## Basic Concepts

> Particle physics studies the elementary "building blocks" of matter and interaction between them.
> Matter consists of particles and fields.
$>$ Particles interact via forces caused by fields.
$>$ Forces are being carried by special particles, called gauge bosons.


$$
\begin{array}{ll}
\text { Forces of nature: } & \\
\text { 1) gravitational } & \\
\text { 2) weak } & n \rightarrow p+e^{-}+\overline{v_{e}} \\
\text { 3) electromagnetic } & e^{+}+e^{-} \rightarrow \gamma+\gamma \\
\text { 4) strong } & \pi^{-}(d \bar{u})+p(u u d) \rightarrow K^{+}(u \bar{s})+\Sigma^{-}(d d s)
\end{array}
$$

## Forces of Nature

|  | Acts on: | Carrier | Range | Strength | Stable <br> systems | Induced reaction |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Gravity | all particles | graviton | long <br> $F \propto 1 / r^{2}$ | $\sim 10^{-39}$ | Solar system | Object falling |
| Weak force | fermions | bosons <br> W and Z | $<10^{-17} m$ | $10^{-5}$ | None | $\beta$-decay |
| Electromagnetism | particles with <br> electric charge | photon | long <br> $F \propto 1 / r^{2}$ | $1 / 137$ | Atoms, <br> stones | Chemical <br> reactions |
| Strong force | quarks and <br> gluons | gluon | $10^{-15} m$ | 1 | Hadrons, <br> nuclei | Nuclear <br> reactions |

- Two people are standing in boats. One person moves their arm and is pushed backwards; a moment later the other person grabs at an invisible object and is driven backwards. Even though you cannot see a basketball, you can assume that one person threw a basketball to the other person because you see its effect on the people.
- It turns out that all interactions which affect matter particles are due to an exchange of force carrier particles, a different type of particle altogether. These particles are like basketballs tossed between matter particles (which are like the basketball players). What we normally think of as "forces" are actually the effects of force carrier particles on matter particles.
> Electromagnetic and weak forces can be described by a single theory $\Rightarrow$ the "Electroweak Theory" was developed in 1960s (Glashow, Weinberg, Salam).
$>$ Theory of strong interactions appeared in 1970s: "Quantum Chromodynamics" (QCD)
> The "Standard Model" (SM) combines both.

Main postulates of SM:


Abdus Salam, Steven Weinberg, Sheldon L. Glashow

1) Basic constituents of matter are quarks and leptons (spin 1/2).
2) They interact by means of gauge bosons (spin 1).
3) Quarks and leptons are subdivided into 3 generations.

## The Standard Model

## Fermions

Bosons

Leptons and Quarks

$$
\text { Spin }=\frac{1}{2} \quad \text { Spin }=1^{*}
$$

Baryons $(\boldsymbol{q q q}) \quad$ Spin $=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots \quad$ Spin $=0,1,2, \cdots$

Force Carrier
Particles

Meson $(q \bar{q})$

Baryons ( $q q q$ ) and Mesons ( $q \bar{q}$ ) are Hadrons
Baryon \# $=(q q q)=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ and $(q \bar{q})=\frac{1}{3}+\left(-\frac{1}{3}\right)=0$

| Lepton | lepton \# | electron \# | muon \# |
| :---: | :---: | :---: | :---: |
| $e^{-}$ | 1 | 1 | 0 |
| $v_{e}$ | 1 | 1 | 0 |
| $\mu$ | 1 | 0 | 1 |
| $v_{\mu}$ | 1 | 0 | 1 |

Lepton numbers are conserved in any reaction (for anti-leptons L = -1)

## Consequence of Lepton-Number Conservation

| reaction | lepton \# | electron \# | muon \# |
| :---: | :---: | :---: | :---: |
| $\nu_{e}+n \rightarrow p+e^{-}$ | $1 \rightarrow 1$ | $1 \rightarrow 1$ | $0 \rightarrow 0$ |
| $\overline{\nu_{e}}+n \rightarrow p+e^{-}$ | $-1 \rightarrow 1$ | $-1 \rightarrow 1$ | $0 \rightarrow 0$ |
| $\mu^{-} \rightarrow e^{-}+\gamma$ | $1 \rightarrow 1$ | $0 \rightarrow 1$ | $1 \rightarrow 0$ |
| $\overline{\nu_{\mu}}+p \rightarrow \mu^{+}+n$ | $-1 \rightarrow-1$ | $0 \rightarrow 0$ | $-1 \rightarrow-1$ |
| $\overline{\nu_{\mu}}+p \rightarrow e^{+}+n$ | $-1 \rightarrow-1$ | $0 \rightarrow-1$ | $-1 \rightarrow 0$ |


"Strangeness" $S=-[N(s)-N(\bar{s})]$
"Charm"
"Bottomness" $\tilde{B}=-[N(b)-N(\bar{b})]$

$$
\begin{array}{ll}
B^{+}=u \bar{b}, & B^{0}=d \bar{b} \\
B^{-}=\bar{u} b, & \bar{B}^{0}=\bar{d} b
\end{array}
$$

No composite hadrons are formed that contain the top (anti) quark

* Majority of hadrons are unstable and tend to decay by the strong interaction to the state with the lowest possible mass ( $\tau \sim 10^{-23} \mathrm{~s}$ )
* Hadrons with the lowest possible mass for each quark number (C, S, etc.) may live much longer before decaying weakly ( $\tau \sim 10^{-7}-10^{13} \mathrm{~s}$ ) or electromagnetically (mesons, $\tau \sim 10^{-16}-10^{-21} s$ )

Some examples of baryons:

| particle | mass <br> $\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ | quark <br> composition | charge <br> (units of e) | S | C | B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p | 0.938 | uud | 1 | 0 | 0 | 0 |
| n | 0.940 | udd | 0 | 0 | 0 | 0 |
| $\Lambda$ | 1.116 | uds | 0 | -1 | 0 | 0 |
| $\Lambda_{c}$ | 2.285 | udc | 1 | 0 | 1 | 0 |

Some examples of mesons:

| particle | mass <br> $\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ | quark <br> composition | charge <br> (units of e) | S | C | B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pi^{+}$ | 0.140 | $u \bar{d}$ | 1 | 0 | 0 | 0 |
| $K^{-}$ | 0.494 | $s \bar{u}$ | -1 | -1 | 0 | 0 |
| $D^{-}$ | 1.869 | $d \bar{c}$ | -1 | 0 | -1 | 0 |
| $D_{s}^{+}$ | 1.969 | $c \bar{s}$ | 1 | 1 | 1 | 0 |
| $B^{-}$ | 5.279 | $b \bar{u}$ | -1 | 0 | 0 | -1 |
| Y | 9.460 | $b \bar{b}$ | 0 | 0 | 0 | 0 |


| Table Bf Baryons |  |  |  |  |  |  |  |  | Mesons |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Symbol | Makeup | Rest mass $\mathrm{MeV} / \mathrm{c}^{2}$ | Spin | B | S | Lifetime (seconds> | Decay Modes | Particle | Symbol | $\begin{array}{l\|l\|} \hline \text { Anti- } \\ \text { particle } \end{array}$ | Makeup | $\begin{array}{\|c\|} \hline \text { Rest mass } \\ \mathrm{MeV} / \mathrm{c}^{2} \end{array}$$\mathrm{S}$ |  | C | B | Lifetime | Decay Modes |
| Particle |  |  |  |  |  |  |  |  | Particle <br> Pion | $\pi^{+}$ | particle | Makeup | $\mathrm{MeV} / \mathrm{c}^{2}$ <br> 139.6 | - | C | 0 | $\begin{array}{\|c\|} \hline \text { Litume } \\ \hline \hline 2.60 \\ \times 10^{-8} \\ \hline \end{array}$ | \| ${ }^{\text {Decay Modes }}$ |
| Proton | p | uud | 938.3 | 1/2 | +1 | 0 | Stable | $\ldots$ | Pion | $\pi^{0}$ | Self | $u \bar{u}-d \bar{d}$ | 135.0 | 0 | 0 |  | $\begin{array}{\|\|c} 0.83 \\ \times 10^{-16} \end{array}$ | $2 \gamma$ |
| Neutron | n | ddu | 939.6 | 1/2 | +1 | 0 | 920 | pe $\underline{v}_{e}$ |  |  |  | $\sqrt{2}$ |  |  |  |  |  |  |
|  | $\Lambda^{0}$ | uds | 1115.6 | 1/2 | +1 | -1 | $\begin{gathered} 2.6 \\ \times 10^{-10} \end{gathered}$ | $\mathrm{p} \pi^{-}, \mathrm{n} \pi^{0}$ | Kaon | $\mathbf{K}^{+}$ | K | uS | 493.7 | +1 | 00 |  | $\begin{gathered} \hline \hline 1.24 \\ \times 10^{-8} \\ \hline \end{gathered}$ | $\mu^{+} v_{\mu}, \pi^{+} \pi^{0}$ |
| Lambda |  |  |  |  |  |  |  |  | Kaon | $\mathrm{K}^{0}{ }_{\text {s }}$ | $\mathbf{K}_{\text {s }}{ }^{\text {a }}$ | 1* | 497.7 | +1 | 00 |  | $\begin{gathered} \hline 0.89 \\ \times 10^{-10} \end{gathered}$ | $\pi^{+} \pi^{-} 2 \pi^{0}$ |
| Sigma | $\Sigma^{+}$ | uus | 1189.4 | $1 / 2$ | +1 | -1 | $\begin{gathered} 0.8 \\ \times 10^{-10} \end{gathered}$ | $\mathrm{p} \pi^{0}, \mathrm{n} \pi^{+}$ | Kaon | $\mathbf{K}_{\text {L }}^{0}$ | $\mathbf{K}_{\text {L }}{ }^{\text {l }}$ | 1* | 497.7 | +1 | 0 | 0 | $\begin{array}{r}5.2 \\ \times 10^{-8} \\ \hline\end{array}$ | $\pi^{+} \mathrm{e}^{-} \underline{v}_{e}$ |
| Sigma | $\Sigma^{0}$ | uds | 1192.5 | 1/2 | $+1$ | -1 | $6 \times 10^{-20}$ | $\Lambda^{0} \gamma$ | Eta | $\eta^{0}$ | Self | 2* | 548.8 | 0 | 0 | 0 | $<10^{-18}$ | $2 \gamma, 3 \mu$ |
|  | $\Sigma^{-}$ | dds | 1197.3 | 1/2 | +1 | -1 |  | $n \pi$ | $\begin{array}{\|\|l\|} \mid \text { Eta prime } \\ \hline \hline \underline{\text { Rho }} \\ \hline \end{array}$ | $\eta^{\prime}$ | Self | 2* | 958 | 0 | 0 | 0 | ... | $\frac{\pi^{+} \pi^{\prime} \eta}{\pi^{+} \pi^{0}}$ |
| Sigma |  |  |  |  |  |  | $\mathrm{x} 10^{-10}$ |  |  | $\rho^{+}$ | $\rho-$ | ud | 770 | 0 | 0 | 0 | 0.4 <br> $\times 10^{-23}$ |  |
| Delta | $\Delta^{+}$ | uuu | 1232 | 3/2 | +1 | 0 | $\begin{gathered} 0.6 \\ \times 10^{-23} \end{gathered}$ | $\mathrm{p} \pi^{+}$ | Rho | $\rho^{0}$ | Self | uı dd | 770 | 0 | 0 | 0 | $\begin{gathered} 0.4 \\ \times 10^{-23} \\ \hline \end{gathered}$ | $\pi^{+} \pi$ |
| Delta | $\Delta^{+}$ | uud | 1232 | 3/2 | +1 | 0 | 0.6 | $\mathrm{p} \pi^{0}$ | Omega | $\omega^{0}$ | Self | u un, dd | 782 | 0 | 0 | 0 | $\begin{gathered} 0.8 \\ \times 10^{-22} \end{gathered}$ | $\pi^{+} \pi^{0}{ }^{0}$ |
| Dela |  |  |  |  |  |  | $\mathrm{x} 10^{-23}$ |  | Phi | $\varphi$ | Self | SS | 1020 | 0 | $0{ }^{0}$ |  | $\begin{gathered} 20 \\ \times 10^{-23} \\ \hline \end{gathered}$ | $\mathrm{K}^{+} \mathrm{K}^{*}, \mathrm{~K}^{0} \underline{K}^{0}$ |
| Delta | $\Delta^{0}$ | udd | 1232 | 3/2 | +1 | 0 | $\begin{array}{r} 0.6 \\ \times 10^{-23} \\ \hline \end{array}$ | $n \pi^{0}$ | \| $\mathrm{D}^{\text {d }}$ | $\mathrm{D}^{+}$ | D | cd | 1869.4 | 0 | +1 | 0 | 10.6 $\times 10^{-13}$ | $\mathrm{K}_{+}{ }_{-}, \mathrm{e}^{+}{ }_{-}$ |
| Delta | $\triangle^{-}$ | ddd | 1232 | 3/2 | +1 | 0 | $\begin{gathered} 0.6 \\ \times 10^{-23} \end{gathered}$ | $n \pi^{-}$ | D | $\mathrm{D}^{0}$ | $\underline{D}^{0}$ | cu | 1864.6 | 0 | +1 | 0 | $\begin{gathered} 4.2 \\ \times 10^{-13} \\ \hline \hline \end{gathered}$ | [ $\mathrm{K}, \mu, \mathrm{e}]+{ }_{+}$ |
|  |  | uss | 1315 | $1 / 2$ | +1 | -2 |  |  | D | $\mathrm{D}^{+}{ }_{\text {s }}$ | $\mathrm{D}_{\text {s }}$ | cs | 1969 | +1 | +1 0 | 0 | 4.7 <br> $\times 10^{-13}$ | K+ |
| Cascade | $\Xi^{0}$ |  |  |  |  |  | $\times 10^{-10}$ | $\Lambda^{0} \pi^{0}$ | J/Psi | J/ $\psi$ | Self | cc | 3096.9 | 0 | 0 | 0 | $\begin{gathered} 0.8 \\ \times 10^{-20} \end{gathered}$ | $\mathrm{e}^{+} e^{*}, \mu^{+} \mu^{\prime} \ldots$ |
| $\begin{gathered} \mathrm{Xi} \\ \text { Cascade } \\ \hline \end{gathered}$ | $\Xi$ | dss | 1321 | 1/2 | +1 | -2 | $\begin{gathered} 1.64 \\ \times 10^{-10} \end{gathered}$ | $\Lambda^{0} \pi^{-}$ | B | B | $\mathrm{B}^{+}$ | bu | 5279 | 0 | 0 | -1 | $\begin{gathered} 1.5 \\ \hline \hline 10^{-12} \\ \hline \end{gathered}$ | $\mathrm{D}^{0}+$ |
| Omega | $\Omega^{-}$ | SSS | 1672 | 3/2 | +1 | -3 | $0.82$ | $\Xi^{0} \pi^{-}, \Lambda^{0} \mathrm{~K}^{-}$ | B | $\mathrm{B}^{0}$ | $\underline{B}^{0}$ | db | 5279 | 0 | 0 | -1 | $\begin{gathered} 1.5 \\ \times 10^{-12} \end{gathered}$ | $\mathrm{D}^{0}+{ }_{-}$ |
|  |  |  |  |  |  |  | $\mathrm{x} 10^{-10}$ |  | $\mathrm{B}_{8}$ | $\mathrm{B}_{8}{ }^{0}$ | $\underline{B_{3}}{ }^{0}$ | sb | 5370 | -1 | 0 | -1 | $\ldots$ | $\mathrm{B}_{\mathrm{s}}+$ |
| Lambda | $\Lambda^{+}{ }_{c}$ | udc | 2281 | $1 / 2$ | $+1$ | 0 | $2 \times 10^{-13}$ | $\ldots$ | Upsilon | $\Upsilon$ | Self | bb | 9460.4 | 0 | 0 | 0 | $\begin{array}{\|} 1.3 \\ \times 10^{-20} \\ \hline \end{array}$ | $\mathrm{e}^{+} e^{-}, \mu^{+} \mu^{-}$. |

## Consequence of Quark-Number Conservation

| reaction | Quark configuration | charge | baryon \# | strangeness \# |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{p}+p \rightarrow \pi^{0}+n$ | $(\bar{u} \bar{u} \bar{d})+(u u d) \rightarrow(u \bar{u}-d \bar{d})+(u d d)$ | yes | no | yes |
| $\pi^{-}+p \rightarrow K^{0}(d \bar{s})+n$ | $(\bar{u} d)+(u u d) \rightarrow(d \bar{s})+(u d d)$ | yes | yes | no |
| $p+p \rightarrow \pi^{+}+n+n$ | $(u u d)+(u u d) \rightarrow(u \bar{d})+(u d d)+(u d d)$ | no | yes | yes |


| decay | Quark configuration |  |
| :---: | :---: | :--- |
| $\Omega^{-} \rightarrow \Lambda^{0}+K^{-}$ | $(s s s) \rightarrow(u d s)+(\bar{u} s)$ | $s \rightarrow u+W^{-}$and $W^{-} \rightarrow \bar{u}+d$ |
| $K^{+} \rightarrow \pi^{+}+\pi^{0}$ | $(u \bar{s}) \rightarrow(u \bar{d})+(u \bar{u}-d \bar{d})$ | $\bar{s} \rightarrow \bar{u}+W^{+}$and $W^{+} \rightarrow u+\bar{d}$ |
| $\Xi^{+} \rightarrow \Lambda^{0}+\pi^{-}$ | $(d s s) \rightarrow(u d s)+(\bar{u} d)$ | $s \rightarrow u+W^{-}$and $W^{-} \rightarrow \bar{u}+d$ |




* Standard Model does not explain neither appearance of the mass nor the reason for existence of 3 generations.

| Force_Particles | Quarks | Charged Leptons | Neutrinos |
| :---: | :---: | :---: | :---: |
| Strong | yes | no | no |
| Electromagnetic | yes | yes | no |
| Weak | yes | yes | yes |

Quarks (hence hadrons) have all types of interactions!

## History of the Universe



## Units and Dimensions

> The energy is measured in electron-Volts:

$$
1 \mathrm{eV}=1.602 \cdot 10^{-19} \mathrm{~J}
$$

$$
1 \mathrm{keV}=10^{3} \mathrm{eV} ; 1 \mathrm{MeV}=10^{6} \mathrm{eV} ; 1 \mathrm{GeV}=10^{9} \mathrm{eV} ; 1 \mathrm{TeV}=10^{12} \mathrm{eV}
$$

$>$ The Planck constant (reduced) is:

$$
\hbar \equiv h / 2 \pi=6.582 \cdot 10^{-22} \mathrm{MeV} \mathrm{~s}
$$

and the "conversion constant" is:

$$
\hbar c=197.327 \cdot 10^{-15} \mathrm{MeV} \mathrm{~m}
$$

$$
\begin{aligned}
\Delta E \cdot \Delta t & =\hbar=\text { energy } * \text { time } \\
\therefore \hbar c & =\text { energy } * \text { time } * \text { velocity } \\
& =\text { energy } * \text { distance }
\end{aligned}
$$

$>$ Charges measured in terms of electronic charges $e=1.6 \cdot 10^{-19} \mathrm{C}$
$>$ Cross sections measured in terms of barns. 1 barn $=10^{-28} \mathrm{~m}^{2}$

## Units and Dimensions

Because $E^{2}=p^{2} c^{2}+m^{2} c^{4}$ where E is the energy, p the momentum, m the rest mass: pc and $\mathrm{mc}^{2}$ have dimensions of energy and it is convenient to measure momentum in units of $\mathrm{GeV} / \mathrm{c}$ and mass in units of $\mathrm{GeV} / \mathrm{c}^{2}$.

$$
\frac{[E(\mathrm{GeV})]^{2}=[p(\mathrm{GeV} / \mathrm{c})]^{2} c^{2}+\left[m\left(\mathrm{GeV} / \mathrm{c}^{2}\right)\right]^{2} c^{4}}{1 \mathrm{eV} / \mathrm{c}^{2}=1.78 \cdot 10^{-36} \mathrm{~kg}}
$$

Because c cancels out we often omit the ci.e. put $c=1$ (and $\hbar=1$ ), so momenta and masses are also measured in GeV .

## Units and Dimensions

This implies, however, that the results of calculations must be translated back to measureable quantities in the end. Conversion factors are the following:

| quantity | conversion factor | natural unit | normal unit |
| :---: | :---: | :---: | :---: |
| mass | $1 \mathrm{~kg}=5.61 \cdot 10^{26} \mathrm{GeV}$ | GeV | $\mathrm{GeV} / \mathrm{c}^{2}$ |
| length | $1 m=5.07 \cdot 10^{15} \mathrm{GeV}^{-1}$ | $\mathrm{GeV}^{-1}$ | $\hbar c / \mathrm{GeV}$ |
| time | $1 s=1.52 \cdot 10^{24} \mathrm{GeV}^{-1}$ | $\mathrm{GeV}^{-1}$ | $\hbar / \mathrm{GeV}$ |
| unit charge | $e=\sqrt{4 \pi \alpha}$ | 1 | $\sqrt{\hbar c}$ |

## Excercise-1:

Derive the conversion factors for mass, length and time in the table above.

Co-moving coordinate systems


$$
S \rightarrow S^{\prime}:\left\{\begin{array}{l}
t^{\prime}=\gamma\left(t-\frac{v x}{c^{2}}\right) \\
x^{\prime}=\gamma(x-v t) \\
y^{\prime}=y \\
z^{\prime}=z
\end{array} \quad S^{\prime} \rightarrow S:\left\{\begin{array}{l}
t=\gamma\left(t^{\prime}+\frac{v x^{\prime}}{c^{2}}\right) \\
x=\gamma\left(x^{\prime}+v t^{\prime}\right) \\
y=y^{\prime} \\
z=z^{\prime}
\end{array}\right.\right.
$$

$$
\gamma \equiv \frac{1}{\sqrt{1-\beta^{2}}} \quad \beta=\frac{v}{c}
$$

Lorentz contraction: $\quad L=L_{0} / \gamma$
Time dilatation:

$$
t=t_{0} \cdot \gamma
$$


non-relativistic
Earth-frame observer

Distance: $L_{0}=10^{4}$ meters
Time: $T=\frac{10^{4} \mathrm{~m}}{(0.98) \cdot\left(3 \cdot 10^{8} \mathrm{~m} / \mathrm{s}\right)}$
$T=34 \cdot 10^{-6} s=21.8$ halflives $\quad=4.36$ halflives
Survival rate:

$$
\frac{I}{I_{0}}=2^{-21.8}=0.27 \cdot 10^{-6} \quad \frac{I}{I_{0}}=2^{-4.36}=0.049
$$

Or only about 0.3 out of a million

Or only about 49000 out of a million

The muon's clock is timedilated, or running slow by a factor $T=\gamma \cdot T_{0}$ so its measured half-life is $5 \cdot 1.56 \mu s=7.8 \mu s$.

non-relativistic

Distance: $L_{0}=10^{4}$ meters
Time: $T=\frac{10^{4} \mathrm{~m}}{(0.98) \cdot\left(3 \cdot 10^{8} \mathrm{~m} / \mathrm{s}\right)}$
$T=34 \cdot 10^{-6} s=21.8$ halflives
Survival rate:

$$
\frac{I}{I_{0}}=2^{-21.8}=0.27 \cdot 10^{-6}
$$

Or only about 0.3 out of a million

Muon-frame observer

$$
\begin{aligned}
& =\frac{2000 \mathrm{~m}}{(0.98) \cdot\left(3 \cdot 10^{8} \mathrm{~m} / \mathrm{s}\right)} \\
& =6.8 \cdot 10^{6} \mathrm{~s} \\
& =4.36 \text { halflives }
\end{aligned}
$$

$$
\frac{I}{I_{0}}=2^{-4.36}=0.049
$$

Or only about 49000 out of a million

The muon sees distance as length-contracted, so that $L=L_{0} / \gamma=0.2 \cdot L_{0}$ $=2 \mathrm{~km}$.

## Relativistic Kinematics

The relativistic relationship between the total energy E, momentum $p$ and rest mass $m$ is

$$
\begin{aligned}
& E^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& \text { or } \\
& E^{2}=p_{x}^{2} c^{2}+p_{y}^{2} c^{2}+{ }_{z}^{2} p c^{2}+m^{2} c^{4}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Non relativistic }(p \ll m) \text { : } \\
& \begin{aligned}
E & =\left(p^{2} c^{2}+m^{2} c^{4}\right)^{1 / 2} \\
& =m c^{2} \cdot\left(1+p^{2} c^{2} / m^{2} c^{4}\right)^{1 / 2} \\
& =m c^{2} \cdot\left(1+p^{2} / 2 m^{2} c^{2}+\cdots\right) \\
& \cong m c^{2}+p^{2} / 2 m \quad(p=m v) \\
& \cong m c^{2}+\frac{1}{2} m v^{2}
\end{aligned}
\end{aligned}
$$

The particle velocity $v=\beta c$ or $\beta=v / c$ and the Lorentz factor

$$
\gamma=\frac{1}{\sqrt{1-v^{2} / c^{2}}}=\frac{1}{\sqrt{1-\beta^{2}}}
$$

So that $\gamma^{2}\left(1-\beta^{2}\right)=1$ or $\gamma^{2}=\gamma^{2} \beta^{2}+1 \quad$ - multiplied by $m^{2} c^{4}$

$$
\begin{aligned}
& \gamma^{2} m^{2} c^{4}=\gamma^{2} \beta^{2} m^{2} c^{4}+m^{2} c^{4} \\
& \text { Compare with } \\
& E^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E=\gamma m c^{2} \text { and } p=\gamma \beta m c \text { or } \gamma=E / m c^{2} \text { and } \beta=p / \gamma m c=p c / E
\end{aligned}
$$

## Invariant Mass



$$
\begin{aligned}
\pi^{0} & \rightarrow \gamma \gamma \quad \text { decay: (two massless particles) } \\
M^{2} & =\left[\left(p_{1}, 0,0, p_{1}\right)+\left(p_{2}, 0, p_{2} \sin \theta, p_{2} \cos \theta\right)\right]^{2}=\left(p_{1}+p_{2}\right)^{2}-p_{2}^{2} \sin ^{2} \theta-\left(p_{1}+p_{2} \cos \theta\right)^{2} \\
& =2 p_{1} p_{2}(1-\cos \theta)
\end{aligned}
$$



$$
\pi^{0} \rightarrow \gamma \gamma
$$

Compute invariant mass $m_{\gamma \gamma}$ for all possible photon pairs

$$
m_{\gamma \gamma}=\sqrt{2 E_{\gamma 1} E_{\gamma 2}\left(1-\cos \theta_{\gamma \gamma}\right)}
$$




Both $\boldsymbol{\sigma}$ and $\boldsymbol{\Gamma}$ are related to the probability for the considered process to occur

## Cross Section

Consider a beam of projectiles of intensity $\boldsymbol{\Phi}_{\mathrm{a}}$ particles/sec which hits a thin foil of target nuclei with the result that the beam is attenuated by reactions in the foil such that the transmitted intensity is $\boldsymbol{\Phi}$ particles/sec.
The fraction of the incident particles disappear from the beam, i.e. react, in passing through the foil is given by

$$
d \Phi=-\Phi \cdot n_{b} \cdot \sigma \cdot d x
$$

The number of reactions that are occurring is the difference between the initial and transmitted flux

$$
\begin{aligned}
\Phi_{\text {initial }}-\Phi_{\text {trans }} & =\Phi_{\text {initial }}\left(1-\exp \left[-n_{b} \cdot d \cdot \sigma\right]\right) \\
& \approx \Phi_{\text {initial }} \cdot N_{b} \cdot \sigma \quad \text { (for thin target) }
\end{aligned}
$$

Example:


A particle current of 1 pnA consists of $6 \cdot 10^{9}$ projectiles/s.
A ${ }^{132}$ Sn target ( $1 \mathrm{mg} / \mathrm{cm}^{2}$ ) consists of $5 \cdot 10^{18}$ nuclei/ $\mathrm{cm}^{2}$

$$
\frac{6 \cdot 10^{23} \cdot 10^{-3} \mathrm{~g} / \mathrm{cm}^{2}}{132 \mathrm{~g}}=4.5 \cdot 10^{18} \quad\left[\frac{\text { target nuclei }}{\mathrm{cm}^{2}}\right]
$$

Luminosity $=$ projectiles $\left[\mathrm{s}^{-1}\right]$ - target nuclei $\left[\mathrm{cm}^{-2}\right]$
Luminosity (projectile $\rightarrow{ }^{132} \mathrm{Sn}$ ) $=3 \cdot 10^{28}\left[\mathrm{~s}^{-1} \mathrm{~cm}^{-2}\right]$

```
Reaction rate \(\left[\mathrm{s}^{-1}\right]=\) luminosity \(\cdot\) cross section \(\left[\mathrm{cm}^{2}\right]\)
    \(=\) projectiles \(\left[\mathrm{s}^{-1}\right] \cdot\) target nuclei \(\left[\mathrm{cm}^{-2}\right] \cdot\) cross section \(\left[\mathrm{cm}^{2}\right]\)
```



In laboratory frame: $\lambda_{\text {decay }}=\gamma \beta c \cdot \tau$


$$
\text { Uncertainty principle } \quad \Gamma \equiv \frac{\hbar}{\tau}
$$

If a particle has a finite lifetime $\tau$, it decays with probability $e^{-t / \tau}$ and a decay width $\Gamma$ can be defined. One can interpret $\Gamma \cdot \tau=\hbar$ as a relationship between uncertainty in energy (mass) and lifetime i.e. $\Delta E \cdot \Delta t=\hbar$.

Strongly decaying particles have very short lifetimes and hence large width. The $\rho(770)$ has $\Gamma=151 \mathrm{MeV}$ and $\tau=4.4 \cdot 10^{-24} \mathrm{~s}$.

Weakly decaying particles have longer lifetimes and hence much smaller widths. The $K^{0}$ meson has $\Gamma=7.3 \cdot 10^{-12} \mathrm{MeV}$ and $\tau=0.9 \cdot 10^{-10} \mathrm{~s}$.

