PHL424: α-decay

Why α-decay occurs?

The mass excess (in MeV/c^2) of ⁴He and its near neighbors

Think classically, occasionally 2 protons and 2 neutrons appear

together at the edge of a nucleus, with outward pointing momentum,

$\downarrow N, Z \rightarrow$	0	1	2	3	4
0	-	7.289	-	-	-
1	8.071	13.136	14.931	25.320	37.996
2	-	14.950	2.425	11.679	18.374
3	-	25.928	11.386	14.086	15.769
4	-	36.834	17.594	14.908	4.942



Compared to its low-A neighbors in the chart of nuclides, ⁴He is bound very strongly.





and bang against the Coulomb barrier.



The energetics of α -decay

The α -decay process is determined by the rest mass difference of the initial state and final state.

 $Q = m(A,Z) - m(A - 4, Z - 2) - m({}_{2}^{4}He)$

 $Q = BE(A - 4, Z - 2) + B_{\alpha}(28.3 \, MeV) - BE(A, Z)$



The Q-value of a reaction or of a decay indicates if it happens spontaneously or if additional energy is needed.



Mass (1u=931.478MeV/c²): 226.0254u \rightarrow 222.0176u + 4.0026u energy gain: 4.87 MeV

Binding energy $[M(A,Z) - Z \cdot M({}^{1}H) - N \cdot M({}^{1}n)]$: -1731.610 MeV \rightarrow -1708.184 MeV – 28.296 MeV energy gain: 4.87 MeV

Mass excess [M(A,Z) - A] : 23.662 MeV \rightarrow 16.367 MeV + 2.425 MeV energy gain: 4.87 MeV





The Q-value and the kinetic energy of α -particles

What is the α -energy for the system $^{214}_{84}Po_{130} \rightarrow ^{210}_{82}Pb_{128} + \alpha$

Mass data: http://nuclear.lu.se/database/masses/

 $BE(^{214}Po) = 1666.0 \text{ MeV} \\BE(^{210}Pb) = 1645.6 \text{ MeV} \\BE(^{4}He) = 28.3 \text{ MeV}$

 $Q_{\alpha} = 7.83 \text{ MeV}$

momentum conservation:

$$m_N \cdot v_N = m_lpha \cdot v_lpha \quad o \quad v_N = \frac{m_lpha}{m_N} \cdot v_lpha$$

energy conservation:

LUNDS

$$Q_{\alpha} = E_{kin}^{N} + E_{kin}^{\alpha}$$

$$= \frac{m_{N}}{2} \cdot v_{N}^{2} + E_{kin}^{\alpha}$$

$$= \frac{m_{N}}{2} \cdot \frac{m_{\alpha}^{2}}{m_{N}^{2}} \cdot v_{\alpha}^{2} + E_{kin}^{\alpha}$$

$$= \frac{m_{\alpha}}{m_{N}} \cdot E_{kin}^{\alpha} + E_{kin}^{\alpha}$$

$$= \frac{m_{\alpha} + m_{N}}{m_{N}} \cdot E_{kin}^{\alpha} \longrightarrow E_{kin}^{\alpha} = Q_{\alpha} \cdot \frac{m_{N}}{m_{N} + m_{\alpha}} = 7.83 MeV \cdot \frac{210}{214} = 7.68 MeV$$

For a typical α -emitter, the recoil energy is ~100-150 keV.



 $E_{kin}(\alpha)$



Energy differences of atomic masses:

 $Q = m(A,Z) - m(A - 4, Z - 2) - m({}_{2}^{4}He)$ $Q = BE(A - 4, Z - 2) + B_{\alpha}(28.3 MeV) - BE(A,Z)$

The Q-value of the decay is the available energy which is distributed as kinetical energy between the two participating particles. Since the mother and daughter nucleus have fixed masses, the α -particles are mono-energetic.

Tracks of α -particles (²¹⁴Po \rightarrow ²¹⁰Pb + α) in a cloud chamber. The constant length of the tracks shows that the α -particles are mono-energetic (E_{α}=7.7 MeV)





a-decay systematics: Geiger-Nuttall law



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Hans-Jürgen Wollersheim - 2018

a-decay schemes / spectra



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a-decay

Proton and neutrons are bound with up to 7 MeV and can not escape out of the nucleus. The emission of a bound system is more probable because of the additional binding energy $E_{\alpha} = 28.3$ MeV.

The Coulomb barrier of the nucleus prevents the α -particles from escaping. The energy needed is generally in the range of about 20-25 MeV.

$$V_C = \frac{2 \cdot (Z - 2) \cdot e^2}{r} = \frac{2 \cdot 82 \cdot 1.44 \text{ MeV } fm}{11.25 \text{ fm}} = 21 \text{ MeV}$$

Classically, the α -particle is reflected on the Coulomb barrier when $E_{\alpha} < V_{C}$, but quantum mechanics allows the penetration through the Coulomb potential via the mechanism of tunneling.





Quantum tunneling and α-decay





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Quantum tunneling



Intrinsic α -wave function 'leaks' out

time independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x) + V(x)\cdot\Psi(x) = E\cdot\Psi(x)$$

with $\int |\Psi(x)|^2 = 1$



general Ansatz for the solutions in area A, B, C

A,C:
$$\Psi''(x) + k^2 \cdot \Psi(x) = 0; \quad k^2 = 2 \cdot m \cdot E/\hbar^2$$

B:
$$\Psi''(x) + k'^2 \cdot \Psi(x) = 0; \quad k'^2 = 2 \cdot m \cdot (E - V_0)/\hbar^2$$

A:
$$\Psi(x) = A_1 \cdot e^{i \cdot k \cdot x} + A_2 \cdot e^{-i \cdot k \cdot x}$$

$$C: \qquad \Psi(x) = C_1 \cdot e^{i \cdot k \cdot x} + C_2 \cdot e^{-i \cdot k \cdot x}$$

B:
$$\Psi(x) = B_1 \cdot e^{\kappa \cdot x} + B_2 \cdot e^{-\kappa \cdot x}; \quad \kappa^2 = 2 \cdot m \cdot (V_0 - E)/\hbar$$





Quantum tunneling

The α -wave function from left will either be reflected or transmitted through the potential.

► In region C the wave is only travelling to the right $\rightarrow C_2 = 0$

With 4 equations for the 5 unknowns A_1 , A_2 , B_1 , B_2 and C_1 4 quantities can be determined with respect to e.g. A_1 .

 $R = \frac{|A_2|^2}{|A_1|^2}$

 $T = \frac{|C_1|^2}{|A_1|^2}$

Reflection coefficient:

Transmission coefficient:



$$\mathbf{X} = -\mathbf{a}:$$

$$A_1 \cdot e^{-i \cdot k \cdot a} + A_2 \cdot e^{i \cdot k \cdot a} = B_1 \cdot e^{-\kappa \cdot a} + B_2 \cdot e^{\kappa \cdot a}$$

$$i \cdot k \cdot A_1 \cdot e^{-i \cdot k \cdot a} - i \cdot k \cdot A_2 \cdot e^{i \cdot k \cdot a} = \kappa \cdot B_1 \cdot e^{-\kappa \cdot a} - \kappa \cdot B_2 \cdot e^{\kappa \cdot a}$$

$$C_1 \cdot e^{i \cdot k \cdot a} = B_1 \cdot e^{\kappa \cdot a} + B_2 \cdot e^{-\kappa \cdot a}$$

X= +**a**:
$$i \cdot k \cdot C_1 \cdot e^{i \cdot k \cdot a} = \kappa \cdot B_1 \cdot e^{\kappa \cdot a} - \kappa \cdot B_2 \cdot e^{-\kappa \cdot a}$$

$$T = \left[1 + \frac{V_0^2}{4 \cdot E \cdot (V_0 - E)} \cdot sinh^2 \sqrt{2 \cdot m \cdot (V_0 - E)} / \hbar \cdot L\right]^{-1}$$

$$T \approx \frac{16 \cdot E \cdot (V_0 - E)}{V_0^2} \cdot exp\left\{-2 \cdot \sqrt{2 \cdot m \cdot (V_0 - E)} \frac{L}{\hbar}\right\}; \quad E \ll V_0$$

$$sinh^2 x = \frac{1}{4} \cdot (e^{2x} + e^{-2x}) - 1/2$$



Quantum tunneling

For a simple box potential one obtains the exact solution for transmission coefficient:

$$T(E) = exp\left\{-\sqrt{2 \cdot m \cdot (V_0 - E)} \frac{2 \cdot L}{\hbar}\right\}$$



In case of a more realistic potential (see below) one has to use step functions (Δx width) as an approximation

$$T(E) = exp\left\{-2\int_{x_i}^{x_a} dx \left(\frac{1}{\hbar} \cdot \sqrt{2 \cdot m \cdot [V(x) - E]}\right)\right\}$$







a-decay systematics: Geiger-Nuttall law

\diamond Geiger-Nuttall law: Relation between the half-life and the energy of α -particles

Ansatz for α -decay probability λ [s⁻¹]:

 $\lambda = S \cdot \omega \cdot P$

- **S** is the probability that an α-particle has been formed inside of the nucleus
- **ω** is the frequency of the α-particle hitting the Coulomb barrier, where the velocity v is given by the kinetic energy and R is the nuclear radius

$$\omega = \frac{1}{\Delta t} = \frac{v}{2R} = \frac{\sqrt{2V_0/m_{\alpha}}}{2R} = \frac{\sqrt{2 \cdot \frac{50MeVc^2}{3727MeV}}}{2 \cdot 10fm} \sim 2 \cdot 10^{21} \, s^{-1}$$

• $\mathbf{P} = e^{-2G}$ is the probability of the tunneling process, where G is the Gamov factor

$$ln\lambda = lnS - ln\Delta t - 2G(E_{\alpha})$$
$$= b(Z) - a\frac{Z}{\sqrt{E_{\alpha}}} \approx -a\frac{Z}{\sqrt{E_{\alpha}}}$$









Chart of Nuclides

- 1. How are the *isotopes* of an element arranged on the chart?
- 2. Nuclides with the same number of neutrons are called *isotones*. How are they arranged on the chart?
- 3. Nuclides with the same mass number are called *isobars*. What would be the orientation of a line connecting an *isobaric* series?
- 4. Begin with the following radioactive parent nuclei, ²³⁵U, ²³⁸U, ²⁴⁴Pu, trace their decay processes and depict the mode and direction of each decay process on the chart. What are the final stable nuclei?

Plutonium Z=94								²²⁸ Pu	²²⁹ Pu	²³⁰ Pu	²³¹ Pu	²³² Pu	²³³ Pu	²³⁴ Pu	²³⁵ Pu	²³⁶ Pu	²³⁷ Pu	²³⁸ Pu	²³⁹ Pu	²⁴⁰ Pu	²⁴¹ Pu	²⁴² Pu	²⁴³ Pu	244Pu		
Nept	unium Z=93	²¹⁹ Np	²²⁰ Np	²²¹ Np	²²² Np	²²³ Np	²²⁴ Np	²²⁵ Np	²²⁶ Np	²²⁷ Np	²²⁸ Np	²²⁹ Np	²³⁰ Np	²³¹ Np	²³² Np	²³³ Np	²³⁴ Np	²³⁵ Np	²³⁶ Np	²³⁷ Np	²³⁸ Np	²³⁹ Np	²⁴⁰ Np	²⁴¹ Np	²⁴² Np	²⁴³ Np
anium Z=92	²¹⁷ U	²¹⁸ U	²¹⁹ U	²²⁰ U	²²¹ U	222U	223U	224U	²²⁵ U	²²⁶ U	227U	228U	²²⁹ U	²³⁰ U	²³¹ U	232U	²³³ U	²³⁴ U	²³⁵ U	²³⁶ U	²³⁷ U	²³⁸ U	²³⁹ U	²⁴⁰ U	²⁴¹ U	²⁴² U
²¹⁵ Pa	²¹⁶ Pa	²¹⁷ Pa	²¹⁸ Pa	²¹⁹ Pa	²²⁰ Pa	²²¹ Pa	²²² Pa	²²³ Pa	224Pa	²²⁵ Pa	²²⁶ Pa	²²⁷ Pa	228Pa	²²⁹ Pa	²³⁰ Pa	²³¹ Pa	²³² Pa	²³³ Pa	²³⁴ Pa	²³⁵ Pa	²³⁶ Pa	²³⁷ Pa	²³⁸ Pa	²³⁹ Pa	²⁴⁰ Pa	²⁴¹ Pa
²¹⁴ Th	²¹⁵ Th	²¹⁶ Th	²¹⁷ Th	²¹⁸ Th	²¹⁹ Th	²²⁰ Th	²²¹ Th	²²² Th	223Th	224Th	225Th	²²⁶ Th	²²⁷ Th	²²⁸ Th	²²⁹ Th	²³⁰ Th	²³¹ Th	²³² Th	²³³ Th	234Th	²³⁵ Th	236Th	²³⁷ Th	238Th	²³⁹ Th	Thoriu Z=90
²¹³ AC	²¹⁴ Ac	²¹⁵ AC	²¹⁶ AC	²¹⁷ Ac	²¹⁸ Ac	²¹⁹ Ac	²²⁰ Ac	²²¹ Ac	²²² Ac	223Ac	²²⁴ Ac	225Ac	228Ac	²²⁷ Ac	²²⁸ Ac	²²⁹ Ac	²³⁰ Ac	²³¹ Ac	²³² Ac	²³³ Ac	²³⁴ Ac	²³⁵ Ac	²³⁶ Ac	²³⁷ Ac	Actiniu Z=89	IM
²¹² Ra	²¹³ Ra	²¹⁴ Ra	²¹⁵ Ra	²¹⁶ Ra	²¹⁷ Ra	²¹⁸ Ra	²¹⁹ Ra	²²⁰ Ra	²²¹ Ra	²²² Ra	²²³ Ra	224Ra	²²⁵Ra	²²⁵Ra	²²⁷ Ra	228Ra	229Ra	²³⁰ Ra	²³¹ Ra	²³² Ra	²³³ Ra	²³₄Ra	235Ra	Radiu Z=88	m	
²¹¹ Fr	²¹² Fr	²¹³ Fr	²¹⁴ Fr	²¹⁵ Fr	²¹⁶ Fr	²¹⁷ Fr	²¹⁸ Fr	²¹⁹ Fr	²²⁰ Fr	²²¹ Fr	²²² Fr	²²³ Fr	224Fr	225Fr	²²⁶ Fr	²²⁷ Fr	²²⁸ Fr	²²⁹ Fr	²³⁰ Fr	²³¹ Fr	²³² Fr	²³³ Fr	Franci Z=87	um		
²¹⁰ Rn	²¹¹ Rn	²¹² Rn	²¹³ Rn	²¹⁴ Rn	²¹⁵Rn	²¹⁶ Rn	²¹⁷ Rn	²¹⁸ Rn	²¹⁹ Rn	²²⁰ Rn	221Rn	²²² Rn	223Rn	224Rn	225Rn	226Rn	227Rn	228Rn	229Rn	230Rn	²³¹ Rn	Rador Z=86	ı			
²⁰⁹ At	²¹⁰ At	²¹¹ At	²¹² At	²¹³ At	²¹⁴ At	²¹⁵ At	²¹⁶ At	²¹⁷ At	²¹⁸ At	²¹⁹ At	²²⁰ At	²²¹ At	222At	²²³ At	224At	225At	²²⁶ At	²²⁷ At	²²⁸ At	229At	Astatiı Z=85	ne				
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²⁰⁷ Bi	²⁰⁸ Bi	²⁰⁹ Bi	²¹⁰ Bi	²¹¹ Bi	²¹² Bi	²¹³ Bi	²¹⁴ Bi	²¹⁵ Bi	²¹⁶ Bi	²¹⁷ Bi	²¹⁸ Bi	²¹⁹ Bi	²²⁰ Bi	²²¹ Bi	²²² Bi	²²³ Bi	²²⁴ Bi	Bismu Z=83	th							
²⁰⁶ Pb	²⁰⁷ Pb	²⁰⁸ Pb	²⁰⁹ Pb	²¹⁰ Pb	²¹¹ Pb	²¹² Pb	²¹³ Pb	²¹⁴ Pb	²¹⁵Pb	²¹⁶ Pb	²¹⁷ Pb	²¹⁸ Pb	²¹⁹ Pb	²²⁰ Pb	Lead Z=82	http://people.physics.anu.edu.au/~ecs103/chart/index.ph										

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