Nuclear Shell Model

The nucleon building blocks...

<table>
<thead>
<tr>
<th>Name</th>
<th>up quark ($u$)</th>
<th>down quark ($d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (MeV)</td>
<td>1.7 – 3.1</td>
<td>4.1 – 5.7</td>
</tr>
<tr>
<td>charge ($e$)</td>
<td>$+2/3$</td>
<td>$-1/3$</td>
</tr>
<tr>
<td>spin</td>
<td>$1/2$</td>
<td>$1/2$</td>
</tr>
</tbody>
</table>

The nuclear building blocks...

<table>
<thead>
<tr>
<th>Name</th>
<th>neutron</th>
<th>proton</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (MeV)</td>
<td>$939.565378(21)$</td>
<td>$938.272046(21)$</td>
</tr>
<tr>
<td>charge ($e$)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>constituents</td>
<td>$2d + 1u$</td>
<td>$2u + 1d$</td>
</tr>
<tr>
<td>$I^\pi$</td>
<td>$1/2^+$</td>
<td>$1/2^+$</td>
</tr>
</tbody>
</table>
Themes and challenges in modern science

- **Complexity out of simplicity – Microscopic**
  How the world, with all its apparent complexity and diversity can be constructed out of a few elementary building blocks and their interactions.

- **Simplicity out of complexity – Macroscopic**
  How the world of complex systems can display such remarkable regularity and simplicity.

Individual excitations of nucleons:

- vibration
- rotation
- fission
The nuclear force is short-range, but does not allow for compression of nuclear matter.

Yukawa – potential:

\[ V_0(r) = g_s \cdot \frac{1}{r} \cdot e^{-\left(\frac{m_\pi c}{h}\right) \cdot r} \]
The deuteron

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (MeV/c²)</td>
<td>1875.61</td>
</tr>
<tr>
<td>Charge (e)</td>
<td>1</td>
</tr>
<tr>
<td>$I^\pi$</td>
<td>1⁺</td>
</tr>
<tr>
<td>Binding energy (MeV)</td>
<td>2.2245</td>
</tr>
<tr>
<td>Magnetic moment ($\mu_N$)</td>
<td>0.8574</td>
</tr>
<tr>
<td>Quadrupole moment (b)</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

The deuteron can not be a pure $s$ state! ~ 96% $s$ and 4% $d$. Not spherical consistent with s/d-ratio = 96/4.

The deuteron is an ideal candidate for tests of our basic understanding of nuclear physics.
Structure of the nuclear force is more complex than e.g. Coulomb force. It results from its structure as residual interaction of the colorless nucleons.

**Central force** $V_0(r)$
results from deuteron properties (96% $^3S_1$ state) $^{2S+1}L_J$

**Spin dependent central force**
results from neutron-proton scattering (spin-spin interaction)

**Not central tensor force**
results from deuteron properties (4% $^3D_1$ state) $^{2S+1}L_J$

**Spin-orbit ($\ell \cdot s$) term**
results from scattering of polarized protons (left/right asymmetry)

\[
V(r) = V_0(r) + V_{ss}(r) \cdot \frac{1}{\hbar^2} + 3 \frac{(s_1 \cdot \vec{x})(s_2 \cdot \vec{x})}{r^2} - \frac{s_1 \cdot s_2}{\hbar^2} + V_{ls}(r) \cdot \frac{1}{\hbar^2}
\]

- $V_0(r)$: central potential
- $V_{ss}(r)$: spin-spin interaction
- $V_T(r)$: tensor force
- $V_{ls}(r)$: spin-orbit interaction
Structure of the nuclear force

- **Spin-spin force:**
  \[ \sim V_{ss}(r) \cdot \vec{s}_1 \cdot \vec{s}_2 / \hbar^2 \]
  - Different eigenvalues for triplet and singlet states
  \[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \quad s = 0, \; \ell = 1 \]
  \[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \quad |\uparrow\rangle \quad s = 1, \; \ell = 0 \]

- **Tensor force:**
  \[ \sim V_T(r) \cdot \frac{3}{\hbar^2} \frac{(\vec{s}_1 \cdot \vec{x})(\vec{s}_2 \cdot \vec{x})}{r^2} - \vec{s}_1 \cdot \vec{s}_2 \]
  - Small deformation of deuterium
  - Maximum magnetic dipole moments
  - Attractive \quad Repulsive

- **\(\ell\cdot s\) coupling:**
  \[ \sim V_{\ell s}(r) \cdot (\vec{\ell} \cdot \vec{s}) \]
  - Scattering of protons on polarized protons
  - Asymmetry of counting rates
  - Left scattering: \(\vec{\ell} \cdot \vec{s} > 0\)
  - Right scattering: \(\vec{\ell} \cdot \vec{s} < 0\)
  - No net contribution in the center of nucleus
  - Radial dependence at the surface of the nucleus
  \[ V_{\ell s}(r) \propto \frac{1}{r} \frac{d\rho}{dr} \]
The force on one nucleon does not only depend on the position of the other nucleons, but also on the distance between the other nucleons! These are called many-body forces.

Remember: Nucleons are finite-mass composite particles, can be excited to resonances. Dominant contribution $\Delta(1232 \text{ MeV})$
The Fermi gas model assumes that protons and neutrons are moving freely within the nuclear volume. They are distinguishable fermions (s = ½) filling two separate potential wells obeying the Pauli principle (↑↓-pair).

The model assumes that all fermions occupy the lowest energy states available to them to the highest occupied state (Fermi energy), and that there is no excitation across the Fermi energy (i.e. zero temperature).

The Fermi energy is common for protons and neutrons in stable nuclei.

If the Fermi energy for protons and neutrons are different then the β-decay transforms one type of nucleons into the other until the common Fermi energy (stability) is reached.
Number of nucleon states

Heisenberg Uncertainty Principle: \( \Delta x \cdot \Delta p \geq \frac{1}{2} \hbar \)

The volume of one particle in phase space: \( 2\pi \cdot \hbar \)

The number of nucleon states in a volume \( V \):

\[
n = \frac{\iiint d^3 r \ d^3 p}{(2\pi \cdot \hbar)^3} = \frac{V \cdot 4\pi \int_0^{p_{\text{max}}} p^2 \ dp}{(2\pi \cdot \hbar)^3}
\]

At temperature \( T = 0 \), i.e. for the nucleus in its ground state, the lowest states will be filled up to the maximum momentum, called the Fermi momentum \( p_F \). The number of these states follows from integration from 0 to \( p_{\text{max}} = p_F \).

\[
n = \frac{V \cdot 4\pi \int_0^{p_F} p^2 \ dp}{(2\pi \cdot \hbar)^3} = \frac{V \cdot 4\pi \cdot p_F^3}{(2\pi \cdot \hbar)^3} \rightarrow n = \frac{V \cdot p_F^3}{6\pi^2 \hbar^3}
\]

Since an energy state can contain two fermions of the same species, we can have

\[
\text{neutrons: } N = \frac{V \cdot (p_F^n)^3}{3\pi^2 \hbar^3} \quad \text{protons: } Z = \frac{V \cdot (p_F^p)^3}{3\pi^2 \hbar^3}
\]

\( p_F^n \) is the Fermi momentum for neutrons, \( p_F^p \) for protons
Fermi momentum

Use \( R = r_0 \cdot A^{1/3} \text{ fm} \)

\[
V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} r_0^3 \cdot A
\]

The density of nucleons in a nucleus = number of nucleons in a volume \( V \):

\[
n = 2 \cdot \frac{V \cdot p_F^3}{6\pi^2 \hbar^3} = 2 \cdot \frac{4\pi}{3} r_0^3 \cdot A \cdot \frac{p_F^3}{6\pi^2 \hbar^3} = \frac{4A}{9\pi} \frac{r_0^3 \cdot p_F^3}{\hbar^3}
\]

Fermi momentum \( p_F \):

\[
p_F = \left( \frac{6\pi^2 \hbar^3 n}{2V} \right)^{1/3} = \left( \frac{9\pi \hbar^3}{4A} \frac{n}{r_0^3} \right)^{1/3} = \left( \frac{9\pi \cdot n}{4A} \right)^{1/3} \frac{\hbar}{r_0}
\]

After assuming that the proton and neutron potential wells have the same radius, we find for a nucleus with \( n = Z = N = A/2 \) the Fermi momentum \( p_F \).

\[
p_F = p_F^n = p_F^p = \left( \frac{9\pi}{8} \right)^{1/3} \frac{\hbar}{r_0} \approx 250 \text{ MeV/c}
\]

The nucleons move freely inside the nucleus with large momenta

Fermi energy: \( E_F = \frac{p_F^2}{2m_N} \approx 33 \text{ MeV} \)

\( m_N = 938 \text{ MeV/c}^2 \) – the nucleon mass
The difference $B'$ between the top of the well and the Fermi level is the average binding energy per nucleon $B/A = 7 – 8 \text{ MeV}$.

→ The depth of the potential $V_0$ and the Fermi energy are independent of the mass number $A$:

$$V_0 = E_F + B' \approx 40 \text{ MeV}$$

Heavy nuclei have a surplus of neutrons. Since the Fermi level of the protons and neutrons in a stable nucleus have to be equal (otherwise the nucleus would enter a more energetically favorable state through $\beta$-decay) this implies that the depth of the potential well as it is experienced by the neutron gas has to be larger than of the proton gas.

Protons are therefore on average less strongly bound in nuclei than neutrons. This may be understood as a consequence of the Coulomb repulsion of the charged protons and leads to an extra term in the potential:

$$V_C = (Z - 1) \frac{\alpha \cdot \hbar c}{R}$$

Protonen: $33\text{MeV} + 7\text{MeV}$, Neutronen: $43\text{MeV} + 7 \text{ MeV}$
The Fermi gas model and the neutron star

**Assumption:** neutron star as cold neutron gas with constant density

- 1.5 sun masses: M = 3 \cdot 10^{30} \text{ kg} (m_N = 1.67 \cdot 10^{-27} \text{ kg}), number of neutrons: n = 1.8 \cdot 10^{57}

Fermi momentum $p_F$ for cold neutron gas:

$$p_F = \left( \frac{9\pi \cdot n}{4} \right)^{1/3} \cdot \frac{\hbar}{R}$$

R is the radius of the neutron star

Average kinetic energy per neutron:

$$\langle E_{\text{kin}}/N \rangle = \frac{3}{5} \cdot \frac{p_F^2}{2m_N} = \left( \frac{9\pi \cdot n}{4} \right)^{2/3} \cdot \frac{3\hbar^2}{10 \cdot m_N \cdot R^2} \cdot \frac{1}{R} = \frac{C}{R^2}$$

Gravitational energy of a star with constant density has an average potential energy per neutron:

$$\langle E_{\text{pot}}/N \rangle = -\frac{3}{5} \cdot \frac{G \cdot n \cdot m_n^2}{R} = -\frac{D}{R}$$

$G = 6.67 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$

Minimum total energy per neutron:

$$\frac{d}{dR} \langle E/N \rangle = \frac{d}{dR} \left[ \langle E_{\text{kin}}/N \rangle + \langle E_{\text{pot}}/N \rangle \right] = 0$$

$$\frac{d}{dR} \left[ \frac{C}{R^2} - \frac{D}{R} \right] = -\frac{2C}{R^3} + \frac{D}{R^2} = 0$$

$$R = \frac{2C}{D} \quad \rightarrow \quad R = \frac{\hbar^2 \cdot (9\pi/4)^{2/3}}{G \cdot m_N^3 \cdot n^{1/3}}$$

radius of a neutron star $\sim 10.7 \text{ km}$
Deviations from the Bethe-Weizsäcker mass formula:

\[
\begin{array}{c|c|c}
\text{mass number } A & B/A (\text{MeV per nucleon}) & \text{especially stable:} \\
\hline
2 & 4 & 4 \text{He}_2 \\
8 & 16 & 16 \text{O}_8 \\
20 & 40 & 40 \text{Ca}_{20} \\
28 & 56 & 56 \text{Ca}_{28} \\
126 & 208 & 208 \text{Pb}_{126}
\end{array}
\]
• deviations from the Bethe-Weizsäcker mass formula: \textit{large binding energies}
2-neutron binding energies = 2-neutron ‘separation’ energies

\[ S_{2n} = BE(N, Z) - BE(N - 2, Z) \]
Shell structure in nuclei

Nuclei with magic numbers of neutrons/protons

- high energies of the first excited $2^+$ state
- small nuclear deformations
  
  transition probabilities measured in single particle units (spu)

\[ E_{21^+} \]

\[ B(E2; 2_1^+ \rightarrow 0^+) \]
Shell structure in nuclei


| Angular Momentum (l=0, 2, 4, 6, 8, 10) | Spin-Orbit Coupling (l, 3/2, 5/2, 7/2, 9/2) | Number of Nucleons
|--------------------------------------|---------------------------------------------|-------------------|
| 7 | 1s^-1 | 1s 1/2 | 16 | (184) | (184)
| 6 | 4s^-1 | 4s 1/2 | 4 | (164) |
| 6 | 3d^-1 | 3d 5/2 | 6 | (142) |
| 6 | 2g^-1 | 2g 9/2 | 10 | (136) |
| 6 | 3t^-1 | 3t 1/2 | 2 | (112) |
| 5 | 3p^-1 | 3p 3/2 | 5 | (106) |
| 5 | 2f^-1 | 2f 5/2 | 4 | (100) |
| 5 | 1h^-1 | 1h 7/2 | 10 | (92) |
| 4 | 3s^-1 | 3s 1/2 | 2 | (70) |
| 4 | 2d^-1 | 2d 3/2 | 4 | (58) |
| 4 | 1g^-1 | 1g 7/2 | \n | \n |
| 3 | 2p^-1 | 2p 3/2 | 4 | (32) |
| 3 | 1f^-1 | 1f 7/2 | \n | \n |
| 2 | 2s^-1 | 2s 1/2 | 2 | (16) |
| 2 | 1d^-1 | 1d 3/2 | \n | \n |
| 1 | 1p^-1 | 1p 1/2 | \n | \n |
| 0 | 1s^-1 | 1s 1/2 | 2 | (2) |

Maria Goeppert-Mayer  J. Hans D. Jensen
Nuclear potential

\[ \hat{H} = \sum_{i=1}^{A} \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j} \hat{V}(r_i, r_j) \]

\[ \hat{H} = \sum_{i=1}^{A} \left[ \frac{\hat{p}_i^2}{2m_i} + \hat{V}(r_i) \right] + \left[ \sum_{i<j} \hat{V}(r_i, r_j) - \sum_{i=1}^{A} \hat{V}(r_i) \right] \]

\[ \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) - \varepsilon \right] \Psi(r) = 0 \]

\[ \Psi(r) = \frac{u_\ell(r)}{r} \cdot Y_{\ell m}(\theta, \phi) \cdot X_{ms} \]

In the average nuclear potential \( V(r) \):

a) harmonic oscillator
b) square well potential
c) Woods-Saxon potential

d) the nucleons move freely

\[ V(r) = \frac{-V_0}{1 + e^{(r-R_0)/a}} \]
\[ \hat{H} = \sum_{i=1}^{A} \left[ \frac{\hat{p}_i^2}{2m_i} + \hat{V}(r_i) \right] \]

Nuclear shell model

harmonic oscillator

square-well potential

realistic potential + spin-orbit coupling

\[ \begin{align*}
6\hbar\omega & \quad 168 \quad 168 \\
5\hbar\omega & \quad 112 \quad 126 \\
4\hbar\omega & \quad 70 \quad 82 \\
3\hbar\omega & \quad 40 \quad 50 \\
2\hbar\omega & \quad 20 \quad 28 \\
1\hbar\omega & \quad 8 \quad 20 \\
0\hbar\omega & \quad 2 \quad 2
\end{align*} \]
The spin-orbit term has its origin in the relativistic description of the single particle motion inside the nucleus.

**Woods-Saxon does not reproduce the correct magic numbers**

\[(2, 8, 20, 40, 70, 112, 168)_{\text{WS}} \quad (2, 8, 20, 28, 50, 82, 126)_{\text{exp}}\]

**Meyer und Jensen (1949): strong spin-orbit interaction**

\[
\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) + V_\ell s(r) \cdot \vec{\ell} \cdot \vec{s} - \epsilon\right] \Psi(r) = 0
\]

\[V_\ell s(r) \sim -\lambda \cdot \frac{1}{r} \cdot \frac{dV}{dr} \quad \text{mit} \quad \lambda > 0\]
The nuclear potential with spin-orbit term:

\[ V(\rho) + \frac{\ell}{2} \cdot V_{ls} \quad \text{for} \quad j = \ell + 1/2 \]

\[ V(\rho) - \frac{\ell + 1}{2} V_{ls} \quad \text{for} \quad j = \ell - 1/2 \]

Spin-orbit interaction leads to a large splitting for large \( \ell \).
**Woods-Saxon potential**

**The spin-orbit term**

- lowers the $j = \ell + 1/2$ orbital from the higher oscillator shell (intruder states)
- reproduces the magic numbers
  - large energy gaps $\rightarrow$ very stable nuclei

**Important consequences:**

- lowering orbitals from higher lying N+1 shell having different parity than orbitals from the N shell
- strong interaction preserves the parity. The lowered orbitals with different parity are rather pure states and do not mix within the shell
Shell model – mass dependence of single-particle energies

- Mass dependence of the neutron energies: \( E \sim R^{-2} \)
- Number of neutrons in each level: \( 2 \cdot (2\ell + 1) \)
Success of the extreme single-particle shell model

<table>
<thead>
<tr>
<th>$Z$</th>
<th>Isotope</th>
<th>Observed $J^\pi$</th>
<th>Shell model $nlj$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$^9$Li</td>
<td>$3/2^-$</td>
<td>1$p_{3/2}$</td>
</tr>
<tr>
<td>5</td>
<td>$^{13}$B</td>
<td>3/2$^-$</td>
<td>1$p_{3/2}$</td>
</tr>
<tr>
<td>7</td>
<td>$^{17}$N</td>
<td>1/2$^-$</td>
<td>1$p_{1/2}$</td>
</tr>
<tr>
<td>9</td>
<td>$^{21}$F</td>
<td>5/2$^+$</td>
<td>1$d_{5/2}$</td>
</tr>
<tr>
<td>11</td>
<td>$^{25}$Na</td>
<td>5/2$^+$</td>
<td>1$d_{5/2}$</td>
</tr>
<tr>
<td>13</td>
<td>$^{29}$Al</td>
<td>5/2$^+$</td>
<td>1$d_{5/2}$</td>
</tr>
<tr>
<td>15</td>
<td>$^{33}$P</td>
<td>1/2$^+$</td>
<td>2$s_{1/2}$</td>
</tr>
<tr>
<td>17</td>
<td>$^{37}$Cl</td>
<td>3/2$^+$</td>
<td>1$d_{3/2}$</td>
</tr>
<tr>
<td>19</td>
<td>$^{41}$K</td>
<td>3/2$^+$</td>
<td>1$d_{3/2}$</td>
</tr>
<tr>
<td>21</td>
<td>$^{45}$Sc</td>
<td>7/2$^-$</td>
<td>1$f_{7/2}$</td>
</tr>
<tr>
<td>23</td>
<td>$^{49}$Va</td>
<td>7/2$^-$</td>
<td>1$f_{7/2}$</td>
</tr>
<tr>
<td>25</td>
<td>$^{53}$Mn</td>
<td>7/2$^-$</td>
<td>1$f_{7/2}$</td>
</tr>
<tr>
<td>27</td>
<td>$^{57}$Co</td>
<td>7/2$^-$</td>
<td>1$f_{7/2}$</td>
</tr>
<tr>
<td>29</td>
<td>$^{61}$Cu</td>
<td>3/2$^-$</td>
<td>2$p_{3/2}$</td>
</tr>
<tr>
<td>31</td>
<td>$^{65}$Ga</td>
<td>3/2$^-$</td>
<td>2$p_{3/2}$</td>
</tr>
<tr>
<td>33</td>
<td>$^{69}$As</td>
<td>(5/2$^-$)</td>
<td>1$f_{5/2}$</td>
</tr>
<tr>
<td>35</td>
<td>$^{73}$Br</td>
<td>(3/2$^-$)</td>
<td>1$f_{5/2}$</td>
</tr>
</tbody>
</table>

➢ *Ground state spin and parity:*  

Every orbital has $2j+1$ magnetic sub-states, completely filled orbitals have spin $J=0$, they do not contribute to the nuclear spin.

For a nucleus with one nucleon outside a completely occupied orbital the nuclear spin is given by the single nucleon.

$$n\ell j \rightarrow J$$

$$(-)^\ell = \pi$$
Success of the extreme single-particle shell model

$^7_{\ 4}Be$

$^1_{\ 9}F$

$^{63}_{\ 28}Ni$

$^{61}_{\ 29}Cu$

$^{91}_{\ 40}Zr$

$^{123}_{\ 51}Sb$

$^{159}_{\ 65}Tb$

$^{183}_{\ 73}Ta$

$^{199}_{\ 81}Tl$

$^{209}_{\ 82}Pb$
Magnetic moments:
The g-factor $g_j$ is given by:

$$
\mu_j = g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s} = g_j \cdot \vec{j} \Rightarrow \mu_j = \left( g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s} \right) \cdot \frac{\vec{j}}{|\vec{j}|} \cdot \frac{\vec{j}}{|\vec{j}|}
$$

with $\vec{l}^2 = (j - s)^2 = j^2 - 2 \cdot j \cdot s + s^2$  
$\vec{s}^2 = (j - \ell)^2 = j^2 - 2 \cdot j \cdot \ell + \ell^2$

$$
\mu_j = \frac{g_\ell \cdot \{j(j+1) + \ell(\ell + 1) - 3/4\} + g_s \cdot \{j(j+1) - \ell(\ell + 1) + 3/4\}}{2 \cdot j(j + 1)} \cdot \vec{j}
$$

$$
g_j = \frac{1}{2} \cdot (g_\ell + g_s) + \frac{1}{2} \cdot \frac{\ell(\ell + 1) - s(s + 1)}{2j(j + 1)} \cdot (g_\ell - g_s)
$$

Simple relation for the g-factor of single-particle states

$$
\frac{\mu}{\mu_N} = g_{\text{nucleus}} = g_\ell \pm \frac{(g_s - g_\ell)}{2\ell + 1} \quad \text{for} \quad j = \ell \pm 1
$$

<table>
<thead>
<tr>
<th>nucleus</th>
<th>state</th>
<th>$J^\pi$</th>
<th>$\mu/\mu_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{15}\text{N}$</td>
<td>p-1$p^{1}_{1/2}$</td>
<td>1/2$^-$</td>
<td>-0.264 \quad -0.283</td>
</tr>
<tr>
<td>$^{15}\text{O}$</td>
<td>n-1$p^{1}_{1/2}$</td>
<td>1/2$^-$</td>
<td>+0.638 \quad +0.719</td>
</tr>
<tr>
<td>$^{17}\text{O}$</td>
<td>n-1$d^{1}_{5/2}$</td>
<td>5/2$^+$</td>
<td>-1.913 \quad -1.894</td>
</tr>
<tr>
<td>$^{17}\text{F}$</td>
<td>p-1$d^{1}_{5/2}$</td>
<td>5/2$^+$</td>
<td>+4.722 \quad +4.793</td>
</tr>
</tbody>
</table>
Success of the extreme single-particle shell model

➤ magnetic moments:
\[
\langle \mu_z \rangle = \begin{cases} 
\frac{g_\ell \cdot \left( j - \frac{1}{2} \right) + \frac{1}{2} \cdot g_s}{j+1} \cdot \mu_N & \text{for } j = \ell + 1/2 \\
\frac{j}{j+1} \cdot \left[ g_\ell \cdot \left( j + \frac{3}{2} \right) - \frac{1}{2} \cdot g_s \right] \cdot \mu_N & \text{for } j = \ell - 1/2 
\end{cases}
\]

➤ g-factor of nucleons:

proton: \( g_\ell = 1; \; g_s = +5.585 \)
neutron: \( g_\ell = 0; \; g_s = -3.82 \)

proton:
\[
\langle \mu_z \rangle = \begin{cases} 
(j + 2.293) \cdot \mu_N & \text{for } j = \ell + 1/2 \\
(j - 2.293) \cdot \frac{j}{j+1} \cdot \mu_N & \text{for } j = \ell - 1/2 
\end{cases}
\]

neutron:
\[
\langle \mu_z \rangle = \begin{cases} 
-1.91 \cdot \mu_N & \text{for } j = \ell + 1/2 \\
+1.91 \cdot \frac{j}{j+1} \cdot \mu_N & \text{for } j = \ell - 1/2 
\end{cases}
\]
Magnetic moments: Schmidt lines

**magnetic moments: neutron**

**magnetic moments: proton**
The three structures of the shell model
Evolution of nuclear structure
(as a function of nucleon number)

- $6^+$
- $4^+$
- $2^+$

- $0^+$, $R_{4/2} < 2$
- $2^+$
- $4^+$
- $6^+$

- Magic (sph. vib.)
- Magic (sph. vib.)
- Mid-shell (ellipsoidal)
Systematics of the Te isotopes (Z=52)

Vibrational

Seniority - short range force

<table>
<thead>
<tr>
<th>Neutron number</th>
<th>Val. Neutr. number</th>
<th>Phonon Number</th>
<th>Seniority</th>
</tr>
</thead>
<tbody>
<tr>
<td>68</td>
<td>14</td>
<td>$^{120}\text{Te}$</td>
<td>0</td>
</tr>
<tr>
<td>70</td>
<td>12</td>
<td>$^{122}\text{Te}$</td>
<td>2</td>
</tr>
<tr>
<td>72</td>
<td>10</td>
<td>$^{124}\text{Te}$</td>
<td>4</td>
</tr>
<tr>
<td>74</td>
<td>8</td>
<td>$^{126}\text{Te}$</td>
<td>6</td>
</tr>
<tr>
<td>76</td>
<td>6</td>
<td>$^{128}\text{Te}$</td>
<td>8</td>
</tr>
<tr>
<td>78</td>
<td>4</td>
<td>$^{130}\text{Te}$</td>
<td>10</td>
</tr>
<tr>
<td>80</td>
<td>2</td>
<td>$^{132}\text{Te}$</td>
<td>12</td>
</tr>
<tr>
<td>82</td>
<td>0</td>
<td>$^{134}\text{Te}$</td>
<td>14</td>
</tr>
</tbody>
</table>