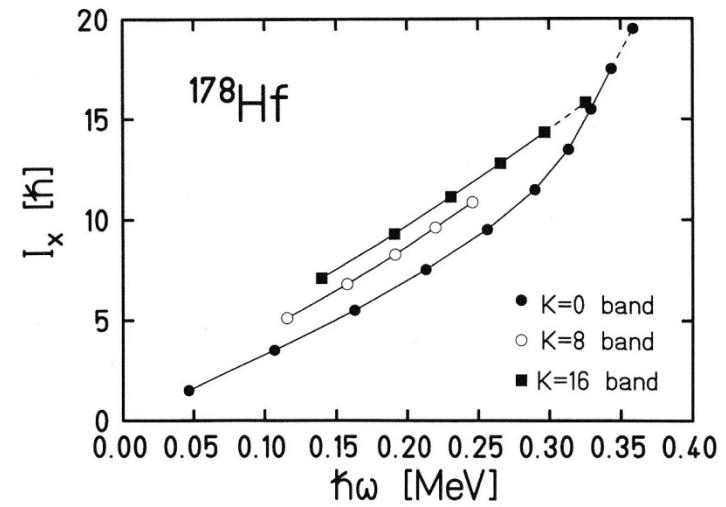
S.M. Mullins et al. Phys. Lett. B393 (1997), 279

Partial Level Scheme of ^{178}Hf

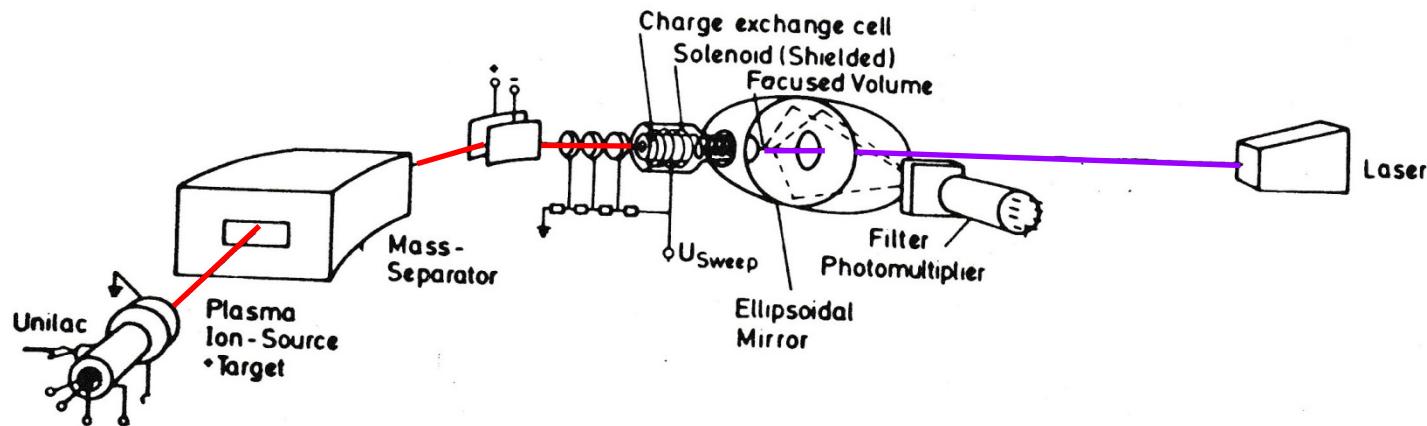


$$I_x = \{(I + 1/2)^2 - K^2\}^{1/2}$$

$$\hbar\omega = \frac{E(I+1) - E(I-1)}{I_x(I+1) - I_x(I-1)}$$

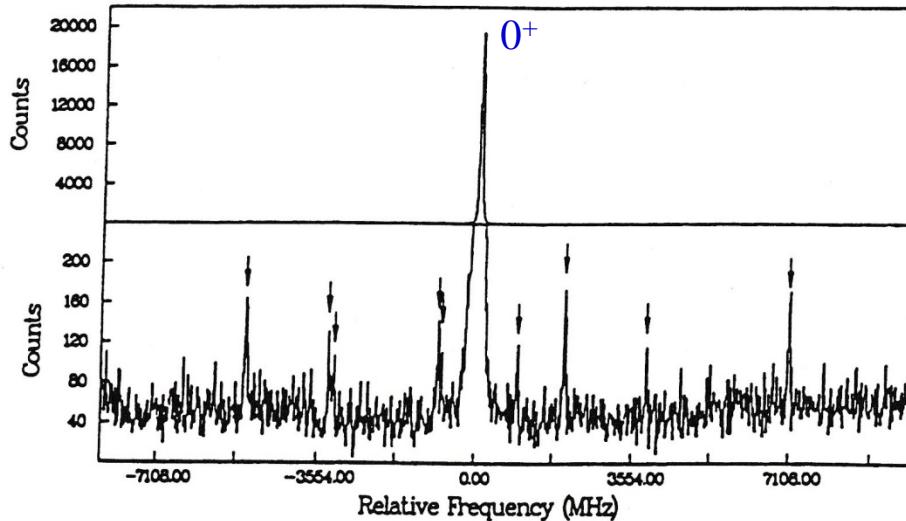


Collinear Laser Spectroscopy



kinetic energy spread in ion source: $\Delta E = mv \cdot \Delta v = \text{few eV}$

residual Doppler width after acceleration: $\Delta v = \frac{\nu_0 \Delta E}{\sqrt{2eU \cdot mc^2}}$ $\Delta v=60 \text{ MHz for } 60 \text{ kV}$



$$\Delta W_{HFS} = \frac{A}{2}K + B \frac{\frac{3}{4}K(K+1) - I(I+1)J(J+1)}{2I(2I-1) \cdot J(2J-1)}$$

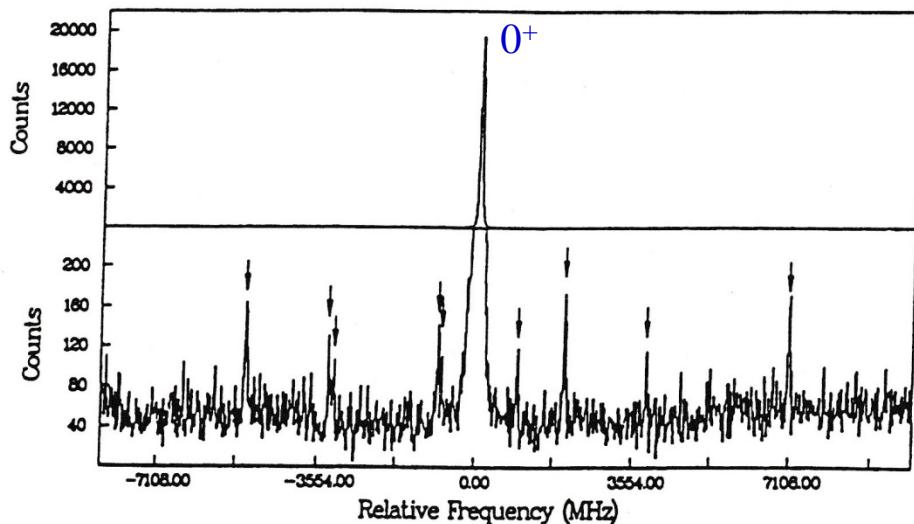
$$K = F(F+1) - I(I+1) - J(J+1)$$

$$A = \frac{\mu_I \cdot \langle H(0) \rangle}{I \cdot J}$$

$$B = eQ_s \cdot \langle V_{zz}(0) \rangle$$

The single peak at 0 MHz corresponds to the ground state ^{178}Hf . The nine hyperfine resonances of ^{178}Hf are marked by arrows.

Collinear Laser Spectroscopy



$$\Delta W_{HFS} = \frac{A}{2} K + B \frac{\frac{3}{4}K(K+1) - I(I+1)J(J+1)}{2I(2I-1) \cdot J(2J-1)}$$

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The single peak at 0 MHz corresponds to the ground state ^{178}Hf .
The nine hyperfine resonances of $^{178}\text{m}^2\text{Hf}$ are marked by arrows.

$$\mu_I^{178m2} = +8.16(4) \text{ nm}$$

$$Q_s^{178m2} = +6.00(7) \text{ b} \quad \rightarrow \quad Q_0 = 7.2(1) \text{ b}$$

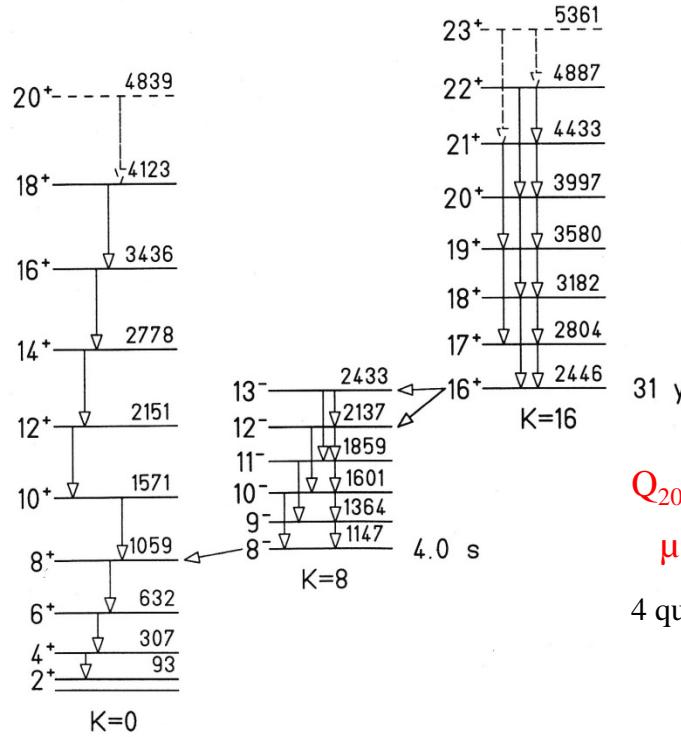
$$\delta \langle r^2 \rangle^{178,178m2} = -0.059(9) \text{ fm}^2$$

strong coupling scheme:

$$Q_s = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} \cdot Q_0 \quad \rightarrow \quad Q_s \ (K=I) = \frac{I \cdot (2I-1)}{(I+1)(2I+3)} \cdot Q_0$$

$$\delta \langle r^2 \rangle = \delta \langle r^2 \rangle_0 + \frac{5}{4\pi} \langle r^2 \rangle_0 \sum_{\lambda} \delta \langle \beta_{\lambda}^2 \rangle$$

Partial Level Scheme of ^{178}Hf



$$Q_{20} = 7.0 \text{ b}$$

$$I_x = \{(I + 1/2)^2 - K^2\}^{1/2}$$

$$\hbar\omega = \frac{E(I+1) - E(I-1)}{I_x(I+1) - I_x(I-1)}$$

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