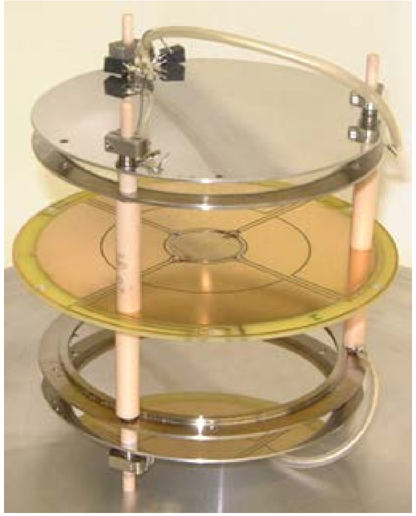
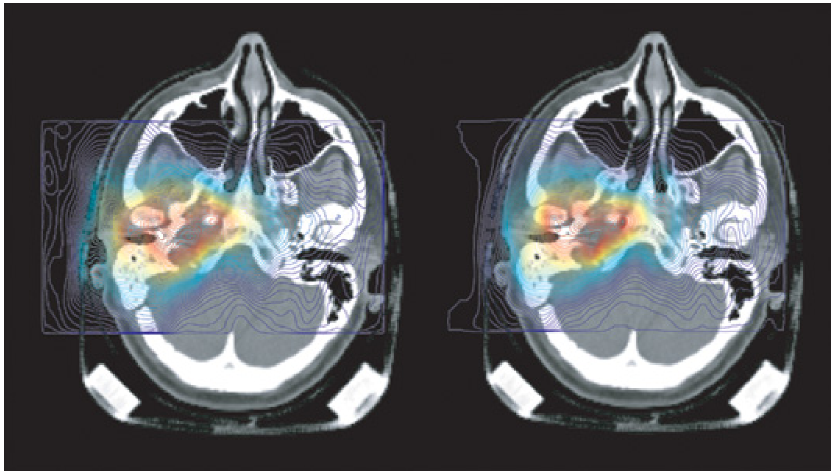
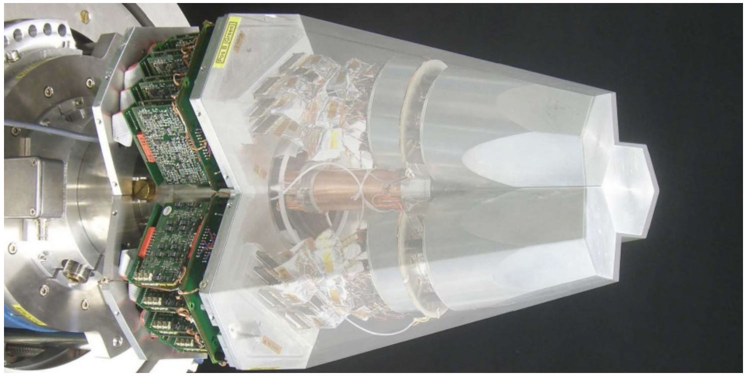
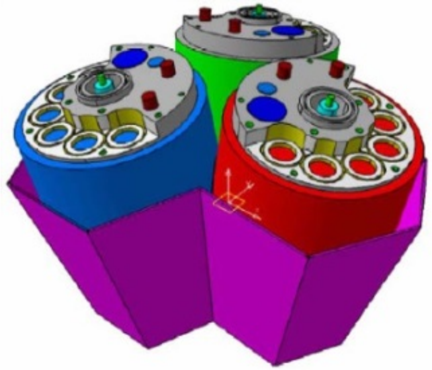
Lecture: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de



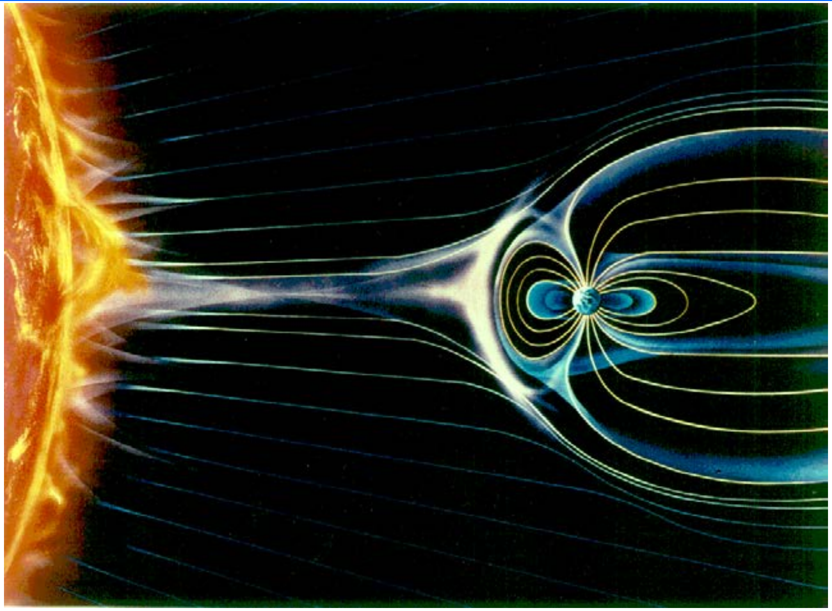






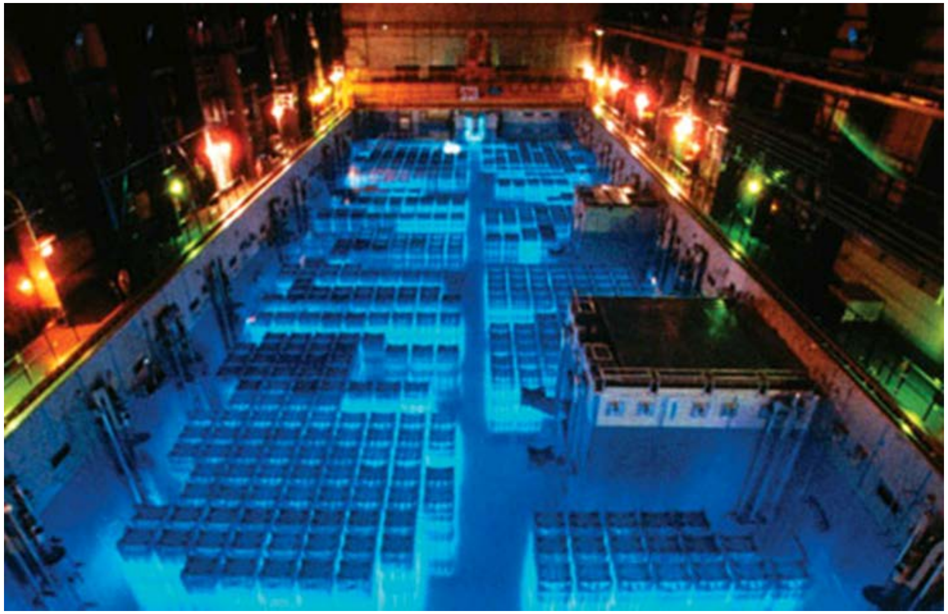
Particle Interaction with Matter

Aurora Borealis



Ionization

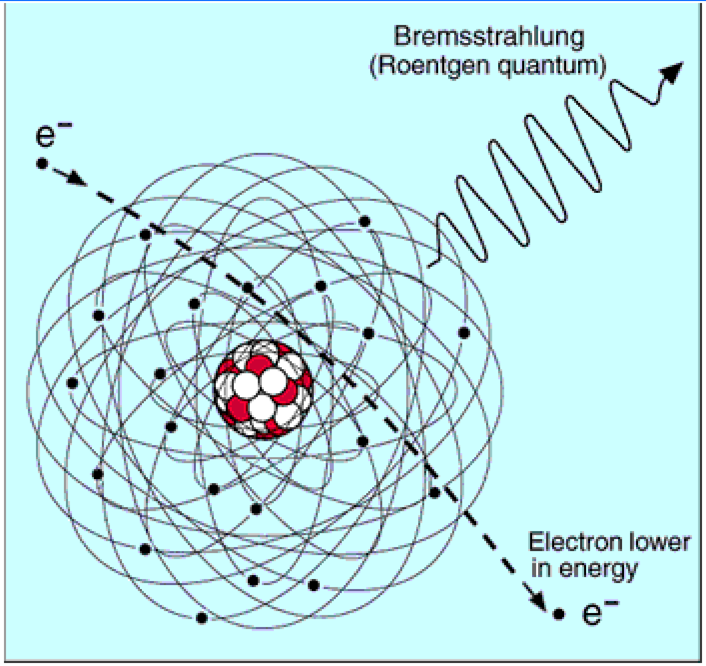
Particle Interaction with Matter



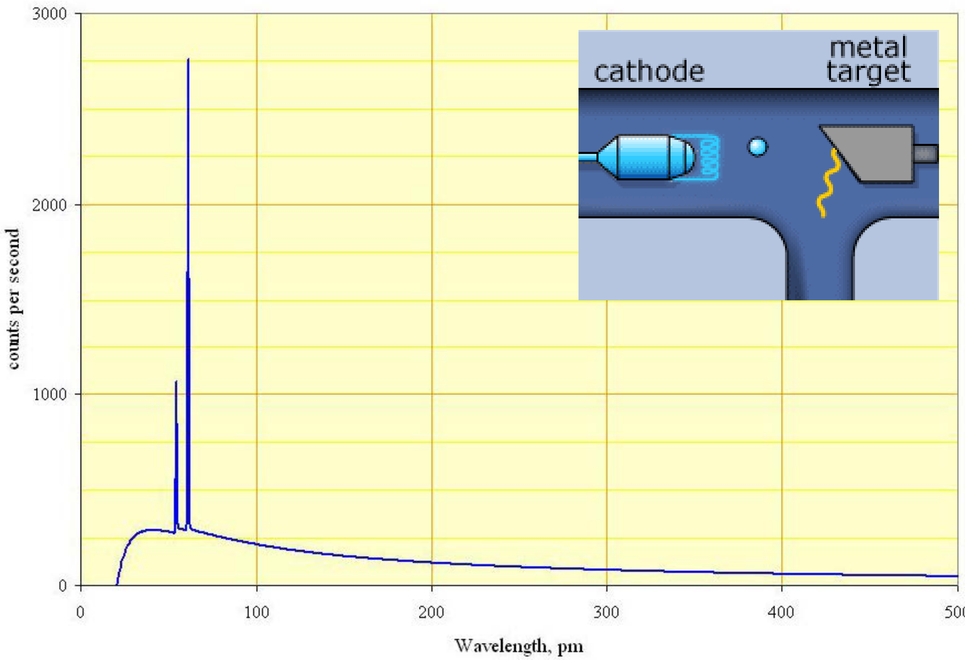
Characteristic glow from a reactor

Cherenkov radiation is an effect similar to sonic booms when the plane exceeds the velocity of sound

Particle Interaction with Matter



Bremsstrahlung or 'braking radiation'



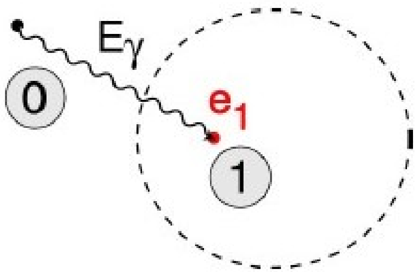
Spectrum of the X-rays emitted by an X-ray tube with a rhodium target, operated at 60 kV. The continuous curve is due to bremsstrahlung, and the spikes are characteristic K lines for rhodium.

Bremsstrahlung

Interaction of gamma rays with matter

~ 100 keV ~1 MeV ~ 10 MeV γ-ray energy

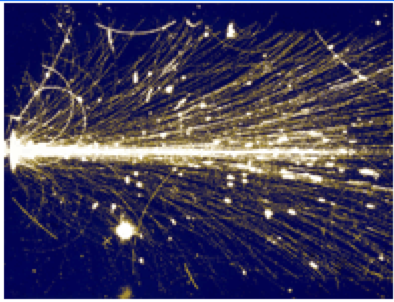
Photoelectric



Isolated hits

Probability of interaction depth

Energy loss – dE/dx



Particles interact differently with matter. Important for detectors is the energy loss per path length. The total energy loss per path length is the sum of all contributions.

$$-\left(\frac{dE}{dx}\right)_{tot} = -\left(\frac{dE}{dx}\right)_{coll} - \left(\frac{dE}{dx}\right)_{rad} - \left(\frac{dE}{dx}\right)_{photoeff} - \left(\frac{dE}{dx}\right)_{compton} - \left(\frac{dE}{dx}\right)_{pair} - \left(\frac{dE}{dx}\right)_{hadron} \cdots$$

Depending on the particle type, the particle energy and the material some processes dominate, other do not occur. For instance only charged particles will interact with electrons of atoms and produce ionization, etc.

Measurement Principles

A measurement requires an interaction of the particle with the material of the detector. The interaction provokes two effects:

1st Creation of a detectable signal, e.g.
ionization → charges
excitation → scintillation
excitation of phonons → heat

2nd Alternation of the particles properties, e.g.
energy loss
change of trajectory due to scattering
absorption

unwanted side effects. They need to be as small as possible and well understood.

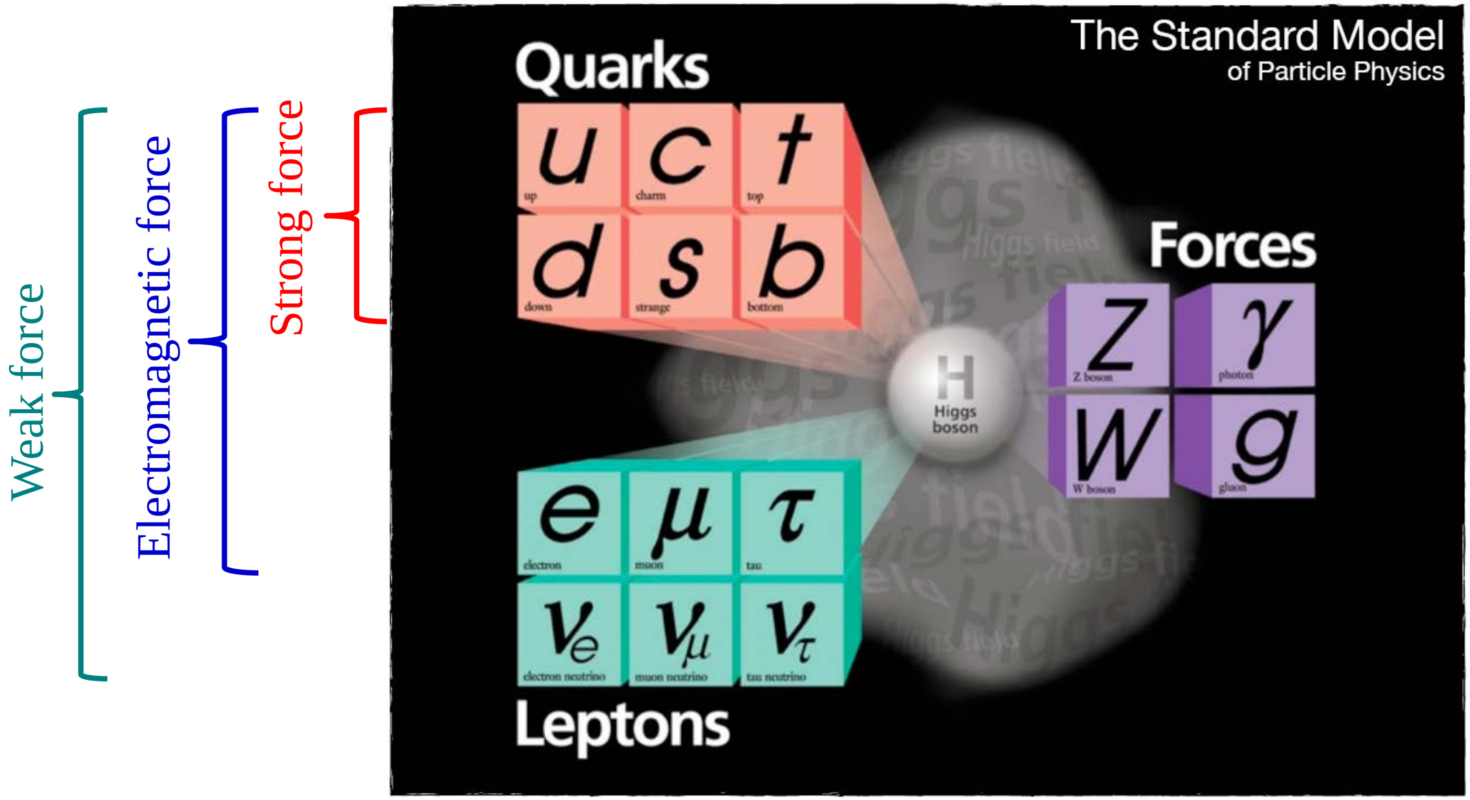
Measurement Principles

A particle detector is an instrument to measure one or more properties of a particle …

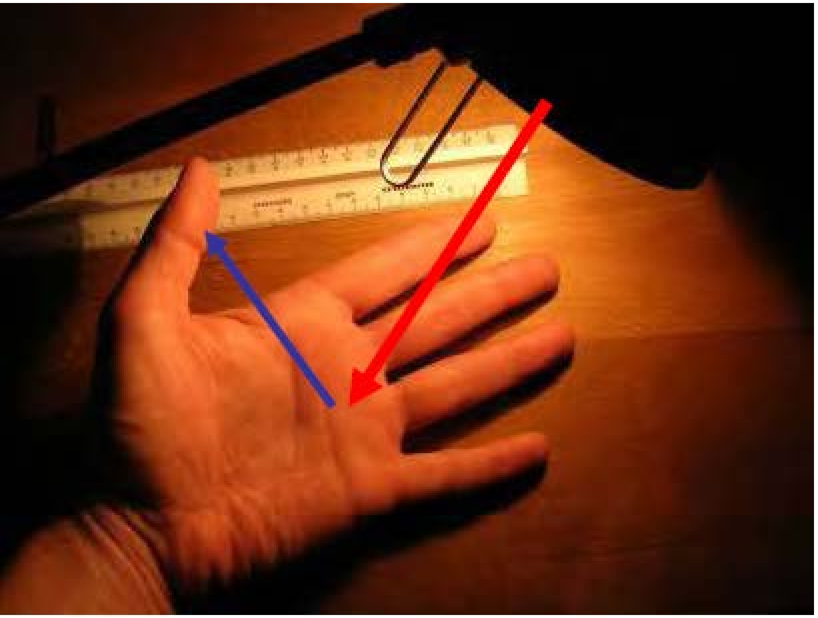
Properties of a particle		Type of detection principle:
- position and direction	$x, \vec{x}$	position and tracking
- momentum	$\lvert\vec{p}\rvert$	tracking in a magnetic field
- energy	E	calorimetry
- mass	m	spectroscopy and PID
- velocity	β	Cherenkov radiation or time of flight
- transition radiation	γ	TRD

- spin, lifetime

What is a Particle?



How do we see Particles?



A light bulb shines on a hand and the different reflections make the fine structure visible.
With a magnifying glass or microscope more details can be seen, but there is a fundamental limit:
The wavelength of the light (1/1000 mm) determines the size of the resolvable objects.

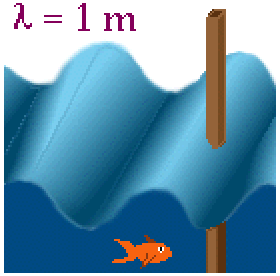
available wavelength

→ electromagnetic waves $E = \frac{hc}{\lambda}$

LW	3000 m	
MW	300 m	
KW	30 m	
UKW	3 m	
GPS	0.3 m	
Infrared	10^{-6} m	
light	$5\cdot10^{-7}$ m	2 eV
UV	10^{-7} m	10 eV
X-ray	10^{-10} m	10^4 eV
γ-ray	10^{-12} m	10^6 eV

light bulb → accelerator
magnifying glass or microscope → detector

Energy, Wavelength and Resolution



wavelength versus resolution

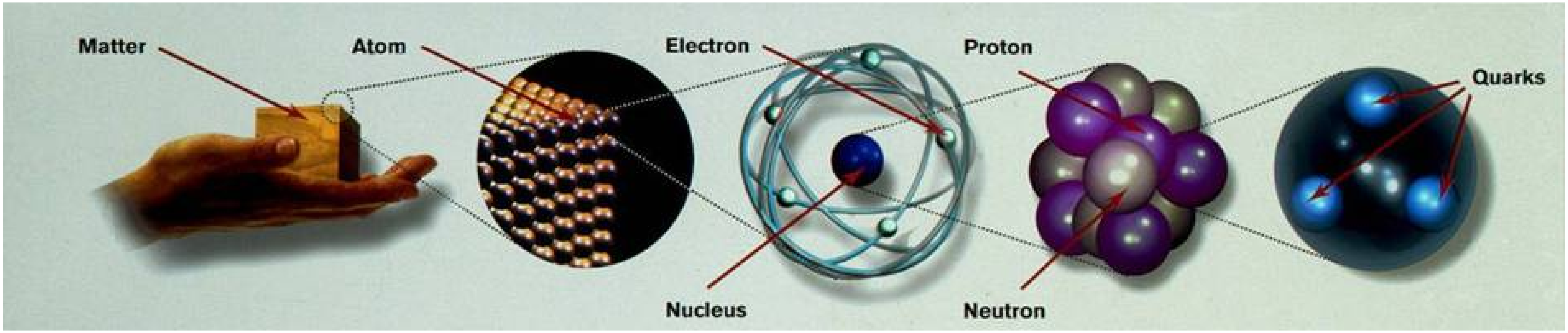
Small objects (smaller than λ) do not disturb the wave
→ small object is not visible
Large objects disturb the wave
→ large object is visible

❖ **all particles have wave properties:**

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E_{kin} \cdot (E_{kin} + 2m_0c^2)}}$$

de Broglie wavelength

Louis de Broglie



10^{-1}m 10^{-9}m 10^{-10}m 10^{-14}m 10^{-15}m

h·c = 1239.84 [MeV fm]

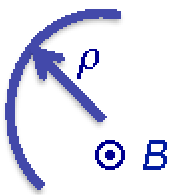
Detection and Identification of Particles

- ❖ Detection = particle counting (is there a particle?)
- ❖ Identification = measurement of mass and charge of the particle

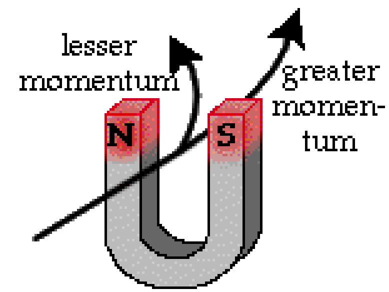
(most elementary particle have $Ze = \pm 1$)

- ❖ How:

\- charged particles are deflected by B fields such that:



$$\rho = \frac{p}{ZeB} \propto \frac{p}{Z} = \frac{\gamma m_0 \beta c}{Z}$$

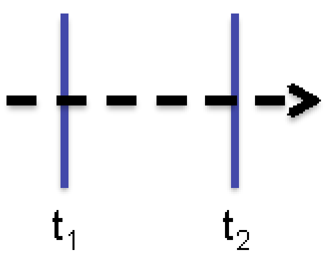


$p =$ *particle momentum*

$m_0 =$ *rest mass*

$\beta c =$ *particle velocity*

\- particle velocity measured with time-of-flight (ToF) method



$$\beta \propto \frac{1}{\Delta t}$$

- ❖ ToF for known distance
- ❖ Ionization $-\frac{dE}{dx} = f(\beta)$
- ❖ Cherenkov radiation
- ❖ Transition radiation

Detection and Identification of Particles

- Detection = particle counting (is there a particle?)
- Identification = measurement of mass and charge of the particle

  (most elementary particle have $Ze = \pm 1$)

- How:

- kinetic energy determined via a calorimetric measurement

$$E_{kin} = (\gamma - 1) m_0 c^2 \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- for Z=1 the mass is extracted from E_{kin} and p
- to determine Z (particle charge) a Z-sensitive variable is e.g. the ionization energy loss

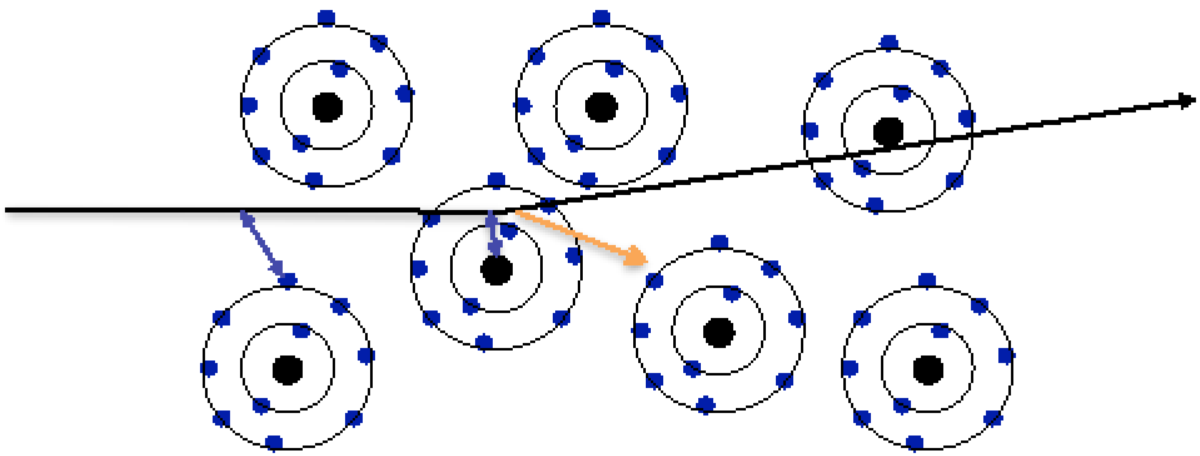
$$\frac{dE}{dx} \propto \frac{z^2}{\beta^2} ln(a \cdot \beta^2 \gamma^2)$$

a = material-dependent constant

Energetic charged particles in matter

Three types of electromagnetic interactions:

1. Ionization (of the atoms of the transversed material)
2. Emission of Cherenkov light
3. Emission of transition radiation



1) Interaction with the atomic electrons. The incoming particle looses energy and the atoms are **excited** or **ionized.**

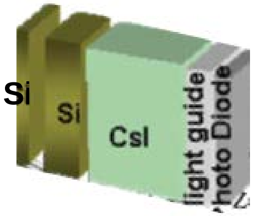
2) Interaction with the atomic nucleus. The particle is deflected (scattered) causing **multiple scattering** of the particle in the material. During this scattering a **Bremsstrahlung** photon can be emitted.

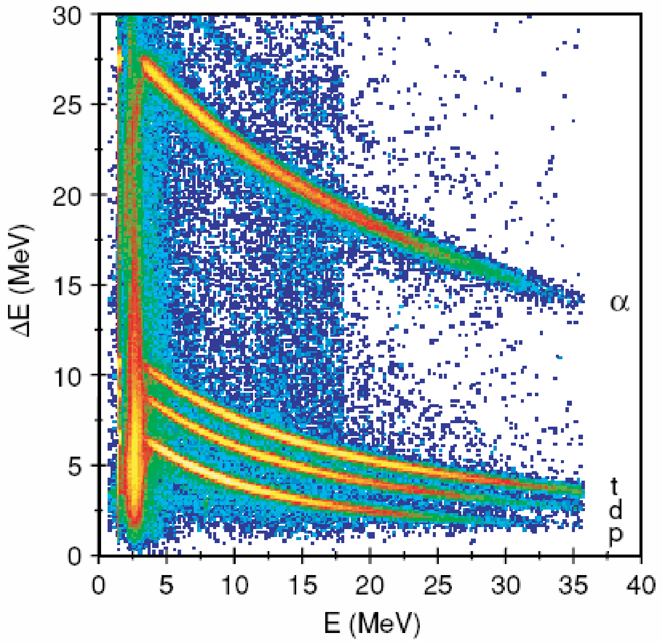
3) In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as **Cherenkov Radiation**. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X-ray photon, called **Transition Radiation**.

Energetic charged particles in matter

$$-\frac{dE}{dx} \propto \frac{mz^2}{E}$$

Charged particle identification with segmented or stacked detectors





$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_0 v^2} nZ \left[\ln \frac{2m_0 v^2}{I} - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2} \right]$$

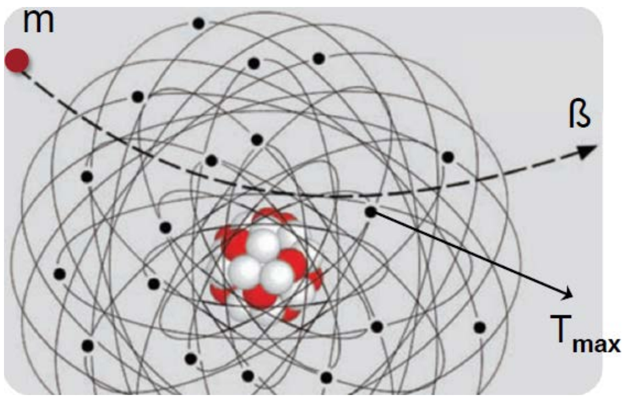
Bethe-Bloch formula

z – projectile atomic number
v – projectile velocity
m_0 - electron mass
e – electron charge

n – target number density
Z – target atomic number
nZ – target electron density
I – average excitation and ionization potential

Energetic charged particles in matter

α-particles are highly ionized and lose their energy very fast by ionization and excitation when passing through matter.



- maximum energy transfer T_{max} of a projectile with mass **m** and velocity β on an electron $\mathbf{m_e}$ at rest

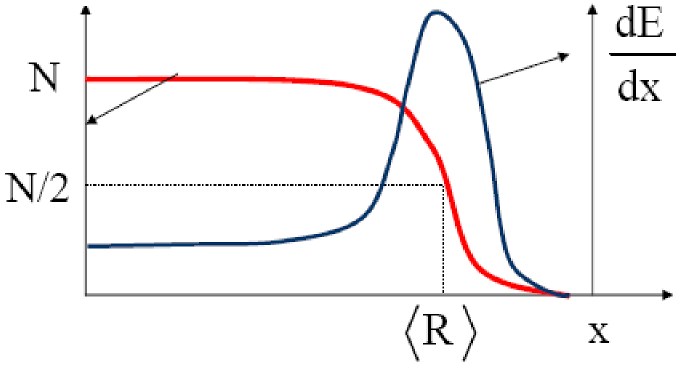
$$T_{max} = \frac{2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2 \cdot m^2}{m^2 + {m_e}^2 + 2 \cdot \gamma \cdot m \cdot m_e}$$

$$\boxed{T_{max} = 2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2}$$

for all heavy primary particles except electrons and positrons

number of α-particles as a function of distance **x**

energy loss of α-particles per distant unit



average range <R> of α-particles with 5 MeV
2,5cm in air, 2,3cm in Al, 4,3cm in tissue

Energy loss and range of charged particles

$$-\frac{dE}{d\varepsilon} = -\frac{1}{\rho}\cdot\frac{dE}{dx} = z^2\cdot\frac{Z}{A}\cdot f(\beta, I)$$

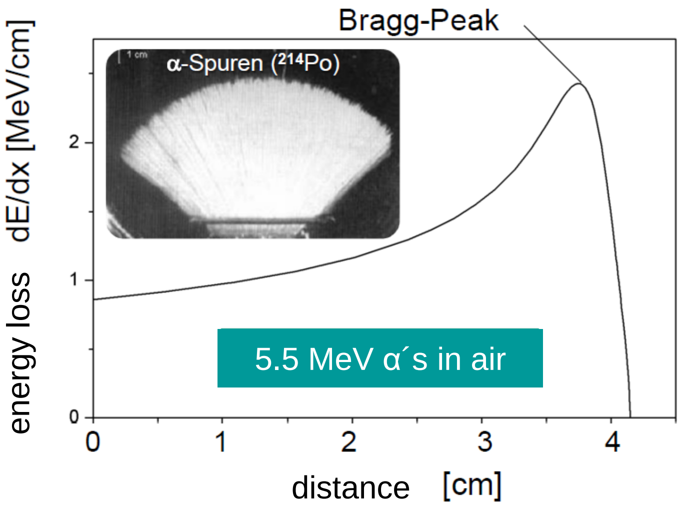
-dE/dε is independent of the material for equal particles

- the average range for particles with kin. energy T is obtained by integration

$$\bar{R} = \int_{E_0}^{0}\left(\frac{dE}{dx}\right)^{-1} dE$$

- 7.7 MeV α-particles in air: $\bar{R}/\rho \approx 7cm$

- range is not exact but there is range straggling, the number of interactions is a statistical process.

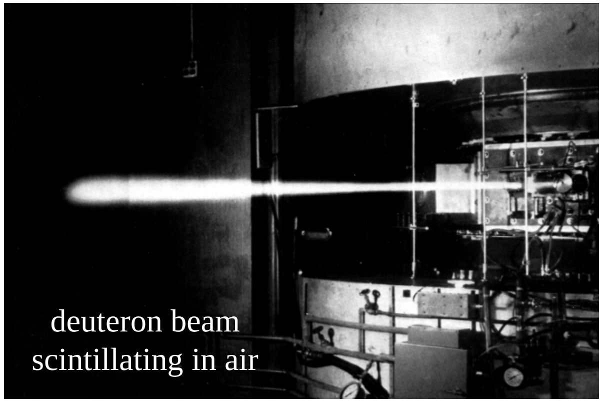


Several empirical and semi-empirical formulae have been proposed to compute range of α-particles in air.

$$R_\alpha^{air}[mm] = \begin{cases} e^{1.61\sqrt{E_\alpha}} & for\ E_\alpha < 4MeV \\ (0.05E_\alpha + 2.85)E_\alpha^{3/2} & for\ 4MeV \le E_\alpha \le 15MeV \end{cases}$$

$$R_\alpha^{air}[cm] = \begin{cases} 0.56E_\alpha & for\ E_\alpha < 4MeV \\ 1.24E_\alpha - 2.62 & for\ 4MeV \le E_\alpha < 8MeV \end{cases}$$

Scaling the range of other materials $R_\alpha^x = 3.37\cdot 10^{-4} R_\alpha^{air}\frac{\sqrt{A_x}}{\rho_x}$



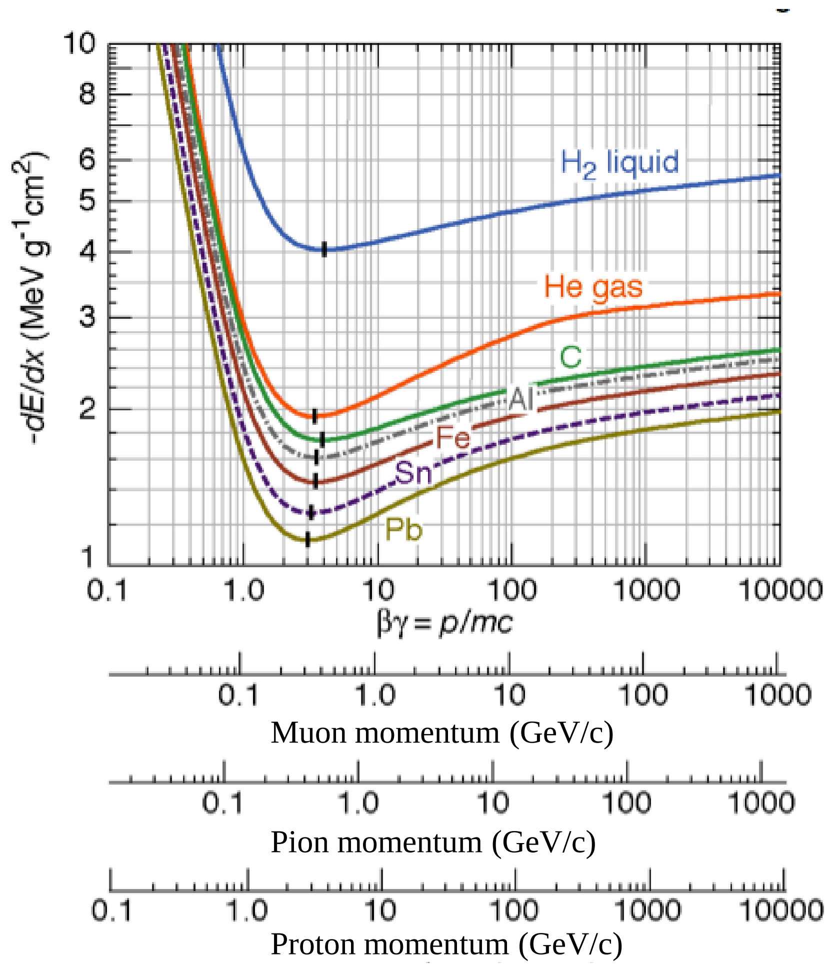
Interaction of charged particles in matter

Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

$$-\frac{dE}{dx} = \underbrace{4\cdot\pi\cdot r_e{}^2\cdot N_a\cdot m_e c^2}_{= 0.3071\ \mathrm{MeV\ g^{-1}cm^2}}\cdot\rho\cdot\frac{Z}{A}\cdot\frac{z^2}{\beta^2}\cdot\left[\frac{1}{2}ln\left(\frac{2\cdot m_e c^2\cdot\gamma^2\cdot\beta^2\cdot T_{max}}{I^2}\right)-\beta^2-\delta-2\cdot\frac{C}{Z}\right] \qquad \approx z^2\cdot\frac{Z}{A}\cdot f(\beta, I)$$

- ❖ Specific energy loss rate $\frac{1}{\rho}\frac{dE}{dx}$ for muons, pions and protons in different materials

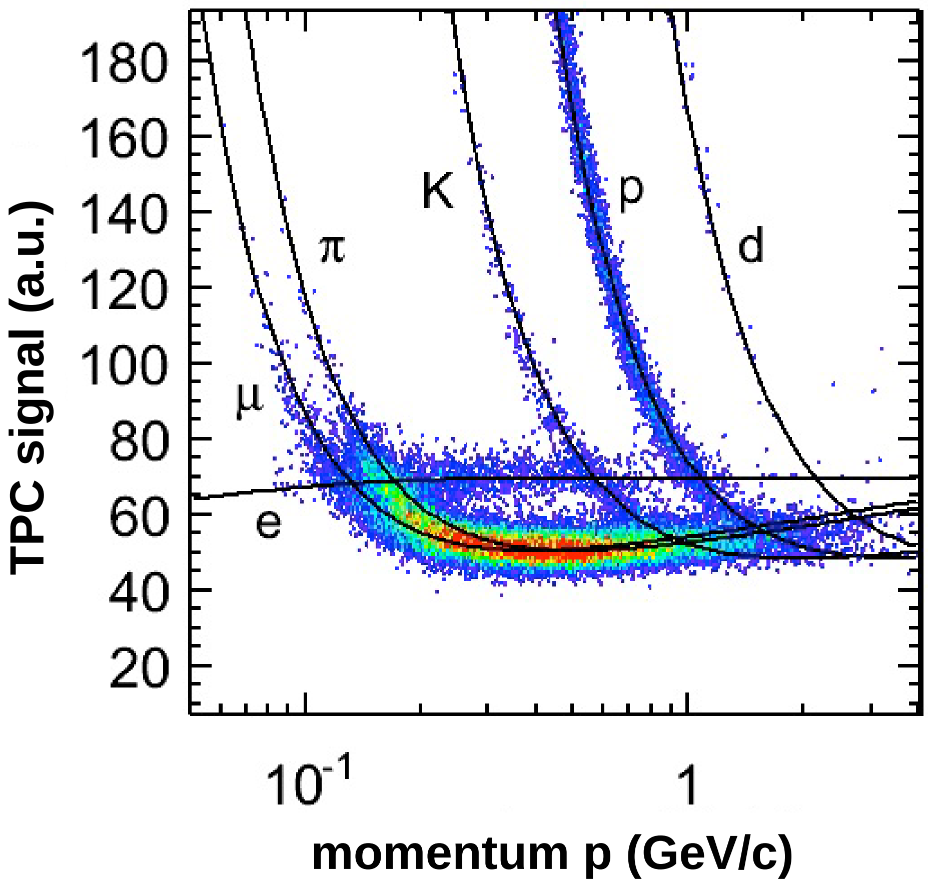
- ❖ Dependence on mass A, charge Z of target nucleus
- ❖ Minimum ionization: 1-2 MeV/g cm^{-2} [H_2: 4 MeV/g cm^{-2}]



Energy loss of charged particles – dE/dx for different particles

- dE/dx for heavy particles in this momentum regime is well described by Bethe-Bloch formula, i.e. the dominant energy loss is collisions with atoms

- dE/dx for electrons does not follow Bethe-Bloch formula. The dominant process is Bremsstrahlung



[ALICE TPC, 2009]

Energy loss for electrons and positrons

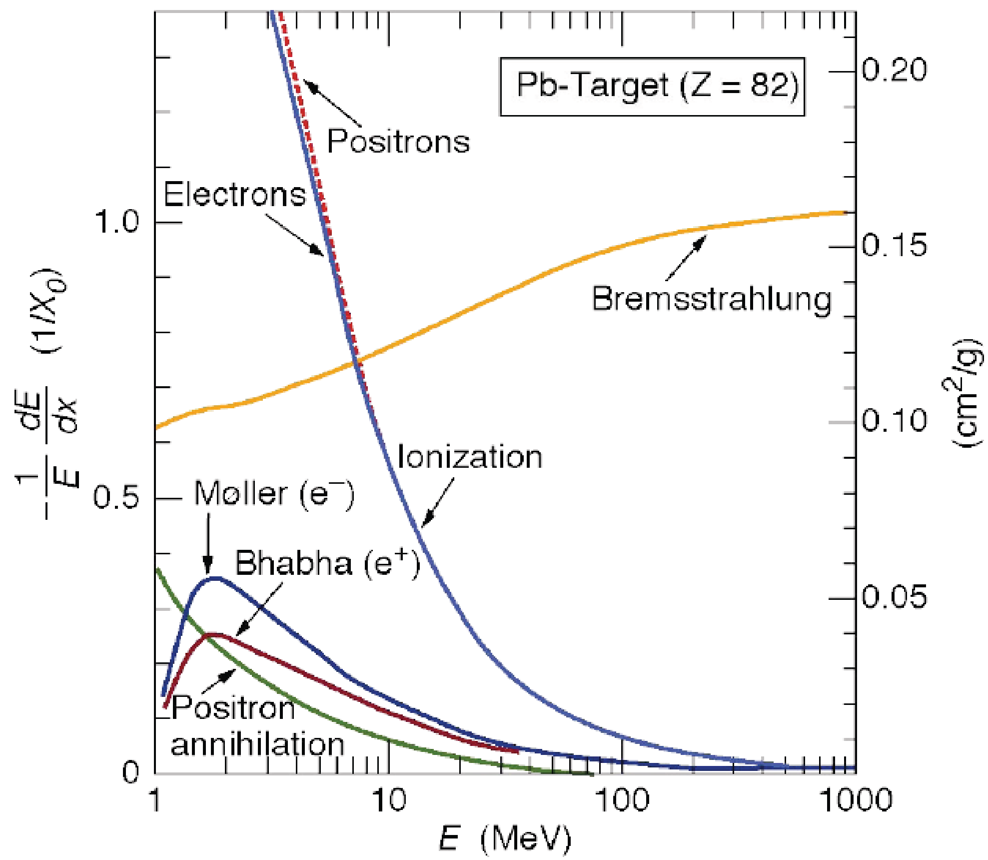
e$^{\pm}$ are exceptional cases due to their low mass.
They will be deflected significantly in each collision.
In addition to the energy loss due to **ionization**, the energy loss due to **Bremsstrahlung** is of importance.

$$-\left(\frac{dE}{dx}\right)_{tot} = -\left(\frac{dE}{dx}\right)_{coll} - \left(\frac{dE}{dx}\right)_{rad}$$

For high energies the energy loss due to Bremsstrahlung is given by

$$-\left(\frac{dE}{dx}\right)_{rad} \propto E \quad \text{and} \quad -\left(\frac{dE}{dx}\right)_{rad} \propto \frac{1}{m^2}$$

Other particles like muons also radiate, especially at higher energies.



Bremsstrahlung

$$-\left(\frac{dE}{dx}\right)_{rad} = \frac{E}{X_0}$$

X_0 is the radiation length. It is the mean distance over which a high-energy electron loses all but 1/e of its energy by Bremsstrahlung

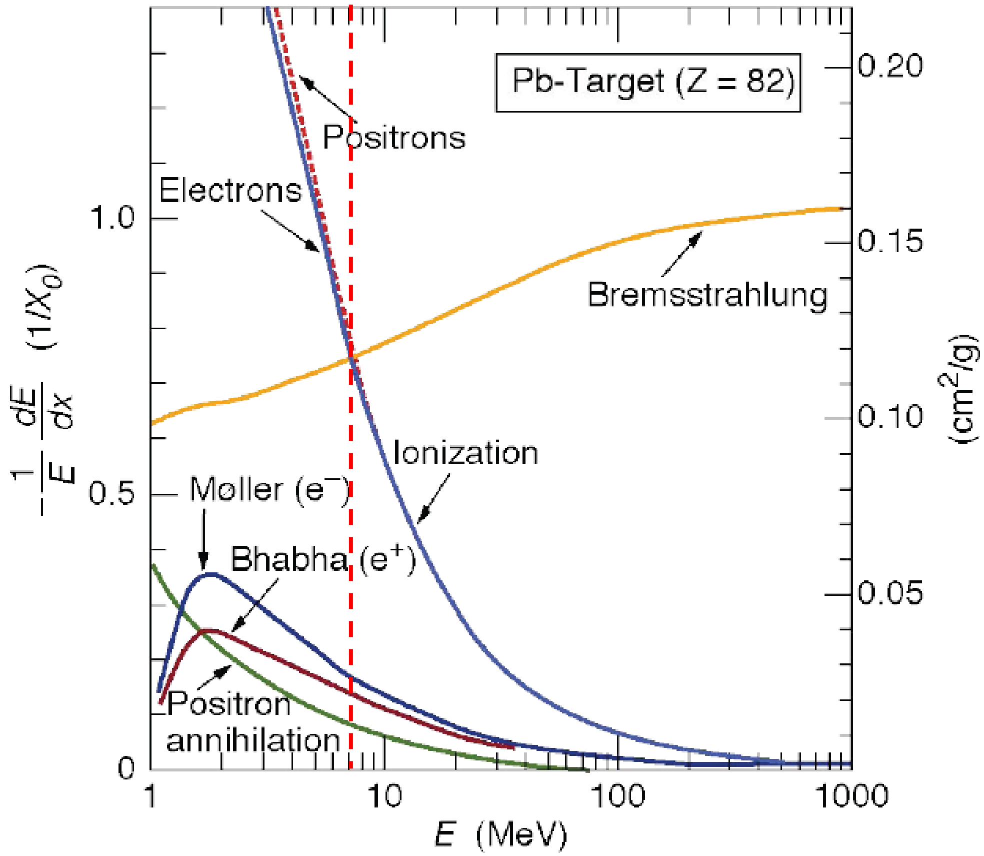
fit to data:

$$X_0 = \frac{716.4 \cdot A}{Z \cdot (Z+1) \cdot ln\left(287/\sqrt{Z}\right)}$$

Usual definition for the critical energy $\mathbf{E_c}$ (electron)

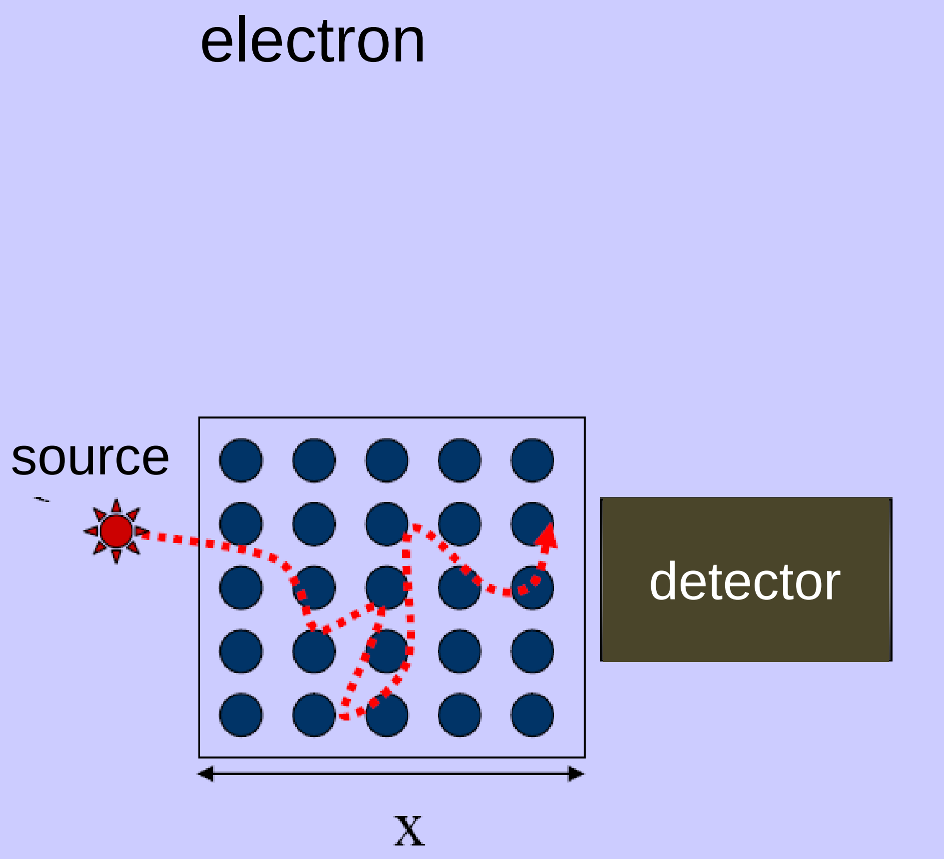
$$\left(\frac{dE}{dx}\right)_{ionization} = \left(\frac{dE}{dx}\right)_{bremsstrahlung}$$

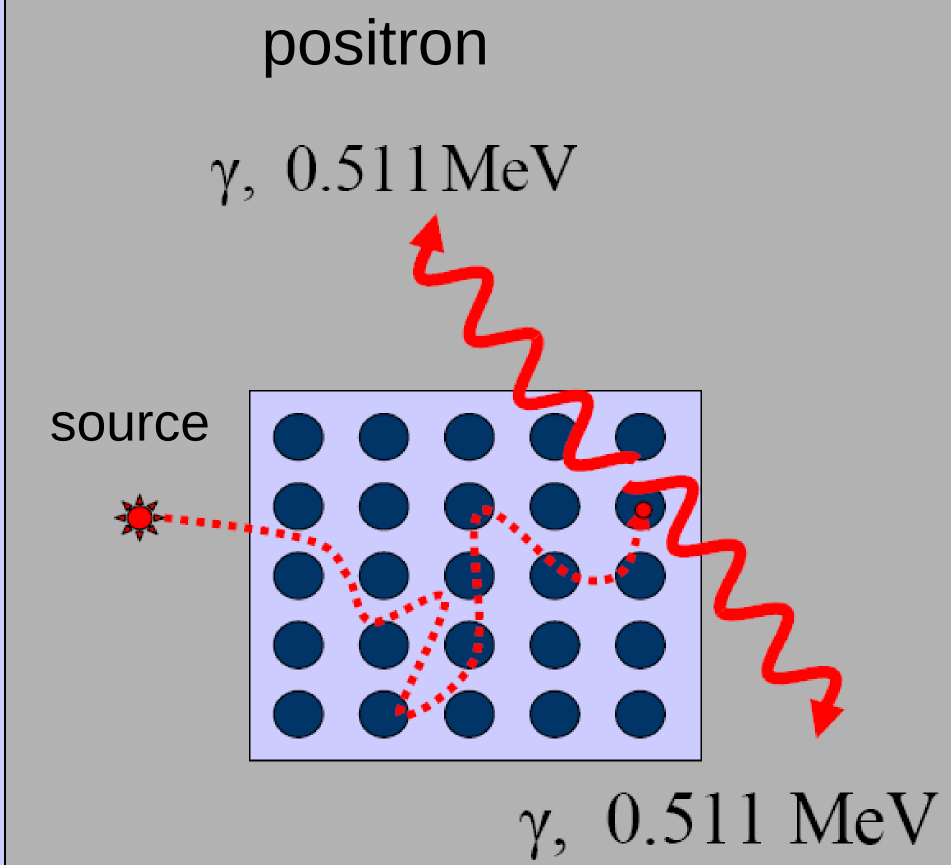
$$E_c(e^-) = \begin{cases} \dfrac{610\ MeV}{Z+1.24} & for\ solids\ and\ liquids \\ \dfrac{710\ MeV}{Z+0.92} & for\ gases \end{cases}$$



example: Pb (Z=82, ρ = 11.34 [g/cm^3] → E_c = 7.34 MeV

Comparison between electrons (β^-) and positrons (β^+) on their way through matter





Cherenkov radiation

$$-\frac{dE}{dx} = \underbrace{4\cdot\pi\cdot {r_e}^2\cdot N_a\cdot m_e c^2}_{= 0.3071\ \text{MeV g}^{-1}\text{cm}^2}\cdot\rho\cdot\frac{Z}{A}\cdot\frac{z^2}{\beta^2}\cdot\left[\frac{1}{2}ln\left(\frac{2\cdot m_e c^2\cdot\gamma^2\cdot\beta^2\cdot T_{max}}{I^2}\right)-\beta^2-\delta-2\cdot\frac{C}{Z}\right]$$

$\delta \equiv$ polarization of medium

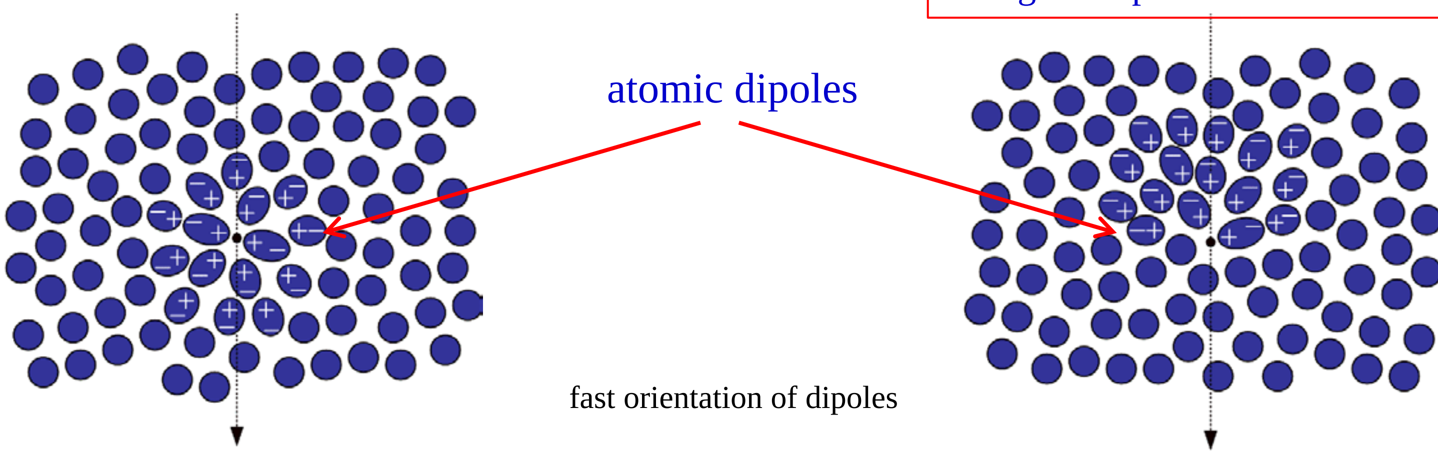
Cherenkov radiation is emitted if the particle velocity v is larger than the velocity of light in the medium.

$$v > \frac{c}{n}$$

$c \cdots$ *speed of light in vacuum*
$n \cdots$ *refraction index of medium*

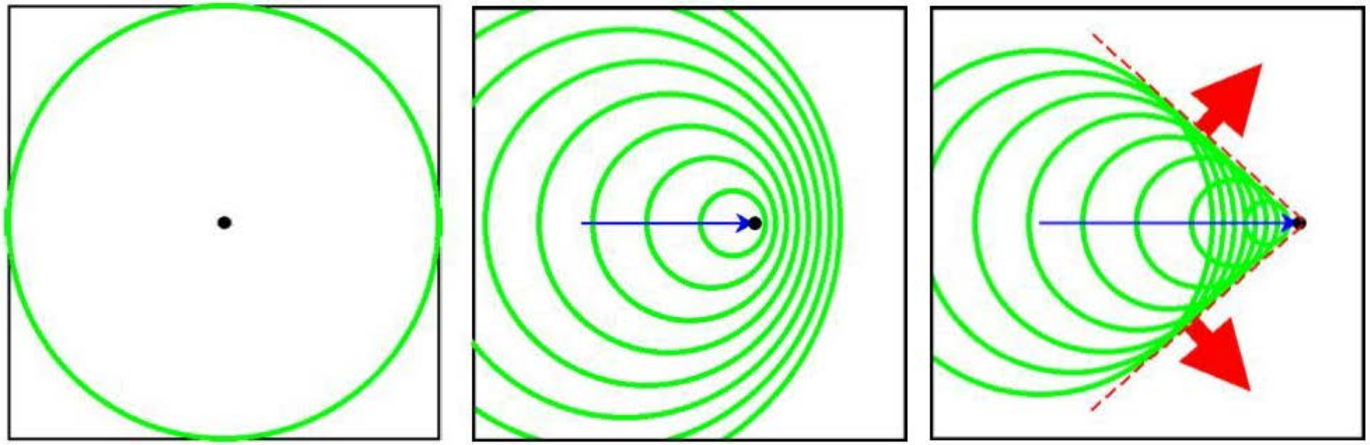
$v < c/n$ symmetric orientation
→ no resulting dipole field

$v > c/n$ asymmetric orientation
→ resulting dipole field
change of dipole field → **radiation**

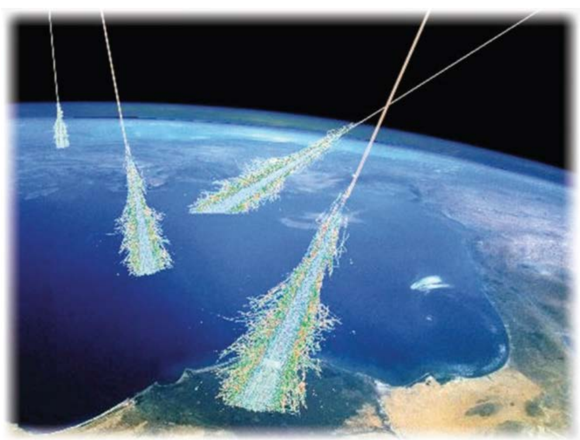


Cherenkov radiation

$\beta = 0$ $\qquad$ $\beta \leq \frac{1}{n}$ $\qquad$ $\beta \geq \frac{1}{n}$

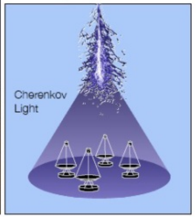


Pavel Alekseyevich Cherenkov

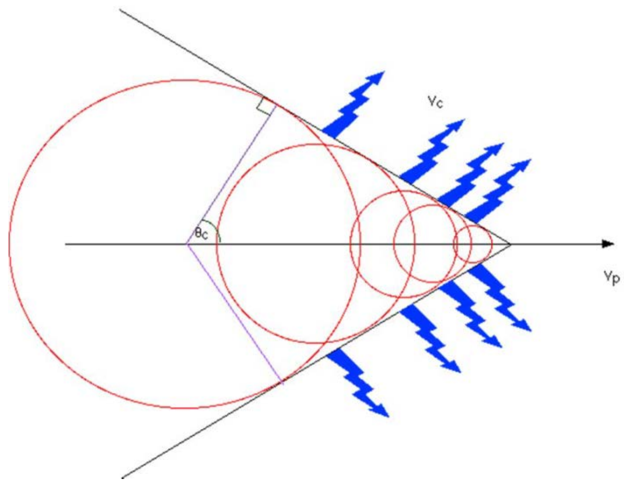


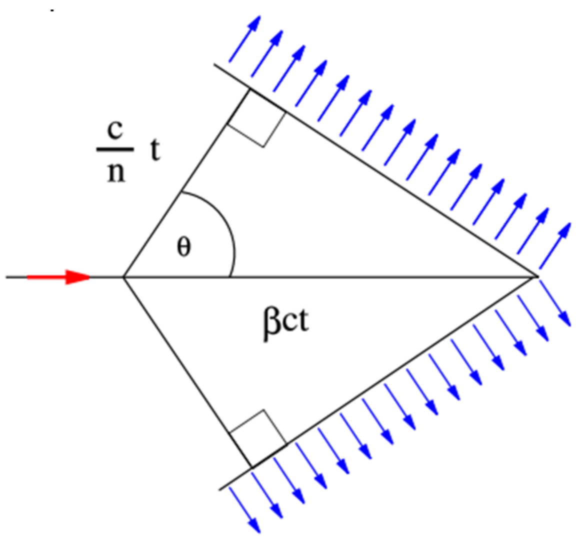
Hess Telescopes Namibia

γ-ray detection



Cherenkov radiation





threshold velocity: $\beta \geq \frac{1}{n}$

$$\gamma = (1-\beta^2)^{-1/2} \geq \frac{n}{\sqrt{n^2-1}}$$

threshold angle: $\cos\theta_C = \frac{1}{\beta \cdot n}$

Parameters of typical radiators

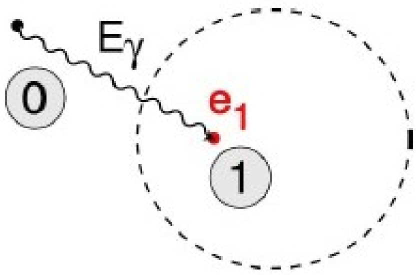
medium	n	β_{thr}	$\theta_{max}(\beta=1)$	$N_{ph}(eV^{-1}cm^{-1})$
air	1.000283	0.9997	1.36	0.208
isobutene	1.00127	0.9987	2.89	0.941
water	1.33	0.752	41.2	160.8
quartz	1.46	0.685	46.7	196.4

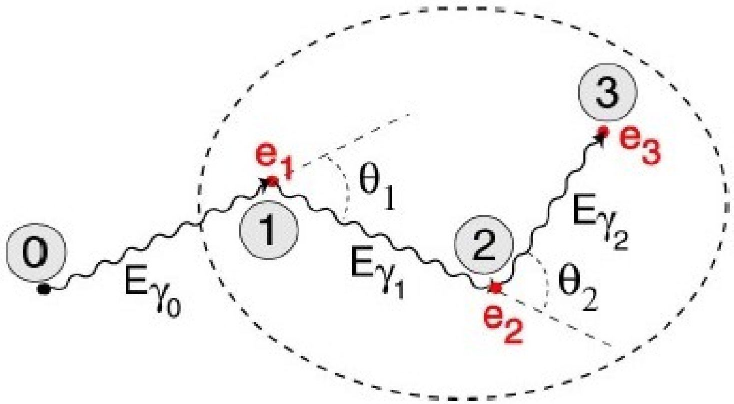
Note: Energy loss due to Cherenkov radiation very small compared to ionization (<1%)

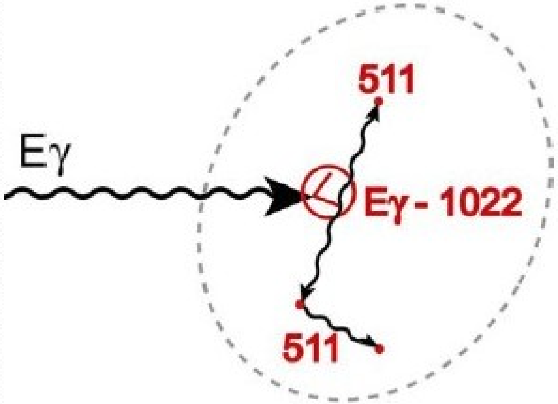
Interaction of gamma rays with matter

~ 100 keV ~1 MeV ~ 10 MeV γ-ray energy

Photoelectric	Compton Scattering	Pair Production

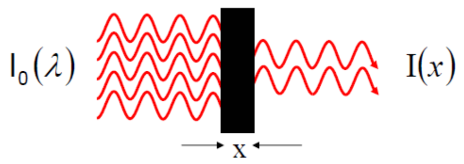






Isolated hits	Angle/Energy	Pattern of hits
Probability of interaction depth	$E_{\gamma'} = \dfrac{E_\gamma}{1+\dfrac{E_\gamma}{m_0c^2}(1-\cos\theta)}$	$E_{1st} = E_\gamma - 2\ mc^2$

Interaction of gamma rays with matter

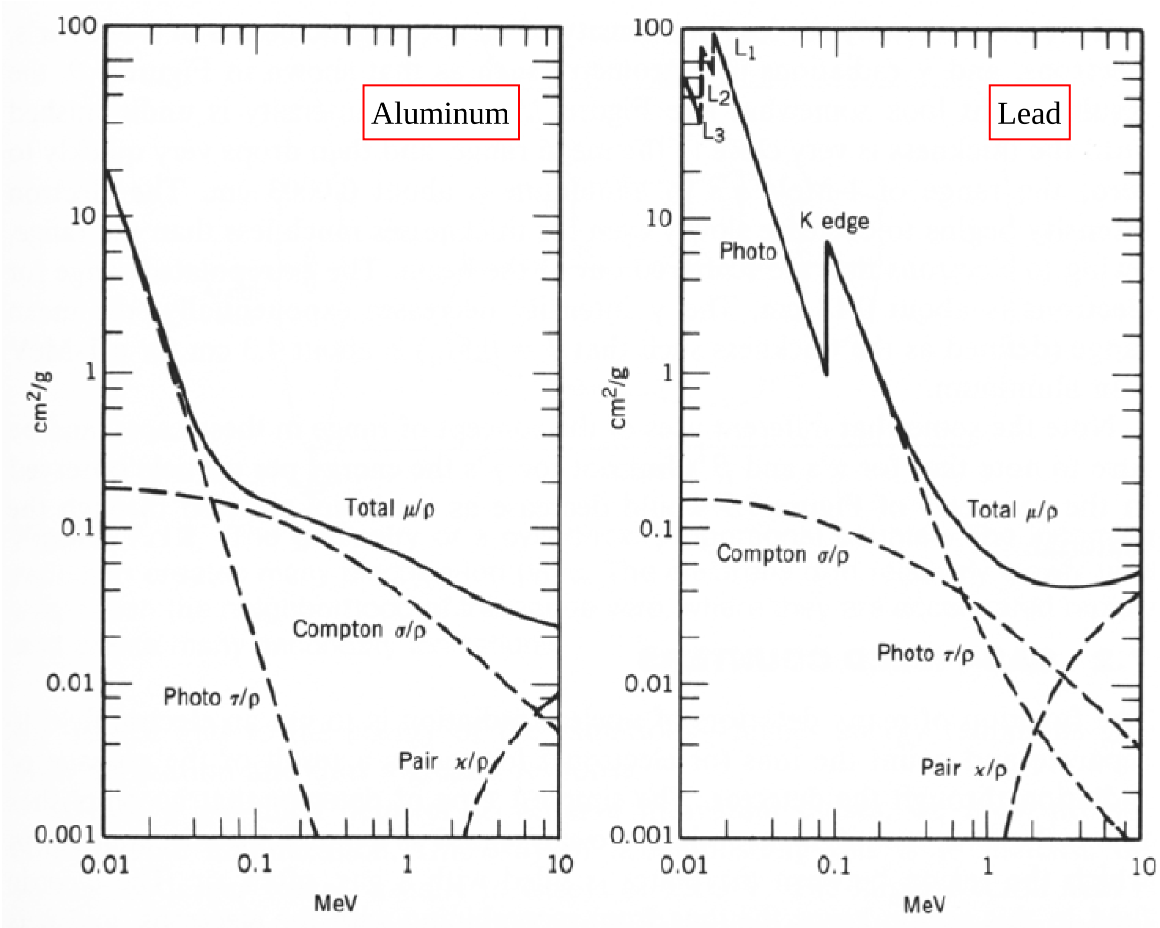


$$I(x) = I_0(\lambda) \cdot e^{-\frac{\mu(\lambda,Z)}{\rho}\rho \cdot x}$$

total absorption coefficient: **μ/ρ** [cm^2/g]

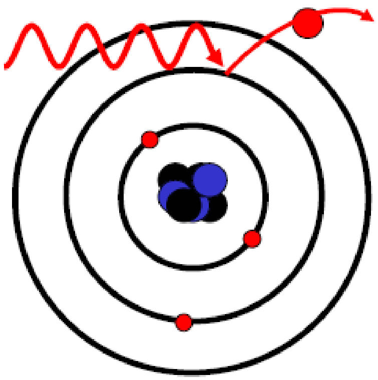
$$\frac{\mu_{total}}{\rho} = \sum_{i=1}^{3} \sigma_i$$

i=1 photoelectric effect
i=2 Compton scattering
i=3 pair production



Interaction of gamma rays with matter

$$E_{e,kin} = h \cdot \nu - E_{Bindung}$$



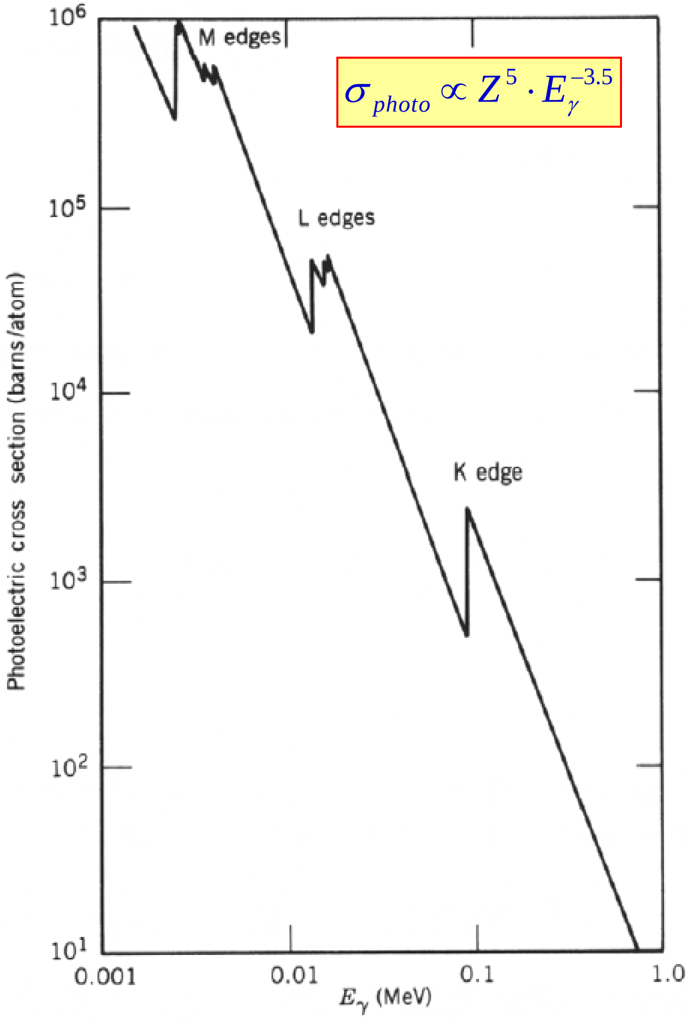
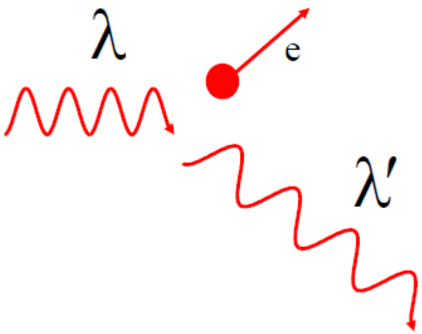


Photo effect:

Absorption of a photon by a bound electron and conversion of the γ-energy in potential and kinetical energy of the ejected electron.

(Nucleus preserves the momentum conservation.)



Compton scattering:
Elastic scattering of a γ-ray on a free electron. A fraction of the γ-ray energy is transferred to the Compton electron. The wave length of the scattered γ-ray is increased: λ‘ > λ.

Maximum energy of the scattered electron:

$$T(e^-)_{\max} = E_\gamma \cdot \frac{2 \cdot E_\gamma}{m_e c^2 + 2 \cdot E_\gamma}$$

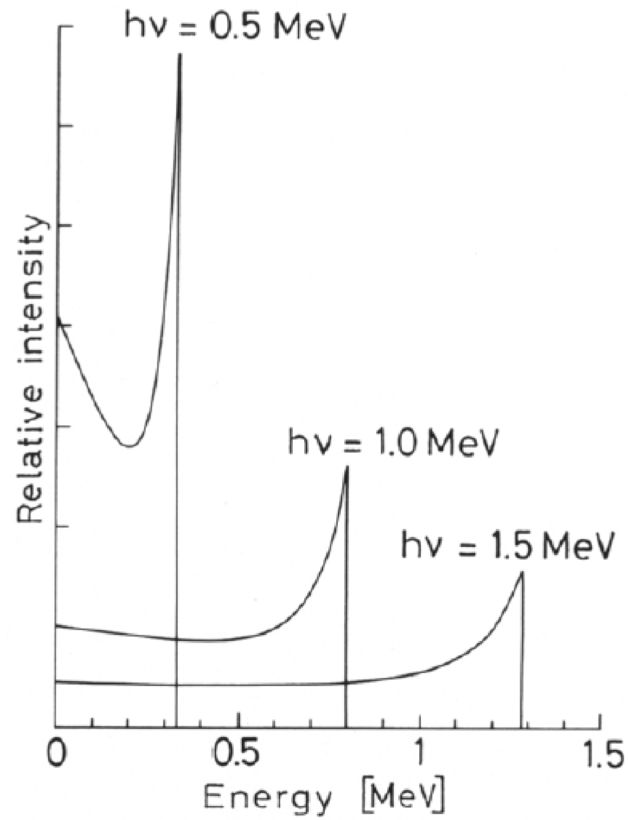
Energy of the scattered γ-photon:

$$E_\gamma' = \frac{E_\gamma \cdot m_e c^2}{m_e c^2 + E_\gamma \cdot (1 - \cos\theta)}$$

$$\cos\theta = 1 + \frac{m_e c^2}{E_\gamma} - \frac{m_e c^2}{E_\gamma'}$$

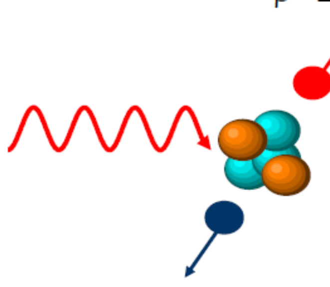
Special case for E >> $m_e c^2$: γ-ray energy after 180^0 scatter is approximately

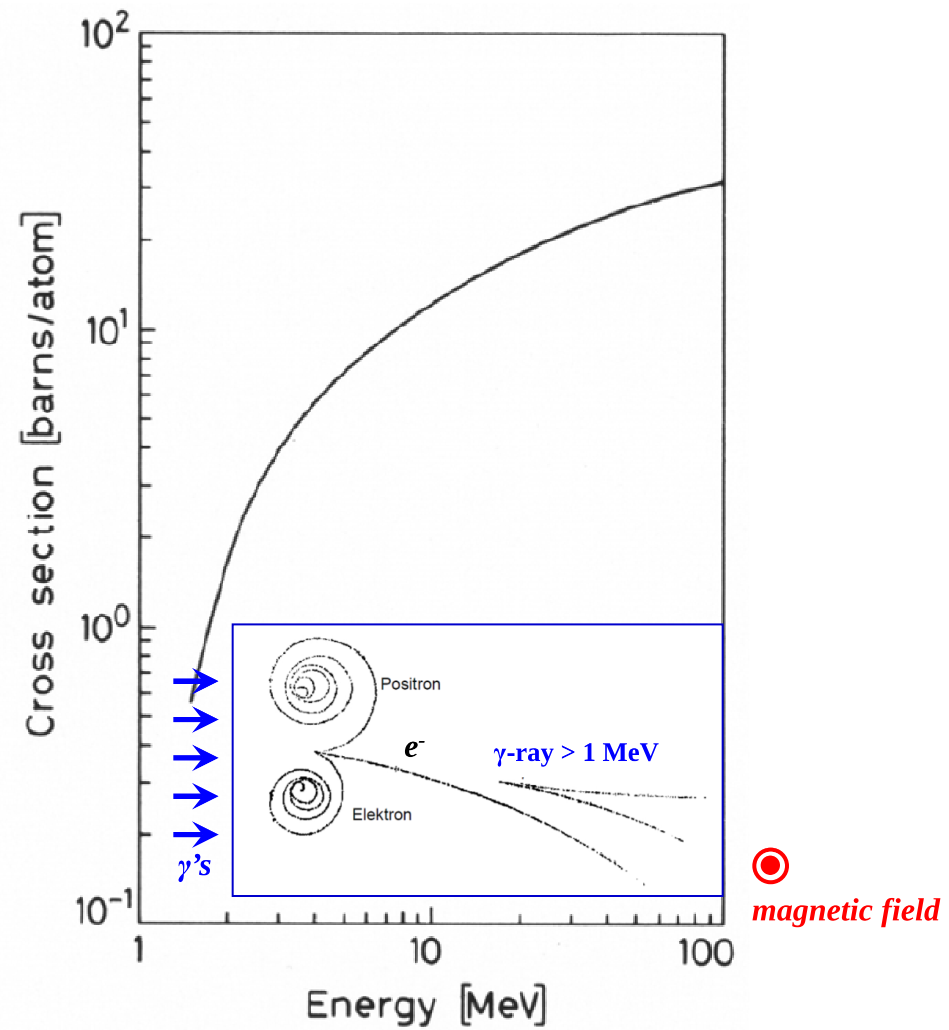
$$E_\gamma' = \frac{m_e c^2}{2} = 256\,keV$$



Gap between the incoming γ-ray and the maximum electron energy.

$$E_{kin}^{\max} = E_\gamma - E_\gamma' = E_\gamma \cdot \frac{2 \cdot E_\gamma / m_e c^2}{1 + 2 \cdot E_\gamma / m_e c^2}$$





Pair production:

If γ-ray energy is >> $2m_0c^2$ (electron rest mass 511 keV), a positron-electron pair can be formed in the strong Coulomb field of a nucleus. This pair carries the γ-ray energy minus $2m_0c^2$.

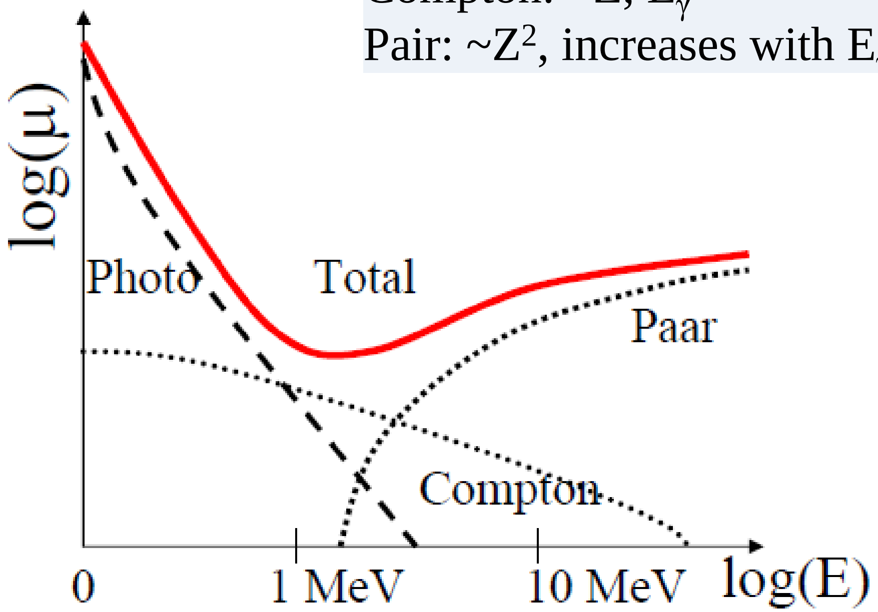
Pair production for $E_\gamma > 2m_ec^2 = 1.022$MeV

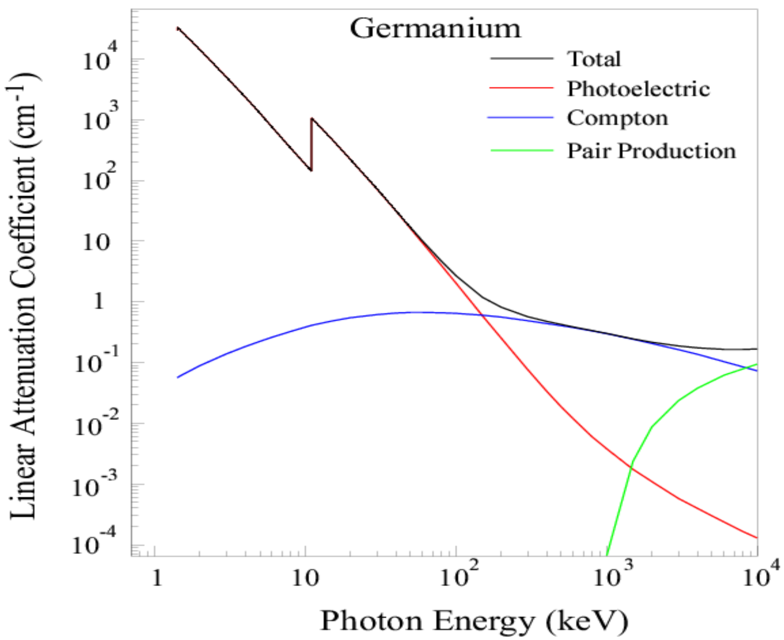
picture of a bubble chamber

Gamma-ray interaction cross section

All three interaction (photo effect, Compton scattering and pair production) lead to an attenuation of the γ-ray or X-ray radiation when passing through matter. The particular contribution depends on the γ-ray energy:

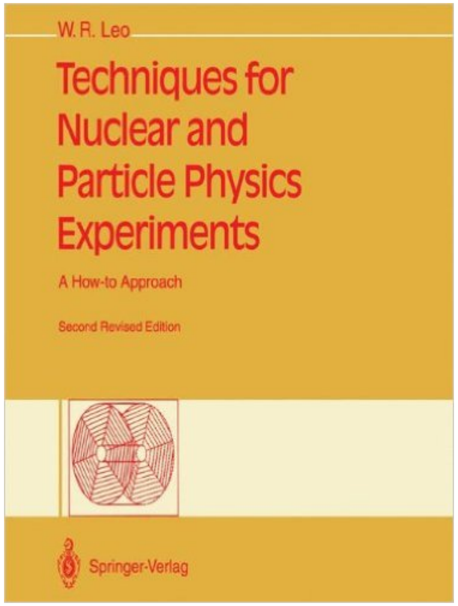
Photo effect: $\sim Z^{4\text{-}5}$, $E_\gamma^{-3.5}$
Compton: $\sim Z$, E_γ^{-1}
Pair: $\sim Z^2$, increases with E_γ



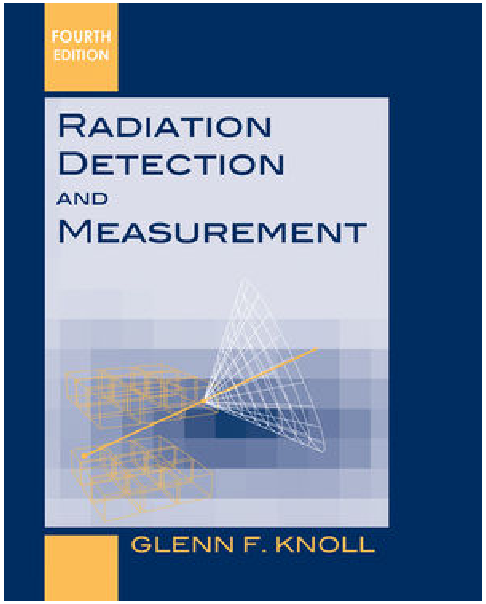


The absorption attenuates the intensity, but the energy and the frequency of the γ-ray and X-ray radiation is preserved!

Literature



- Recommended Textbook



- Recommended Textbook

Some Nuclear Units

Nuclear energies are very high compared to atomic processes, and need larger units. The most commonly used unit is the MeV.

1 electron Volt = 1 eV = $1.6\cdot10^{-19}$ Joules

1 MeV = 10^{6} eV; 1 GeV = 10^{9} eV; 1 TeV = 10^{12} eV

However, the nuclear size are quite small and need smaller units:

Atomic sizes are on the order of 0.1 nm = 1 Angstrom = 10^{-10} m. Nuclear sizes are on the order of femtometers which in the nuclear context are usually called fermis:

1 fermi = 1 fm = 10^{-15} m

Atomic masses are measured in terms of atomic mass units with the carbon-12 atom defined as having a mass of exactly 12 amu. It is also common practice to quote the rest mass energy $E=m_0c^2$ as if it were the mass. The conversion to amu is:

1 u = $1.66054\cdot10^{-27}$ kg = 931.494 MeV/c^2

electron mass = 0.511 MeV/c^2; proton mass = 938.27 MeV/c^2; neutron mass = 939.56 MeV/c^2

NNDC National Nuclear Data Center

Mass data: www.nndc.bnl.gov/qcalc/

Some Nuclear Units

Quantity	HEP units	SI Units
length	1 fm	10^{-15} m
energy	1 GeV	$1.602 \cdot 10^{-10}$ J
mass	1 GeV/c^2	$1.78 \cdot 10^{-27}$ kg
$\hbar = h/2$	$6.588 \cdot 10^{-25}$ GeV s	$1.055 \cdot 10^{-34}$ Js
c	$2.988 \cdot 10^{23}$ fm/s	$2.988 \cdot 10^{8}$ m/s
$\hbar c$	0.1973 GeV fm	$3.162 \cdot 10^{-26}$ Jm

Natural units ($\hbar = c = 1$)	
mass	1 GeV
length	$1\ \mathrm{GeV}^{-1} = 0.1973$ fm
time	$1\ \mathrm{GeV}^{-1} = 6.59 \cdot 10^{-25}$ s

Relevant Formulae

The relevant formulae are calculated if A_1, Z_1 and A_2, Z_2 are the mass number (amu) and charge number of the projectile and target nucleus, respectively, and T_{lab} is the laboratory energy (MeV)

$$E = T_{lab} + m_0 \cdot c^2$$

$$m \cdot c^2 = T_{lab} + m_0 \cdot c^2$$

$$\frac{m_0 \cdot c^2}{\sqrt{1-\beta^2}} = T_{lab} + m_0 \cdot c^2$$

beam velocity:

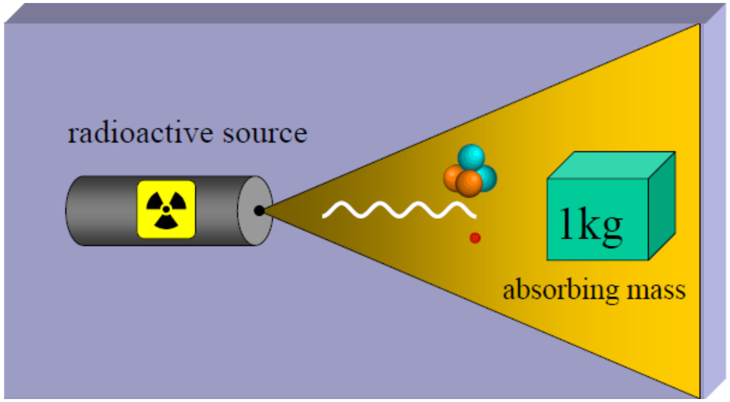
$$\beta = \frac{\sqrt{{T_{lab}}^2 + 1863 \cdot A_1 \cdot T_{lab}}}{931.5 \cdot A_1 + T_{lab}}$$

Lorentz contraction factor:

$$\gamma = (1-\beta^2)^{-1/2}$$

$$\gamma = \frac{931.5 \cdot A_1 + T_{lab}}{931.5 \cdot A_1}$$

$$\beta \cdot \gamma = \frac{\sqrt{{T_{lab}}^2 + 1863 \cdot A_1 \cdot T_{lab}}}{931.5 \cdot A_1}$$



$$absorbed\ \ dose = \frac{absorbed\ \ energy}{mass}$$

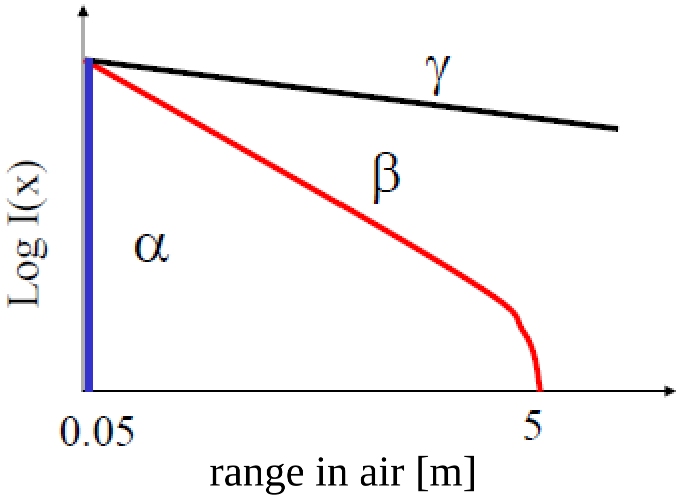
$$1\ \frac{Joule}{kg} = 1\ Gray\ (Gy)$$

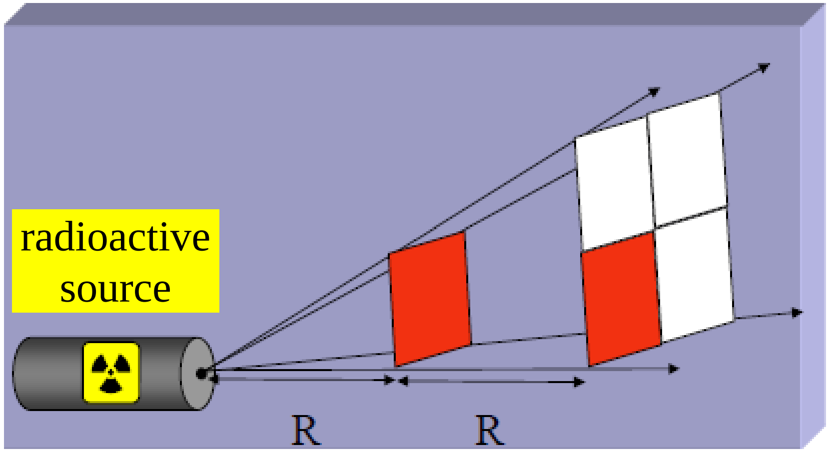
The biological dose, sometimes also known as the dose equivalent is expressed in units of Sieverts [Sv]. This dose reflects the fact that the biological damage caused by a particle depends not only on the total energy deposited but also on the rate of energy loss per unit distance traversed by the particle.

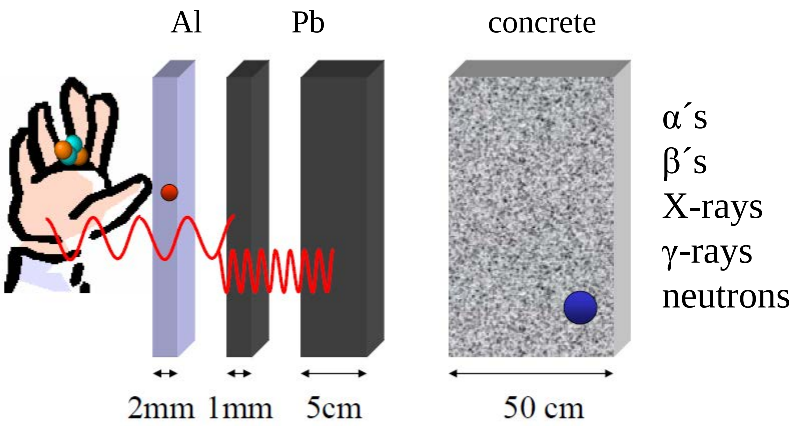
equivalent dose = energy dose · quality factor Q

radiation	quality factor Q
X-ray, γ, β	1
thermal neutrons	2.3
fast neutrons	10
α-particles, heavy ions	20

Radiation protection

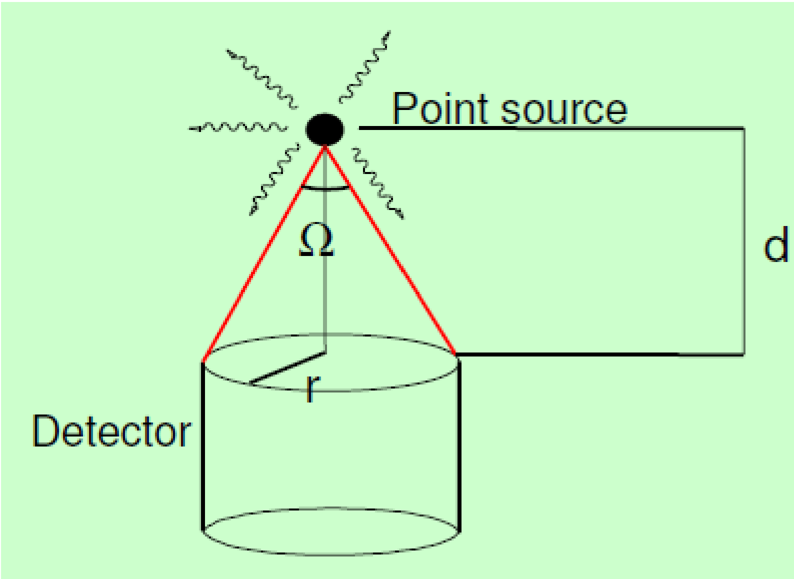




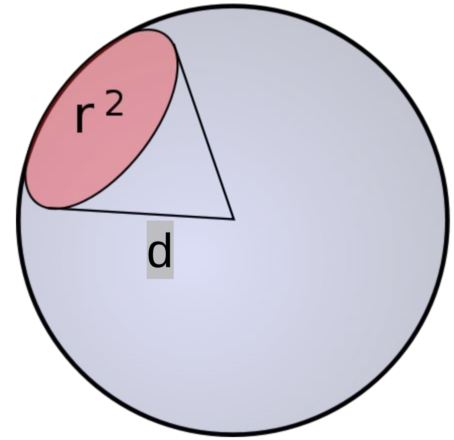


$$\Omega = \frac{a^2}{4\pi \cdot R^2} = \frac{\pi \cdot r^2}{4\pi \cdot R^2}$$

Solid angle



1 Ci = $3.7 \cdot 10^{10}$ Bq



$$\frac{\Omega}{4\pi} \cong \frac{\pi r^2}{4\pi d^2} = \left(\frac{r}{2d}\right)^2$$

Ω = solid angle between source and detector (sr)

For a point source:

$$\frac{\Omega}{4\pi} = \frac{1}{2} \cdot \left(1 - \frac{d}{\sqrt{d^2 + r^2}}\right)$$

d (cm) r = 3cm	$\Omega/4\pi$ [%]	$\Omega/4\pi$ [%]
5	7.13	55
10	2.11	2.25
15	0.97	1