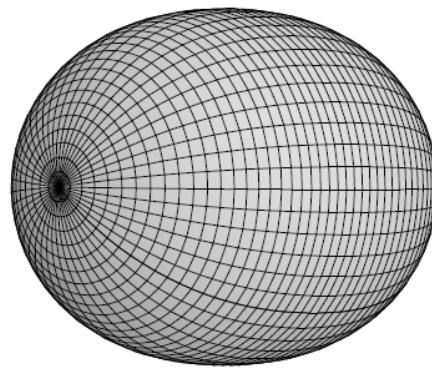


Shape parameterization

$$R(\theta, \phi) = R_0 \cdot \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} \cdot Y_{\lambda\mu}(\theta, \phi) \right]$$

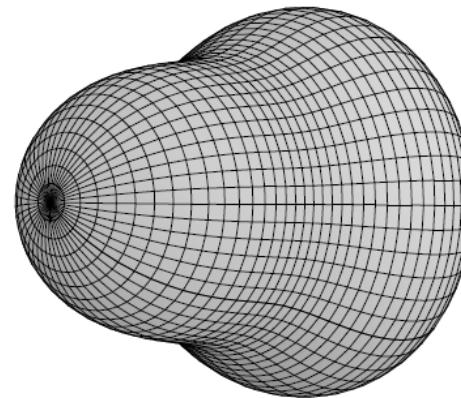
axially symmetric quadrupole



$$\lambda=2$$

$$\alpha_{20} \neq 0, \alpha_{2\pm 1} = \alpha_{2\pm 2} = 0$$

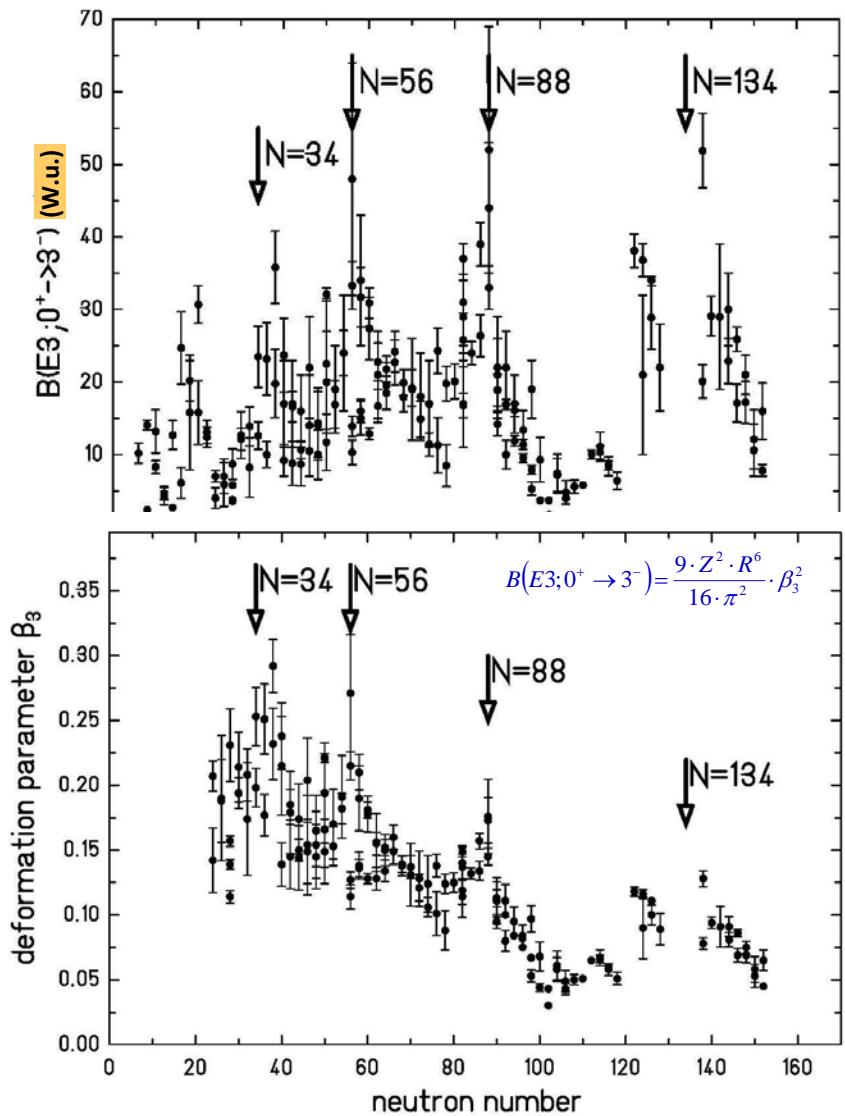
axially symmetric octupole



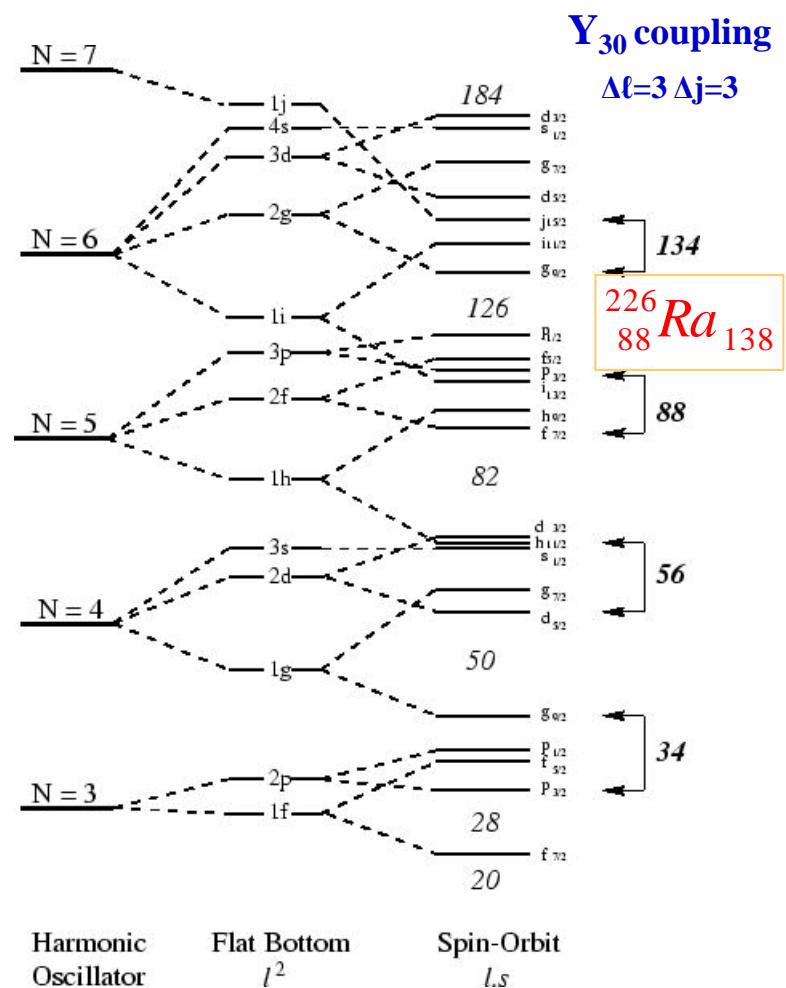
$$\lambda=3$$

$$\begin{aligned} \alpha_{30} &\neq 0, \alpha_{3\pm 1,2,3} = 0 \\ \alpha_{20} &\neq 0, \alpha_{2\pm 1,2} = 0 \end{aligned}$$

Octupole collectivity



R.H. Spear At. Data and Nucl. Data Tables 42 (1989), 55



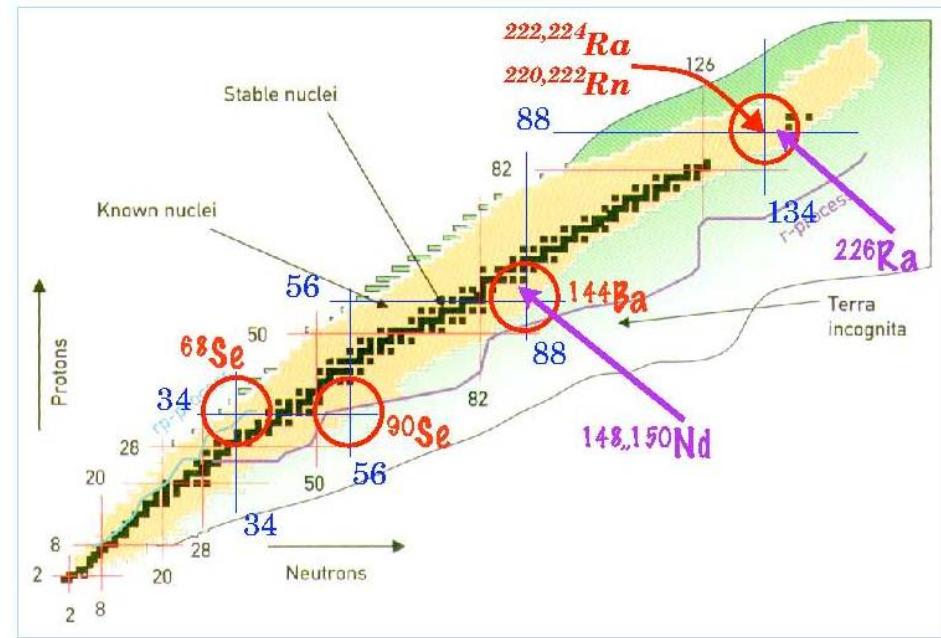
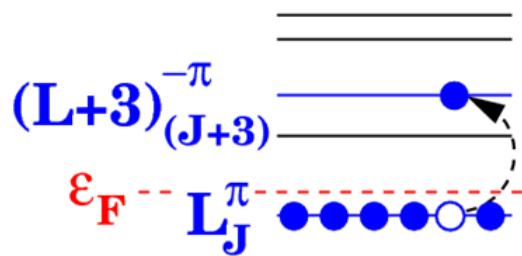
some subshells interact via the r^3Y_{30} operator
e.g. in light actinide nuclei one has an interaction between
 $j_{15/2}$ and $g_{9/2}$ neutron orbitals
 $i_{13/2}$ and $f_{7/2}$ proton orbitals

Octupole collectivity

Octupole correlations enhanced at the magic numbers: **34, 56, 88, 134**

Microscopically ...

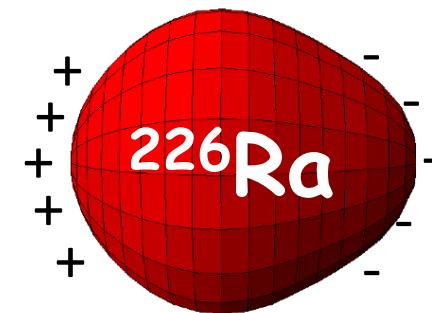
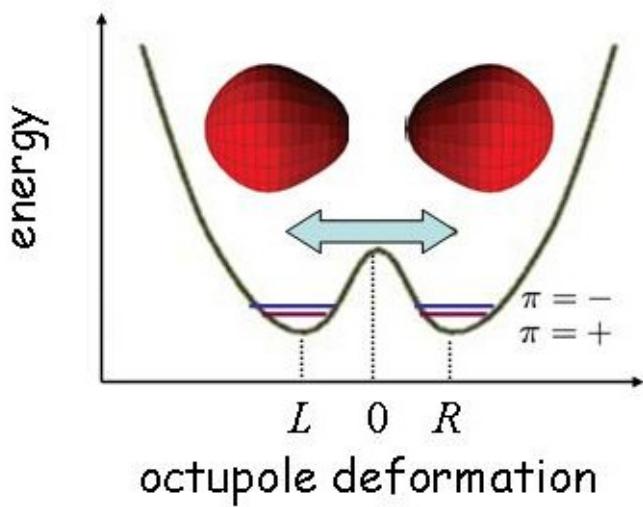
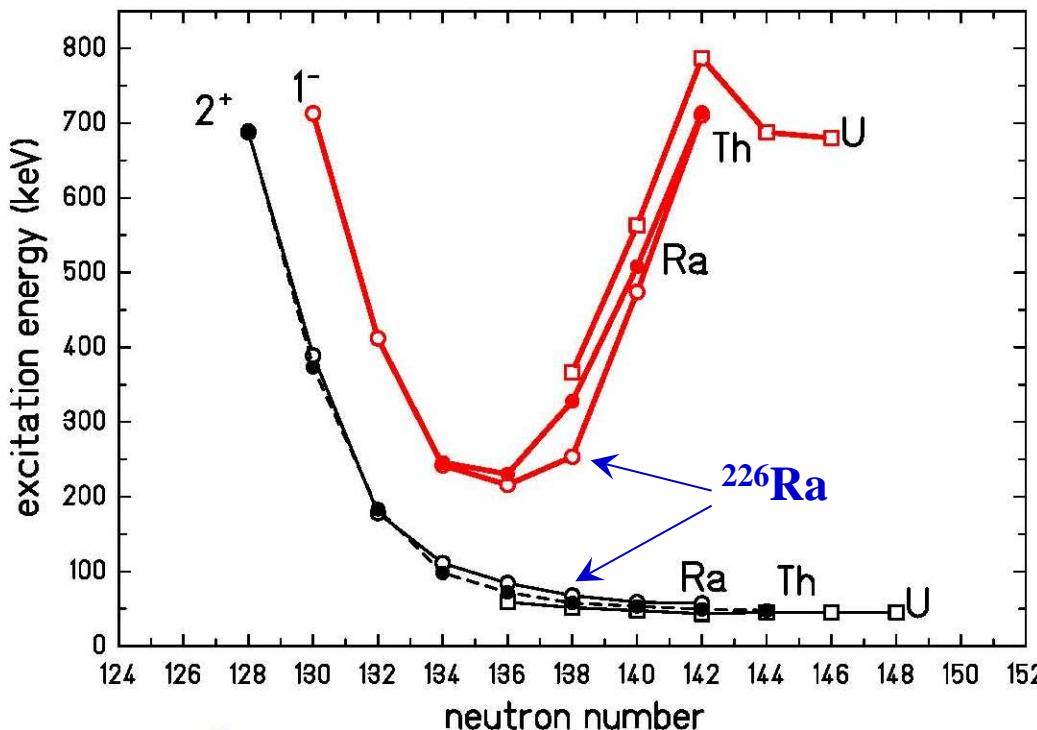
Intruder orbitals of opposite parity and $\Delta J, \Delta L = 3$ close to Fermi level



^{226}Ra close to $Z=88$ $N=134$

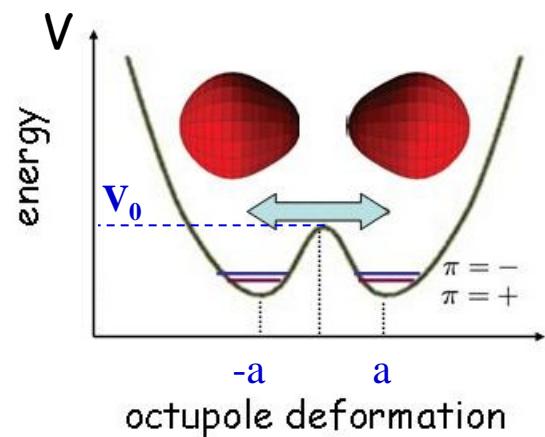
$$\pi(f_{7/2} \rightarrow i_{13/2}) \quad \nu(g_{9/2} \rightarrow j_{15/2})$$

Octupole collectivity



In an **octupole** deformed nucleus the center of mass and center of charge tend to separate, creating a non-zero **electric dipole moment**.

The double oscillator

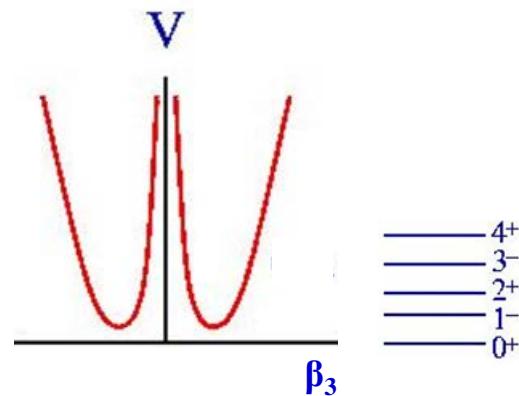


$$H \cdot \psi = -\frac{\hbar^2}{2 \cdot B} \frac{\partial^2 \psi}{\partial \beta_3^2} + \frac{V_0}{a^2} \cdot (\lvert \beta_3 \rvert - a)^2 \cdot \psi = E \cdot \psi$$

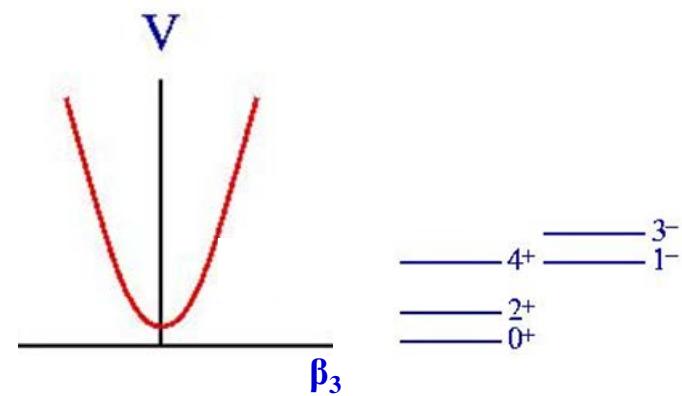
$$E_{even} = \hbar\omega \cdot \left(\nu_{even} + \frac{1}{2} \right) = \hbar\omega \cdot \left(\frac{1}{2} - \sqrt{\frac{2 \cdot V_0}{\hbar\omega \cdot \pi}} \cdot e^{-\frac{2V_0}{\hbar\omega}} \right)$$

$$E_{odd} = \hbar\omega \cdot \left(\nu_{odd} + \frac{1}{2} \right) = \hbar\omega \cdot \left(\frac{1}{2} + \sqrt{\frac{2 \cdot V_0}{\hbar\omega \cdot \pi}} \cdot e^{-\frac{2V_0}{\hbar\omega}} \right)$$

$$P |\Psi\rangle \neq |\Psi\rangle$$

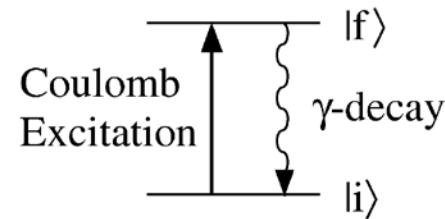
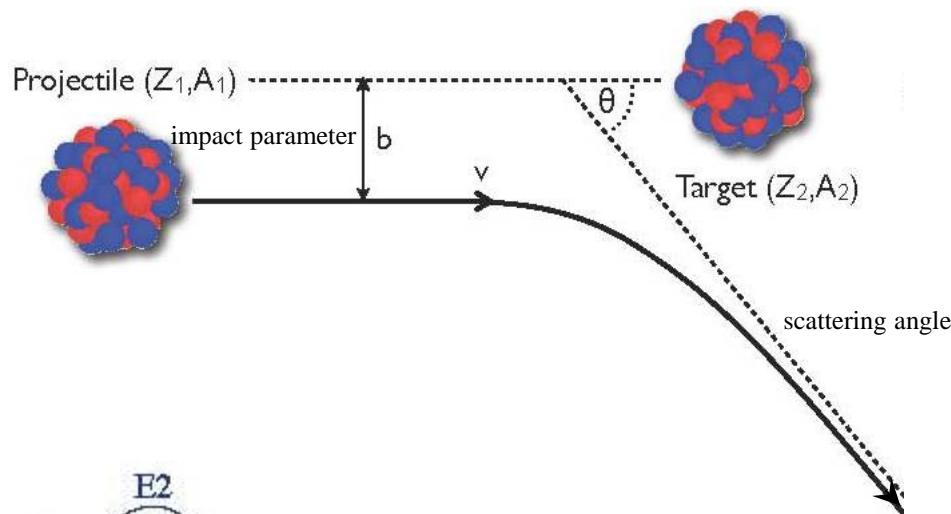


octupole deformed



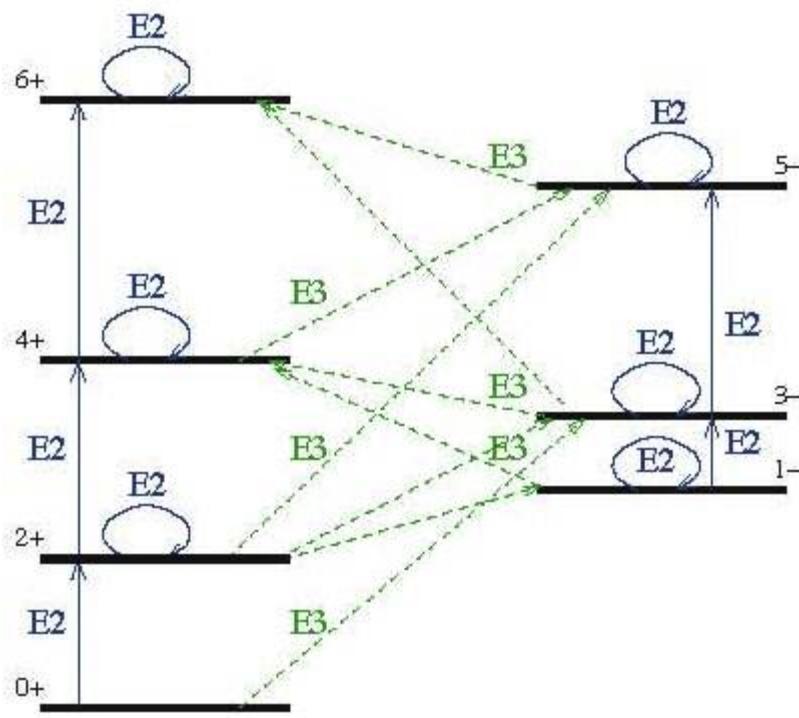
octupole vibrational

Coulomb excitation

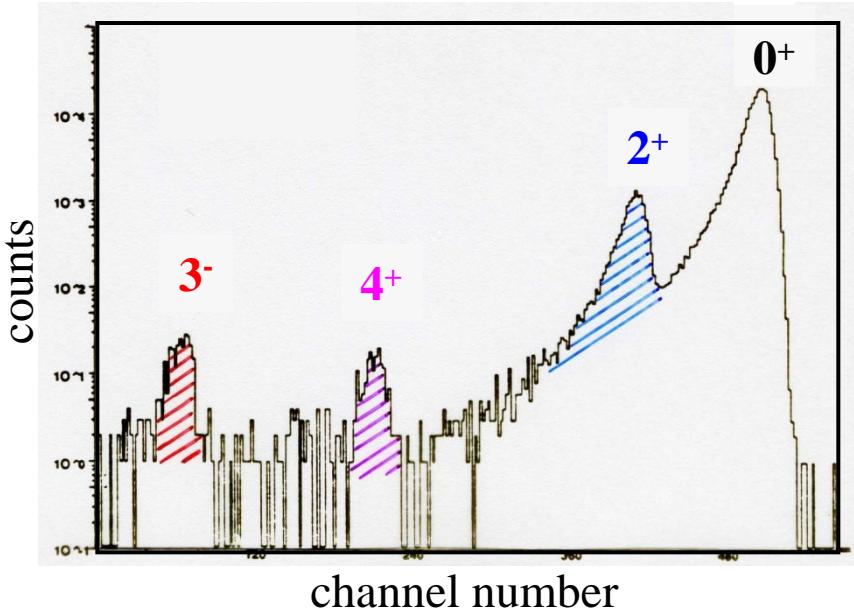
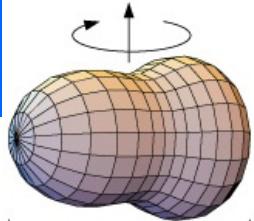


$$\frac{d\sigma_{i \rightarrow f}}{d\Omega_{cm}} = P_{i \rightarrow f} \cdot \frac{d\sigma_{Ruth}}{d\Omega_{cm}}$$

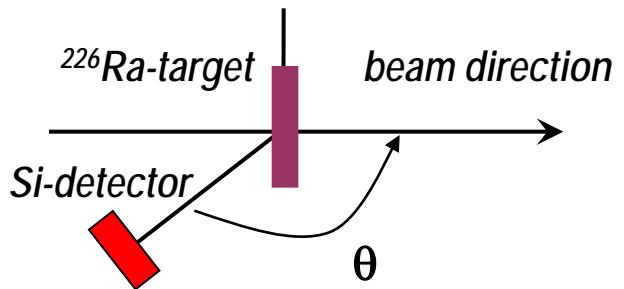
$$d\sigma_{E2} \cong 4.819 \cdot \left(1 + \frac{A_1}{A_2}\right)^{-2} \cdot \frac{A_1}{Z_2^2} \cdot E_{MeV} \cdot B(E2; I_i \rightarrow I_f) \cdot df_{E2}(\eta, \xi) [b]$$



Scattered α -spectrum of ^{226}Ra



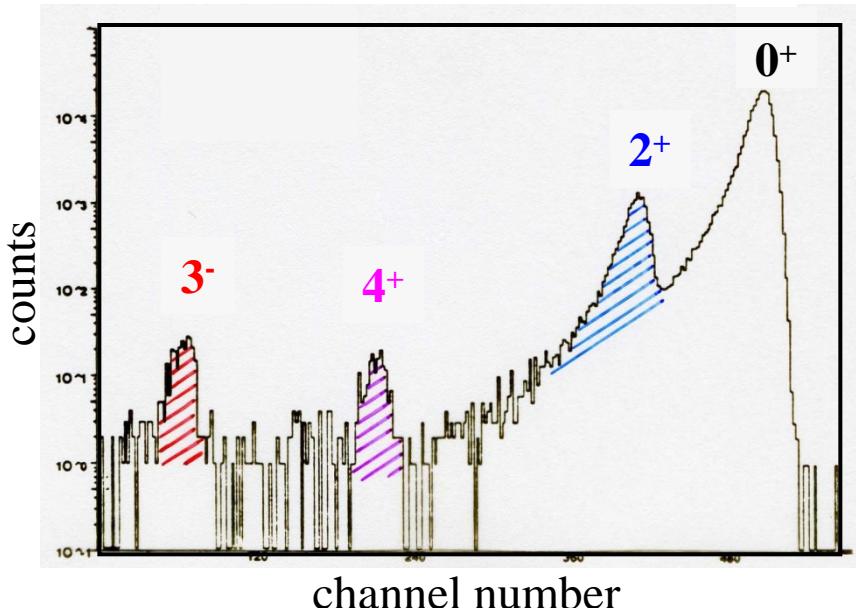
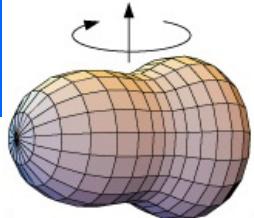
$^4\text{He} \rightarrow ^{226}\text{Ra}$
 $E_\alpha = 16 \text{ MeV}$
 $\theta_{\text{lab}} = 145^\circ$



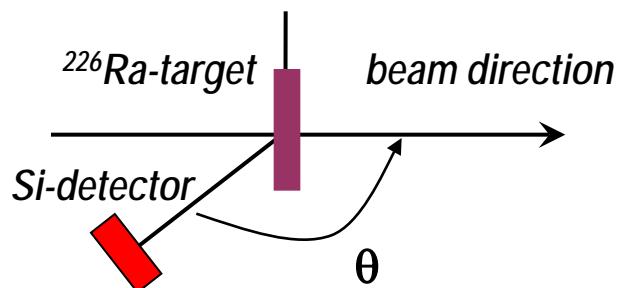
$$P_{i \rightarrow f} = \frac{d\sigma_{i \rightarrow f}}{d\sigma_{el}} \cong \frac{d\sigma_{i \rightarrow f}}{d\sigma_{Ruth}}$$

λ	$\langle \lambda \ M(E\lambda) \ 0 \rangle [eb^{\lambda/2}]$	$\beta_\lambda (\text{exp})$	$\beta_\lambda (\text{theo})$
2	2.27 (3)	0.165 (2)	0.164
3	1.05 (5)	0.104 (5)	0.112
4	1.04 (7)	0.123 (8)	0.096

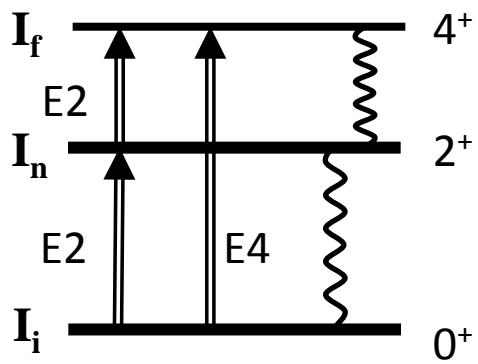
Scattered α -spectrum of ^{226}Ra



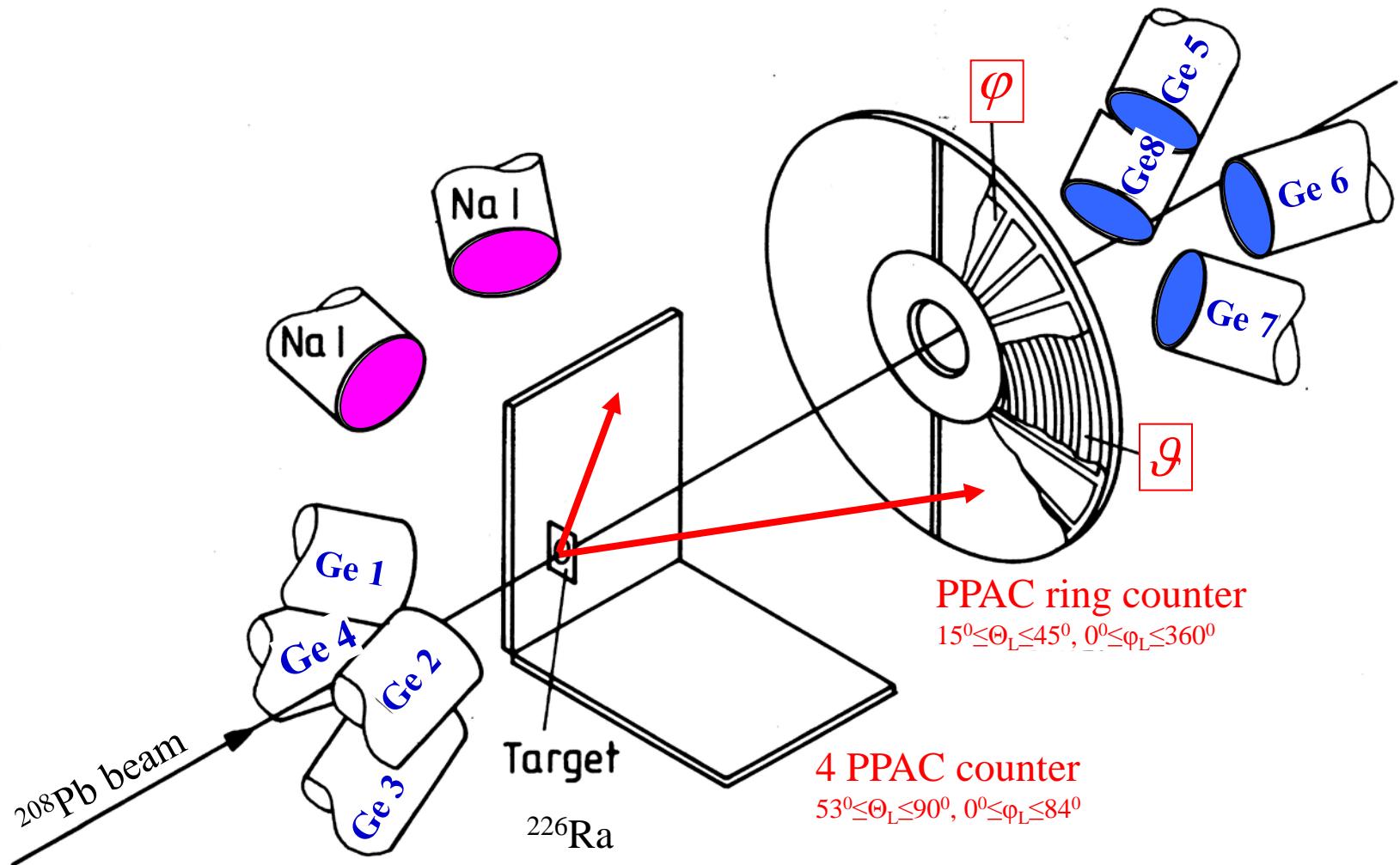
$^4\text{He} \rightarrow ^{226}\text{Ra}$
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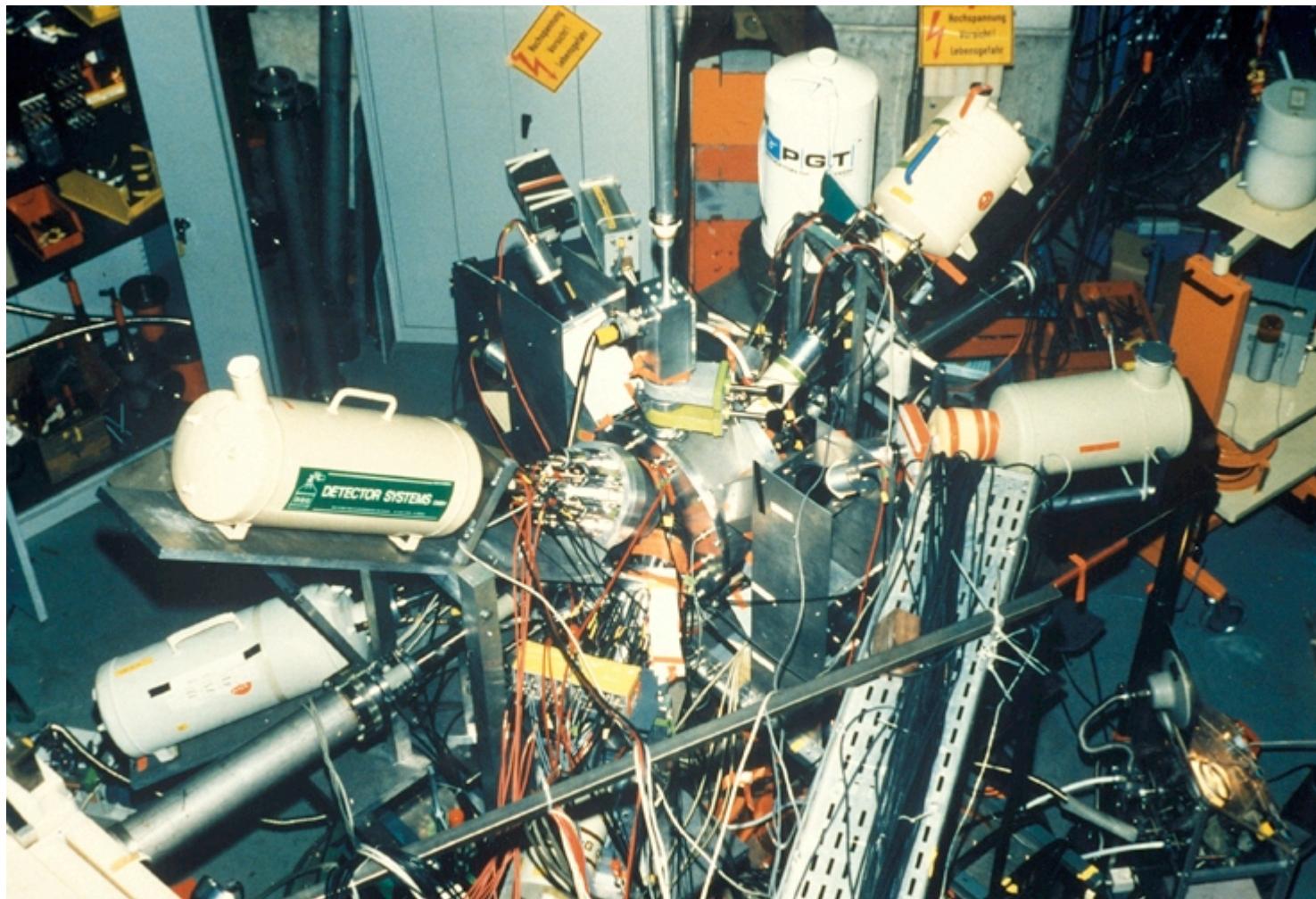


Experimental set-up

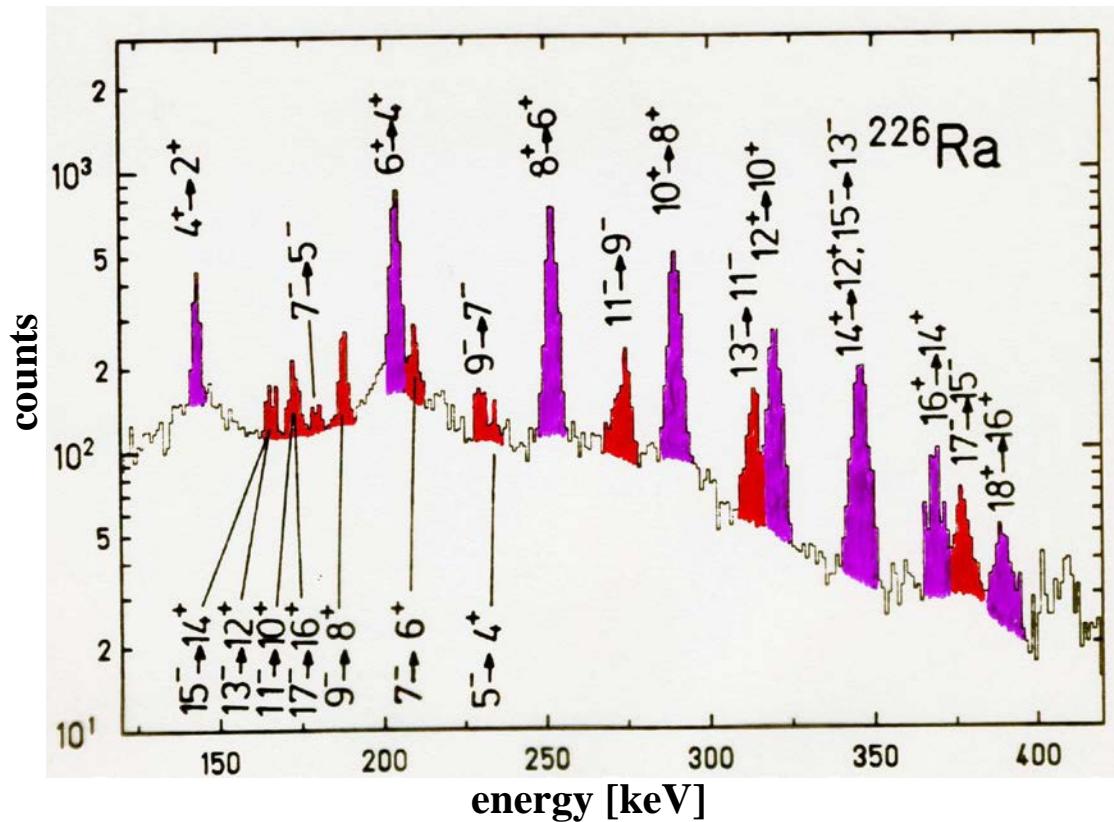


$^{226}\text{RaBr}_2$ ($400 \mu\text{g}/\text{cm}^2$) on C-backing ($50 \mu\text{g}/\text{cm}^2$) and covered by Be ($40 \mu\text{g}/\text{cm}^2$)

Coulomb excitation of ^{226}Ra



γ -ray spectrum of ^{226}Ra



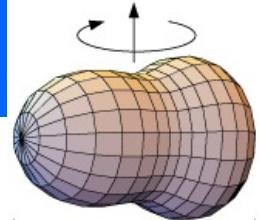
$^{208}\text{Pb} \rightarrow ^{226}\text{Ra}$

$E_{\text{lab}} = 4.7 \text{ AMeV}$

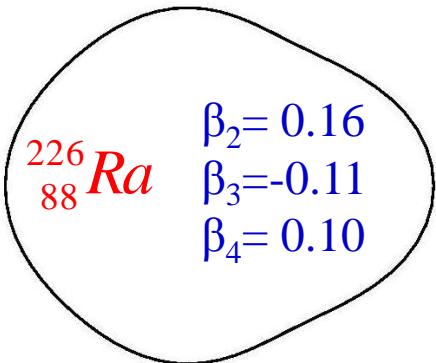
$15^0 \leq \theta_{\text{lab}} \leq 45^0$

$0^0 \leq \phi_{\text{lab}} \leq 360^0$

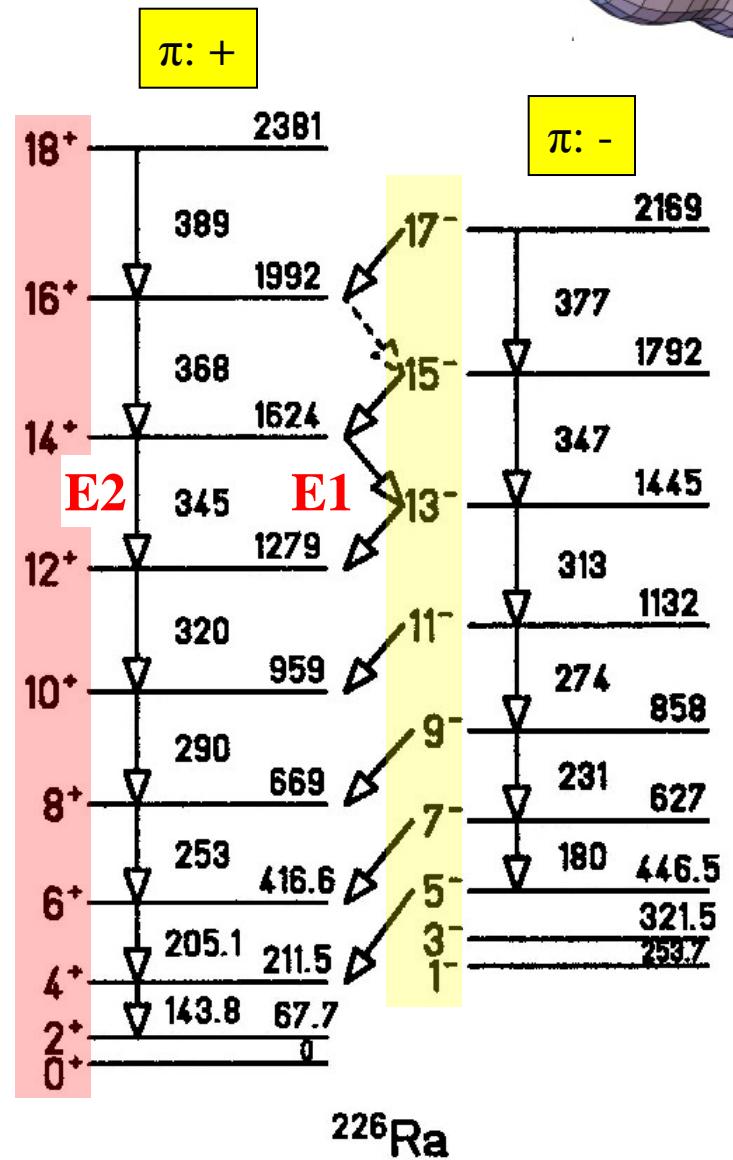
Signature of an octupole deformed nucleus



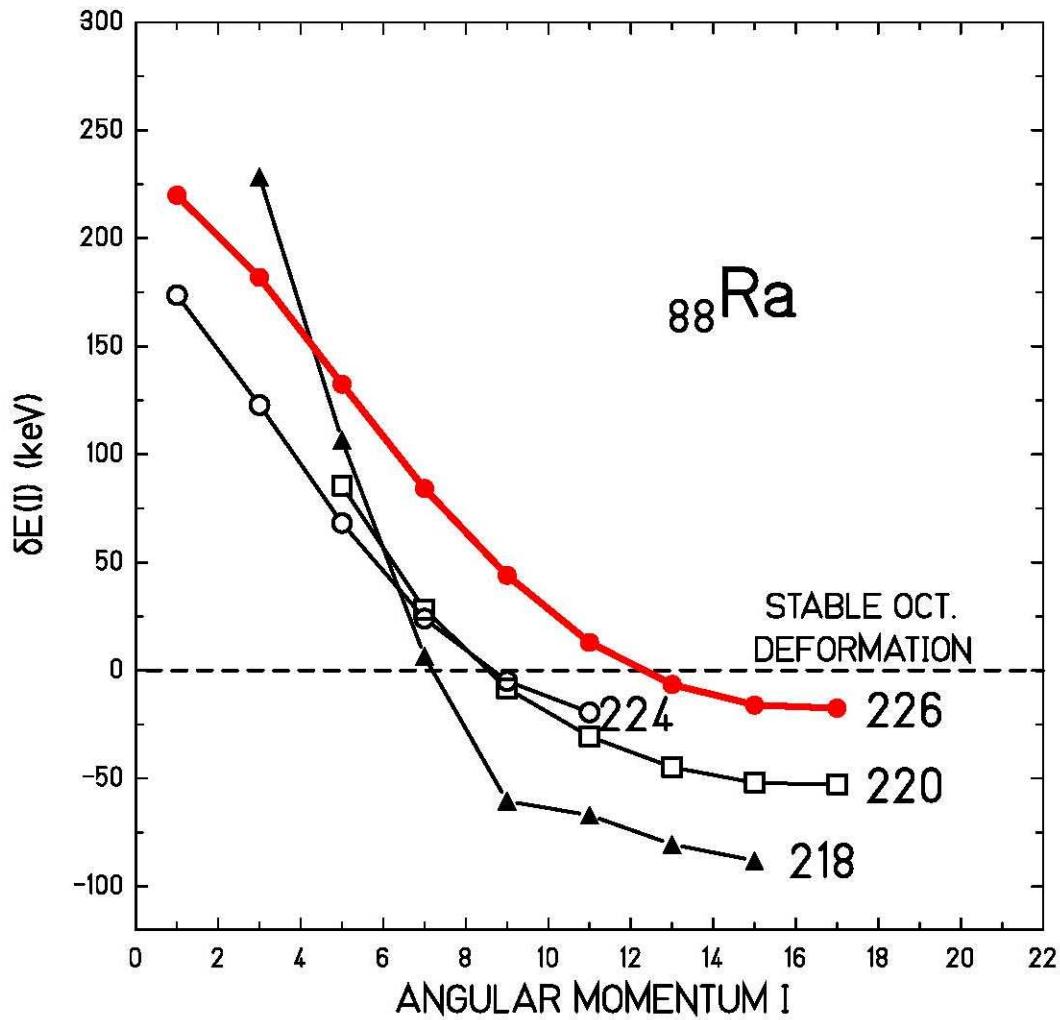
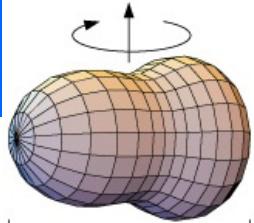
$$R(\theta) = R_0 \cdot [1 + \beta_2 \cdot Y_{20}(\theta) + \beta_3 \cdot Y_{30}(\theta) + \beta_4 \cdot Y_{40}(\theta)]$$



Single rotational band with spin sequence:
I = 0+, 1-, 2+, 3-, ...
excitation energy **E ~ I·(I+1)**
competition between intraband **E2** and
interband **E1** transitions
E1 transition strength **10⁻² W.u.**



Signature of an octupole deformed nucleus

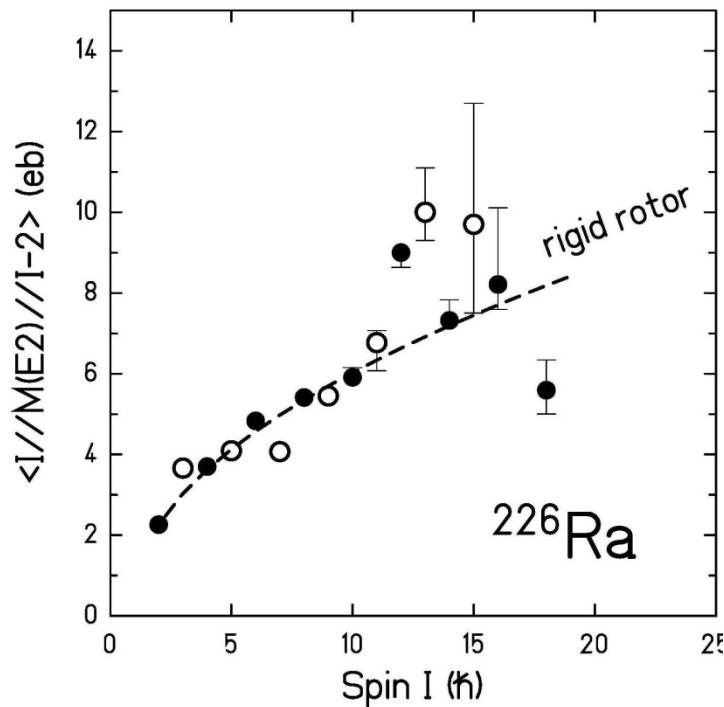


Energy displacement δE between the positive- and negative-parity states if they form a single rotational band

$$\delta E(I) = E(I)^- - \frac{E(I+1)^+ + E(I-1)^+}{2}$$

$$= -\frac{\hbar^2}{2\beta} \text{ for rigid rotor}$$

Electric transition quadrupole moments in ^{226}Ra



○ negative parity states
● positive parity states

rigid rotor model:

$$\langle I - 2 \| M(E2) \| I \rangle = \sqrt{\frac{15}{32 \cdot \pi}} \cdot \sqrt{\frac{I \cdot (I-1)}{2I-1}} \cdot Q_2 \cdot e$$

liquid drop:

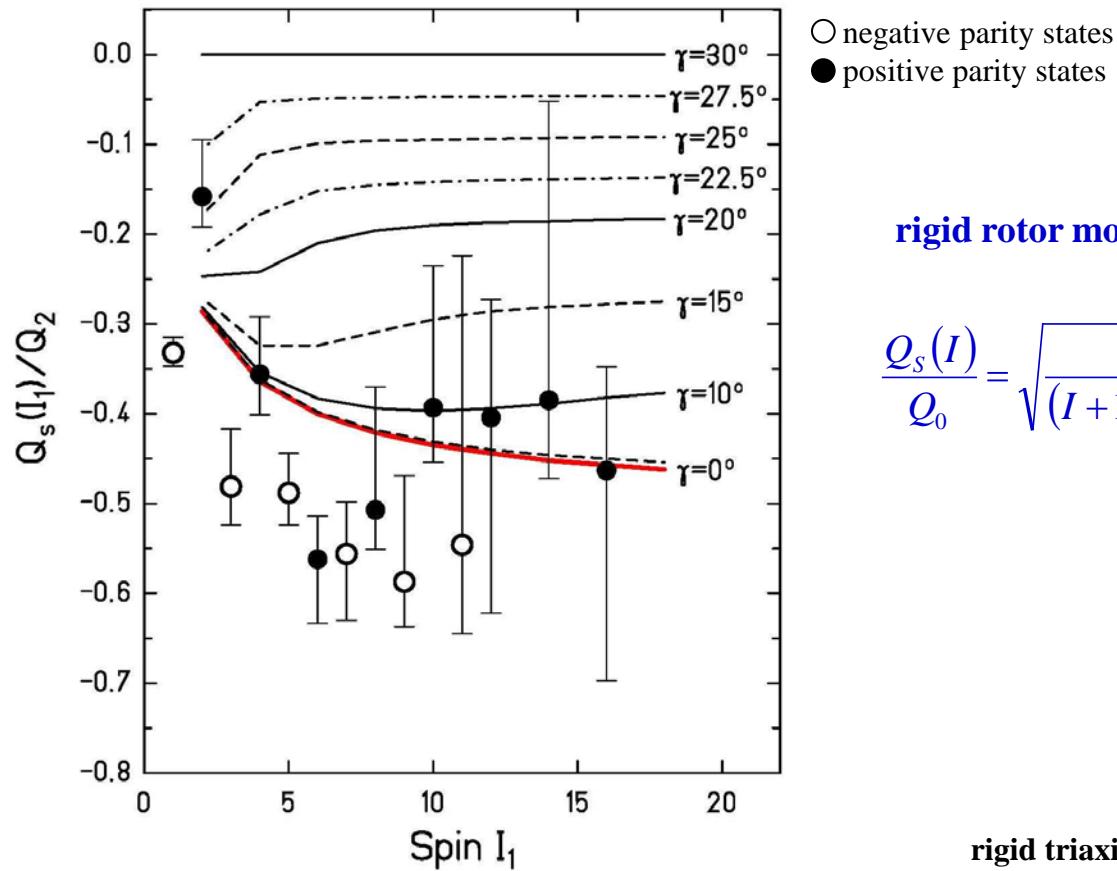
$$Q_2 = \frac{3 \cdot Z \cdot R_0^2}{\sqrt{5 \cdot \pi}} \cdot (\beta_2 + 0.360\beta_2^2 + 0.336\beta_3^2 + 0.328\beta_4^2 + 0.967\beta_2\beta_4) \quad [\text{fm}^2]$$

$Q_2(\text{exp}) = 750 \text{ fm}^2$

$\beta_2 = 0.21$

$Q_2(\text{theo}) = 680 \text{ fm}^2$

Static quadrupole moments in ^{226}Ra



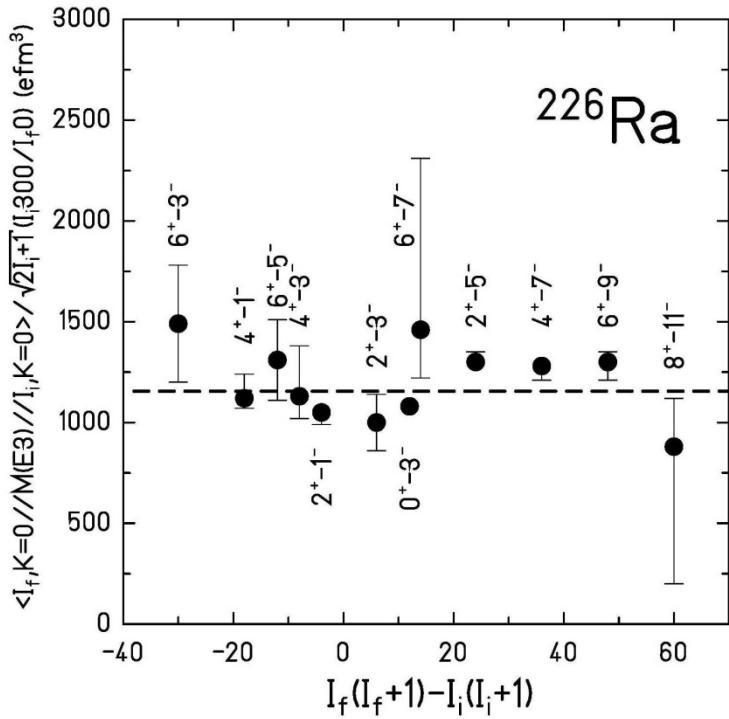
rigid rotor model:

$$\frac{Q_s(I)}{Q_0} = \sqrt{\frac{I \cdot (2I-1)}{(I+1) \cdot (2I+1) \cdot (2I+3)}} \cdot \frac{\langle I | M(E2) | I \rangle}{\langle 2_1 | M(E2) | 0_1 \rangle}$$

rigid triaxial rotor model:

$$\frac{Q_s(2_1)}{Q_0} = -\frac{6 \cdot \cos(3\gamma)}{7 \cdot \sqrt{9 - 8 \cdot \sin^2(3\gamma)}}$$

Electric transition octupole moments in ^{226}Ra

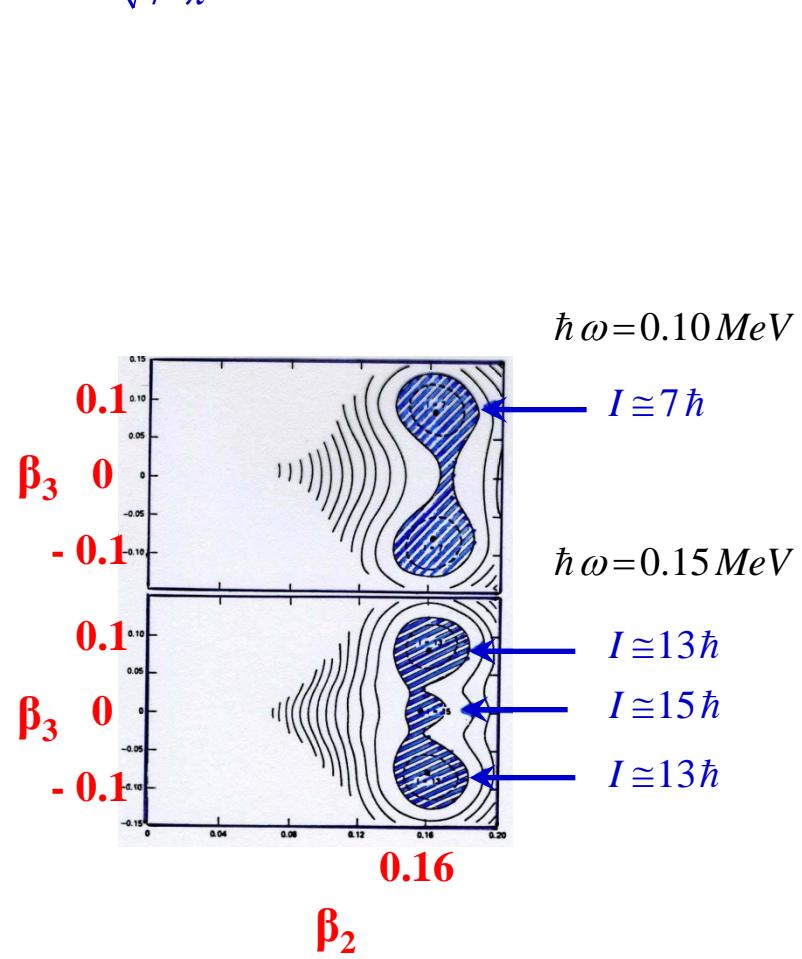


$$\langle I-3 | M(E3) | I \rangle = -\sqrt{\frac{35}{32\pi}} \cdot \sqrt{\frac{I \cdot (I-1) \cdot (I-2)}{(2I-3) \cdot (2I+3)}} \cdot Q_3 \cdot e$$

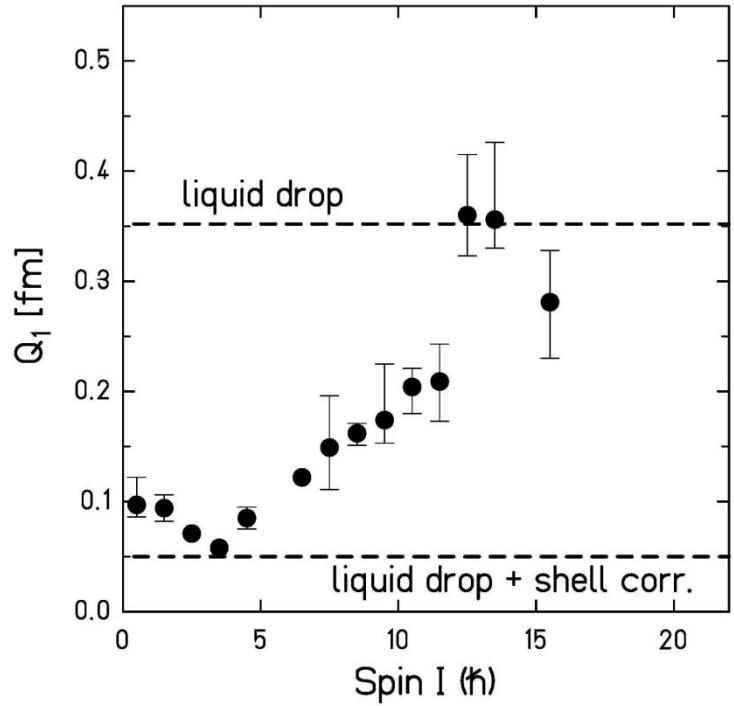
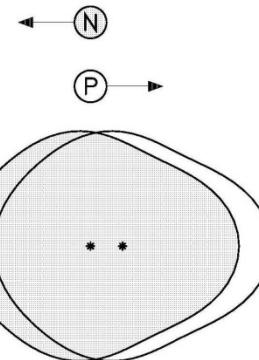
$$\langle I-1 | M(E3) | I \rangle = \sqrt{\frac{21}{32\pi}} \cdot \sqrt{\frac{(I-1) \cdot I \cdot (I+1)}{(2I-3) \cdot (2I+3)}} \cdot Q_3 \cdot e$$

liquid drop:

$$Q_3 = \frac{3 \cdot Z \cdot R_0^3}{\sqrt{7 \cdot \pi}} \cdot (\beta_3 + 0.841\beta_2\beta_3 + 0.769\beta_3\beta_4) [fm^3]$$



Intrinsic electric dipole moments in ^{226}Ra



liquid-drop contribution:

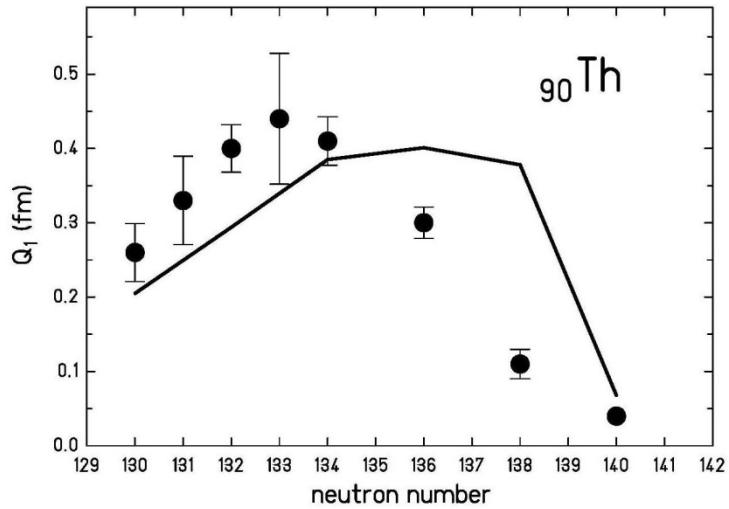
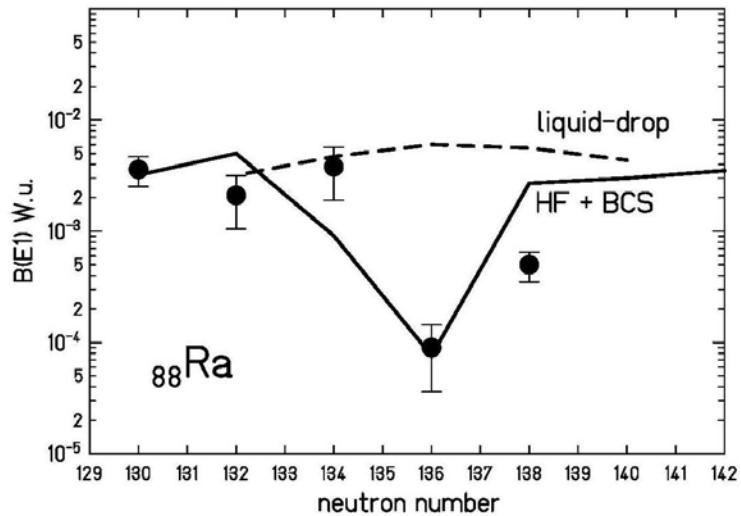
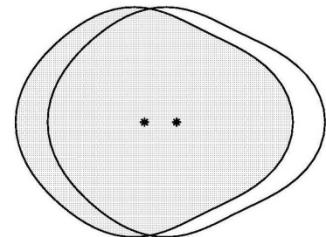
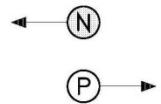
$$Q_1^{LD} = C_{LD} \cdot A \cdot Z \cdot (\beta_2 \beta_3 + 1.458 \cdot \beta_3 \beta_4)$$

with $C_{LD} = 5.2 \cdot 10^{-4} [\text{fm}]$

rigid rotor model:

$$\langle I - 1 | M(E1) | I \rangle = -\sqrt{\frac{3}{4\pi}} \cdot \sqrt{I} \cdot Q_1 \cdot e$$

Intrinsic electric dipole moments in Ra / Th



liquid-drop contribution:

$$Q_1^{LD} = C_{LD} \cdot A \cdot Z \cdot (\beta_2 \beta_3 + 1.458 \cdot \beta_3 \beta_4)$$

with $C_{LD} = 5.2 \cdot 10^{-4}$ [fm]

G. Leander et al.; Nucl. Phys. A453 (1986) 58

Center of mass conservation

$$0 = \begin{Bmatrix} \int \rho_0 \cdot x \cdot d\tau \\ \int \rho_0 \cdot y \cdot d\tau \\ \int \rho_0 \cdot z \cdot d\tau \end{Bmatrix} = \int \rho_0 \cdot \vec{r} \cdot d\tau$$

The coordinates (x, y, z) can be expressed by

$$r \cdot Y_{1m}(\theta, \phi) = \begin{cases} \frac{1}{2} \cdot \sqrt{\frac{3}{\pi}} \cdot z & m=0 \\ \mp \sqrt{\frac{3}{2\pi}} \cdot (x \pm iy) & m=\pm 1 \end{cases}$$

$$0 = \sqrt{\frac{4\pi}{3}} \cdot \int \rho_0 \cdot r \cdot Y_{10}(\theta, \phi) \cdot d\tau = \iint r^3 \cdot dr \cdot Y_{10} \cdot d\Omega$$

$$0 = \frac{R_0^4}{4} \cdot \int \left\{ 1 + 4 \sum_{\ell_1 m_1} \alpha_{\ell_1 m_1}^* Y_{\ell_1 m_1} + 6 \sum_{\ell_1 m_1 \ell_2 m_2} \alpha_{\ell_1 m_1}^* \alpha_{\ell_2 m_2}^* Y_{\ell_1 m_1} Y_{\ell_2 m_2} + \dots \right\} \cdot Y_{10} \cdot d\Omega$$

$$0 = 4 \cdot \alpha_{10}^* + 6 \sum_{\ell_1 m_1 \ell_2 m_2} \alpha_{\ell_1 m_1}^* \alpha_{\ell_2 m_2}^* \cdot \left[\frac{(2\ell_1+1) \cdot (2\ell_2+1) \cdot 3}{4\pi} \right]^{1/2} \cdot \begin{pmatrix} \ell_1 & \ell_2 & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \cdot \begin{pmatrix} \ell_1 & \ell_2 & \mathbf{1} \\ m_1 & m_2 & \mathbf{0} \end{pmatrix}$$

$$\alpha_{10}^* = -\frac{3}{2} \sum_{\ell_1 m_1 \ell_2 m_2} \alpha_{\ell_1 m_1}^* \alpha_{\ell_2 m_2}^* \cdot \left[\frac{(2\ell_1+1) \cdot (2\ell_2+1) \cdot 3}{4\pi} \right]^{1/2} \cdot \begin{pmatrix} \ell_1 & \ell_2 & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \cdot \begin{pmatrix} \ell_1 & \ell_2 & \mathbf{1} \\ m_1 & m_2 & \mathbf{0} \end{pmatrix}$$

The dipole coordinate is not an independent quantity. It is non-zero for nuclear shapes with both quadrupole and octupole degrees of freedom.

$$\beta_1 = -\sqrt{\frac{3}{4\pi}} \cdot \frac{9}{\sqrt{35}} \cdot \beta_2 \cdot \beta_3$$

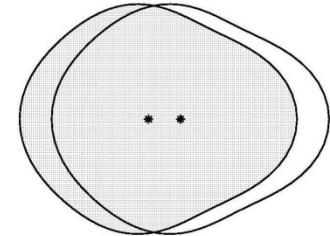
$$\beta_1 = -\frac{3}{2} \cdot \left[\frac{5 \cdot 7 \cdot 3}{4\pi} \right]^{1/2} \cdot \begin{pmatrix} 2 & 3 & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}^2 \cdot \{ \beta_2 \cdot \beta_3 + \beta_3 \cdot \beta_2 \}$$

$$\begin{pmatrix} 2 & 3 & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} = -\sqrt{\frac{3}{35}}$$

Intrinsic electric dipole moment

 \mathbf{N}

 \mathbf{P}



$$Q_1^{LD} = e \cdot \int z \cdot \rho_{proton} \cdot d\tau = \sqrt{\frac{4\pi}{3}} \cdot e \cdot \iiint \rho_p \cdot r^3 \cdot dr \cdot Y_{10}(\theta, \phi) \cdot d\Omega$$

The local volume polarization of electric charge can be derived from the requirement of a minimum in the energy functional. (Myers Ann. of Phys. (1971))

$$\frac{\rho_{proton} - \rho_{neutron}}{\rho_{proton} + \rho_{neutron}} = -\frac{1}{4C_{LD}} \cdot e \cdot V_C(r) \quad V_C(r) = \left\{ \frac{3}{2} - \frac{1}{2} \cdot \left(\frac{r}{R_0} \right)^2 + \sum_{\ell=1}^{\infty} \frac{3}{2\ell+1} \left(\frac{r}{R_0} \right)^{\ell} \cdot \beta_{\ell} \cdot Y_{\ell 0} \right\} \cdot \frac{Ze}{R_0}$$

where ρ_p and ρ_n are the proton and neutron densities, C_{LD} is the volume symmetry energy coefficient of the liquid drop model and V_C is the Coulomb potential generated by ρ_p inside the nucleus ($r < R_0$)

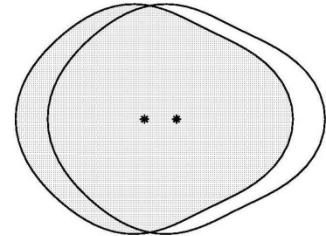
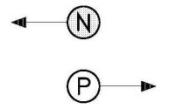
$$\rho_p = -\frac{\rho_0}{4 \cdot C_{LD}} \cdot e \cdot V_C(r) + \rho_n \quad \rho_0 = \rho_p + \rho_n \quad \rho_p = \frac{\rho_0}{2} \cdot \left[1 - \frac{1}{4 \cdot C_{LD}} \cdot e \cdot V_C(r) \right]$$

$$V_C(r) = \left\{ \frac{3}{2} - \frac{1}{2} \cdot \left(\frac{r}{R_0} \right)^2 + \left(\frac{r}{R_0} \right) \cdot \beta_1 \cdot Y_{10} + \frac{3}{5} \cdot \left(\frac{r}{R_0} \right)^2 \cdot \beta_2 \cdot Y_{20} + \frac{3}{7} \cdot \left(\frac{r}{R_0} \right)^3 \cdot \beta_3 \cdot Y_{30} \right\} \cdot \frac{Ze}{R_0}$$

Keeping the center of gravity fixed, the integral

$$0 = \iint \rho_0 \cdot r^3 \cdot dr \cdot Y_{10} \cdot d\Omega \quad \beta_1 = -\sqrt{\frac{3}{4\pi}} \cdot \frac{9}{\sqrt{35}} \cdot \beta_2 \cdot \beta_3$$

Intrinsic electric dipole moment



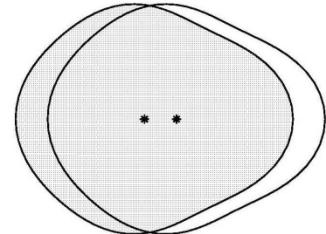
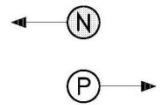
$$Q_1^{LD} = \sqrt{\frac{4\pi}{3}} \cdot e \cdot \iint \frac{\rho_0}{2} \cdot \left[1 - \frac{1}{4C_{LD}} \cdot e \cdot V_C(r) \right] \cdot r^3 \cdot dr \cdot Y_{10}(\theta, \phi) \cdot d\Omega$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \frac{3 \cdot A}{8 \cdot \pi \cdot R_0^3} \cdot \frac{1}{4C_{LD}} \frac{Z}{R_0} \cdot \iint \left\{ \frac{3}{2} - \frac{1}{2} \cdot \left(\frac{r}{R_0} \right)^2 + \left(\frac{r}{R_0} \right) \cdot \beta_1 \cdot Y_{10} + \frac{3}{5} \cdot \left(\frac{r}{R_0} \right)^2 \cdot \beta_2 \cdot Y_{20} + \frac{3}{7} \cdot \left(\frac{r}{R_0} \right)^3 \cdot \beta_3 \cdot Y_{30} \right\} \cdot r^3 \cdot dr \cdot Y_{10} \cdot d\Omega$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \frac{3 \cdot A}{8 \cdot \pi \cdot R_0^3} \cdot \frac{1}{4C_{LD}} \cdot \frac{Z}{R_0} \cdot \left\{ \begin{aligned} & \frac{3}{2} \frac{R_0^4}{4} \int [1 + 4 \cdot (\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30}) + 6 \cdot (\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30})^2 + \dots] \cdot Y_{10} \cdot d\Omega \\ & - \frac{1}{2} \frac{R_0^4}{6} \int [1 + 6 \cdot (\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30}) + 15 \cdot (\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30})^2 + \dots] \cdot Y_{10} \cdot d\Omega \\ & + \frac{R_0^4}{5} \int [\beta_1 Y_{10} + 5 \cdot (\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30}) \cdot \beta_1 Y_{10} + \dots] \cdot Y_{10} \cdot d\Omega \\ & + \frac{3}{5} \frac{R_0^4}{6} \int [\beta_2 Y_{20} + 6 \cdot (\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30}) \cdot \beta_2 Y_{20} + \dots] \cdot Y_{10} \cdot d\Omega \\ & + \frac{3}{7} \frac{R_0^4}{7} \int [\beta_3 Y_{30} + 7 \cdot (\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30}) \cdot \beta_3 Y_{30} + \dots] \cdot Y_{10} \cdot d\Omega \end{aligned} \right\}$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \frac{3 \cdot A}{8 \cdot \pi \cdot R_0^3} \cdot \frac{1}{4C_{LD}} \cdot \frac{Z}{R_0} \cdot \left\{ \begin{aligned} & \frac{3}{2} \frac{R_0^4}{4} [4\beta_1 + 6 \cdot \int (2\beta_1\beta_2 Y_{10} Y_{20} + 2\beta_2\beta_3 Y_{20} Y_{30}) \cdot Y_{10} \cdot d\Omega] \\ & - \frac{1}{2} \frac{R_0^4}{6} [6\beta_1 + 15 \cdot \int (2\beta_1\beta_2 Y_{10} Y_{20} + 2\beta_2\beta_3 Y_{20} Y_{30}) \cdot Y_{10} \cdot d\Omega] \\ & + \frac{R_0^4}{5} [\beta_1 + 5 \cdot \int \beta_1\beta_2 Y_{10} Y_{20} \cdot Y_{10} \cdot d\Omega] \\ & + \frac{3}{5} \frac{R_0^4}{6} [6 \cdot \int (\beta_1\beta_2 Y_{10} Y_{20} + \beta_2\beta_3 Y_{20} Y_{30}) \cdot Y_{10} \cdot d\Omega] \\ & + \frac{3}{7} \frac{R_0^4}{7} [7 \cdot \int \beta_2\beta_3 Y_{20} Y_{30} \cdot Y_{10} \cdot d\Omega] \end{aligned} \right\}$$

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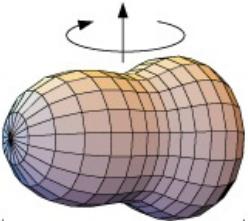
$$\int Y_{20} Y_{30} Y_{10} \cdot d\Omega = \sqrt{\frac{5 \cdot 7 \cdot 3}{4\pi}} \begin{pmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \end{pmatrix}^2 = \sqrt{\frac{3}{4\pi}} \frac{3}{\sqrt{35}}$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \cdot \frac{3 \cdot A \cdot Z}{32 \cdot \pi \cdot C_{LD}} \cdot \left\{ \begin{array}{l} \frac{3}{2}\beta_1 + \frac{9}{2}\sqrt{\frac{3}{4\pi}} \frac{3}{\sqrt{35}} \beta_2 \beta_3 \\ -\frac{1}{2}\beta_1 + \frac{5}{2}\sqrt{\frac{3}{4\pi}} \frac{3}{\sqrt{35}} \beta_2 \beta_3 \\ \quad + \frac{1}{5}\beta_1 \\ + \frac{3}{5}\sqrt{\frac{3}{4\pi}} \frac{3}{\sqrt{35}} \beta_2 \beta_3 \\ + \frac{3}{7}\sqrt{\frac{3}{4\pi}} \frac{3}{\sqrt{35}} \beta_2 \beta_3 \end{array} \right\}$$

$$Q_1^{LD} = e^3 \cdot \frac{3 \cdot A \cdot Z}{32 \cdot \pi \cdot C_{LD}} \cdot \frac{60}{35 \cdot \sqrt{35}} \cdot \beta_2 \cdot \beta_3$$

$$Q_1^{LD} = 0.01245 \cdot \frac{e \cdot A \cdot Z}{C_{LD}} \cdot \beta_2 \cdot \beta_3 \quad [fm]$$

$$C_{LD} \approx 20 \text{ MeV}$$



- single rotational band for $I > 10\hbar$
- no backbending observed
- $\beta_2, \beta_3, \beta_4$ deformation parameters are in excellent agreement with calculated values
- octupole deformation is three times larger than in octupole-vibrational nuclei
- equal transition quadrupole moments for positive- and negative-parity states
- static quadrupole moments are in excellent agreement with an axially symmetric shape
- electric dipole moments are close to liquid-drop value ($I > 10\hbar$)

- octupole deformation seems to be stabilized with increasing rotational frequency

Coulomb excitation of ^{226}Ra



^{226}Ra target broken after 8 hours



Christoph Fleischmann