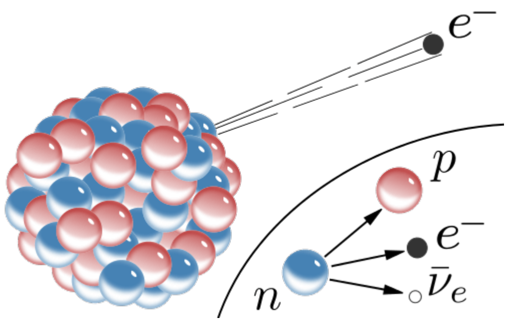
PHL424: β-decay

Beta Decay: universal term for all weak-interaction transitions between two neighboring isobars

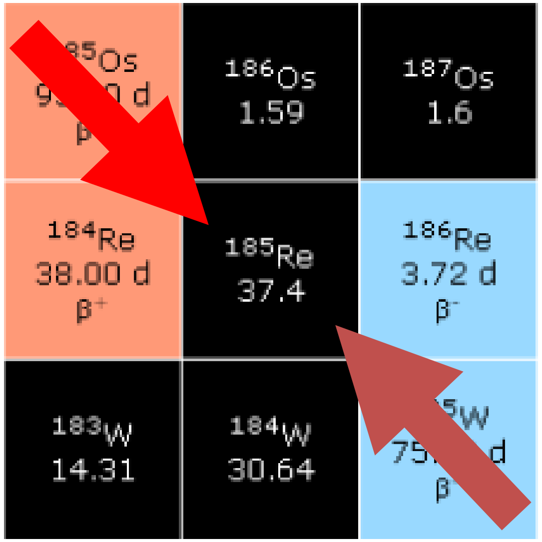


three different forms

β^-, β^+ & EC (capture of an atomic electron)

β⁺: $p \rightarrow n + e^+ + \nu$

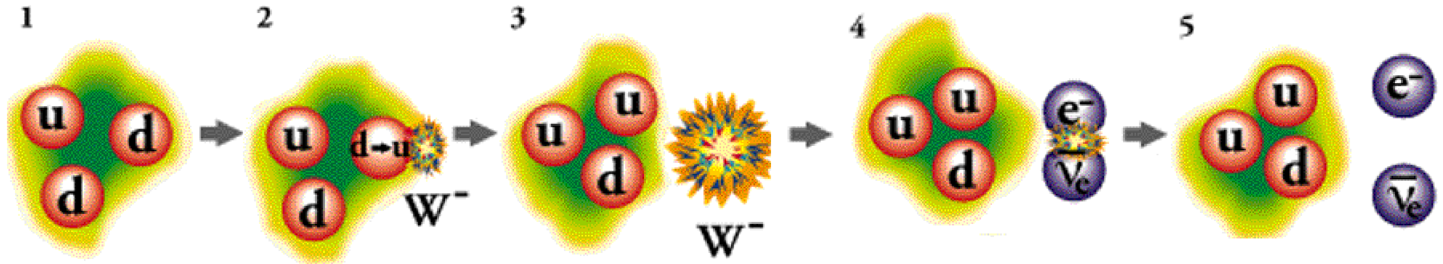
EC: $p + e^- \rightarrow n + \nu$



β⁻: $n \rightarrow p + e^- + \bar{\nu}$

a nucleon inside the nucleus is transformed into another

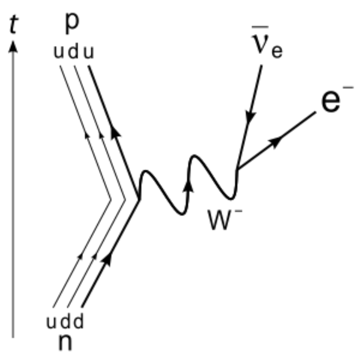
β-decay of a free neutron



neutron (udd, q=0)

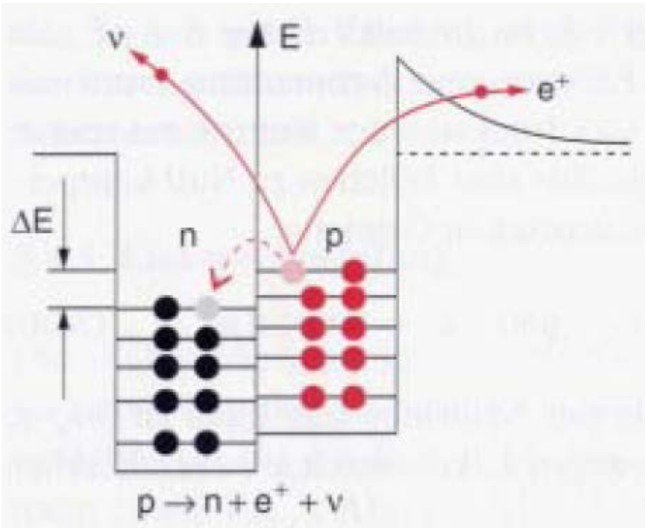
d-quark (q= -1/3) → u-quark (q=2/3) + virtual W⁻ boson (q= -1)

proton (uud) and W⁻ boson separate and an electron (q=-1) and an anti-neutrino is created out of the virtual W⁻ boson.



$m_p = 1.673 \cdot 10^{-27}$ kg
938.272MeV/c^2

$m_n = 1.675 \cdot 10^{-27}$ kg
939.565MeV/c^2



The mean lifetime τ of free neutrons is 885.8(7) [s]

$$n \rightarrow p + e^{-} + \bar{\nu}_e + 0.782\ MeV$$

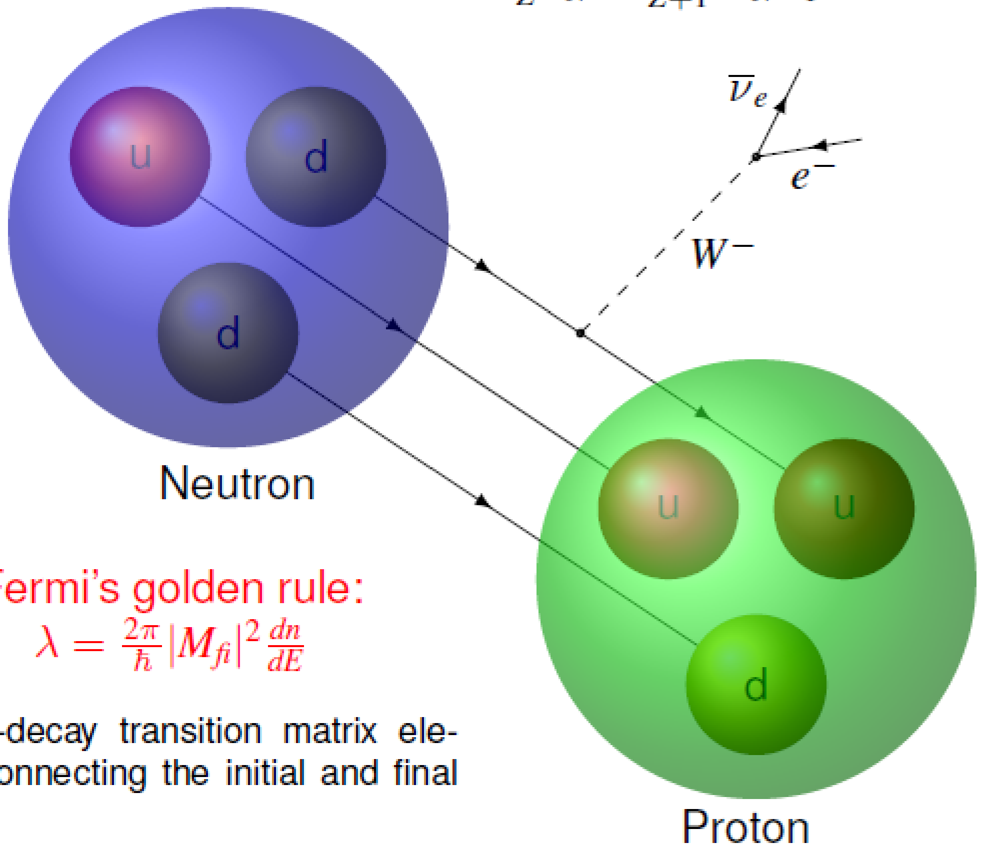
The β^{+}-decay in the nucleus is only possible because the neutron mass is greater than the proton mass.

β-decay of a neutron

β^- Decay

For β^- decay:

$${}^{A}_{Z}\mathrm{X}_N \rightarrow {}^{A}_{Z+1}\mathrm{Y}_{N-1} + e^- + \overline{\nu}_e$$



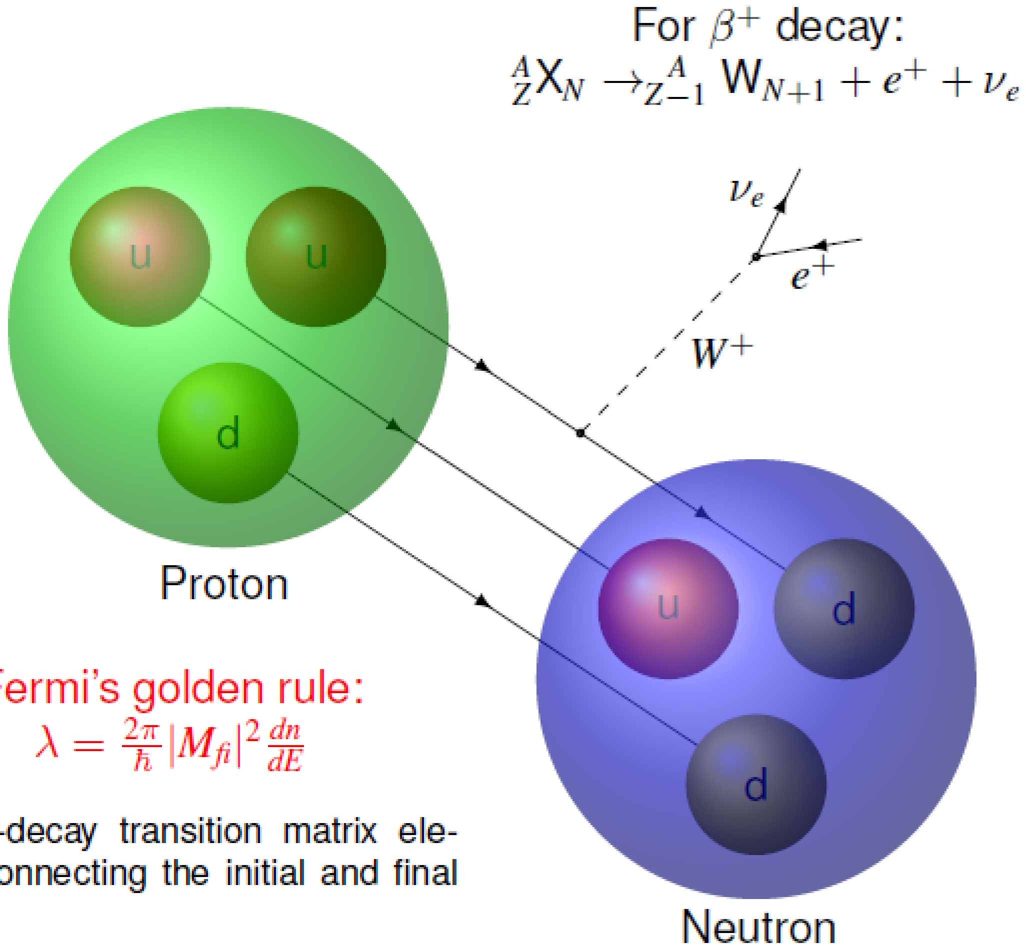
Fermi's golden rule:

$$\lambda = \frac{2\pi}{\hbar} |M_{fi}|^2 \frac{dn}{dE}$$

M_{fi}: β-decay transition matrix element connecting the initial and final states

β-decay of a proton

β^+ Decay



Fermi's golden rule:

$$\lambda = \frac{2\pi}{\hbar}|M_{fi}|^2\frac{dn}{dE}$$

M_{fi}: β-decay transition matrix element connecting the initial and final states

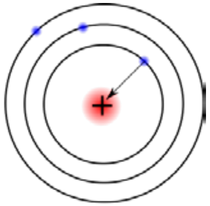
$^{A}Z \rightarrow A(Z\pm1)$ β-decay
$^{A}Z \rightarrow {}^{A}(Z\pm2)$ double β-decay

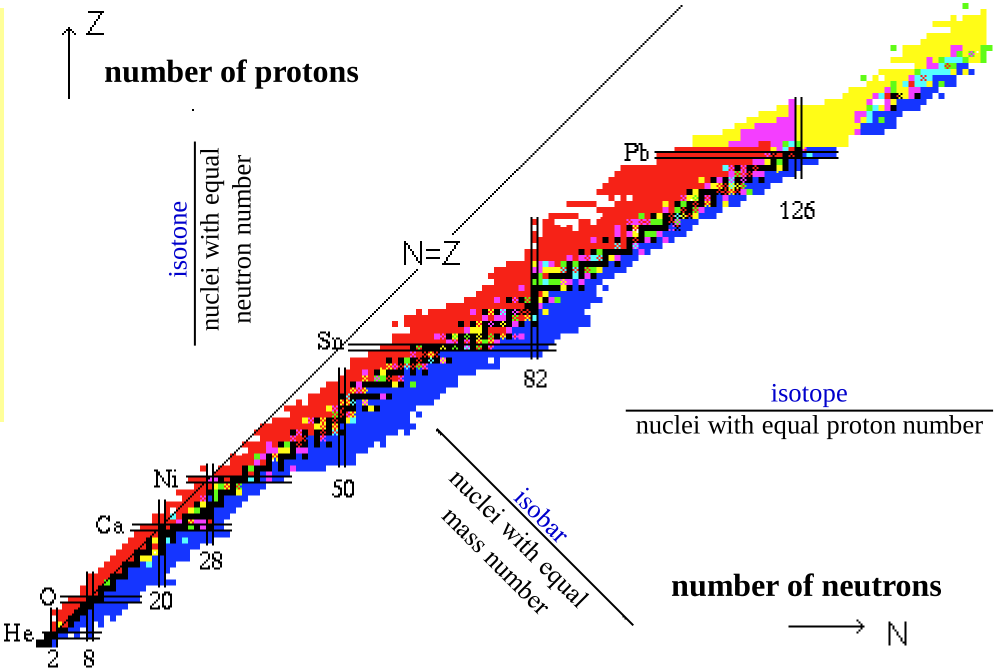
β^{-}-decay $n \rightarrow p + e^{-} + \bar{\nu}_e$ if $Q_\beta > 0$

β^{+}-decay $p \rightarrow n + e^{+} + \nu_e$ if $Q_\beta > 1.02$ MeV

EC $p + e^{-} \rightarrow n + \nu_e$ if $Q_\beta > 0$

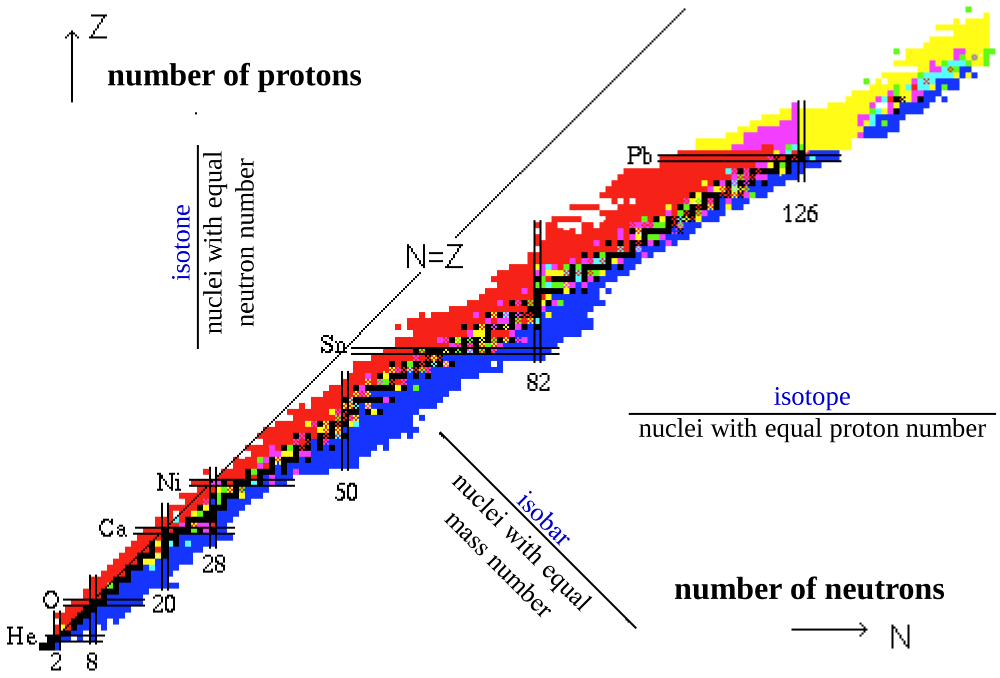
Electron capture: proton-rich nucleus absorbs an inner electron.

EC 



Motivation:
For the description of the nuclear synthesis in astrophysical environment one needs excellent knowledge about the β-decay properties in unstable nuclei far away from the valley of stability.

➢ For isotopes with substantial neutron excess it is energetically favorable, if a neutron changes into a proton.
➢ For neutron deficient nuclei one has the opposite process: a proton changes into a neutron.



β⁻-decay

$$n \to p + e^- + \bar{\nu}_e$$
$$d \to u + e^- + \bar{\nu}_e$$

β⁺-decay

$$p \to n + e^+ + \nu_e$$
$$u \to d + e^+ + \nu_e$$

Necessary conditions for the different β-decays:

$m(A,Z) > m(A,Z+1)$ β⁻-decay (the created electron is already considered)

$m(A,Z) > m(A,Z-1) + 2 \cdot m_e \cdot c^2$ β⁺-decay (the mother nucleus has one electron more and a positron is created)

$m(A,Z) > m(A,Z-1) + \epsilon$ electron capture EC (ε is the excitation energy of the hole in the daughter nucleus, EC can transform protons into neutrons at a lower energy)

Liquid drop mass formula

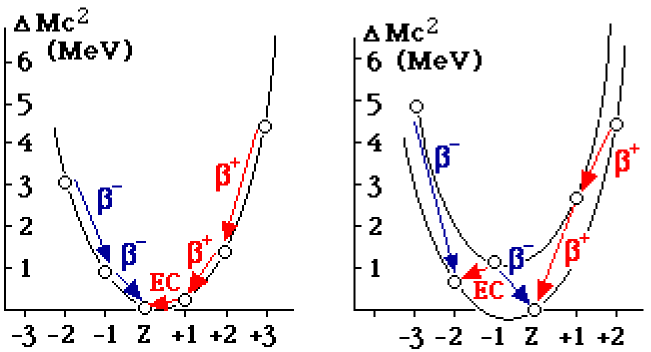
$$m(A,Z) = Z \cdot m_H + (A-Z) \cdot m_n - a_V \cdot A + a_S \cdot A^{2/3} + a_C \cdot \frac{Z \cdot (Z+1)}{A^{1/3}} + a_{asym} \cdot \frac{(Z-A/2)^2}{A} \pm \delta$$

1935 Carl von Weizsäcker

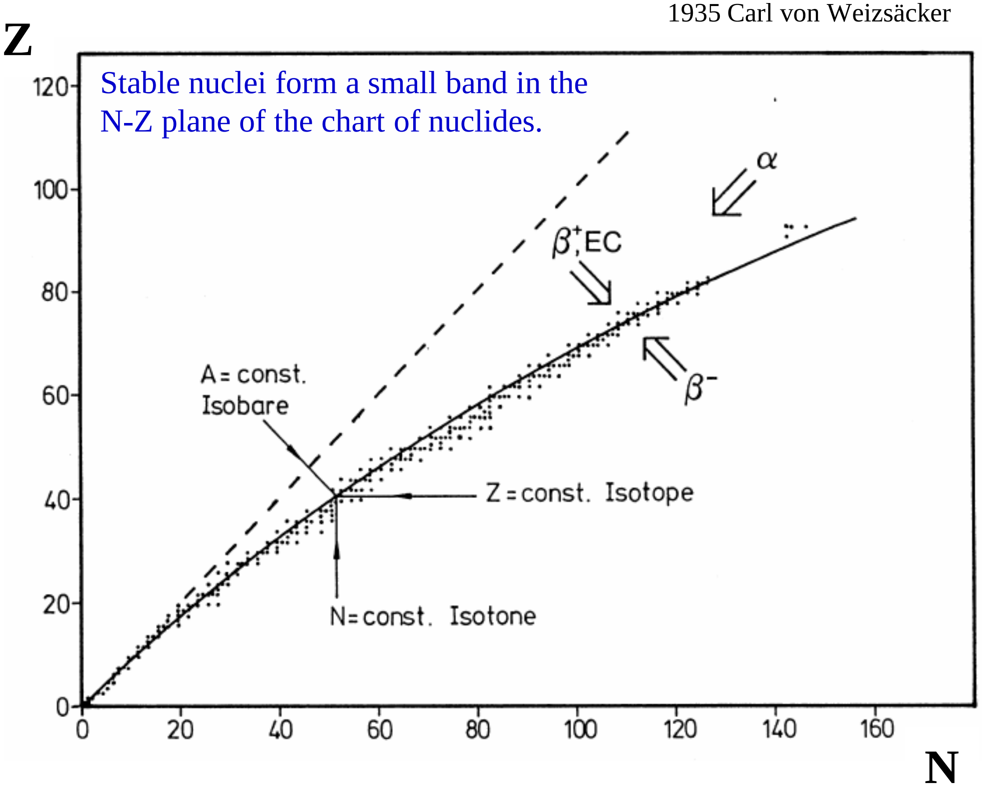
From the mass formula one obtains

$$\boxed{m(A,Z) = \gamma \cdot Z^2 - \beta \cdot Z + \alpha \cdot A \pm \delta}$$

For isobar chains (nuclei with constant mass) m(A,Z) is a quadratic function of Z.

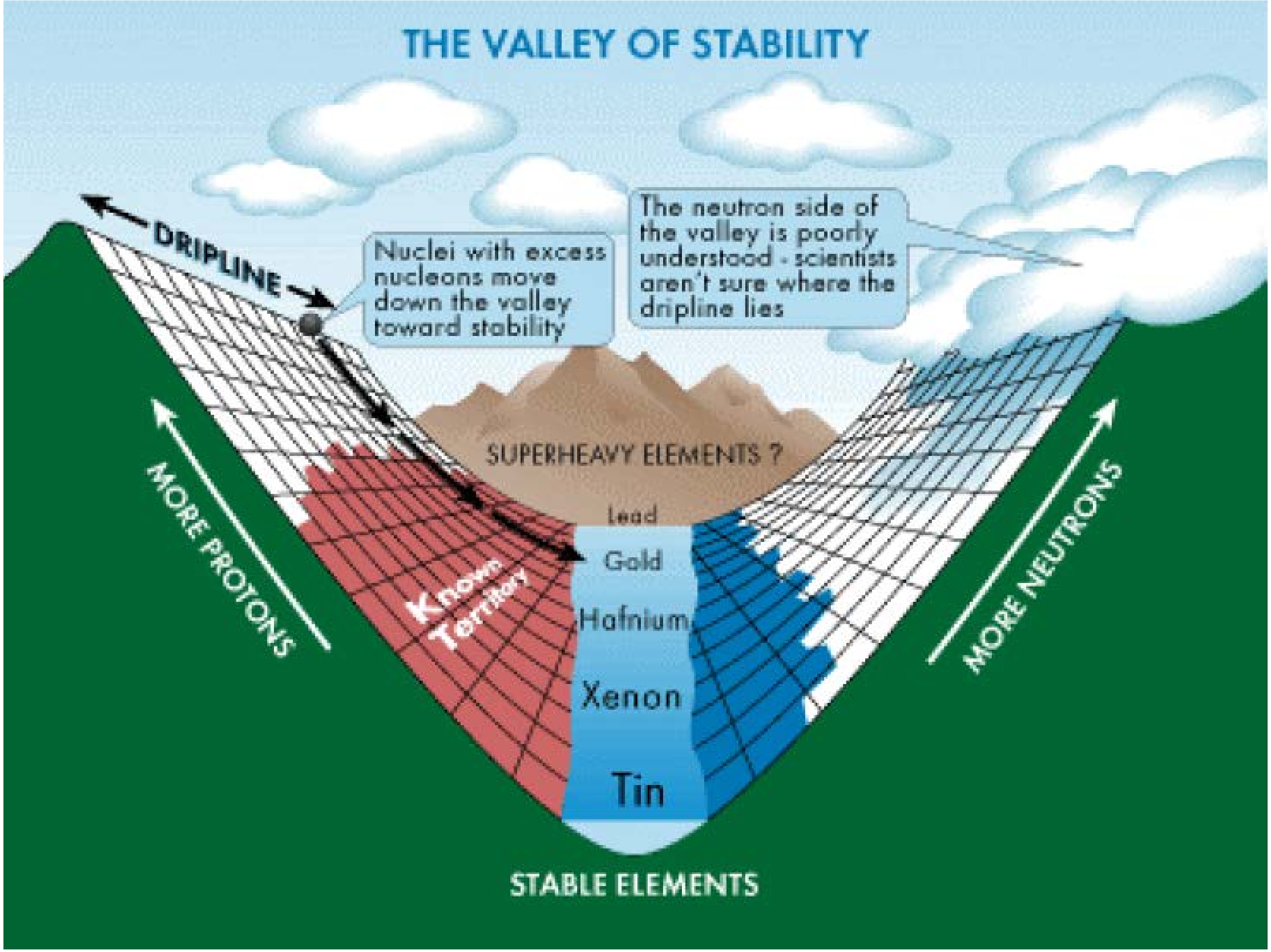


Odd A   Even A



$$\alpha = \left(m_n - a_V + \frac{a_S}{A^{1/3}} + \frac{a_{asym}}{4}\right) \qquad \beta = \left[a_{asym} + m_n - m(^1H)\right] \qquad \gamma = \left(\frac{a_C}{A^{1/3}} + \frac{a_{asym}}{A}\right)$$

Liquid drop mass parabola



Liquid drop mass parabola for odd-A isobars

In β⁻ decay a neutron changes into a proton. $n \rightarrow p + e^- + \bar{\nu}_e$

the mass balance for β⁻ decay is given by: $Q_\beta = \{[m(A,Z) - Z \cdot m_e] - [m(A,Z+1) - (Z+1) \cdot m_e + m_e]\} \cdot c^2$

$$= \{m_N(A,Z) - m_N(A,Z+1)\} \cdot c^2$$

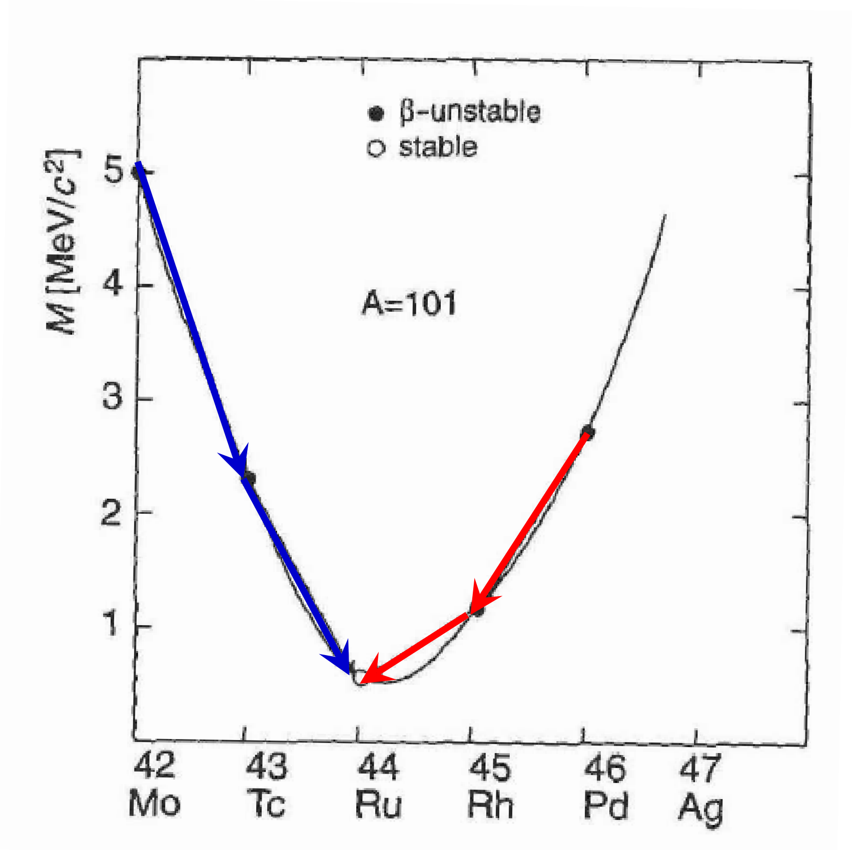
The rest mass of an anti-neutrino < 7 eV/c² can be neglected.

β⁻ decay is possible under the following condition:

$$m(A,Z) > m(A,Z+1)$$

example: mass parabola for A=101 isobars

$${}^{101}_{42}Mo \rightarrow {}^{101}_{43}Tc + e^- + \bar{\nu}_e$$
$${}^{101}_{43}Tc \rightarrow {}^{101}_{44}Ru + e^- + \bar{\nu}_e$$
$${}^{101}_{44}Ru \; is \; stable$$



What are the decays of ^{101}Pd und ^{101}Rh?

β^+- decay

In neutron deficient nuclei a proton changes into a neutron:

$$p \rightarrow n + e^+ + \nu_e$$

a positron e^+, the anti-particle of an electron (positive charge, equal mass) and an electron-neutrino ν_e will be emitted.

The mass balance of the β^+ decay is given by:

$$Q_\beta = \{[m(A,Z) - Z \cdot m_e] - [m(A,Z-1) - (Z-1) \cdot m_e + m_e]\} \cdot c^2$$

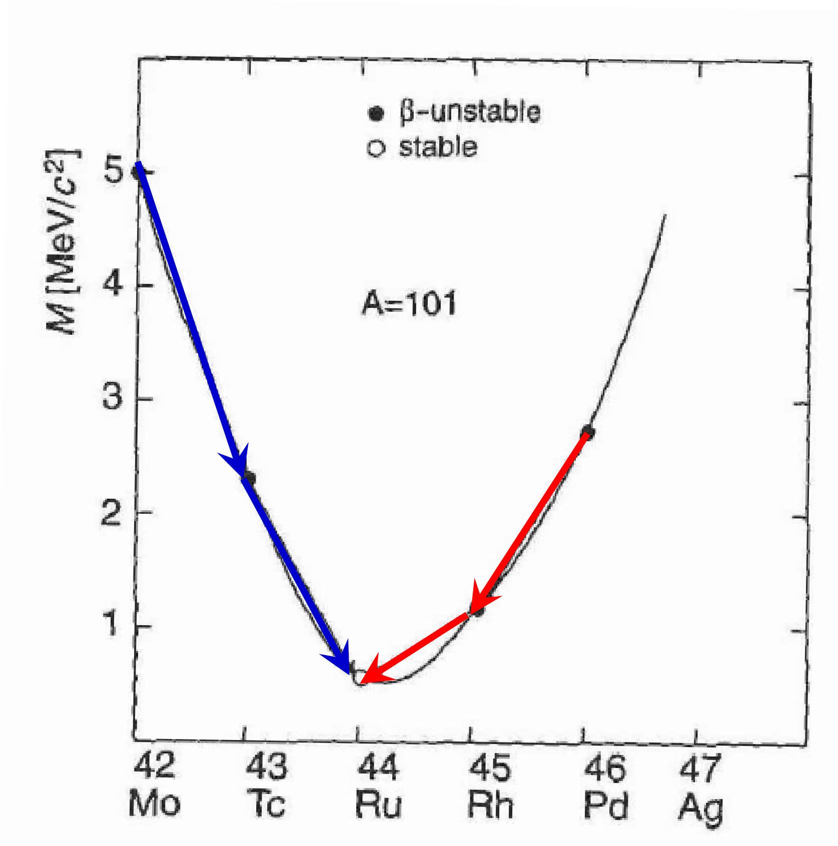
$$= \{m_N(A,Z) - m_N(A,Z-1) - 2 \cdot m_e\} \cdot c^2$$

Condition for β^+ decay: $m(Z,A) > m(Z-1,A) + 2 \cdot m_e$

The term $2 \cdot m_e$ considers that a positron is created and an an electron is left over from the mother nucleus.

The following β^+ decays are observed for A=101:

$$^{101}_{46}Pd \rightarrow {}^{101}_{45}Rh + e^+ + \nu_e$$
$$^{101}_{45}Rh \rightarrow {}^{101}_{44}Ru + e^+ + \nu_e$$
$$^{101}_{44}Ru \textit{ is stable}$$



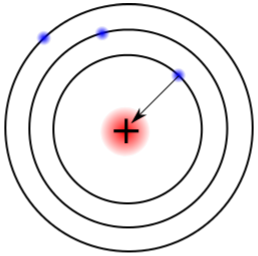
β^+- decay versus electron capture

In neutron deficient nuclei a proton can be converted into a neutron:

$$p \rightarrow n + e^+ + \nu_e$$

A positron e^+, the anti-particle of an electron (positive charge, equal mass) and an electron-neutrino ν_e will be emitted.

- Condition for a β^+ decay: $m(Z,A) > m(Z\text{-}1,A) + 2 \cdot m_e$



$m_e = 0.00055$ [u]

Electron capture EC

In competition to the β^+ decay is the electron capture process which is energetically more favorable to change protons into neutrons.

$$p + e^- \rightarrow n + \nu_e$$

K-electrons have a high probability inside of the nucleus and can easily captured.

Condition for K-electron capture: $m(Z,A) > m(Z\text{-}1,A) + \varepsilon$

$\varepsilon \sim 10^{-8}$ [u] is the excitation energy of the hole in the daughter nucleus.

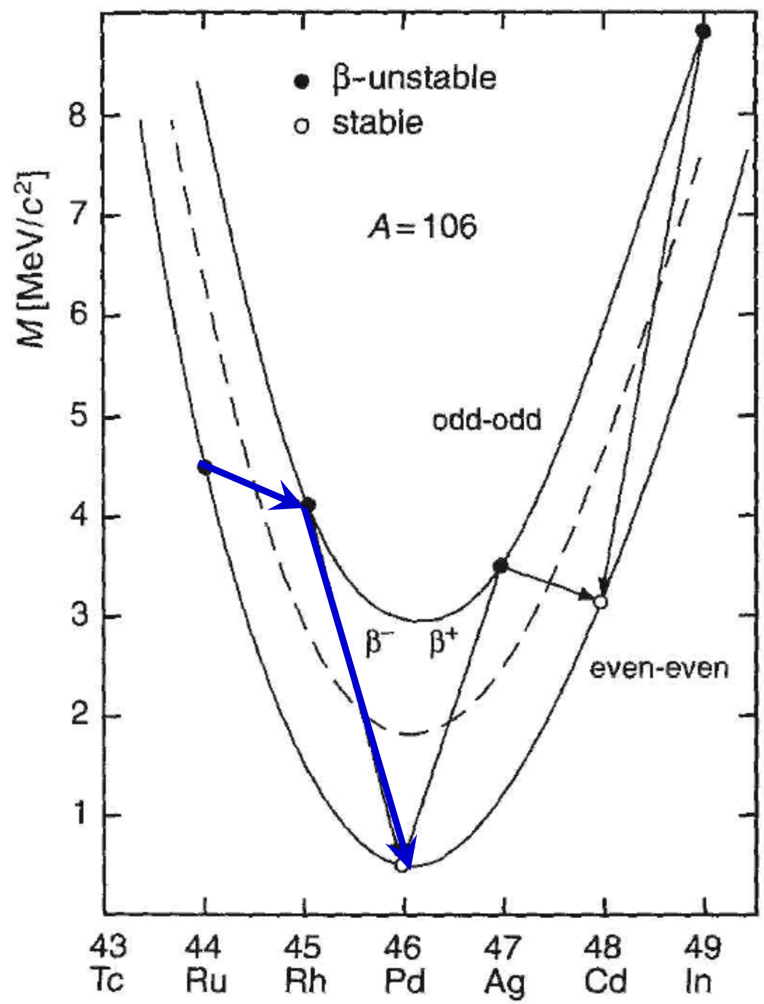
The Auger-effect is an alternative process to X-ray emission when a hole is filled in a stronger bound electron shell.

Liquid drop mass parabola for even-A isobars

In isobars with even mass number two separate parabola exist because of the pairing energy: one for even-even nuclei and one higher lying for odd-odd nuclei. The difference is 2δ, twice the pairing energy.

Consequence:

- All odd-odd nuclei have at least one stronger bound even-even nucleus (isobar) and are hence instable.
- Exceptions: ^{2}H, ^{6}Li, ^{10}B, ^{14}N because of the higher asymmetry energy.



Example: mass parabola der A=106 isobars

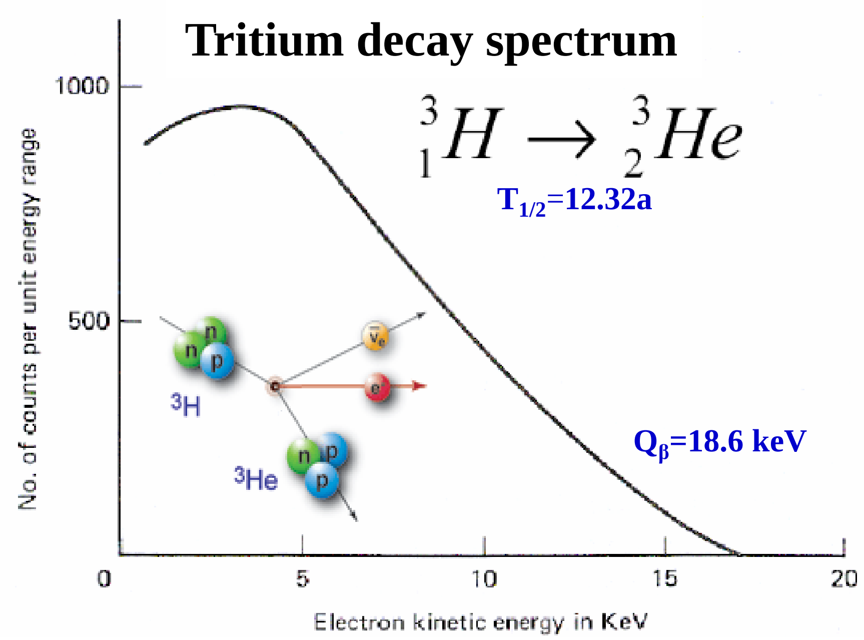
$$^{106}_{44}Ru \rightarrow {}^{106}_{45}Rh + e^- + \overline{\nu}_e$$
$$^{106}_{45}Rh \rightarrow {}^{106}_{46}Pd + e^- + \overline{\nu}_e$$
$$^{106}_{46}Pd \text{ is stable}$$

In the β^- decay an electron and an electron anti-neutrino will be emitted simultaneously.

$$n \rightarrow p + \boxed{e^- + \bar{\nu}_e}$$

The β^- decay energy is given by the mass difference between mother and daughter nucleus.

This energy will be distributed as kinetic energy on the emitting particles, the electron and the anti-neutrino.
Hence, the electron spectrum is continuous.
It starts at zero energy and ends at the maximum possible energy $E_{max} = E_0 - m_\nu \cdot c^2$ (= Q_β).



β decays have a long lifetime and a small decay probability, the related interaction is small compared to other interactions in the nucleus, therefore time dependent perturbation theory is a good approximation.

Fermi´s golden rule

Fermi´s golden rule:

$$W_{if} = \frac{2\pi}{\hbar} \cdot \left|\langle \Psi_f | H_{int} | \Psi_i \rangle\right|^2 \cdot \rho(E_f)$$

it depends on the transition matrix element and on the level density of final states (phase space).

The decay rate with the emission of an electron in the energy range between E_e and $E_e + dE_e$ and an anti-neutrino in the energy range between $E_{\bar{\nu}}$ and $E_{\bar{\nu}} + dE_{\bar{\nu}}$ is given by

$$W_{if}(E_e, E_{\bar{\nu}})dE_e dE_{\bar{\nu}} = \frac{2\pi}{\hbar}\left|\langle \Psi_f | H_{int} | \Psi_i \rangle\right|^2 \rho_f \cdot \delta(E_0 - E_R - E_e - E_{\bar{\nu}})dE_e dE_{\bar{\nu}}$$

E_0 decay energy
E_R recoil energy of the nucleus, which is neglected since it is small due to its large mass.
ρ_f level density of the final states
$\langle\Psi_f|H_{int}|\Psi_i\rangle$ matrix element of the weak interaction

The state of a particle is determined by its position and its momentum

$$\Delta x \cdot \Delta y \cdot \Delta z \cdot \Delta p_x \cdot \Delta p_y \cdot \Delta p_z \approx h^3$$

Each particle has a volume of h^3 in phase space. The number of states in momentum shell $\Delta p = 4\pi \cdot p^2 dp$ is given by

$$dn = \frac{V \cdot 4\pi \cdot p^2}{h^3} dp$$

level density of free particles

$\Delta x \cdot \Delta y \cdot \Delta z = V$

matrix element must be small compared to the energy intervals in the system (otherwise no perturbation theory)

Fermi´s golden rule

number of final states: $$\rho_f dN_e dN_{\bar{\nu}} = \frac{V^2 \cdot (4\pi)^2 \cdot p_e^2 dp_e \cdot p_{\bar{\nu}}^2 dp_{\bar{\nu}}}{(2\pi \cdot \hbar)^6}$$

For the absolute values of the momenta one uses relativistic energy and momentum relations:

$$E_e = \sqrt{p_e^2 \cdot c^2 + m_e^2 \cdot c^4} \qquad E_{\bar{\nu}} = p_{\bar{\nu}} \cdot c$$

$$p_e dp_e = E_e dE_e / c^2 \qquad dp_{\bar{\nu}} = dE_{\bar{\nu}} / c$$

number of states in the energy interval dE_e and $dE_{\bar{\nu}}$

$$\rho_f dN_e dN_{\bar{\nu}} = \frac{V^2 \cdot (4\pi)^2}{(2\pi \cdot \hbar)^6 \cdot c^6} E_e \sqrt{E_e^2 - m_e^2 \cdot c^4}\, E_{\bar{\nu}}^2\, dE_e dE_{\bar{\nu}}$$

Since the anti-neutrino is not measured, one obtains with $E_{\bar{\nu}} = E_0 - E_e$ after the integration over the anti-neutrino energy $E_{\bar{\nu}}$ the decay rate for the emission of an electron with an energy between E_e and $E_e + dE_e$.

$$W_{if}(E_e) dE_e = \frac{2\pi}{\hbar} \left| \langle \Psi_f | H_{int} | \Psi_i \rangle \right|^2 \cdot \frac{V^2 \cdot (4\pi)^2}{(2\pi \cdot \hbar)^6 \cdot c^6} \; E_e \sqrt{E_e^2 - m_e^2 \cdot c^4} (E_0 - E_e)^2 \; dE_e$$

The phase space factor, which yields from the level density, determines the essentially shape of the energy spectrum!

β-decay spectrum: matrix element

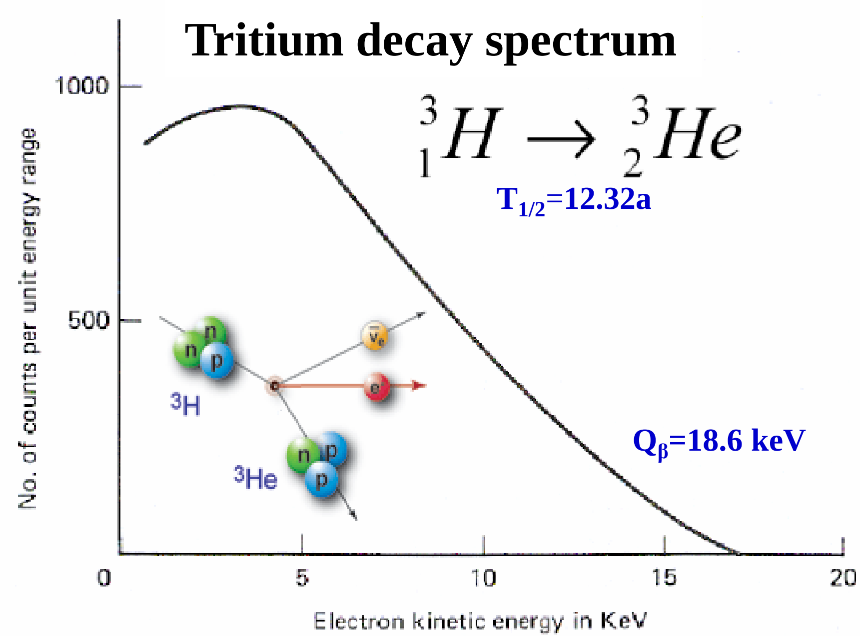
The phase space factor $E_e\sqrt{E_e^2 - m_e^2 \cdot c^4}(E_0 - E_e)^2$, which results from the level density, determines essentially the shape of the energy spectrum.

The matrix element is obtained by integrating over the position and spin variables for the electron, anti-neutrino and the nucleon in the nucleus.

Electron and anti-neutrino are described by plane waves (de Broglie wave length of electron $2 \cdot 10^{-13}$ m > $R_{nucleus}$). For the small nucleus one performs an expansion around r = 0.

$$\left|\langle\Psi_f\middle|H_{int}\middle|\Psi_i\rangle\right|^2 = |\Psi_e(0)|^2 \cdot |\Psi_{\bar{\nu}}(0)|^2 \cdot \left|M_{fi}\right|^2$$

M_{fi} is the nuclear matrix element that does not depend on the electron energy.



β-decay spectrum: Coulomb interaction

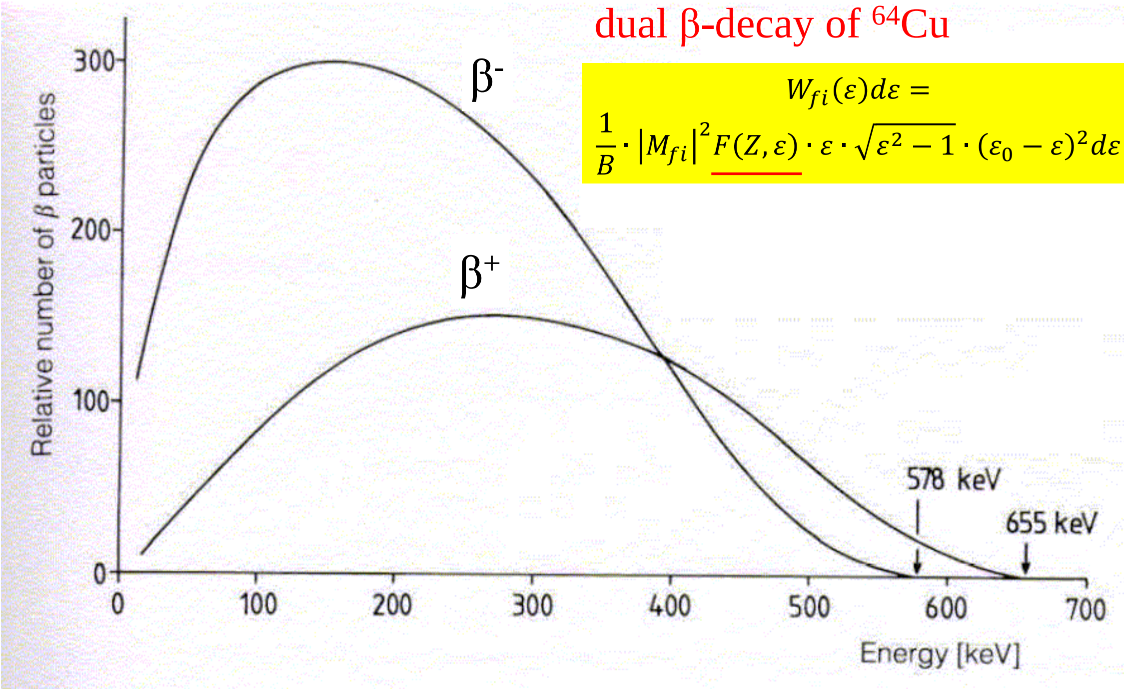
The electron feels the Coulomb interaction of the nucleus with the electrons of the atomic shell. The influence of the Coulomb interaction with the proton will be considered:

$$F(Z,E_e) = \frac{|\Psi_e(0)_{Coul}|^2}{|\Psi_e(0)_{free}|^2}$$

The function $F(Z,E_e)$ is tabulated.

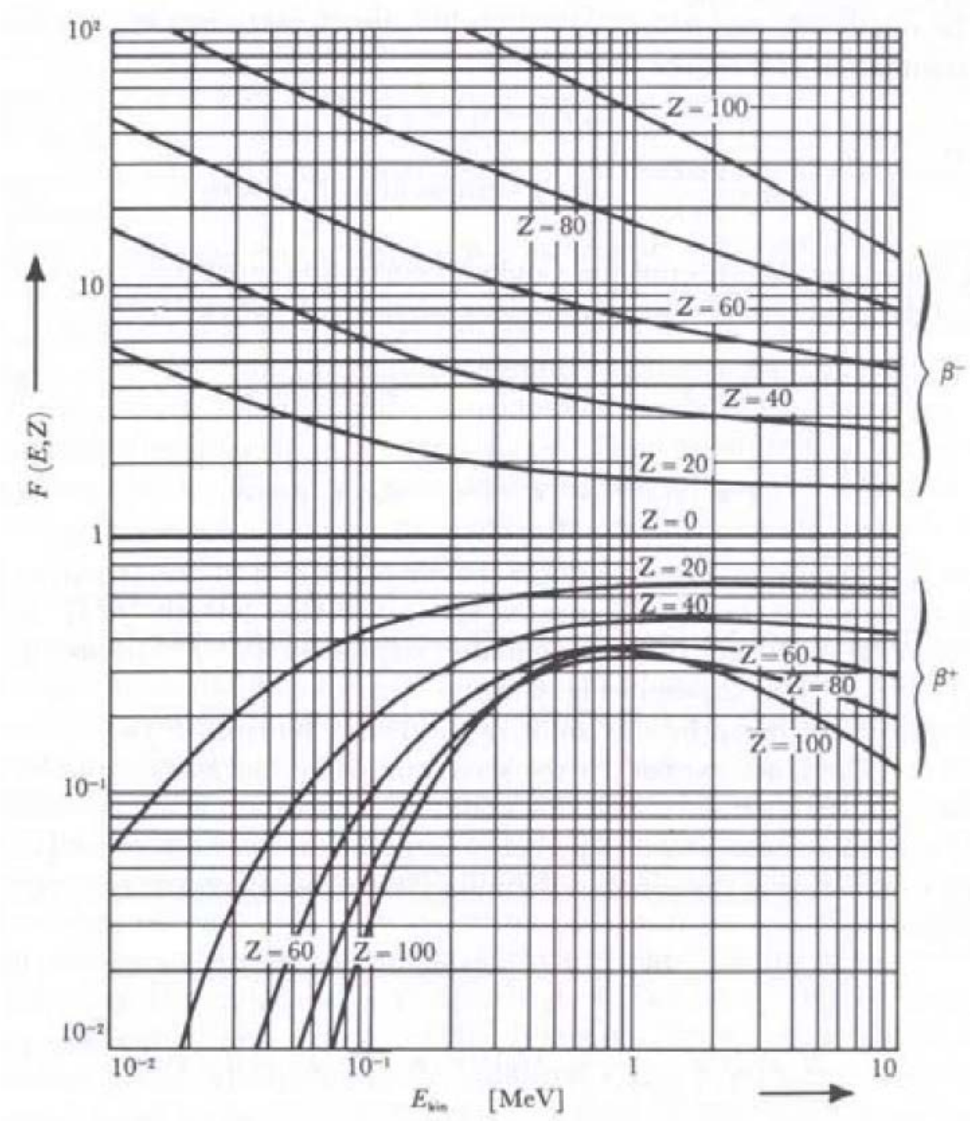
dual β-decay of ^{64}Cu

$$W_{fi}(\varepsilon)d\varepsilon = \frac{1}{B}\cdot|M_{fi}|^2 F(Z,\varepsilon)\cdot\varepsilon\cdot\sqrt{\varepsilon^2-1}\cdot(\varepsilon_0-\varepsilon)^2 d\varepsilon$$



β^+ vs. β^- spectrum under the influence of the Coulomb field.

β-decay spectrum: Fermi function



β-decay probability

For the electron spectrum one obtains:

$$W_{fi}(\varepsilon)d\varepsilon = \frac{1}{B}\cdot\left|M_{fi}\right|^2 F(Z,\varepsilon)\cdot\varepsilon\cdot\sqrt{\varepsilon^2-1}\cdot(\varepsilon_0-\varepsilon)^2 d\varepsilon$$

where B contains only natural constants and ε as well as ε_0 are in units of the electron mass $m_e c^2$

$$\varepsilon = E_e/m_e c^2 \qquad \varepsilon_0 = E_0/m_e c^2$$

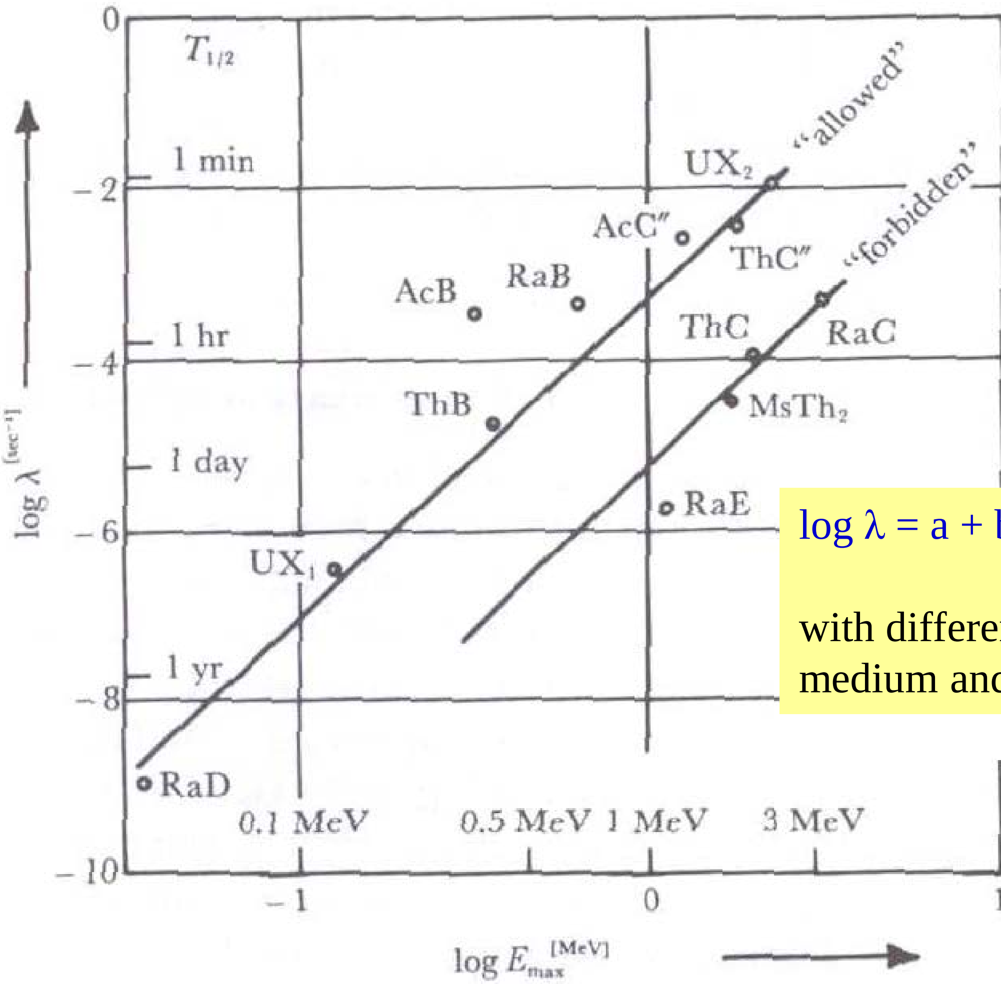
The decay probability is obtained by integrating over the electron energy

$$\lambda = \int_0^{E_0} W_{fi}(E_e)\,dE_e = \frac{1}{B}\cdot\left|M_{fi}\right|^2\cdot\int_0^{\varepsilon_0} F(Z,\varepsilon)\cdot\varepsilon\cdot\sqrt{\varepsilon^2-1}\cdot(\varepsilon_0-\varepsilon)^2\,d\varepsilon$$

with $f(Z,\varepsilon_0) = \int_0^{\varepsilon_0} F(Z,\varepsilon)\cdot\varepsilon\cdot\sqrt{\varepsilon^2-1}\cdot(\varepsilon_0-\varepsilon)^2 d\varepsilon$

$$\boxed{f(Z,\varepsilon_0) \propto \varepsilon_0^5}$$

Sargent diagram



$\log \lambda = a + b \cdot \log E_{\beta,max}$

with different a, b for light, medium and heavy nuclei

The higher the energy of the fastest electron, the larger the decay constant

β-decay probability

For the electron spectrum one obtains:

$$W_{fi}(\varepsilon)d\varepsilon = \frac{1}{B} \cdot \left|M_{fi}\right|^2 F(Z,\varepsilon) \cdot \varepsilon \cdot \sqrt{\varepsilon^2 - 1} \cdot (\varepsilon_0 - \varepsilon)^2 d\varepsilon$$

where B contains only natural constants and ε as well as ε_0 are in units of the electron mass $m_e c^2$

$$\varepsilon = E_e / m_e c^2 \qquad \varepsilon_0 = E_0 / m_e c^2$$

The decay probability is obtained by integrating over the electron energy

$$\lambda = \int_0^{E_0} W_{fi}(E_e)\, dE_e = \frac{1}{B} \cdot \left|M_{fi}\right|^2 \cdot \int_0^{\varepsilon_0} F(Z,\varepsilon) \cdot \varepsilon \cdot \sqrt{\varepsilon^2 - 1} \cdot (\varepsilon_0 - \varepsilon)^2\, d\varepsilon$$

with $f(Z,\varepsilon_0) = \int_0^{\varepsilon_0} F(Z,\varepsilon) \cdot \varepsilon \cdot \sqrt{\varepsilon^2 - 1} \cdot (\varepsilon_0 - \varepsilon)^2 d\varepsilon$

$$\lambda = \frac{ln2}{T_{1/2}} = \frac{\left|M_{fi}\right|^2}{B} \cdot f(Z,\varepsilon_0) \qquad or \qquad f(Z,\varepsilon_0) \cdot T_{1/2} = \frac{B \cdot ln2}{\left|M_{fi}\right|^2}$$

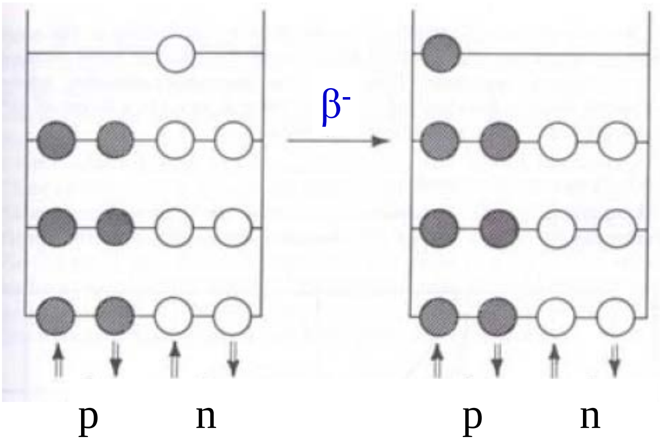
Typically one obtains very large numbers, therefore log ft-values.

β-decay: log ft - values

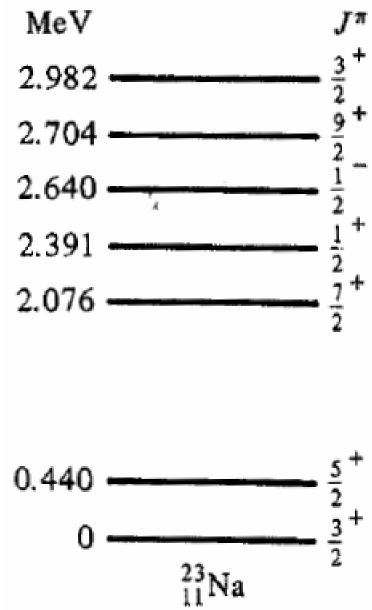
$$\log f \cdot T_{1/2} = log(B \cdot ln2) - log|M_{fi}|^2$$

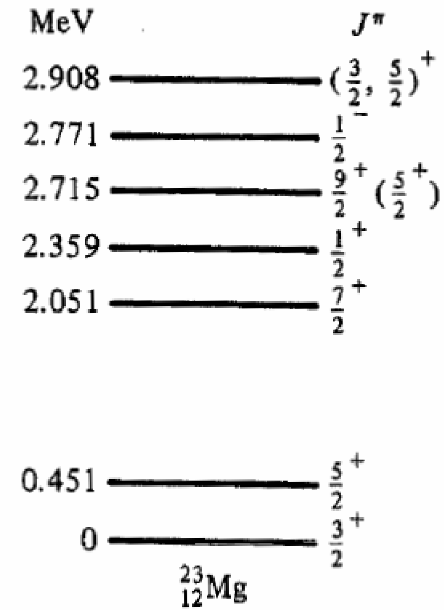
The size of log ft values are very different. It depends on the nuclear matrix element and the selection rule which determine the decay.

One distinguishes super-allowed, allowed, unique- and multiple-forbidden decays by means of the selection rules for momentum (I) and parity (π).

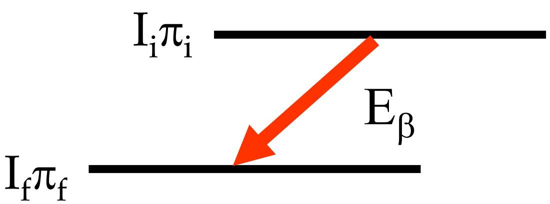


mirror nuclei





Classification of β-decay transitions



$$I_i = I_f + L_\beta + S_\beta \qquad \pi_i \pi_f = (-1)^{L_\beta}$$

$$L_\beta = l_{\beta-(\beta+)} + l_{\bar{\nu}(\nu)} \qquad S_\beta = s_{\beta-(\beta+)} + s_{\bar{\nu}(\nu)} = \begin{cases} 0 & \uparrow\downarrow \\ 1 & \downarrow\downarrow \text{ or } \uparrow\uparrow \end{cases}$$

$L_\beta = n$ defines the degree of forbiddenness (n)

allowed

when $L_\beta = n = 0$ and $\pi_i\pi_f = +1$

$$\Delta I = \left| I_i - I_f \right| \equiv 0,1$$

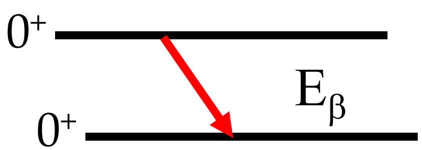
forbidden

when $L_\beta = n > 0$ and/or $\pi_i\pi_f = -1$

Classification of β-decay transitions

$$(\pi_i \pi_f = +1)$$

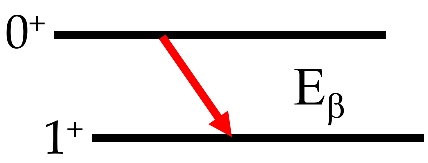
Fermi



$$\Delta I = \left| I_i - I_f \right| \equiv 0$$

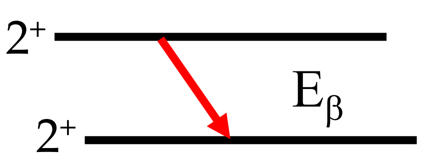
$$L_\beta = 0 \qquad S_\beta = 0 \downarrow\uparrow$$

Gamow-Teller



$$\Delta I = \left| I_i - I_f \right| \equiv 1$$

$$L_\beta = 0 \quad S_\beta = 1 \uparrow\uparrow or \downarrow\downarrow$$



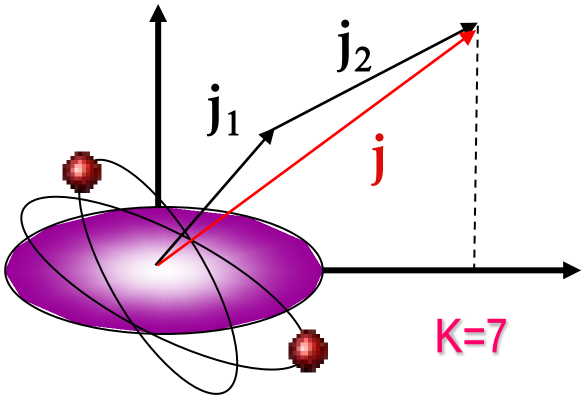
mixed Fermi & Gamow-Teller

$$\Delta I = \left| I_i - I_f \right| \equiv 0 \qquad I_i \neq 0$$

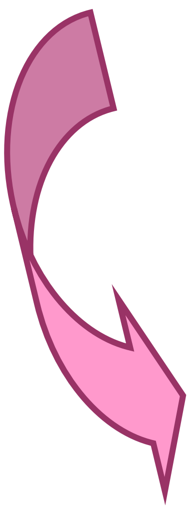
Classification of β-decay transitions

Type of transition	Order of forbiddenness	ΔI	$\pi_i \pi_f$
Allowed		0,+1	+1
Forbidden unique	1	∓ 2	-1
	2	∓ 3	+1
	3	∓ 4	-1
	4	∓ 5	+1
	.	.	.
Forbidden	1	0, ∓ 1	-1
	2	∓ 2	+1
	3	∓ 3	-1
	4	∓ 4	+1
	.	.	.

Nuclear structure is important

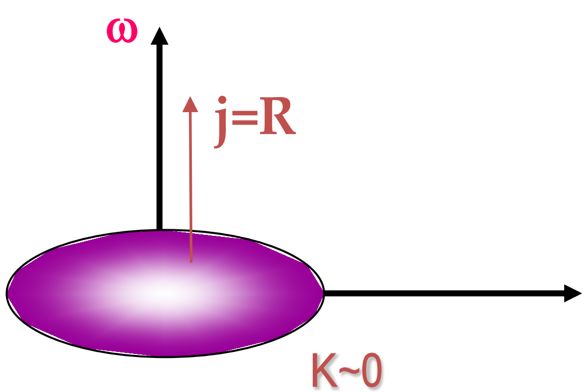


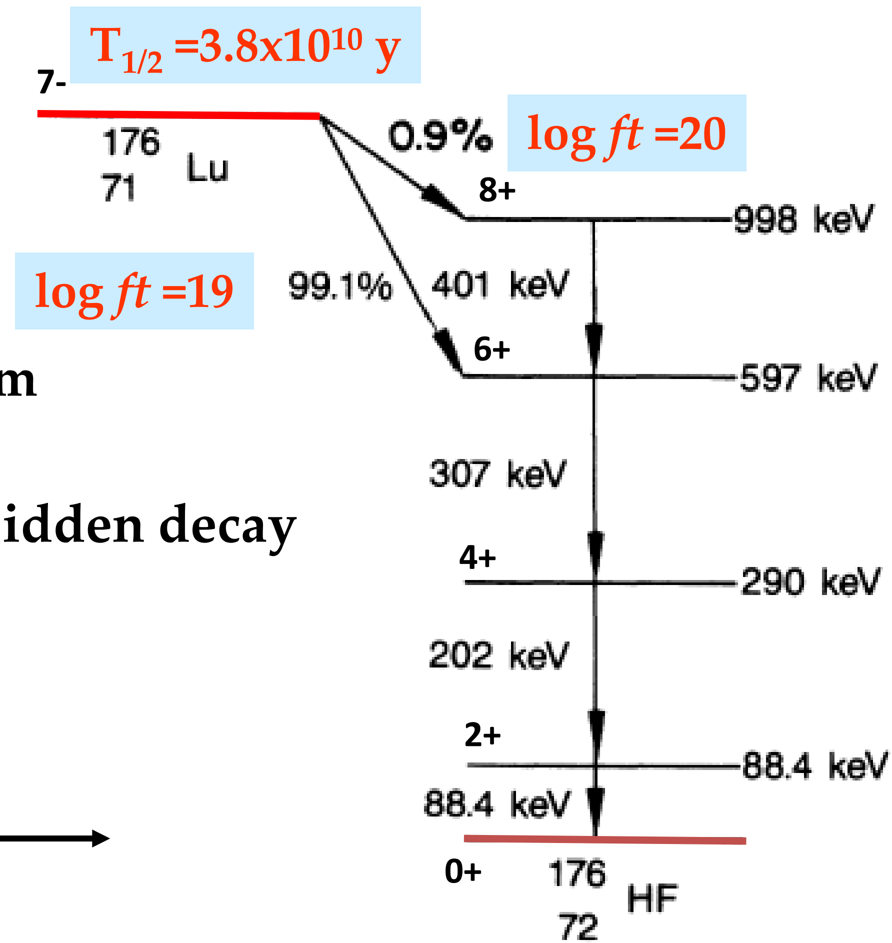
First forbidden → 5 < log *ft* < 10



large angular momentum
re-orientation

K-forbidden decay



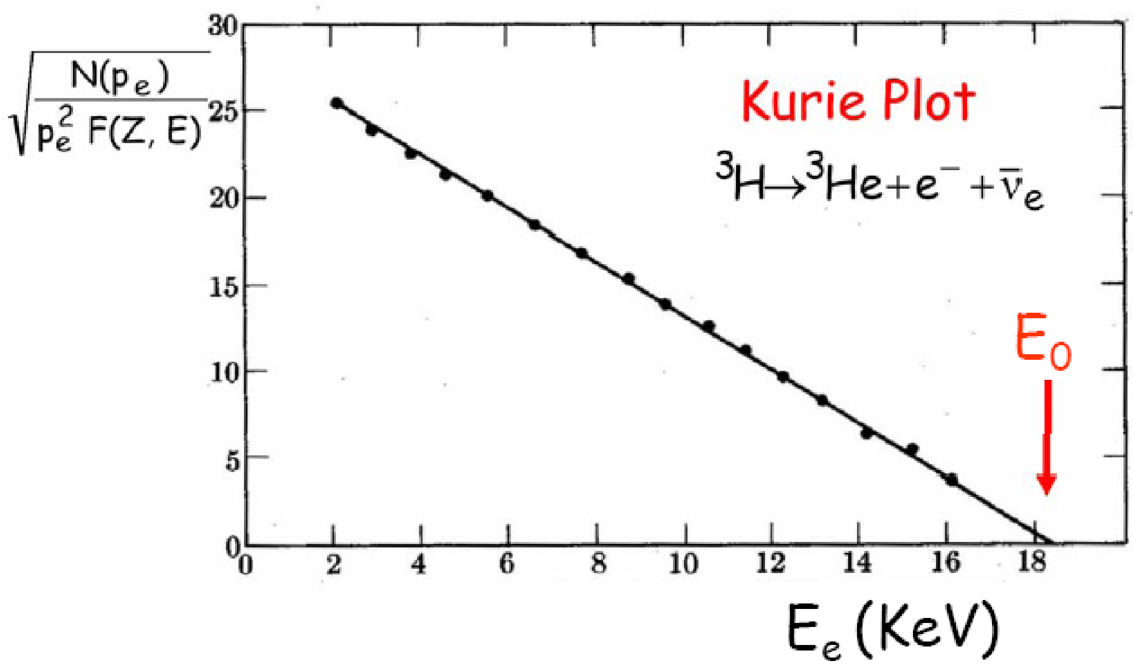


Counting rate as a function of electron momentum:

$$N(p_e) \sim F(Z,\varepsilon) \cdot p_e^2 \cdot (\varepsilon_0 - \varepsilon)^2$$

Kurie-plot:

$$\sqrt{\frac{N(p_e)}{F(Z,\varepsilon) \cdot p_e^2}} = const \cdot (\varepsilon_0 - \varepsilon)$$



Kurie plot and neutrino mass

$m_\nu < 3\ eV/c^2$ (c.f. $m_e = 0.511\ MeV/c^2$)

$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$