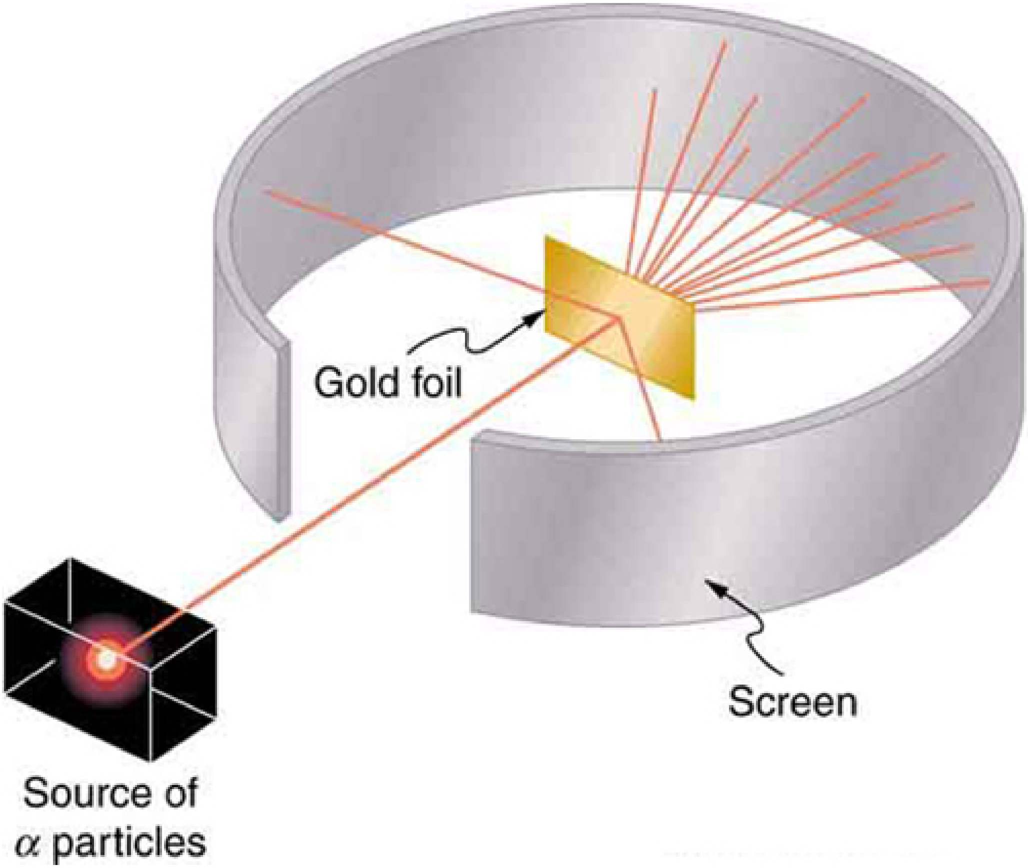
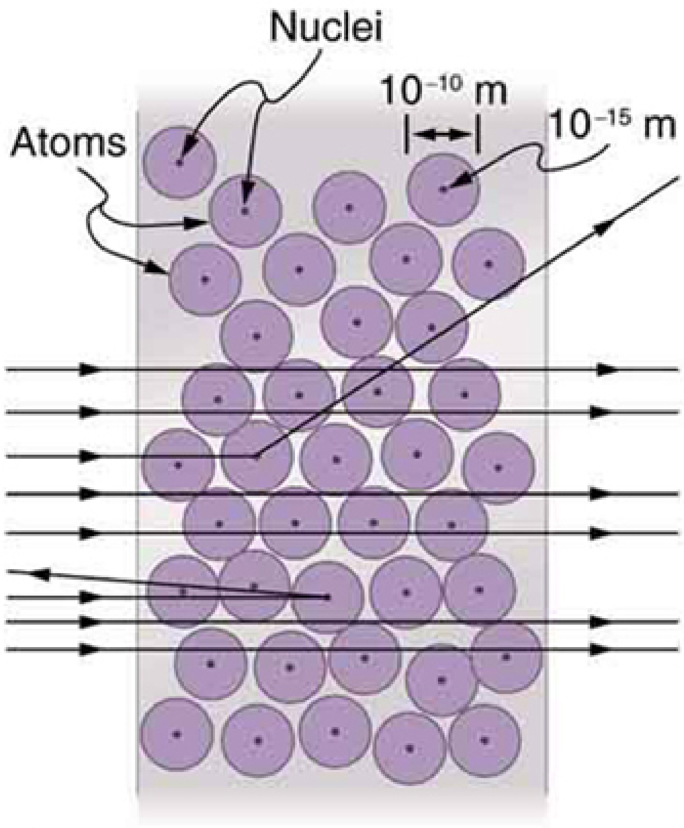
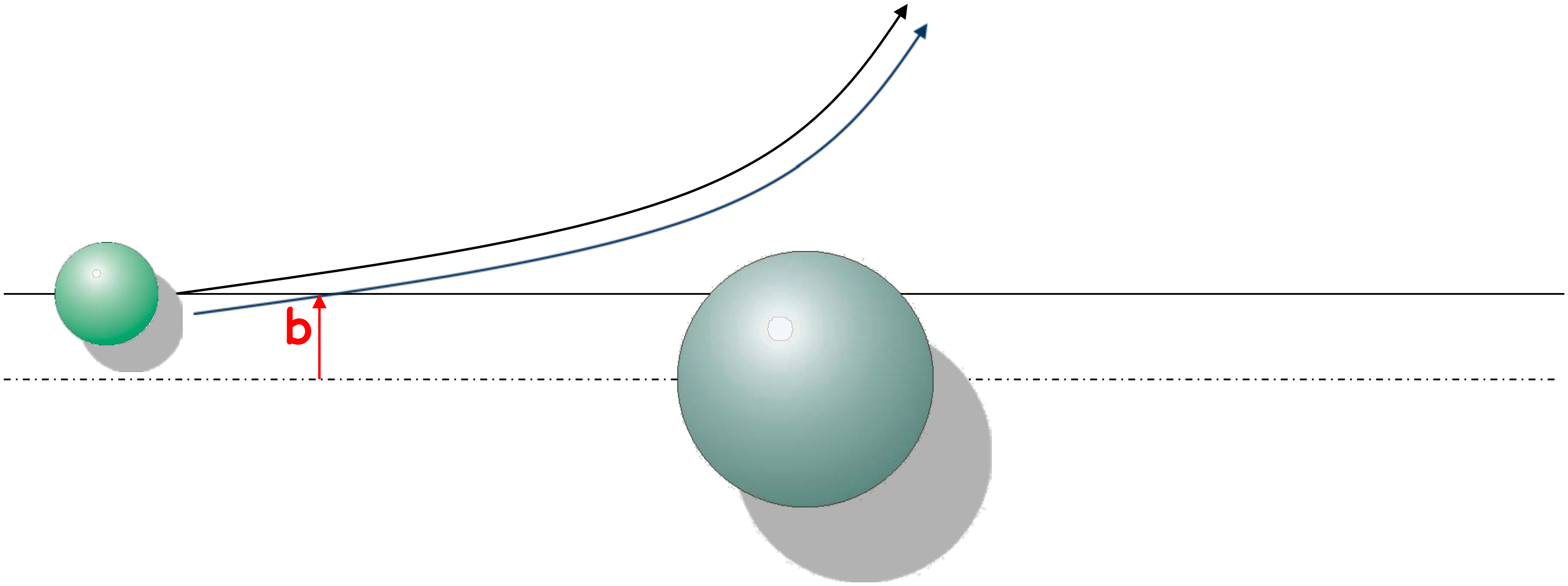
Semi-classical reaction theory

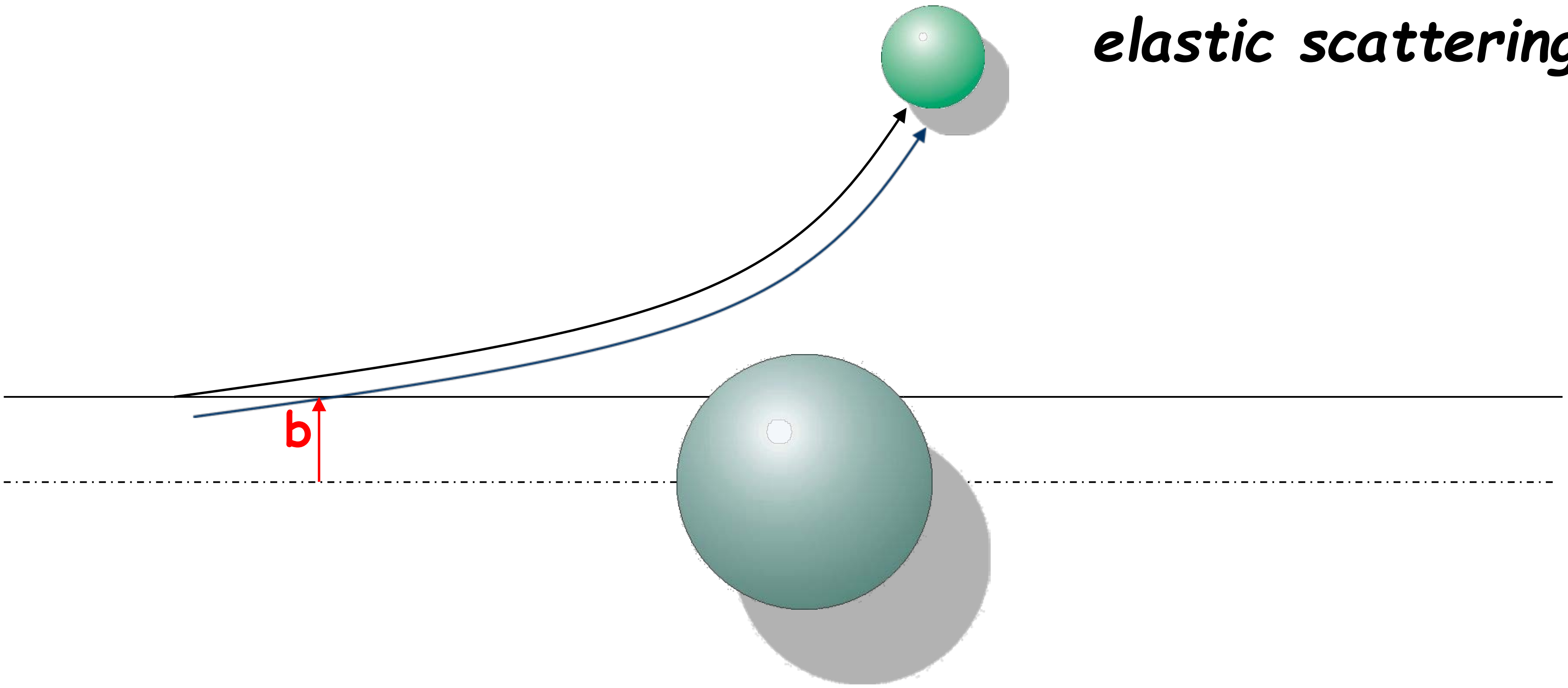




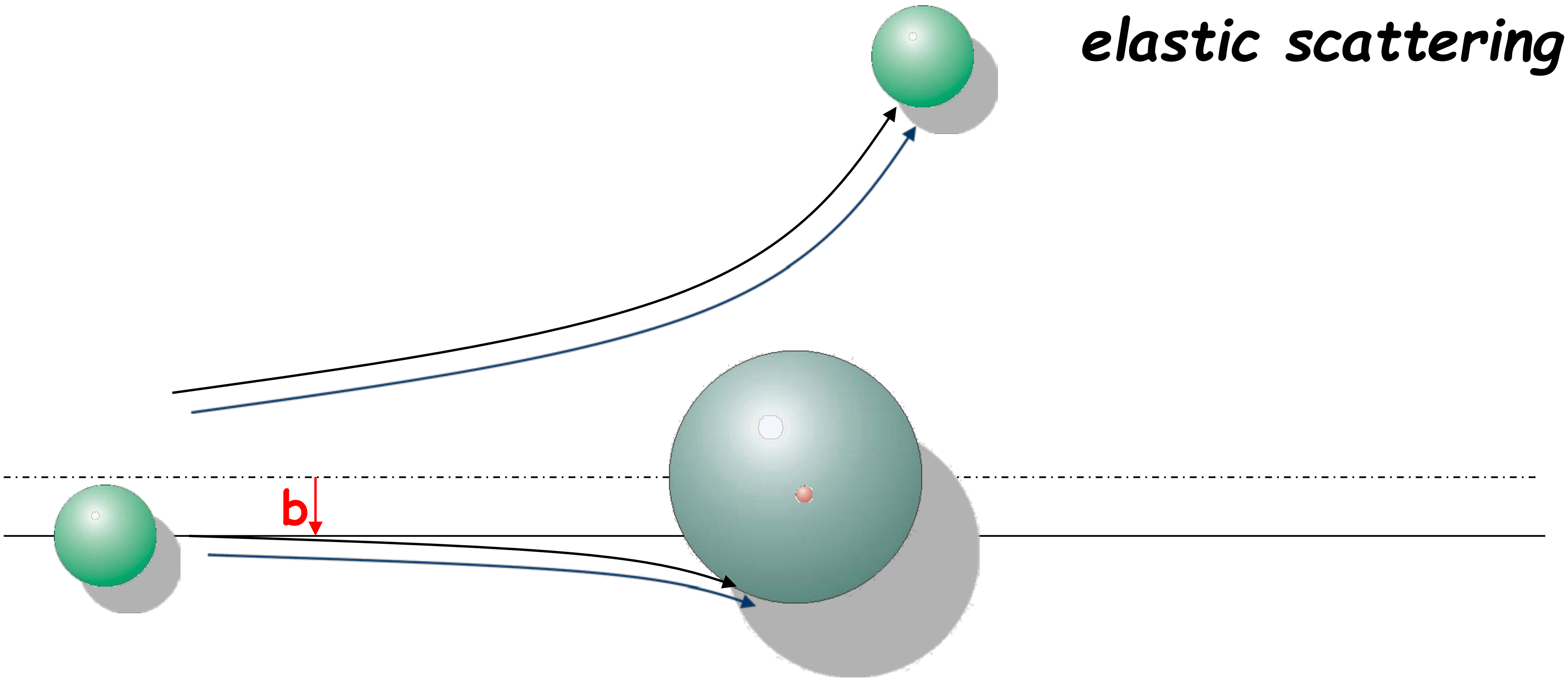
elastic scattering



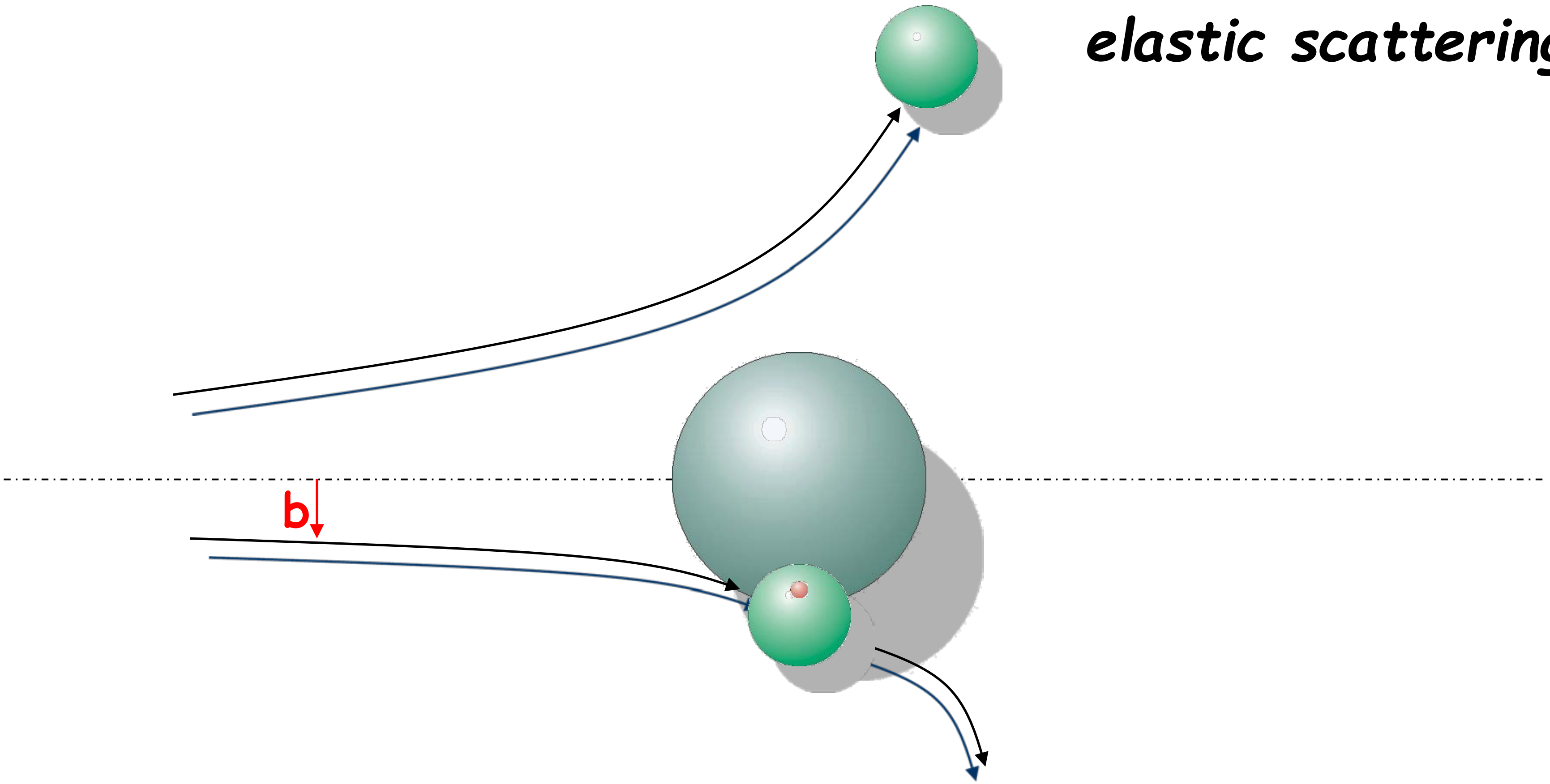
Semi-classical reaction theory



elastic scattering


elastic scattering
b

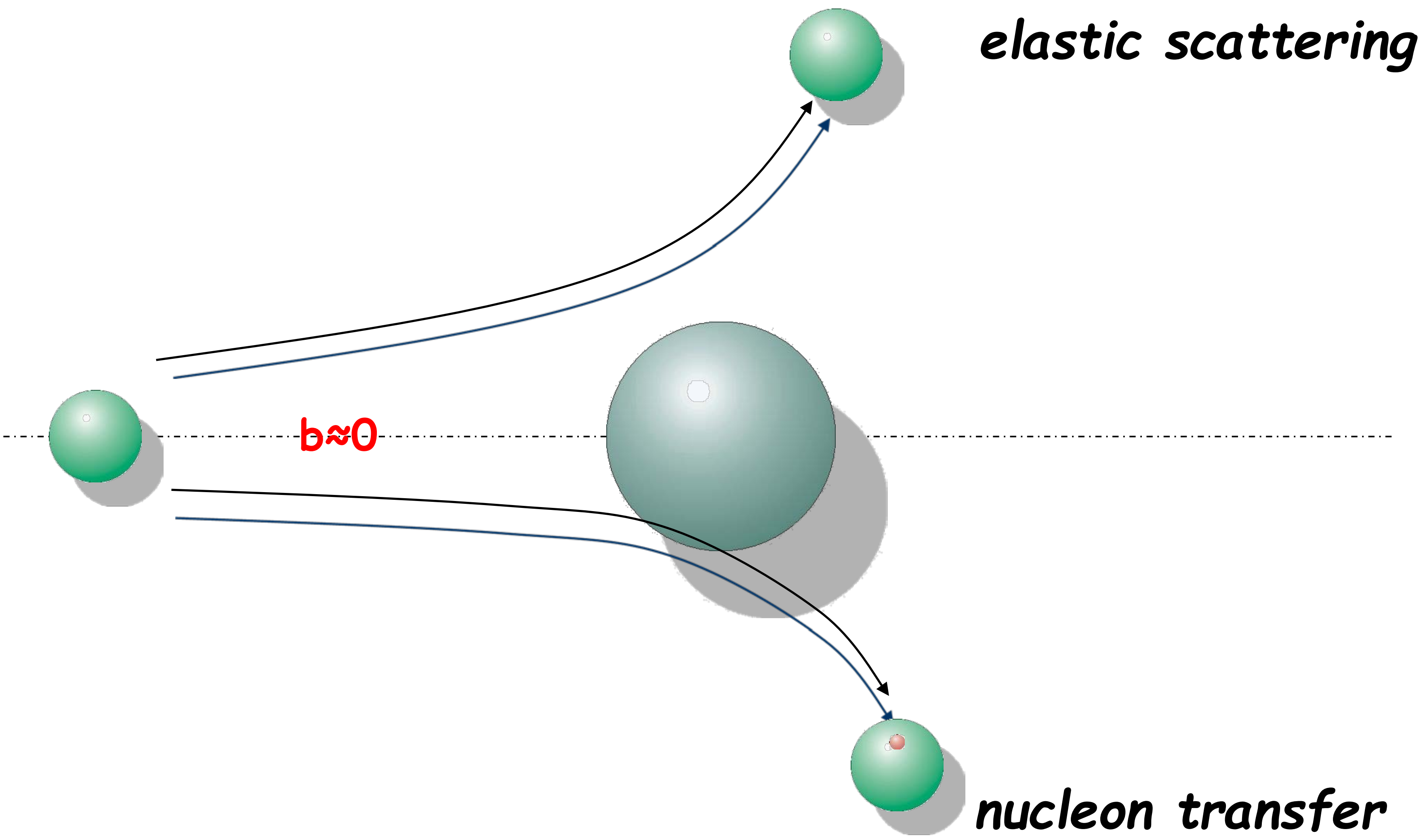
Semi-classical reaction theory



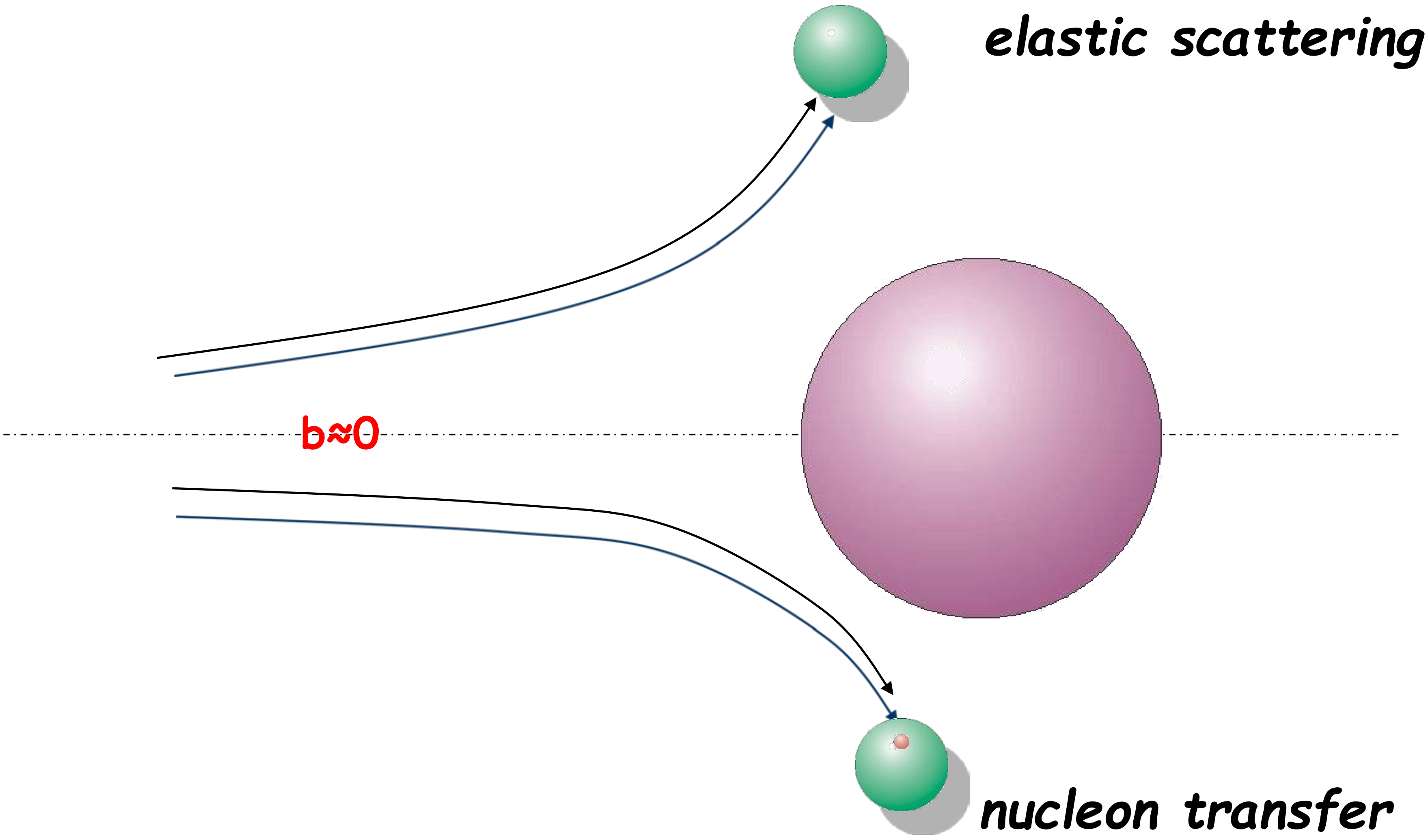
elastic scattering

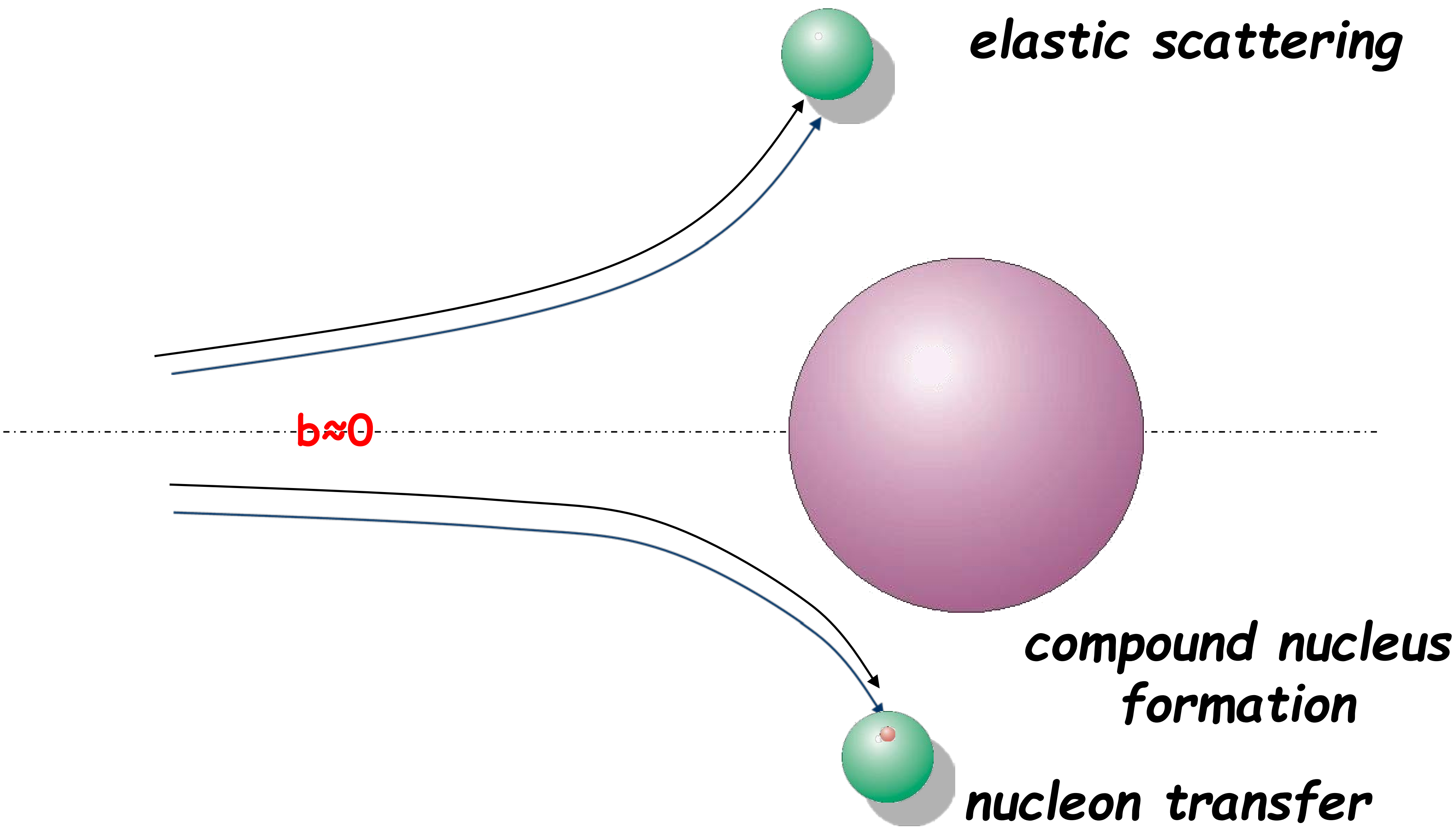
nucleon transfer

Semi-classical reaction theory



Semi-classical reaction theory




elastic scattering
b≈0
compound nucleus formation
nucleon transfer

Nuclear reaction cross section

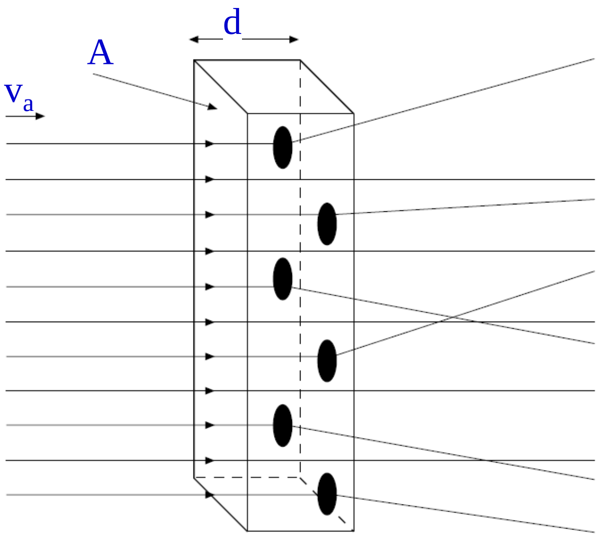
Consider a beam of projectiles of intensity $\mathbf{\Phi_a}$ particles/sec which hits a thin foil of target nuclei with the result that the beam is attenuated by reactions in the foil such that the transmitted intensity is $\mathbf{\Phi}$ particles/sec.
The fraction of the incident particles disappear from the beam, i.e. react, in passing through the foil is given by

$$d\Phi = -\Phi \cdot n_b \cdot \sigma \cdot dx$$

The number of reactions that are occurring is the difference between the initial and transmitted flux

$$\Phi_{initial} - \Phi_{trans} = \Phi_{initial}(1 - exp[-n_b \cdot d \cdot \sigma])$$

$$\approx \Phi_{initial} \cdot N_b \cdot \sigma \qquad \text{(for thin target)}$$



$\Phi_a = n_a \cdot v_a$ $\qquad N_b = n_b \cdot A \cdot d$

Example:

A particle current of 1 pnA consists of $6 \cdot 10^9$ projectiles/s.

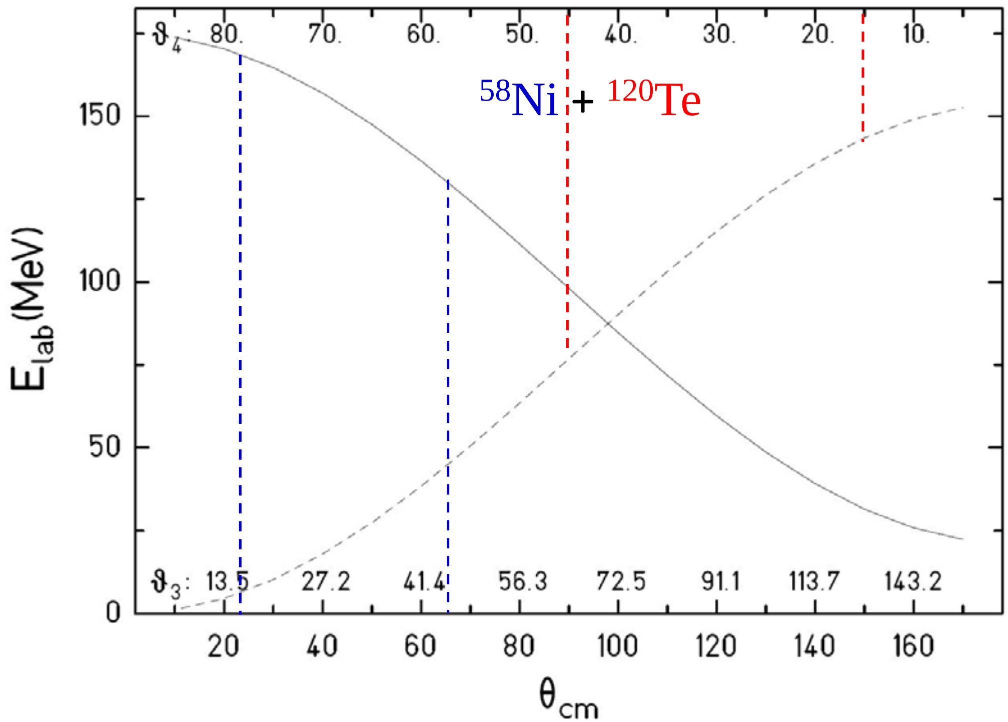
A ^{132}Sn target (1 mg/cm^2) consists of $5 \cdot 10^{18}$ nuclei/cm^2

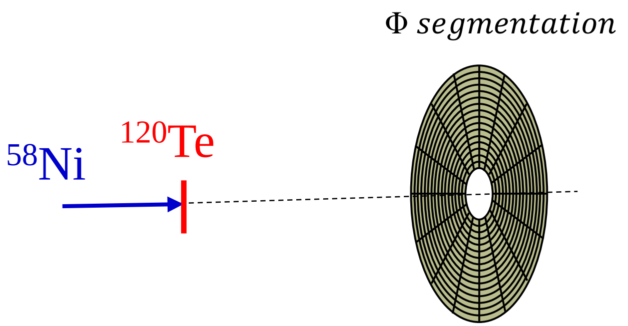
$$\frac{6 \cdot 10^{23} \cdot 10^{-3}\ g/cm^2}{132 g} = 4.5 \cdot 10^{18} \quad \left[\frac{target\ nuclei}{cm^2}\right]$$

Luminosity = projectiles [s^{-1}] · target nuclei [cm^{-2}]

Luminosity (projectile → ^{132}Sn) = $3 \cdot 10^{28}$ [s^{-1}cm^{-2}]

Reaction rate [s^{-1}] = luminosity · cross section [cm^2]
= projectiles [s^{-1}] · target nuclei [cm^{-2}] · cross section [cm^2]





projectile and target nucleus
are measured in PPAC

electrode is made of concentric rings
(proportional to polar angle ϑ)

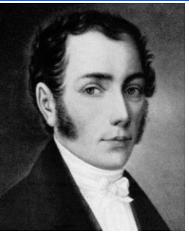
$$15^0 \leq \vartheta \leq 45^0$$

Solid angle: $\Omega = \iint sin\vartheta \, d\vartheta \, d\varphi$

$$\Omega = 2\pi \cdot (1 - cos\vartheta)$$

PPAC: 1.626 sr

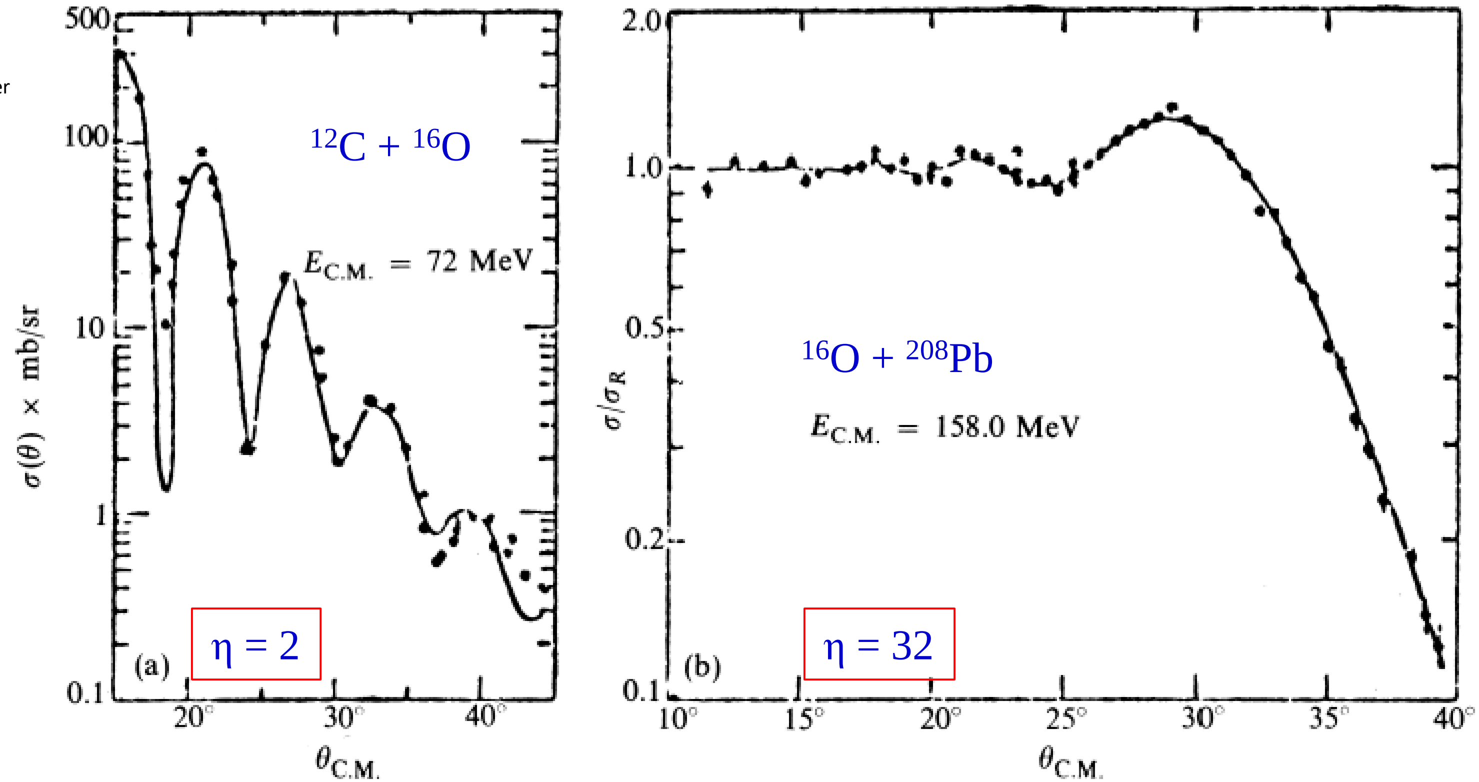
Elastic Scattering



Joseph von Fraunhofer
1787 – 1826

Fraunhofer (left) **and Fresnel** (right) **diffraction**

Augustin Jean Fresnel
1788 -1827



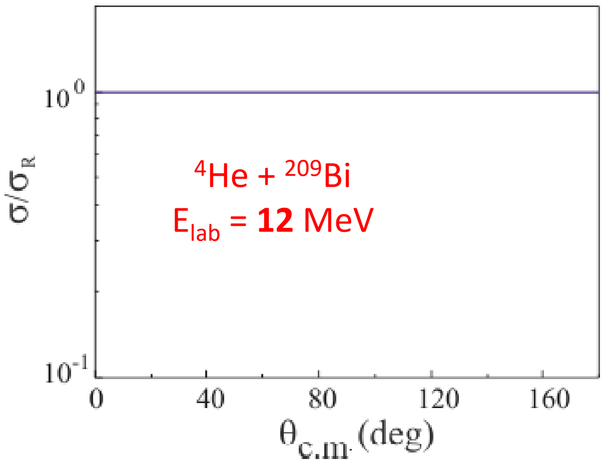
Born approximation (quantum description) or ***classical description***: $\eta = \frac{a}{ƛ}$

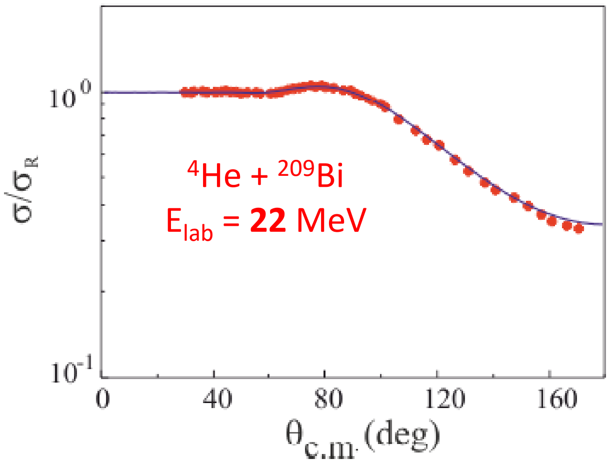
half distance of closest approach for head-on collision $a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2} \quad [fm]$

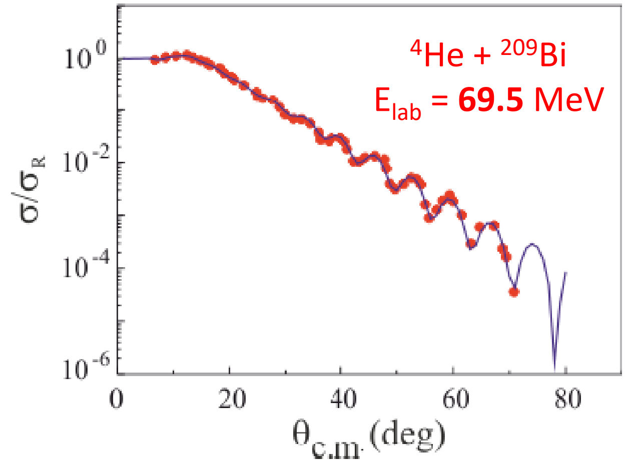
$$\eta = k_\infty \cdot a = 0.157 \cdot Z_1 Z_2 \cdot \sqrt{\frac{A_1}{T_{lab}}}$$

wave length of projectile $ƛ = (k_\infty)^{-1}$ $k_\infty = 0.219 \cdot \frac{A_2}{A_1 + A_2} \cdot \sqrt{A_1 \cdot T_{lab}} \quad [fm^{-1}]$

Elastic Scattering





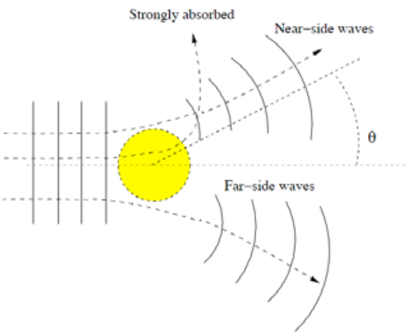


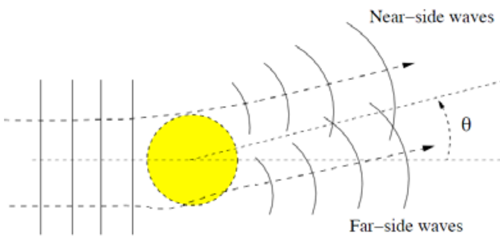
Rutherford scattering
$\eta = 15$

Fresnel scattering
$\eta = 11$

Fraunhofer scattering
$\eta = 6$

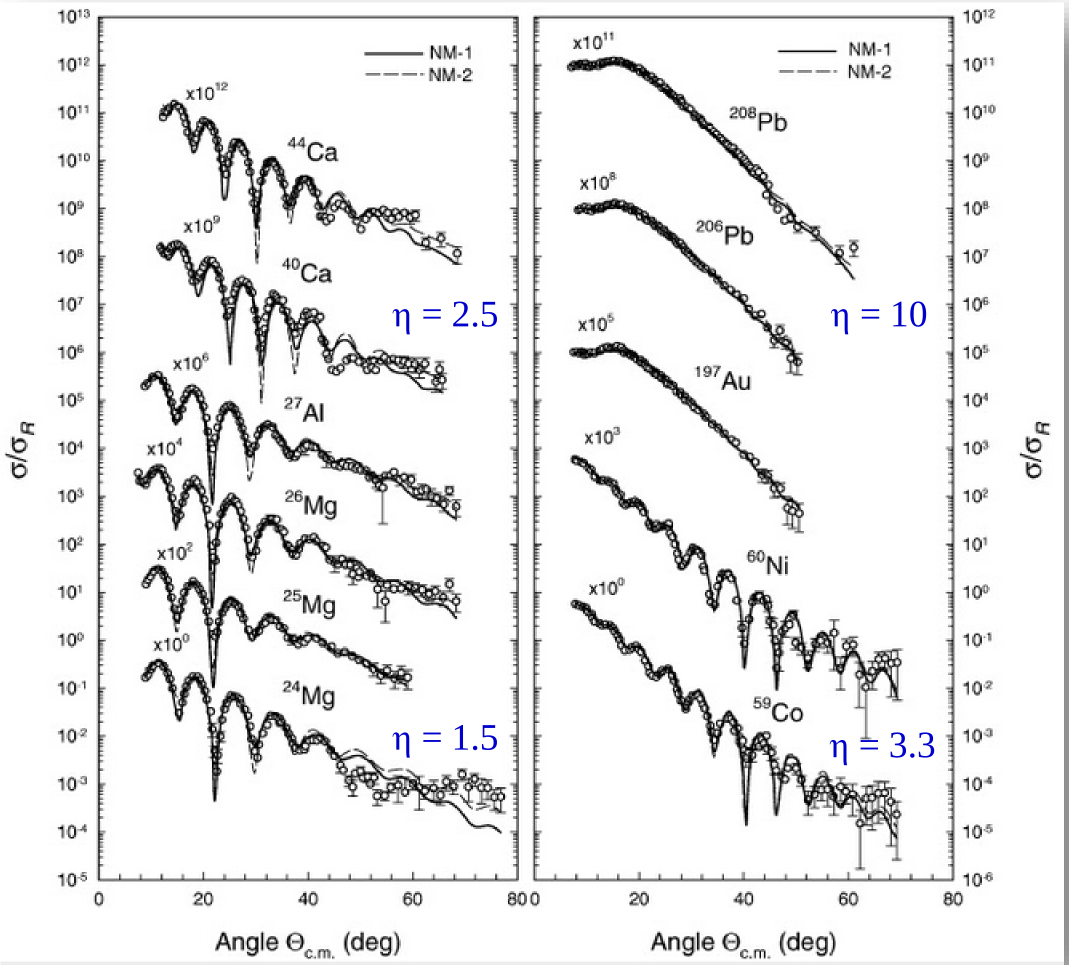
Transition from classical (optical) picture to quantum picture





Elastic Scattering

^{6}Li elastic scattering @ 88 MeV



particle

Fresnel scattering (η≥10)

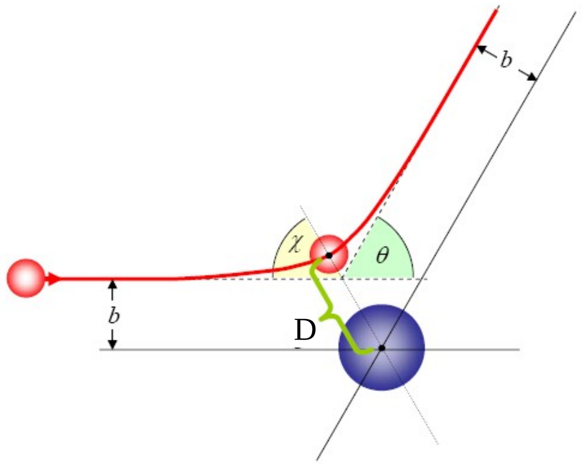
wave

Fraunhofer scattering (η<10)

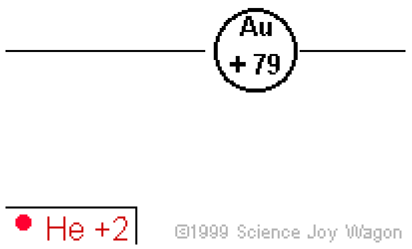
Oscillation in angular distribution → good angular resolution required

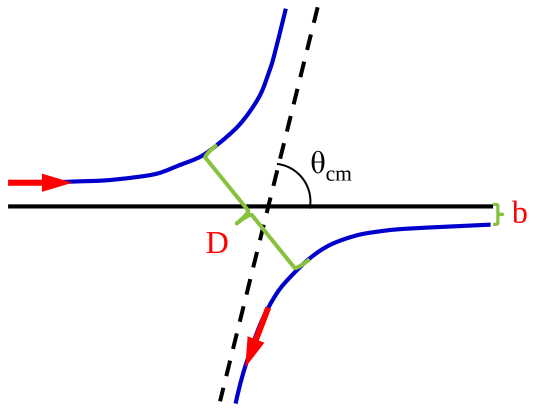
S. Hossain et al. Phys. Scr. 87 (2013) 015201

Scattering parameters



$$\theta = \pi - 2\chi$$





center of mass system

impact parameter:

$$b = a \cdot cot \frac{\theta_{cm}}{2}$$

distance of closest approach:

$$D = a \cdot \left[sin^{-1} \frac{\theta_{cm}}{2} + 1 \right]$$

orbital angular momentum:

$$\ell = k_\infty \cdot b = \eta \cdot cot \frac{\theta_{cm}}{2}$$

half distance of closest approach in a head-on collision ($\theta_{cm}=180^0$):

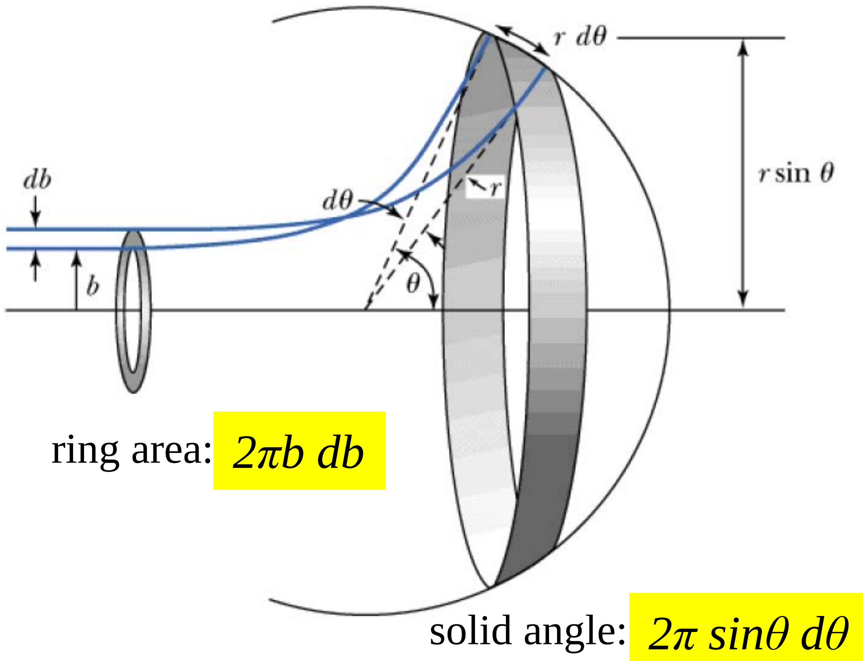
$$a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2} \quad [fm]$$

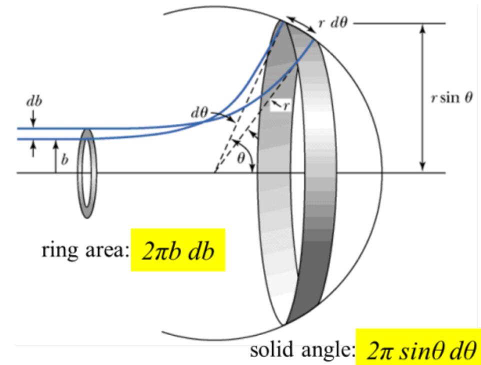
asymptotic wave number:

$$k_\infty = 0.219 \cdot \frac{A_2}{A_1 + A_2} \cdot \sqrt{A_1 \cdot T_{lab}} \quad [fm^{-1}]$$

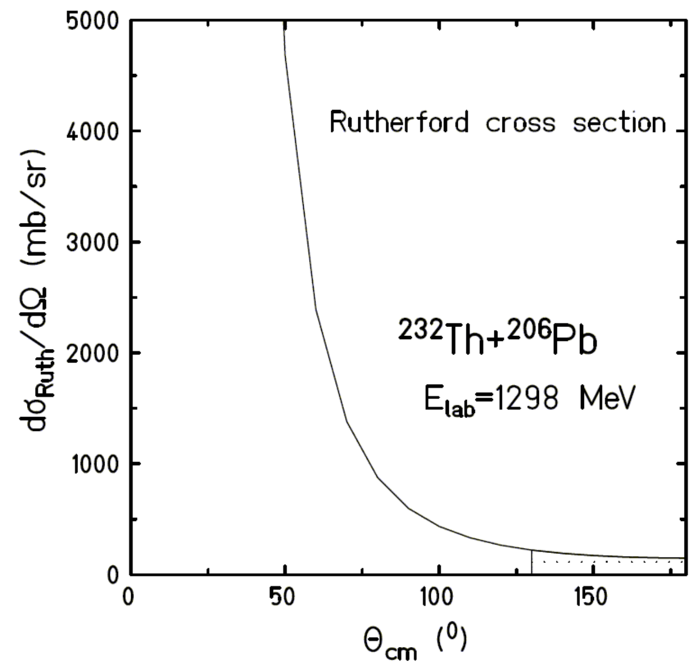
Sommerfeld parameter:

$$\eta = k_\infty \cdot a = 0.157 \cdot Z_1 Z_2 \cdot \sqrt{\frac{A_1}{T_{lab}}}$$

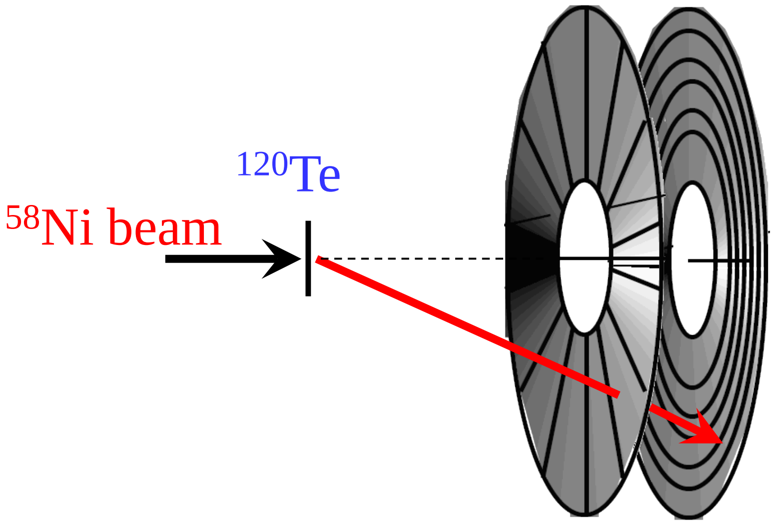




$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$

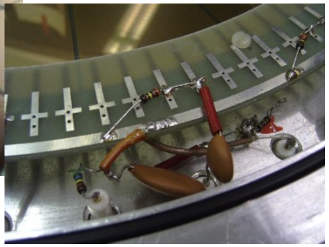


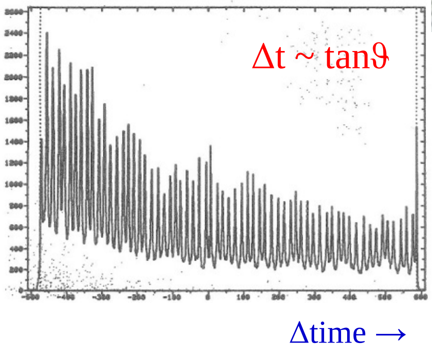
Annular gas-filled parallel-plate avalanche counter (PPAC)

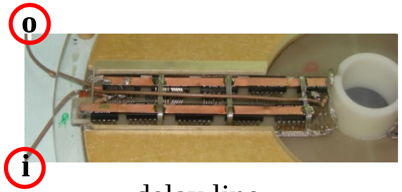


V_0 ~ 500 V
p = 5-10 Torr
gap ~ 3 mm (anode-cathode)

$$\varphi_p \qquad \approx \tan \vartheta_p$$



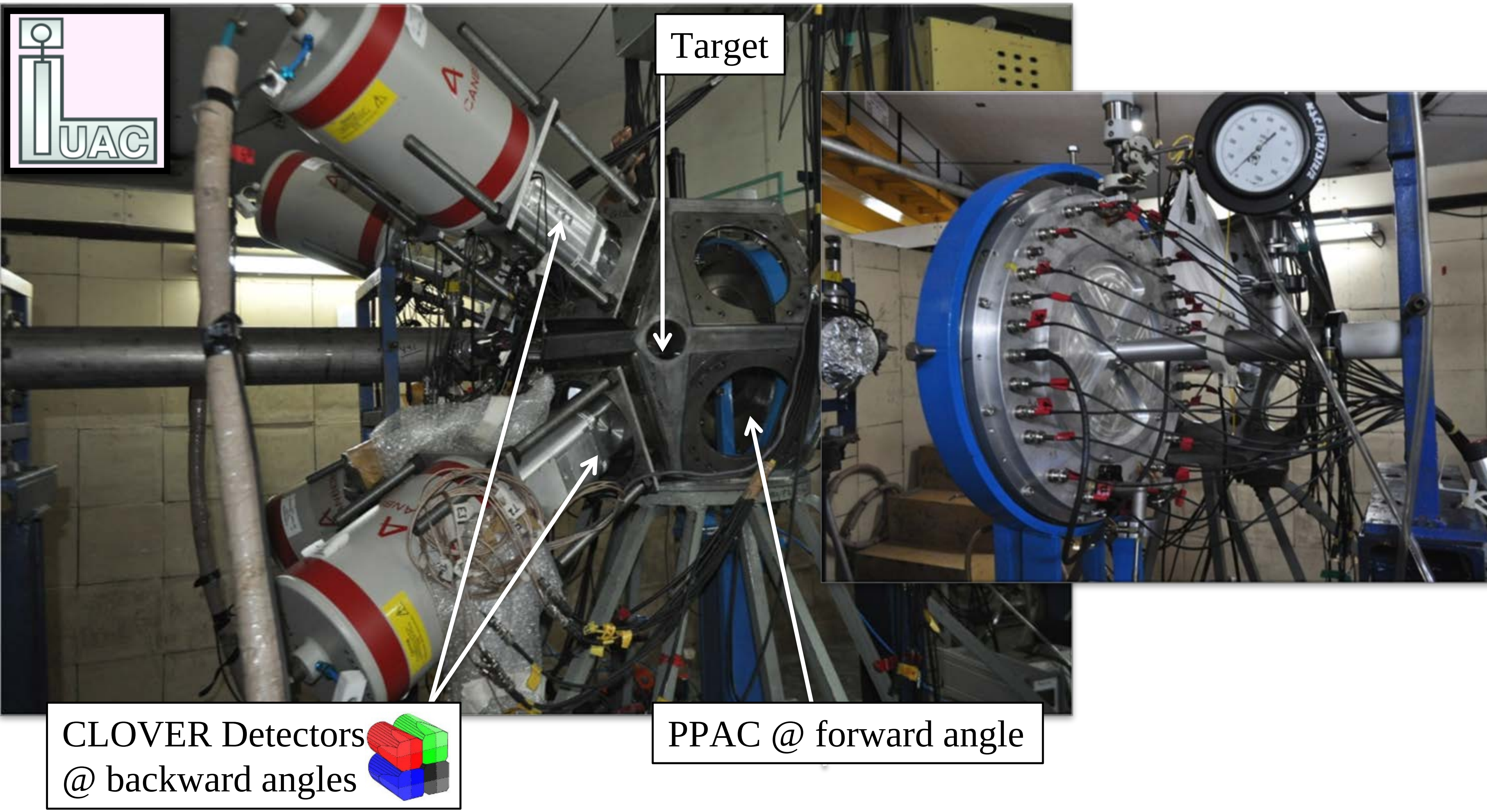




delay line

A. Jhingan detector laboratory at IUAC

Experimental set-up at IUAC

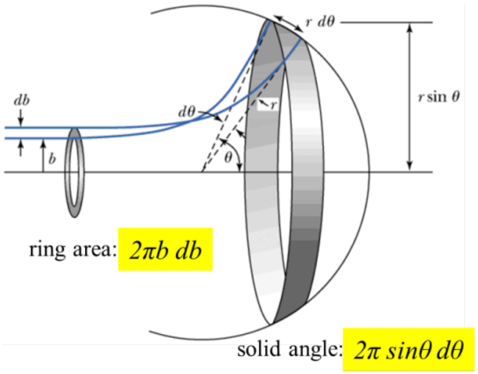


Summary

- impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2} \qquad a = \frac{Z_p \cdot Z_t \cdot e^2}{2 \cdot E_{cm}}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$



- angular momentum and scattering angle:

$$\ell = \eta \cdot cot\frac{\theta}{2} \qquad \eta = k_\infty \cdot a \qquad k_\infty = \frac{\sqrt{2 \cdot m \cdot E_{cm}}}{\hbar}$$

$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$

- distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

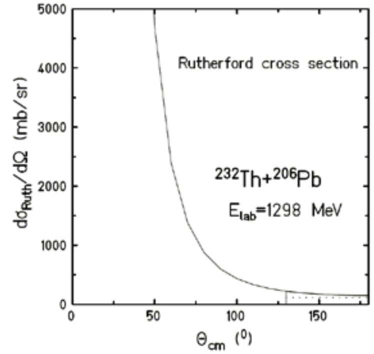
$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$

Summary

❖ impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$



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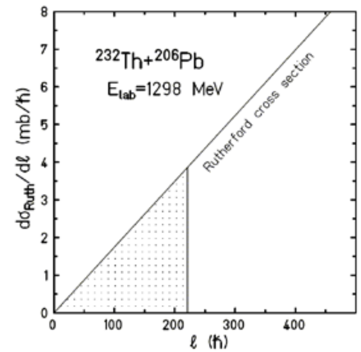

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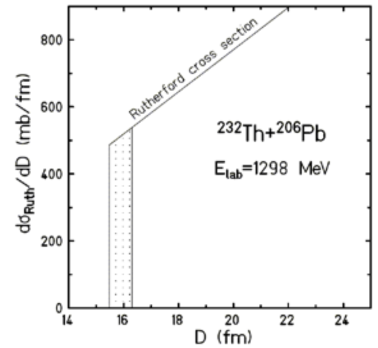
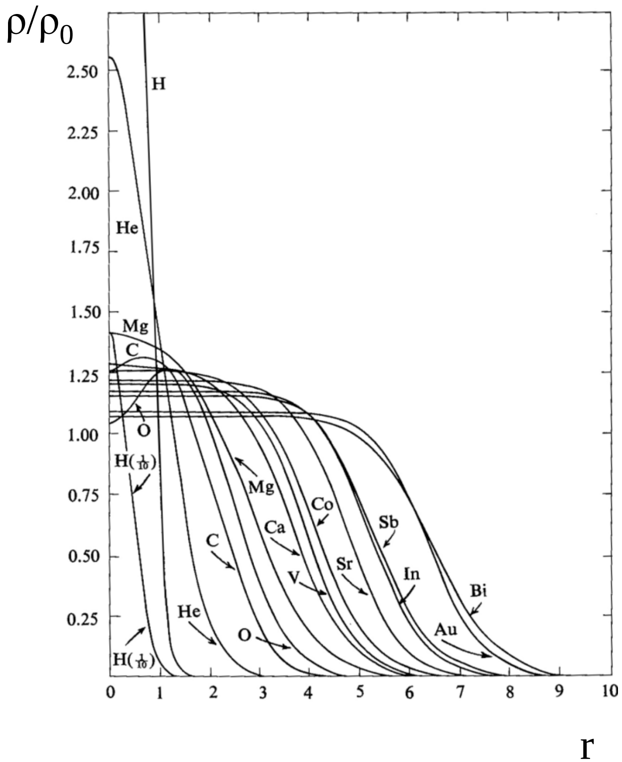
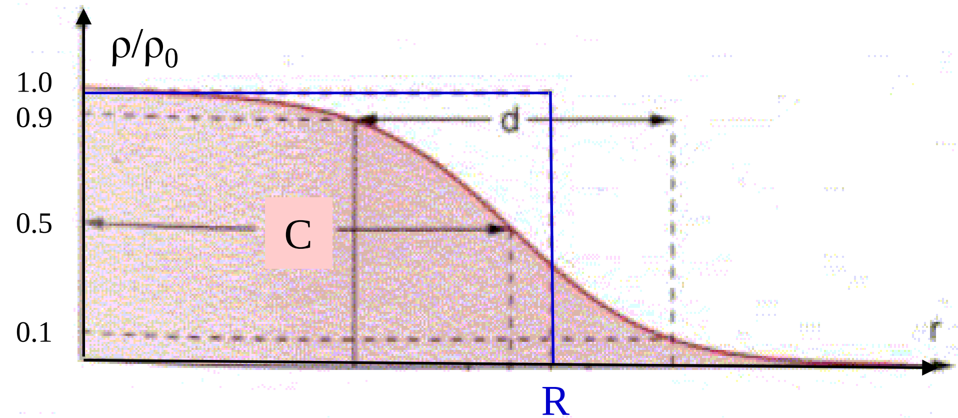


Reminder: Nuclear radius





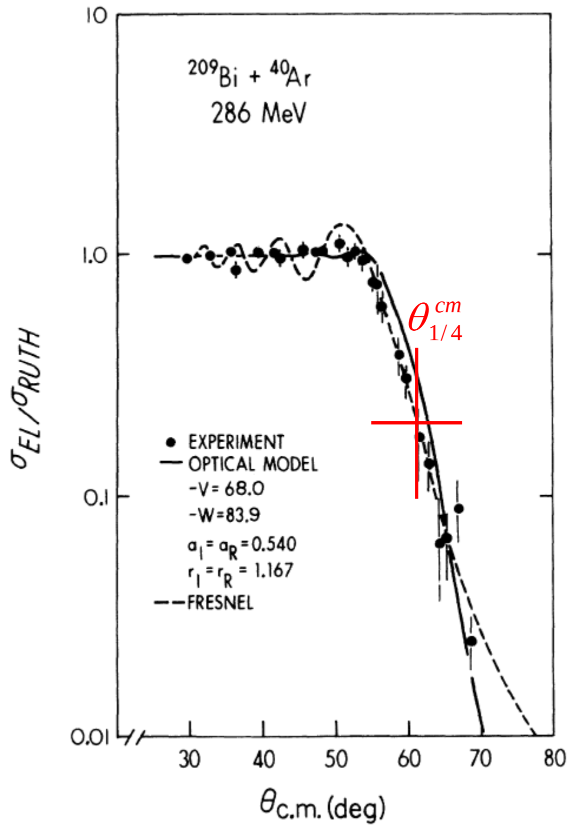
nuclear radius of a homogenous charge distribution:

$$R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$$

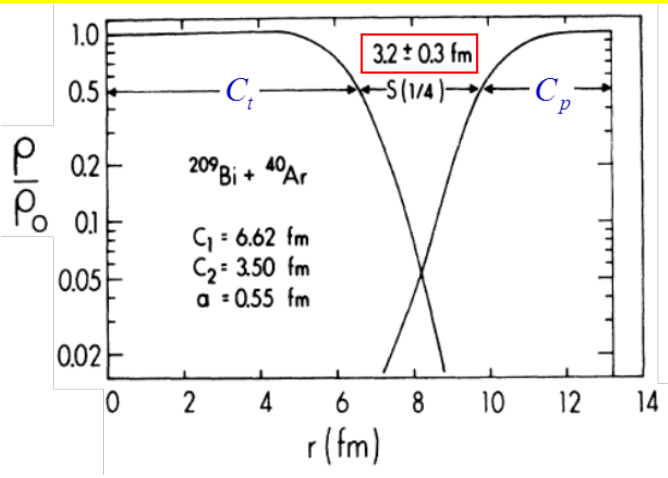
nuclear radius of a Fermi charge distribution:

$$C_i = R_i \cdot \left(1 - R_i^{-2}\right) \quad [fm]$$

Elastic scattering and the nuclear radius



Nuclear density distributions at the nuclear interaction radius



$$R_{int} = C_p + C_t + 4.49 - \frac{C_p + C_t}{6.35} \quad [fm]$$

$C_i = R_i \cdot (1 - R_i^{-2}) \quad [fm]$ $\qquad R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$

$\theta_{1/4} = 60^0 \rightarrow R_{int} = 13.4$ [fm]

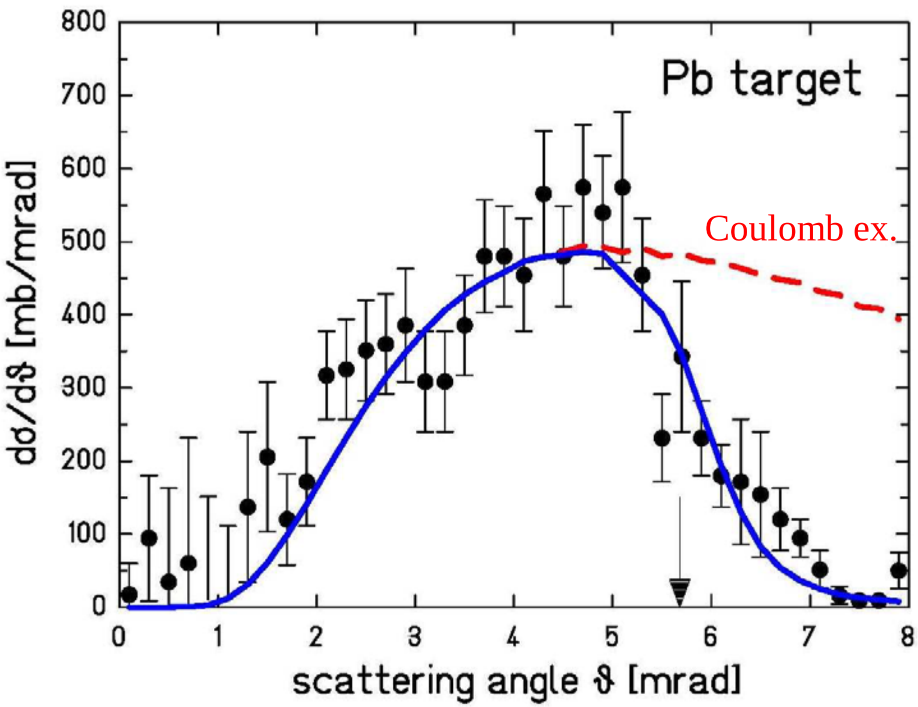
$\rightarrow \ell_{gr} = 152$ [ħ]

Nuclear interaction radius: (distance of closest approach)

$$R_{int} = D = a \cdot \left[sin^{-1} \frac{\theta_{1/4}}{2} + 1 \right]$$

High-energy Coulomb excitation

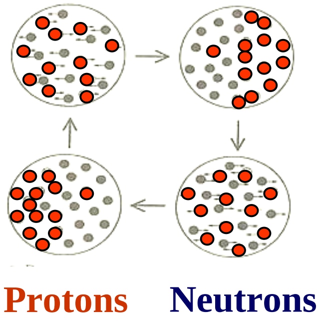
grazing angle

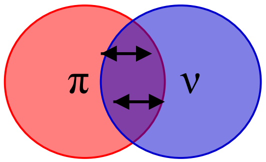


^{136}Xe on ^{208}Pb at 700 MeV/u

excitation of giant dipole resonance

$R_{\text{int}} = 15.0\ fm \ \rightarrow\ \vartheta_{1/4} = 5.7\ mrad$





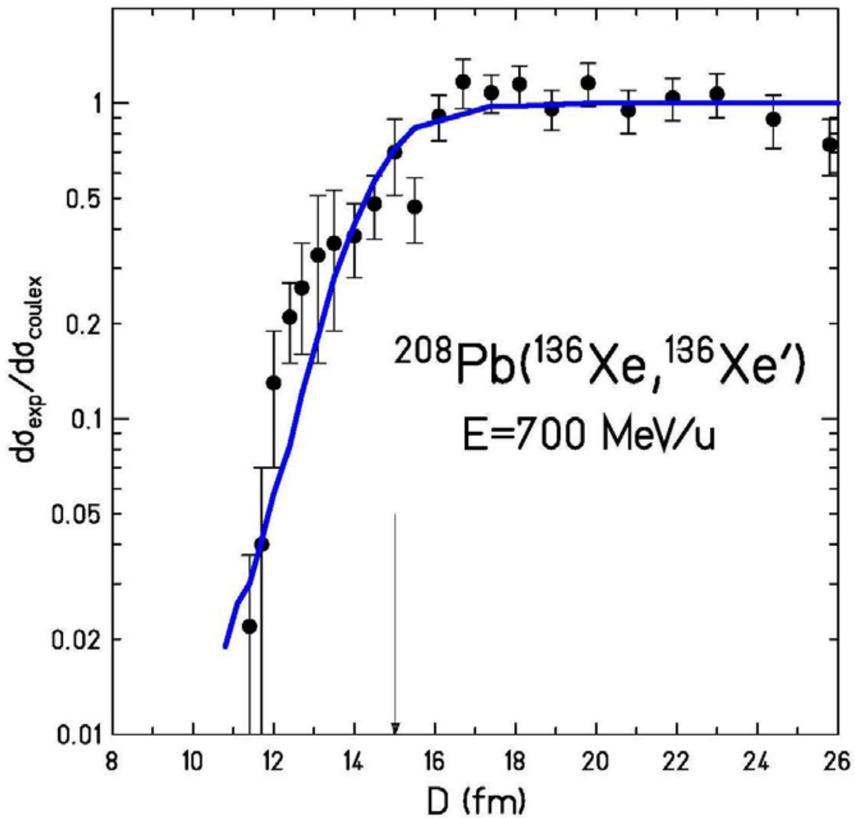
For relativistic projectiles ($\theta_{cm} \approx \vartheta_{lab}$):

$$D = \frac{2 \cdot Z_P Z_T e^2}{m_0 c^2 \beta^2 \gamma} \cdot \frac{1}{\vartheta}$$

A.Grünschloß et al., Phys. Rev. C60 051601 (1999)

High-energy Coulomb excitation

grazing angle



^{136}Xe on ^{208}Pb at 700 MeV/u

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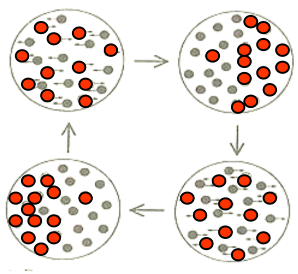
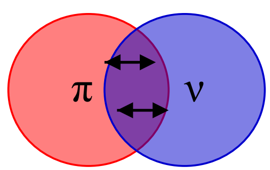


Protons **Neutrons**



For relativistic projectiles ($\theta_{cm} \approx \vartheta_{lab}$):

$$D = \frac{2 \cdot Z_P Z_T e^2}{m_0 c^2 \beta^2 \gamma} \cdot \frac{1}{\vartheta}$$

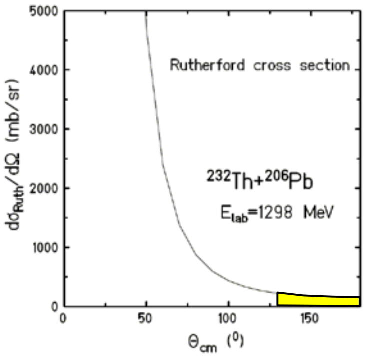
A.Grünschloß et al., Phys. Rev. C60 051601 (1999)

Elastic scattering and nuclear reactions

- impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$

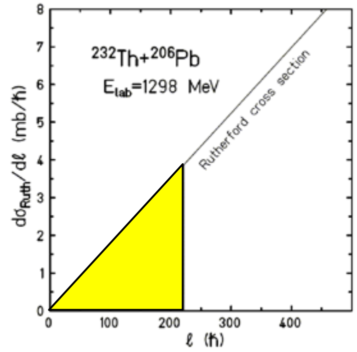


$\theta_{1/4} = 132^0$

- angular momentum and scattering angle:

$$\ell = \eta \cdot cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$

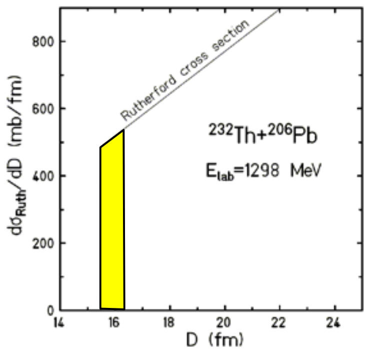


$\ell_{gr} = 206\ \hbar$

- distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$



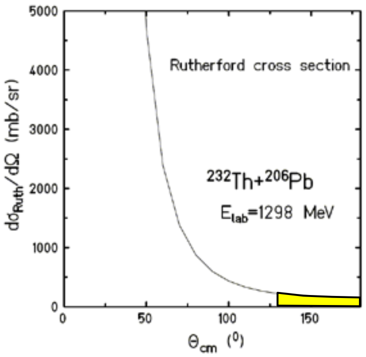
$R_{int} = 16.2$ fm

Elastic scattering and nuclear reactions

❖ impact parameter and scattering angle:

$$b = a \cdot cot\,\frac{\theta}{2}$$

$$\sigma_{reaction} = 2\pi a^2 \cdot \left[\left(1 - cos\ \theta_{1/4}^{cm}\right)^{-1} - 0.5\right]$$

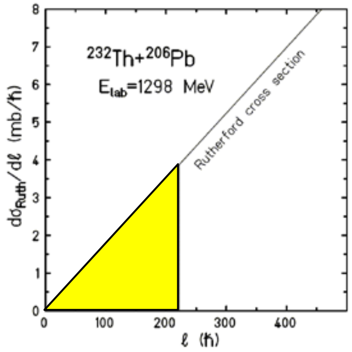


$\theta_{1/4} = 132^0$ $a = 7.73$ fm

❖ angular momentum and scattering angle:

$$\ell = \eta \cdot cot\,\frac{\theta}{2}$$

$$\sigma_{reaction} = \frac{\pi}{k_\infty^2} \cdot \ell_{gr}(\ell_{gr} + 1)$$

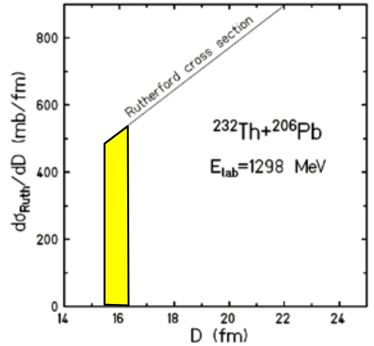


$\ell_{gr} = 206$ ħ $k_\infty = 59.9$ fm^{-1}

❖ distance of closest approach and scattering angle:

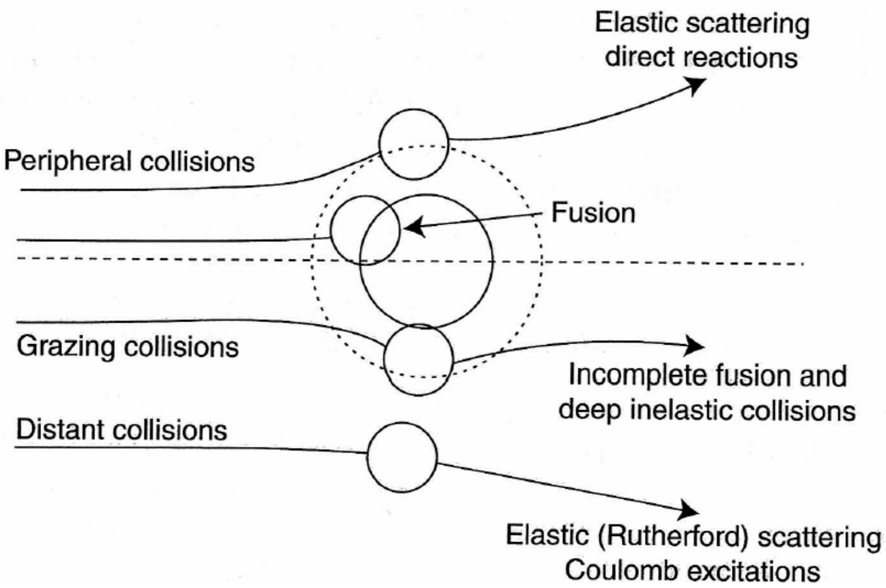
$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

$$\sigma_{reaction} = \pi \cdot R_{int}^2 \cdot \left(1 - \frac{V_C(R_{int})}{E_{cm}}\right)$$

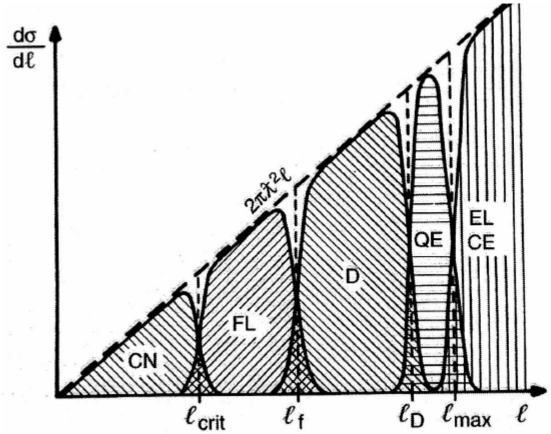


$R_{int} = 16.2$ fm $V_C(R_{int}) = 656$ MeV

Classification of heavy ion collisions



partial cross section vs. angular momentum



- CN: compound nucleus
- FL: fusion-like
- D: deep inelastic
- QE: quasi elastic
- CE: Coulomb excitation
- EL: elastic