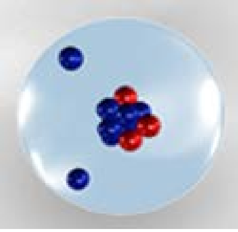
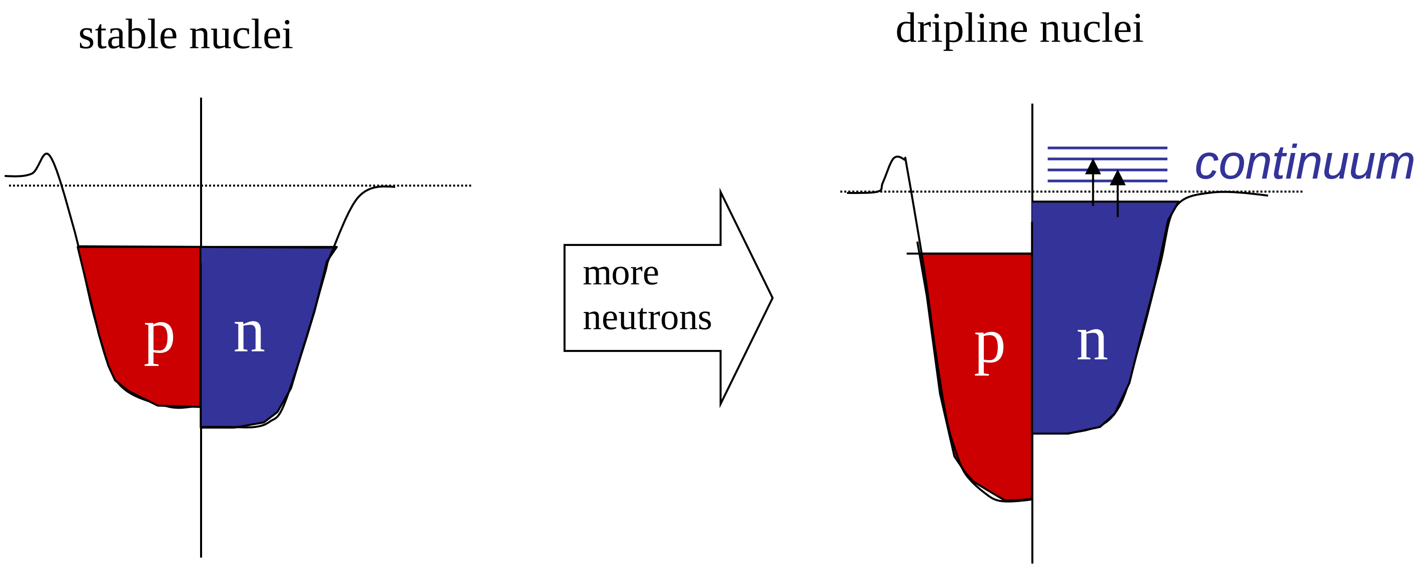
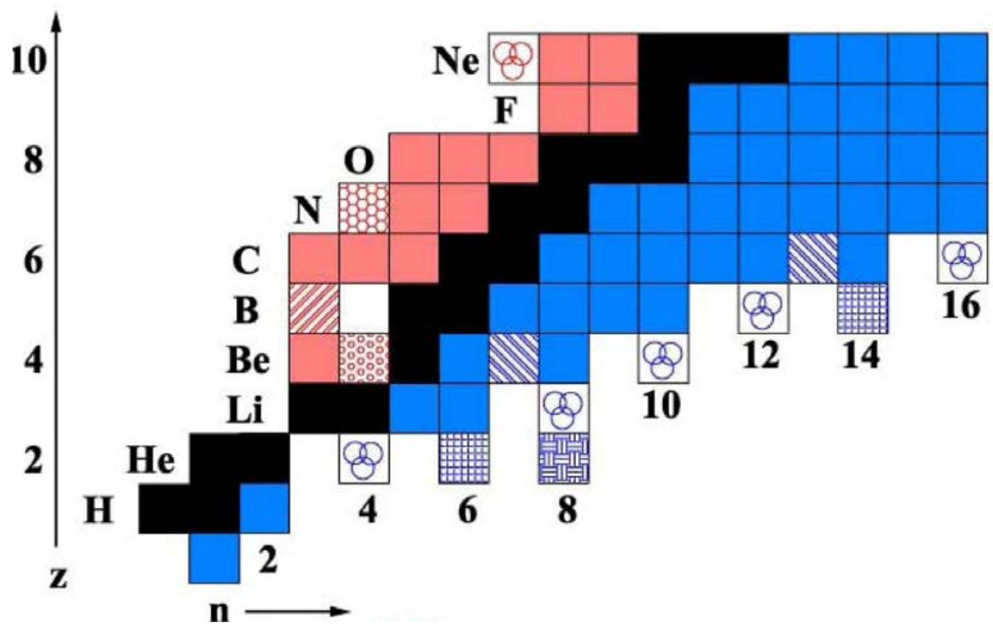
Limits of stability: halo nuclei







Measurement of the total reaction cross section

- *800 MeV/u ^{11}B primary beam*
- *Fragmentation*
- *FRagment Separator FRS*

$$\sigma_I(p,t) = \pi \cdot [R_I(p) + R_I(t)]^2$$

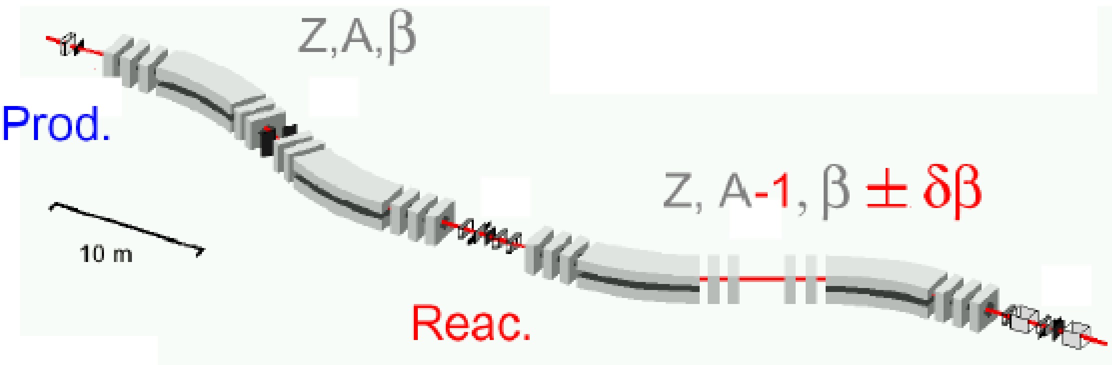
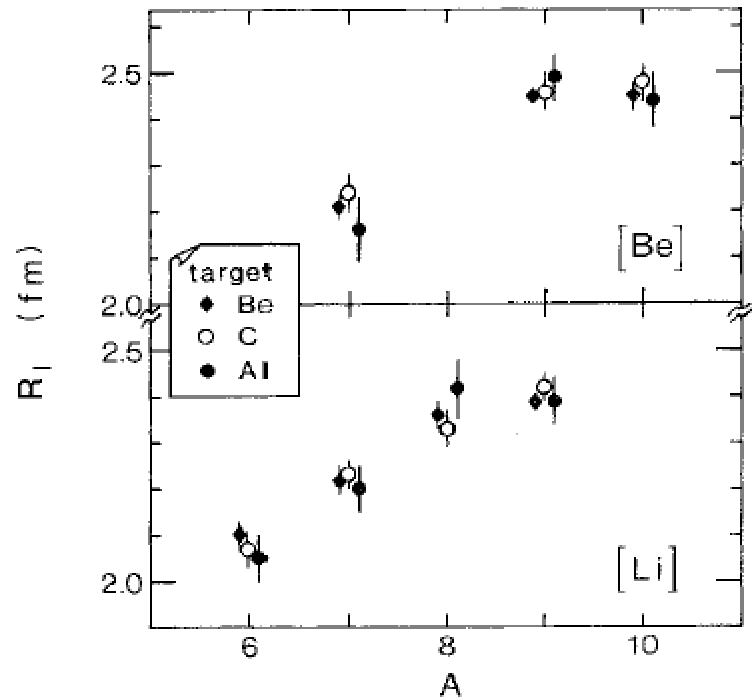
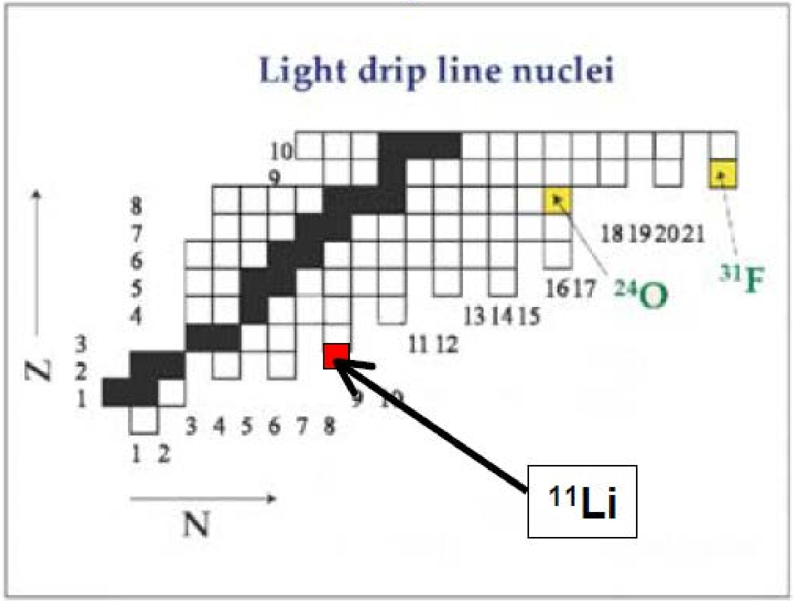


TABLE I. Interaction cross sections (σ_I) in millibarns.

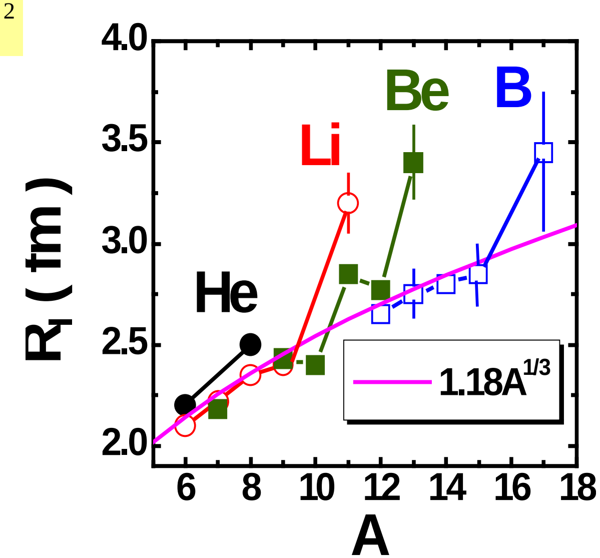
Beam	Target Be	C	Al
^{6}Li	651 ± 6	688 ± 10	1010 ± 11
^{7}Li	686 ± 4	736 ± 6	1071 ± 7
^{8}Li	727 ± 6	768 ± 9	1147 ± 14
^{9}Li	739 ± 5	796 ± 6	1135 ± 7
^{7}Be	682 ± 6	738 ± 9	1050 ± 17
^{9}Be	755 ± 6	806 ± 9	1174 ± 11
^{10}Be	755 ± 7	813 ± 10	1153 ± 16



Measurement of the total reaction cross section



$$\sigma_I(p,t) = \pi \cdot [R(p) + R(t)]^2$$



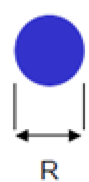
^{11}Li is the heaviest bound Li isotope

^{10}Li not bound

$S_{2n}(^{11}Li)$ = 295(35) keV

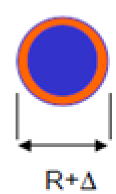
only bound in its ground state

reason for larger radius?



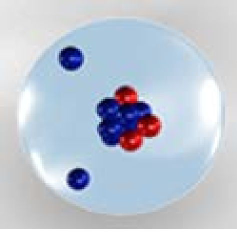
$$\sigma = \pi R^2$$

deformation



extended wave function

$$\sigma = \pi (R + \Delta)^2$$



At the limit of the strong force – halo nuclei

reason for larger radius?

deformation

extended wave function

$\Rightarrow$ measurements of magnetic moment and quadrupole moment

$$\mu\left({}^{11}Li\right) = 3.667(3)\cdot\mu_N$$

$$\mu_{sp}\left(\pi\, p_{3/2}\right) = 3.79\cdot\mu_N$$

^{11}Li consists in its ground state of paired neutrons and a $p_{3/2}$ proton

- ***g-factor of nucleons:***

proton: $g_\ell = 1$; $g_s = +5.585$

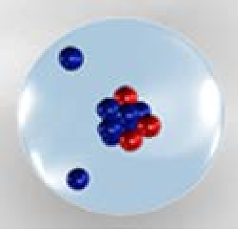
neutron: $g_\ell = 0$; $g_s = -3.82$

proton:

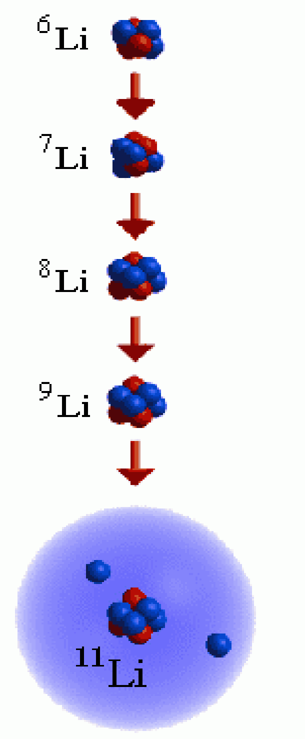
$$\langle\mu_z\rangle = \begin{cases} (j+2.293)\cdot\mu_K & \text{für} \quad j=\ell+1/2 \\ (j-2.293)\cdot\dfrac{j}{j+1}\cdot\mu_K & \text{für} \quad j=\ell-1/2 \end{cases}$$

neutron:

$$\langle\mu_z\rangle = \begin{cases} -1.91\cdot\mu_K & \text{für} \quad j=\ell+1/2 \\ +1.91\cdot\dfrac{j}{j+1}\cdot\mu_K & \text{für} \quad j=\ell-1/2 \end{cases}$$



At the limit of the strong force – halo nuclei



reason for larger radius?
deformation
extended wave function

⇒ measurements of magnetic moment and quadrupole moment

$$\mu\left({}^{11}Li\right) = 3.667(3)\cdot\mu_N$$

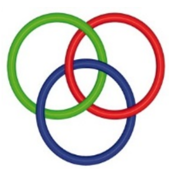
$$\mu_{sp}\left(\pi\, p_{3/2}\right) = 3.79\cdot\mu_N$$

^{11}Li consists in its ground state of paired neutrons and a $p_{3/2}$ proton

$$\frac{Q\left({}^{11}Li\right)}{Q\left({}^{9}Li\right)} = 1.09(5) \qquad Q\left({}^{11}Li\right) = 0.0312(45)\, b$$

→ spherical and large radius not because of deformation

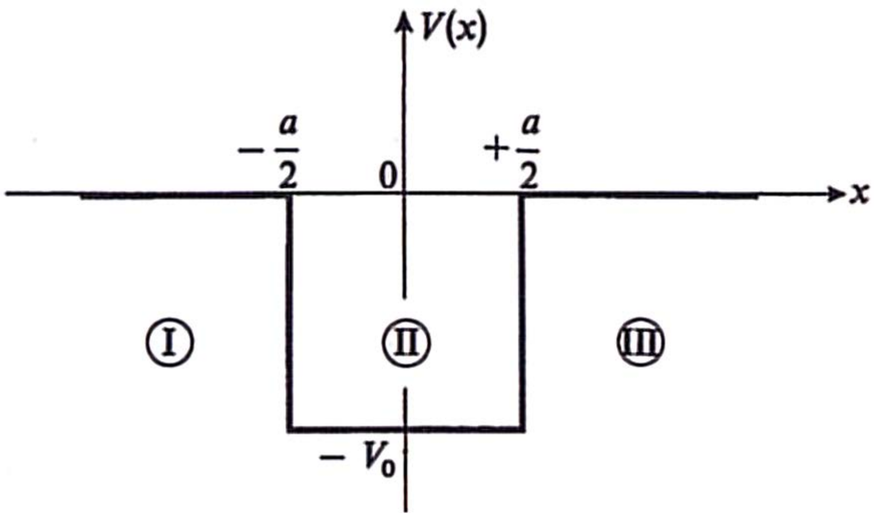
3 borromean rings



HALO:

Exotic nuclei with large neutron excess form nuclei with halo-structure: ^{11}Li nuclei consist of a normal ^{9}Li nucleus with a halo of two neutrons. Halo nuclei form borromean states, they are interlocked in such a way that breaking any cycle allows the others to disassociate.

Single particle potential



outside of the square-well potential:

$$\left\{\frac{d^2}{dx^2}+\frac{2\cdot m}{\hbar^2}\cdot E\right\}\phi(x)=0 \qquad \kappa^2=-\frac{2\cdot m}{\hbar^2}\cdot E$$

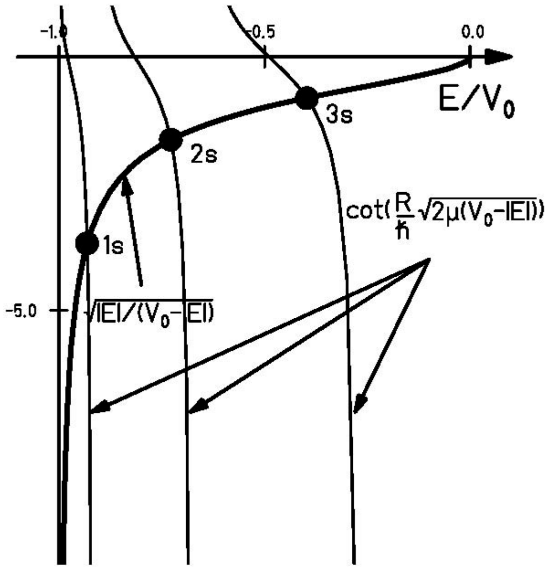
Lösung: $\phi_a(x)=A\cdot e^{-\kappa\cdot x}+B\cdot e^{+\kappa\cdot x}$

inside of the square-well potential:

$$\left\{\frac{d^2}{dx^2}+\frac{2\cdot m}{\hbar^2}\cdot(E+V_0)\right\}\phi(x)=0 \qquad k^2=\frac{2\cdot m}{\hbar^2}\cdot(E+V_0)$$

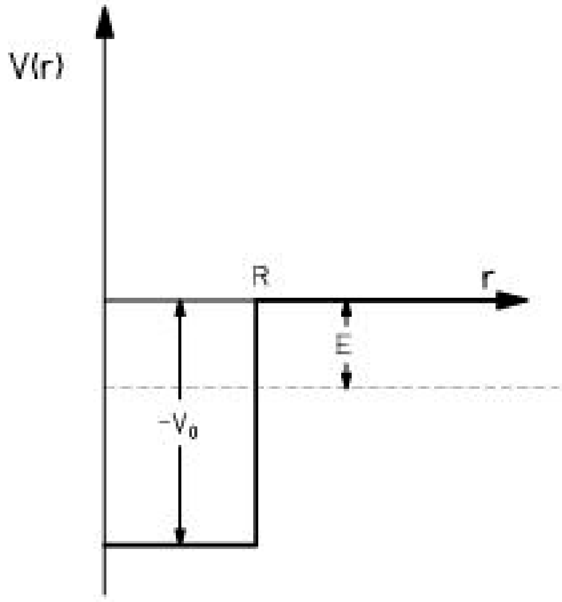
Lösung: $\phi_i(x)=C\cdot\cos(k\cdot x)+D\cdot\sin(k\cdot x)$

continuity of the wave function: $\cot\left(k\cdot\frac{a}{2}\right)=-\frac{\kappa}{k}$



graphical solution of the eigenvalue problem

Energy eigenvalues



Orbital $n\ell$	$E_{n\ell}$ (MeV) ^{36}Ca R=3.96fm
1s	13.16
1p	26.90
1d	44.26
2s	52.61
1f	65.08

Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\cdot\mu}\nabla^2+V(r)\right]\Psi(r)=E\ \Psi(r)$$

$$\Psi(r)=u_{n\ell}(r)\cdot Y_{\ell m}(\vartheta,\varphi)$$

$$\frac{d^2u}{dr^2}+\frac{2}{r}\frac{du}{dr}+\left[\frac{2\cdot\mu}{\hbar^2}(E-V(r))-\frac{\ell\cdot(\ell+1)}{r^2}\right]u(r)=0$$

$$with\quad \mu=\frac{m_{p,n}\cdot(M_A-m_{p,n})}{M_A}\cong m_{p,n}\quad\left(\approx 931.478\frac{MeV}{c^2}\right)$$

ℓ=0 energies:

$$\xi\cdot\cot\xi=-\eta\ \Rightarrow\ \cot\left(0.2187\cdot R\cdot\sqrt{V_0-|E_{ns}|}\right)=-\sqrt{\frac{|E_{ns}|}{V_0-|E_{ns}|}}$$

ℓ=1 energies:

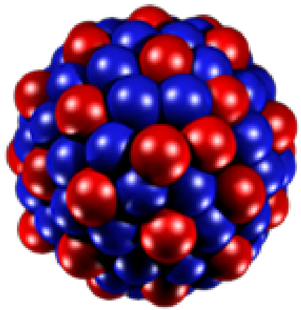
$$k\cdot R\cdot\cot(k\cdot R)=1+\frac{k^2}{\kappa^2}\cdot(1+\kappa\cdot R)$$

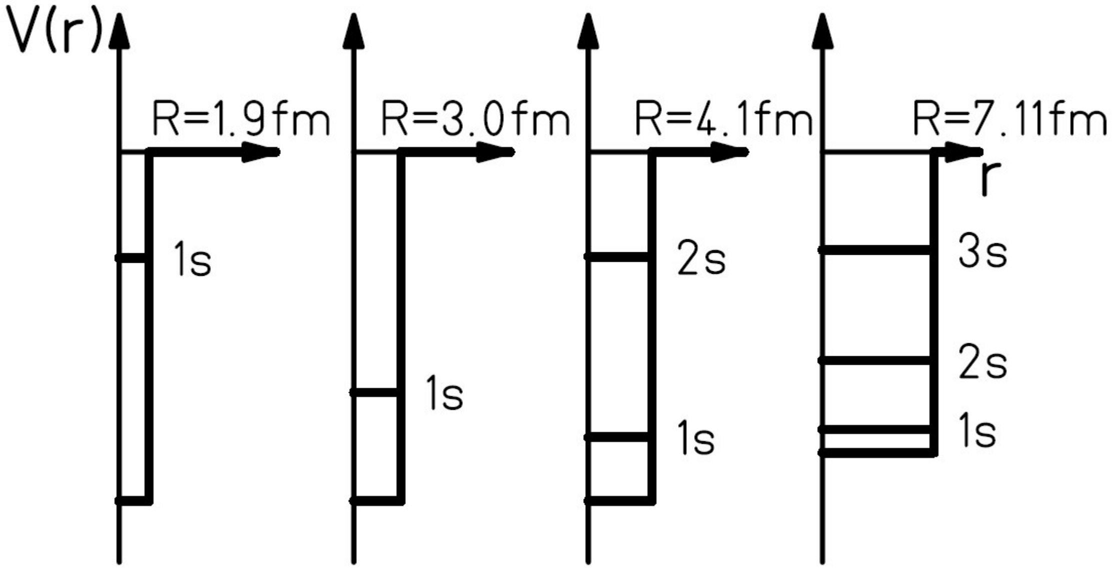
ℓ=2 energies:

$$\frac{1}{1-k\cdot R\cdot\cot(k\cdot R)}=\frac{3}{k^2\cdot R^2}\cdot\left(1+\frac{k^2}{\kappa^2}\right)+\frac{1}{1+\kappa\cdot R}$$

$$V_0=\left[51-33.1\cdot\frac{N-Z}{A}\right]\ MeV\qquad R=1.2\cdot A^{1/3}\ \ [fm]$$

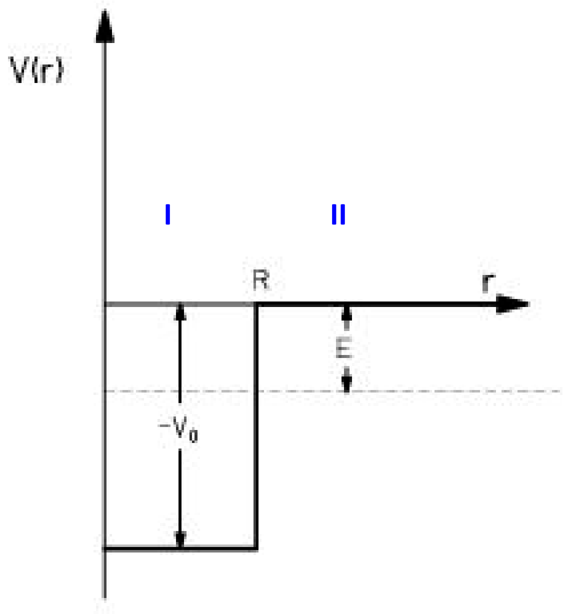
Energy eigenvalues





Energy eigenvalues for ℓ=0 in ^{4}He, ^{16}O, ^{40}Ca und ^{208}Pb

Wave function of the deuteron



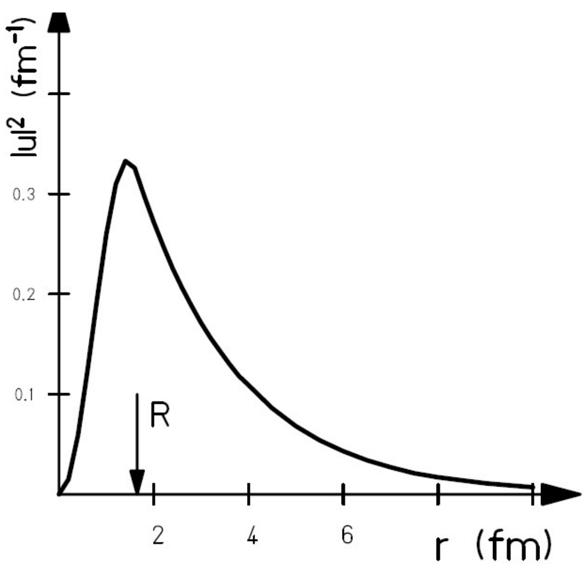
$$u_{n,s}^{I}(r) = A \cdot \sin(k \cdot r) \quad \text{with} \quad k = \frac{\sqrt{2 \cdot \mu \cdot (V_0 - |E_{n,s}|)}}{\hbar}$$

$$u_{n,s}^{II}(r) = B \cdot e^{\kappa \cdot r} \quad \text{with} \quad \kappa = \frac{\sqrt{2 \cdot \mu \cdot |E_{n,s}|}}{\hbar}$$

normalization: $\int_0^\infty |u_{n,\ell}(r)|^2 \, dr = 1$

$$A = \sqrt{\frac{2 \cdot \kappa}{\kappa \cdot R - \frac{\kappa}{k} \cdot \sin(k \cdot R) \cdot \cos(k \cdot R) + \sin^2(k \cdot R)}}$$

$$B = A \cdot e^{+\kappa \cdot R} \cdot \sin(k \cdot R)$$



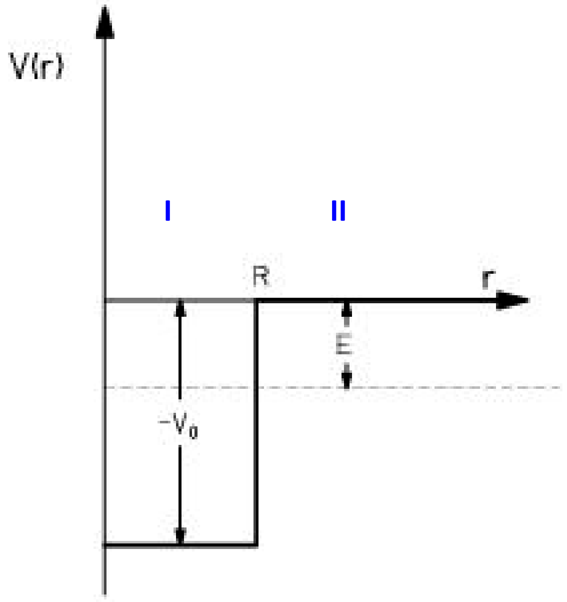
$V_0 = 51 \ MeV$

$R = 1.65 \ fm$

$\mu = 0.5$

$|E_{1,s}| = 2.224 \ MeV$

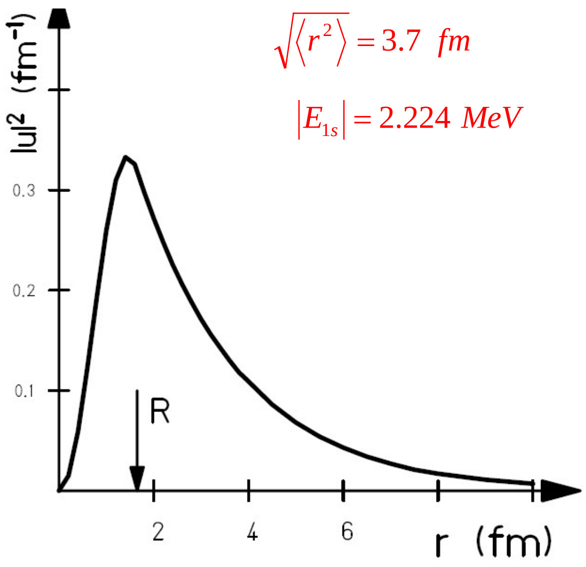
Radius of the deuteron



$$\langle r^2\rangle = \frac{\int \Psi^* \cdot r^2 \cdot \Psi \cdot r^2 \cdot dr\, d\Omega}{\int \Psi^* \cdot \Psi \cdot r^2 \cdot dr\, d\Omega} = \int_0^R A^2 \cdot r^2 \cdot \sin^2(kr)dr + \int_R^\infty B^2 \cdot r^2 \cdot e^{-2\cdot\kappa\cdot r} dr$$

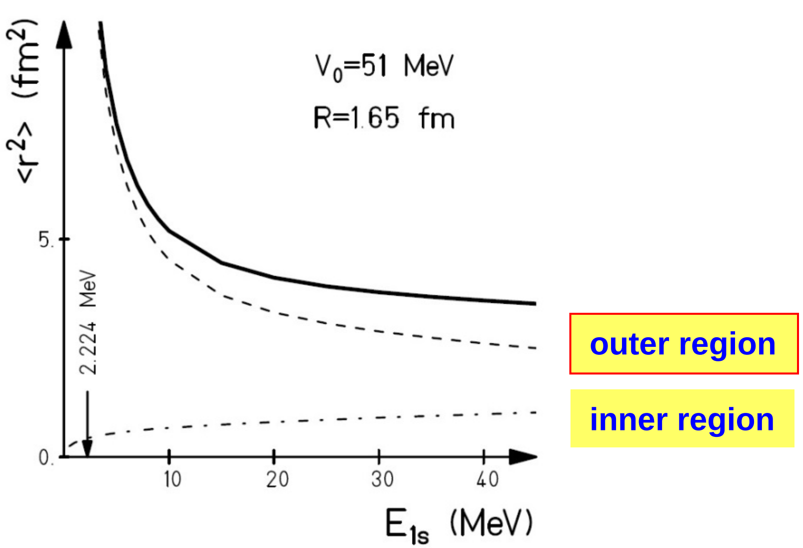
$$A = \sqrt{\frac{2\cdot\kappa}{\kappa\cdot R - \frac{\kappa}{k}\cdot\sin(k\cdot R)\cdot\cos(k\cdot R) + \sin^2(k\cdot R)}}$$

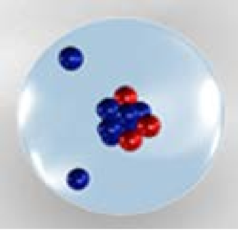
$$B = A\cdot e^{+\kappa\cdot R}\cdot\sin(k\cdot R)$$



$$\sqrt{\langle r^2\rangle} = 3.7\ fm$$

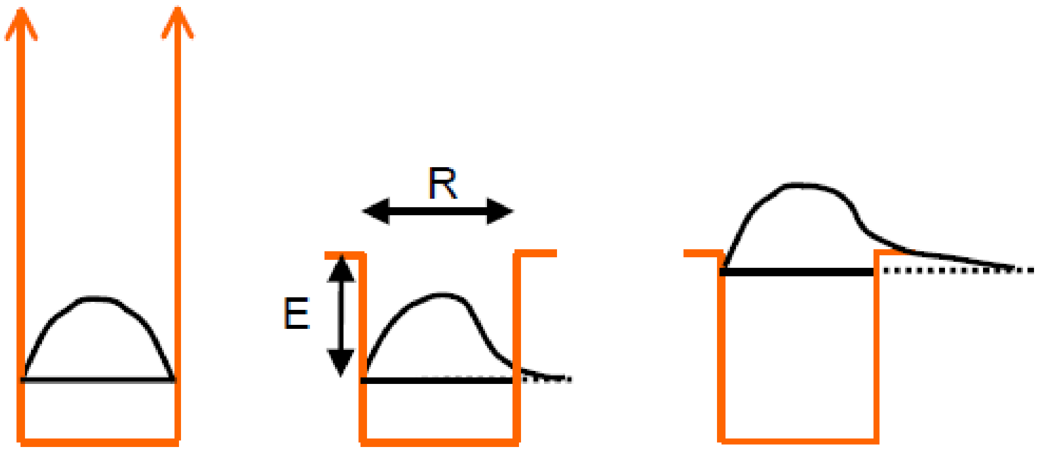
$$|E_{1s}| = 2.224\ MeV$$





Limits of stability: halo nuclei

What can one expect at the neutron-dripline?



wave function outside of the potential

$$\Psi(r) \propto \frac{e^{-\kappa r}}{r}$$

$$\kappa^2 = \frac{2 \cdot \mu \cdot E}{\hbar^2} \approx 0.05 \cdot E(MeV) \quad [fm^{-2}]$$

$$\langle r^2 \rangle = \frac{\int r^4\, dr \left(e^{-\kappa \cdot r} / \kappa \cdot r\right)^2}{\int r^2\, dr \left(e^{-\kappa \cdot r} / \kappa \cdot r\right)^2}$$

The smaller the binding energy, the more extended the wave function

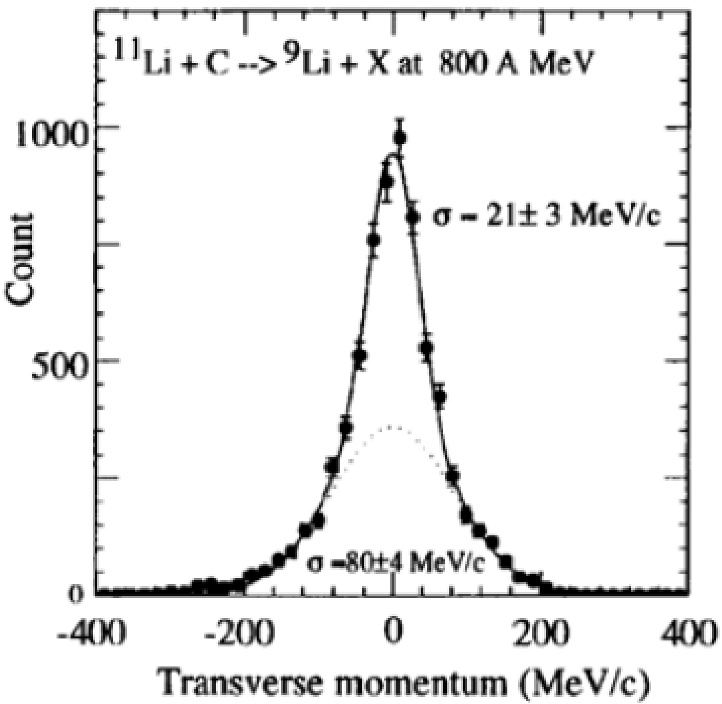
$$\langle r^2 \rangle = \frac{1}{2 \cdot \kappa^2} \cdot (1 + \kappa \cdot R) \approx \frac{\hbar^2}{4 \cdot \mu \cdot S_n}$$

E	κ^2	κ	$1/\kappa \sim r$
7 MeV	0.35 fm^{-2}	0.6 fm^{-1}	1.7 fm
1 MeV	0.05 fm^{-2}	0.2 fm^{-1}	4.5 fm
0.1 MeV	0.005 fm^{-2}	0.07 fm^{-1}	14 fm

Fourier-transform:

$$|F(p)|^2 = \hbar \cdot \kappa \cdot \frac{1}{\pi^2 \cdot \left(\kappa^2 \cdot \hbar^2 + p^2\right)^2}$$

Limits of stability: halo nuclei



One can use the arguments of an extended wave function with an exponential decline:

S_{2n}=250(80) keV

$$\Psi(r) \propto \frac{e^{-\kappa r}}{r} \qquad \kappa^2 = \frac{2 \cdot \mu_{2n} \cdot S_{2n}}{\hbar^2}$$

test of the extended wave function

momentum distribution:

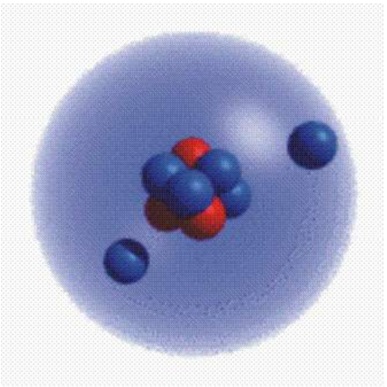
- wider momentum distribution for strongly bound particles
- narrow momentum distribution for weakly bound particles

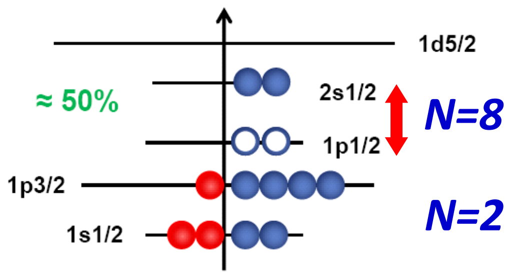
$$\underline{\Delta p} \cdot \underline{\Delta x} \geq \hbar$$

small → large

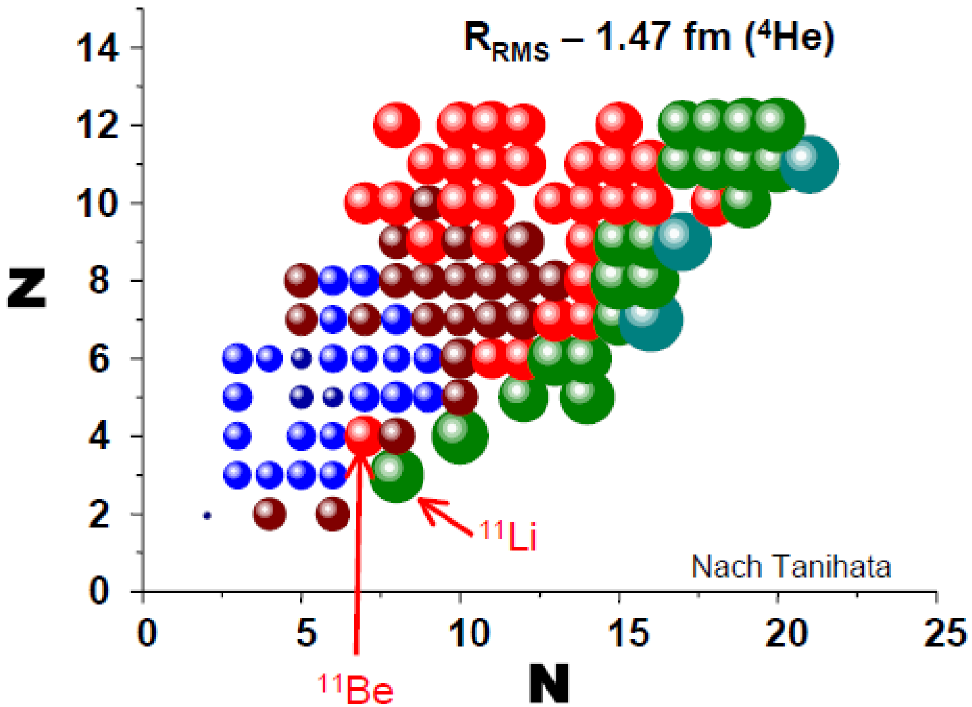
interpretation:

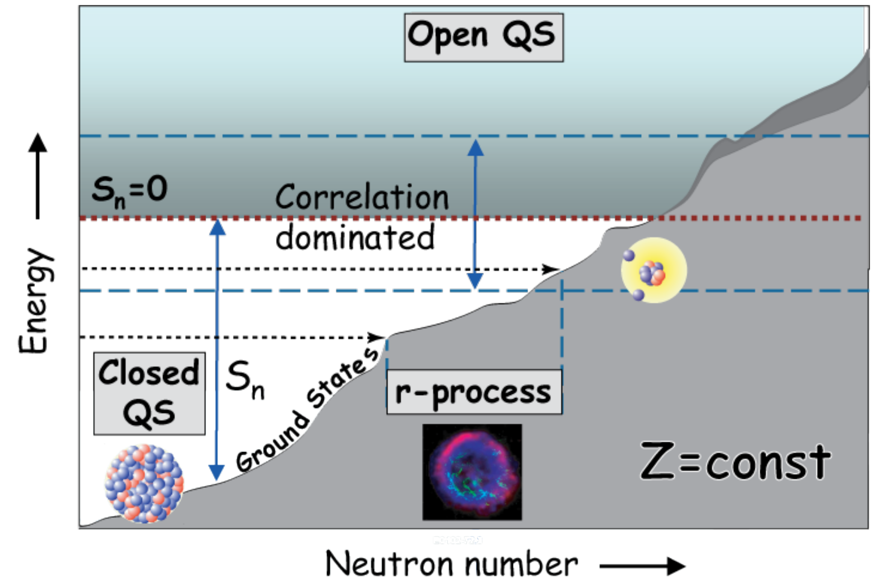
One can simplify ^{11}Li by describing it as a ^{9}Li core plus a di-neutron





radii of lighter nuclei





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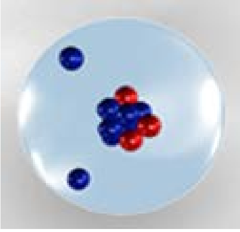
Berechnen sie den Radius der 2-Neutron Wellenfunktion für ^{11}Li $\sqrt{\langle r_{2n}^2 \rangle} \approx \frac{1}{\kappa}$



- ^{10}Li ist nicht gebunden
- Man kann ^{11}Li sehr vereinfacht beschreiben als ^{9}Li plus einem Di-Neutron.

$S_{2n}(^{11}\text{Li}) = 0.295(35)$ MeV