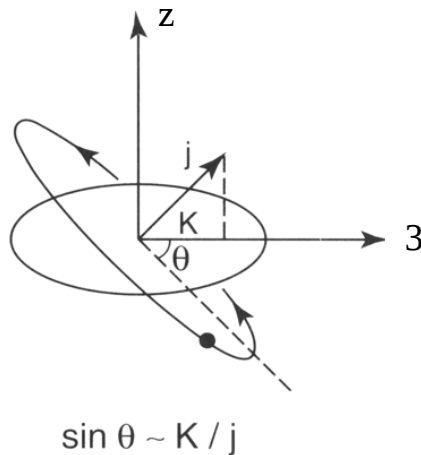


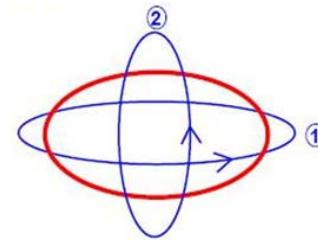
Deformed (Nilsson) Shell Model



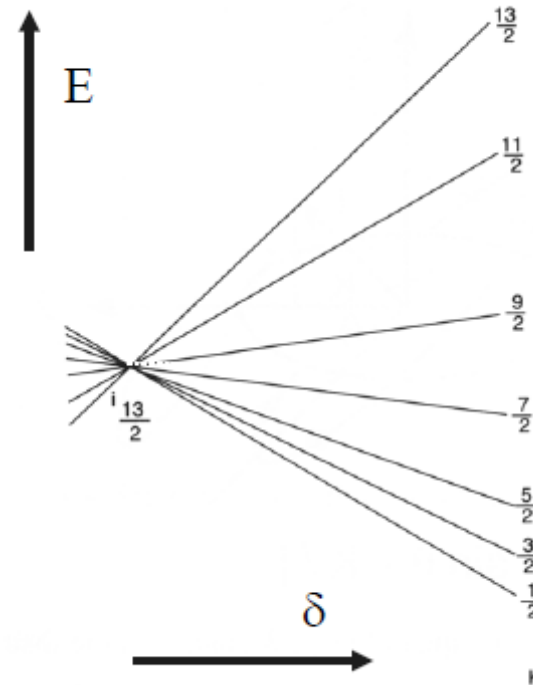
- spherical nucleus: $R = R_0$
- deformed nucleus: $R = R_0 \cdot \left[1 + \sum_{\lambda=2,\mu} \alpha_{2\mu} \cdot Y_{2\mu} \right]$
→ can rotate



- Separation of laboratory system and body-fixed (intrinsic) system
- K = projection of the single-particle angular momentum onto the symmetry axis
- Rotation perpendicular to the symmetry axis will not change the K -quantum number

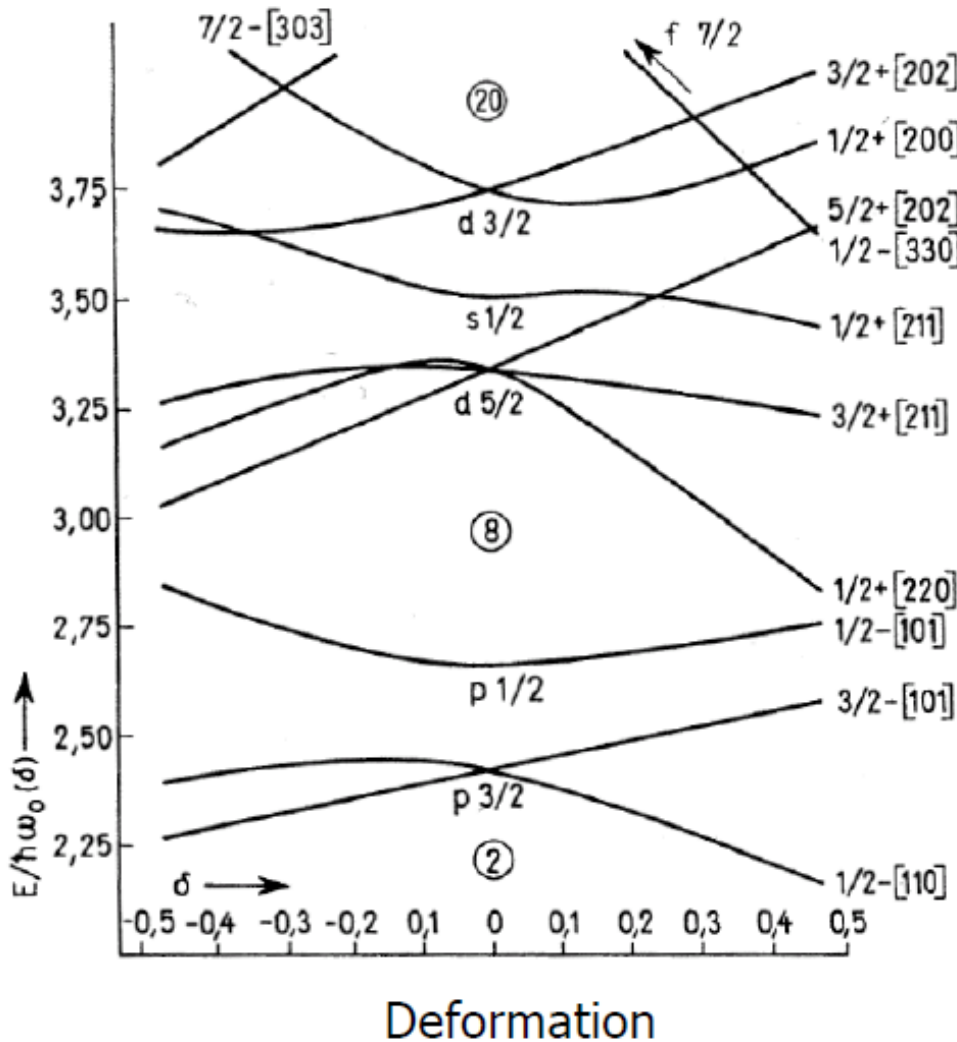


orbit 1 is closer to the center of gravity than orbit 2.
The energy of orbit 1 is the lowest.



Nilsson Model (quadrupole interaction)

Sven Gösta Nilsson



Nilsson Model is a single-particle model for deformed nuclei.

$$H = \frac{p^2}{2m} + \frac{m \cdot [\omega_x^2(x^2 + y^2) + \omega_z^2 z^2]}{2} + C \cdot \ell \cdot s + D \cdot \ell^2$$

with $\omega_x^2 = \omega_0^2 \cdot \left(1 + \frac{2}{3} \cdot \delta\right)$ $\omega_z^2 = \omega_0^2 \cdot \left(1 - \frac{4}{3} \cdot \delta\right)$

The labelling of the orbits is: $\Omega^\pi[Nn_z\Lambda]$.

Ω projection of the total particle angular momentum on the symmetry axis

π parity of the wave function

N the principal quantum number of the major oscillator shells

n_z the number of quanta associated with the wave function moving along the z-direction

Λ projection of the orbital angular momentum onto the z-axis

Nilsson Model for small deformations

$$H = \underbrace{-\frac{\hbar^2}{2m}\Delta + \frac{m}{2}\omega_0^2 r^2 + C \cdot \vec{L} \cdot \vec{S} + D \cdot \vec{L}^2}_{\text{shell model with H.O. potential}} - \underbrace{m\omega_0^2 r^2 \delta \frac{4}{3} \sqrt{\frac{4\pi}{5}} Y_{20}(\theta, \Phi)}_{H_{\text{def}}}$$

shell model with H.O. potential

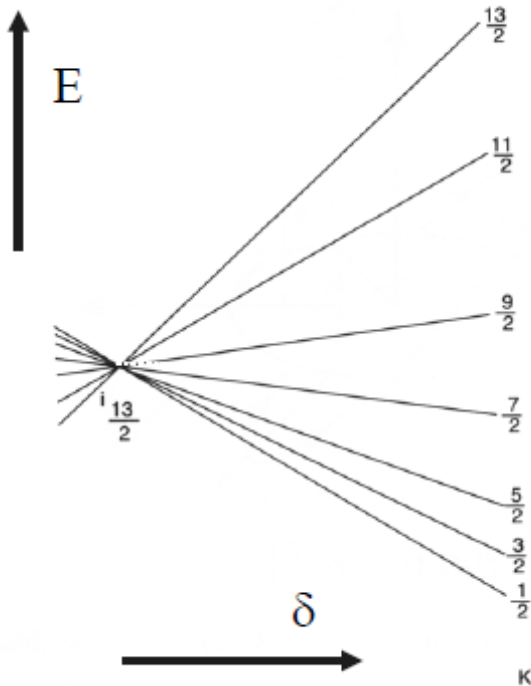
H_{def}

$$\Delta E(N\ell j K) = -\frac{4}{3} \sqrt{\frac{4\pi}{5}} m\omega_0^2 \cdot \delta \cdot \langle N\ell j m | r^2 Y_{20} | N\ell j m \rangle$$

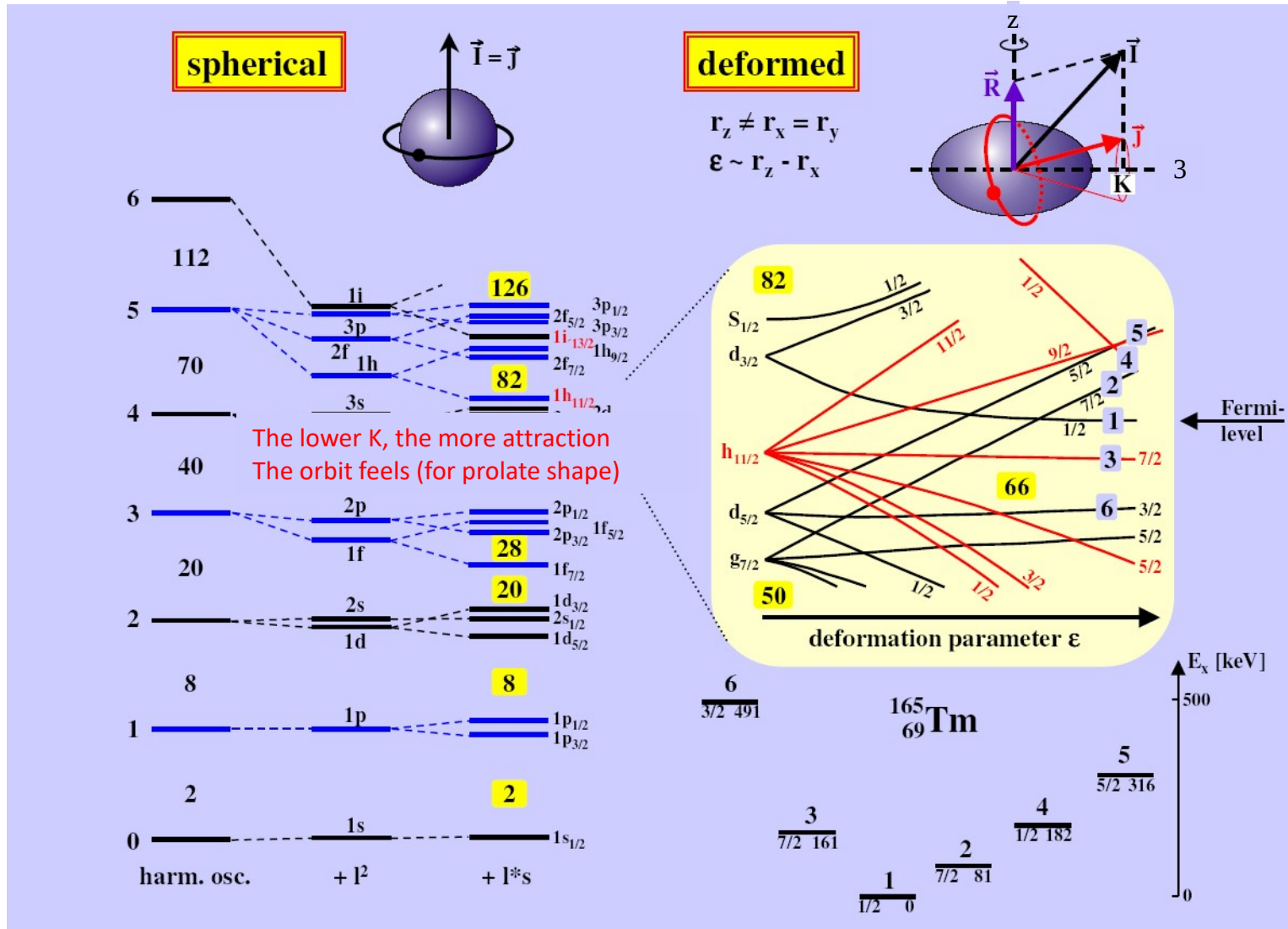
$$\Delta E(N\ell j K) = -\frac{2}{3} \hbar\omega_0 \left(N + \frac{3}{2}\right) \cdot \delta \cdot \frac{[3K^2 - j(j+1)] \cdot [\frac{3}{4} - j(j+1)]}{(2j-1)j(j+1)(2j+3)}$$

results for small deformations δ :

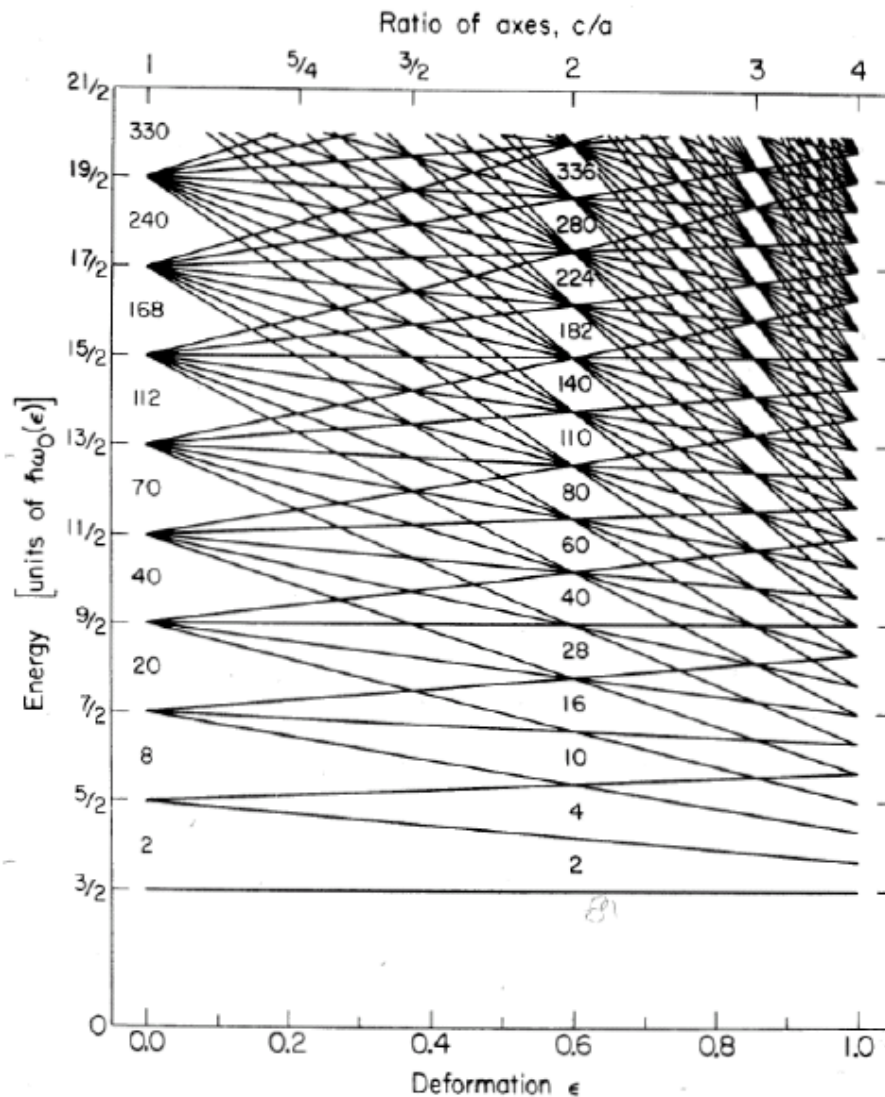
- $\Delta E \propto \delta \approx \beta$
- $\Delta E \propto K^2 - j(j+1)$
- $\Delta E \propto N$



Spherical Shell Model → Nilsson Model



Shell closures for large deformations

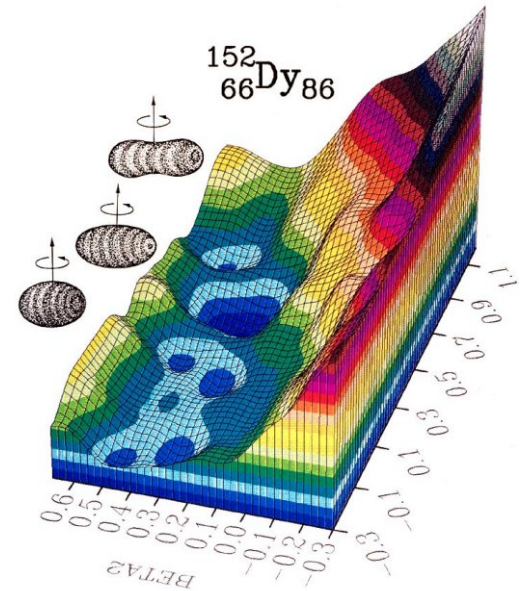
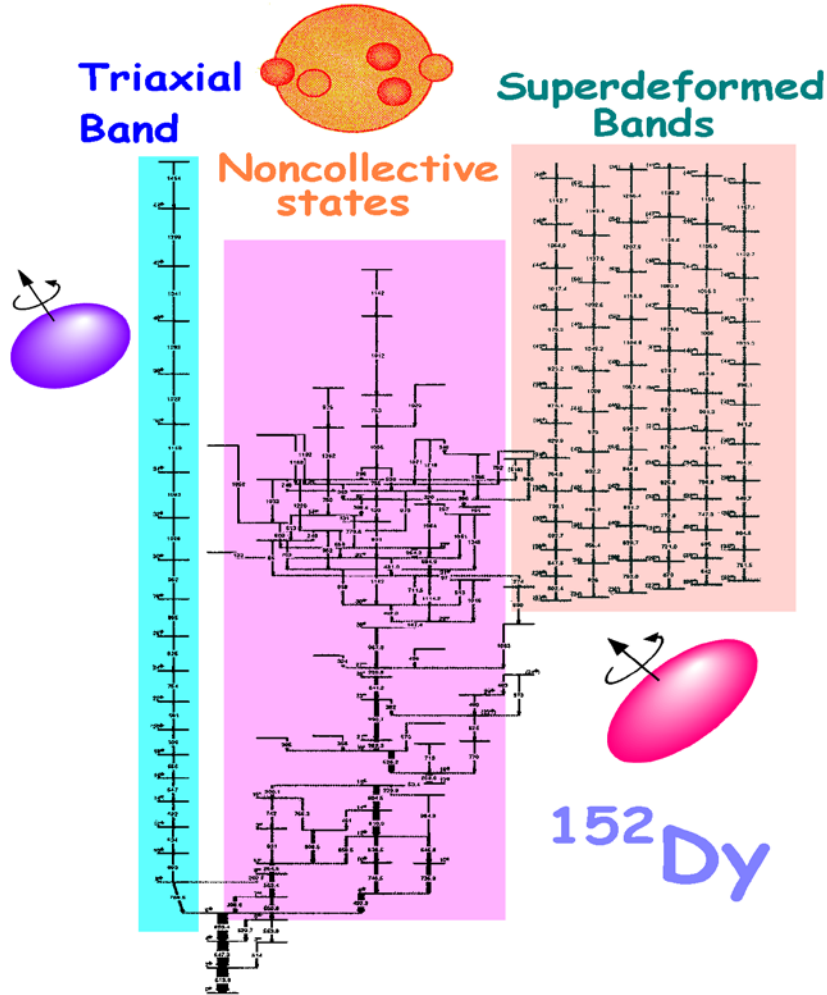


For large deformation the $\vec{\ell} \cdot \vec{s}$ and $\vec{\ell}^2$ terms can be neglected.

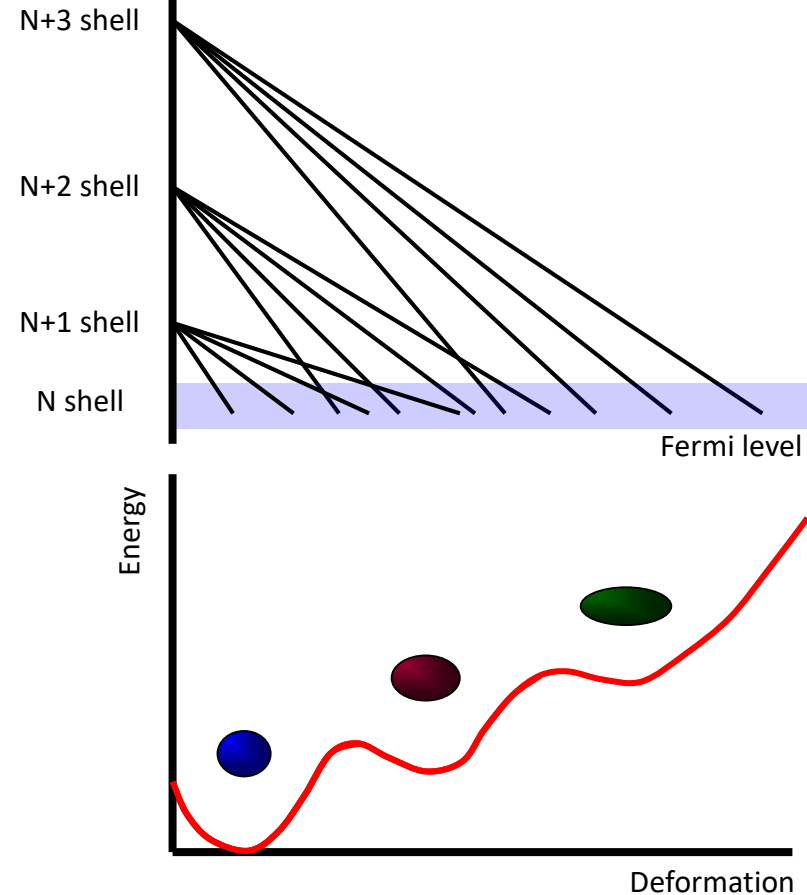
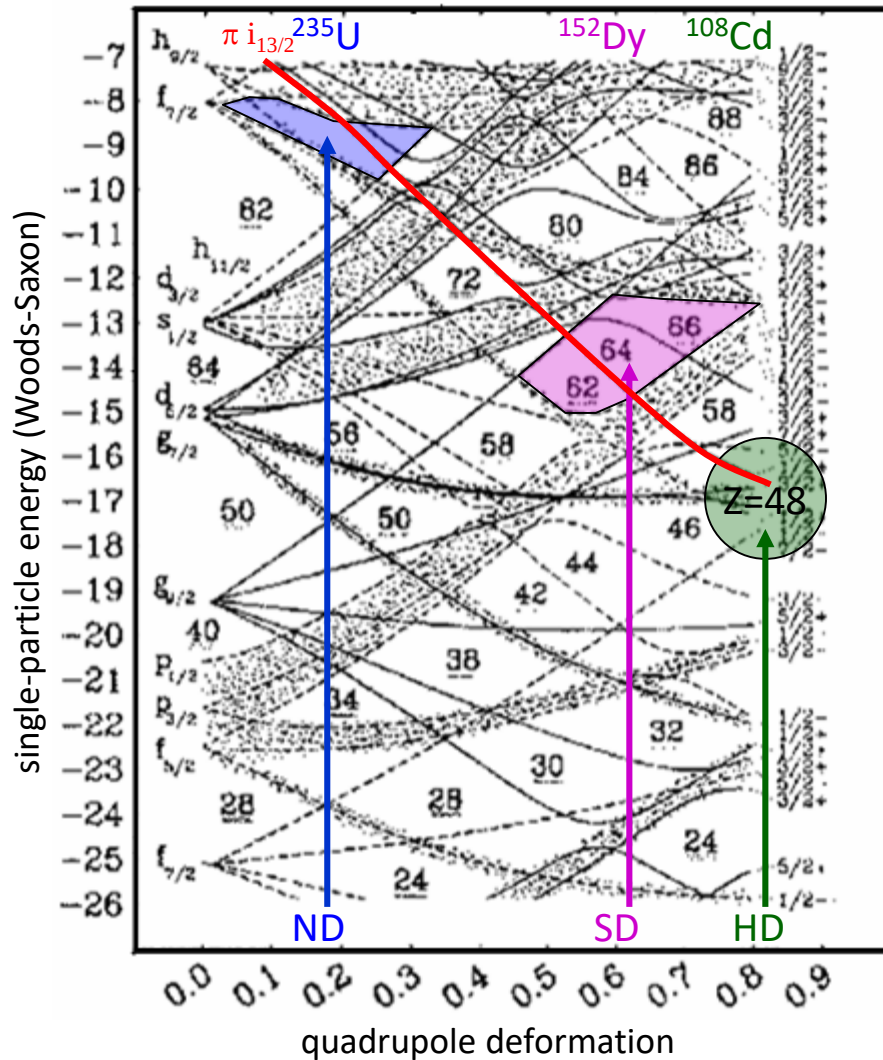
spectrum of a prolate deformed harmonic oscillator as a function of the deformation parameter ϵ

Nuclear Structure of ^{152}Dy : hadronic field theory in nuclei

Coexistence of collective and noncollective motion

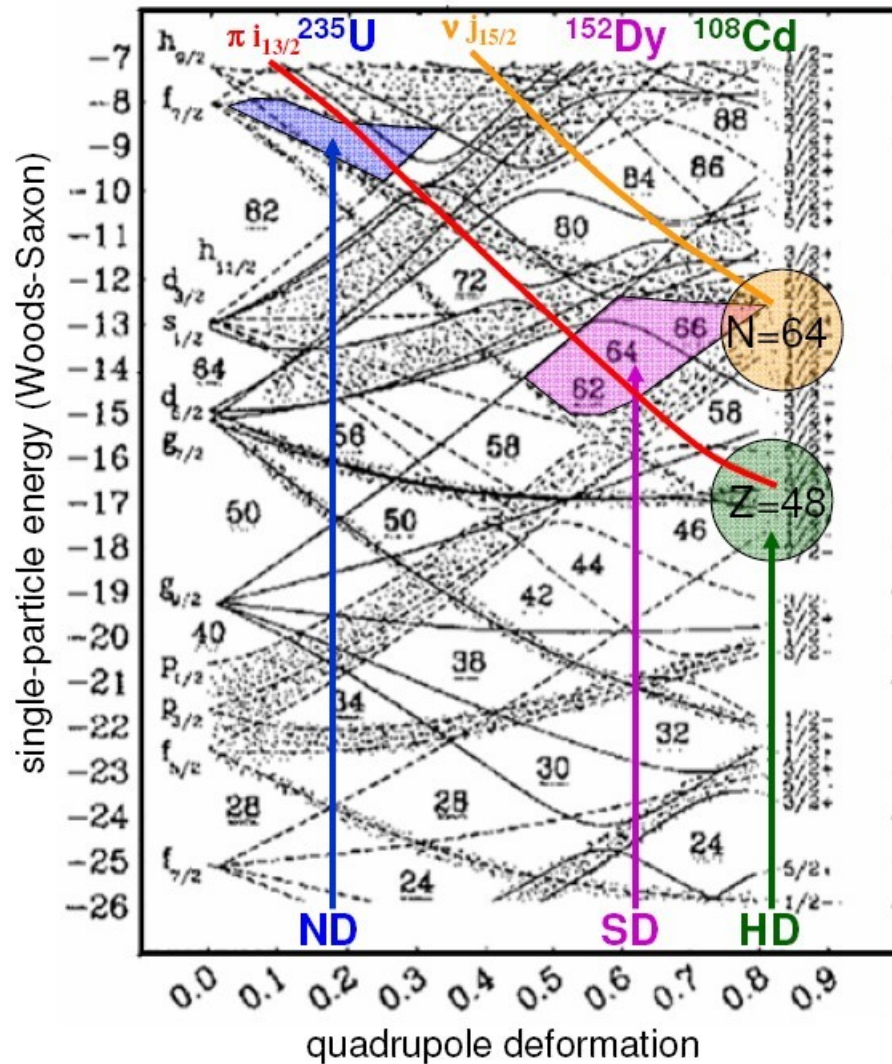


Nuclear structure and intruder states



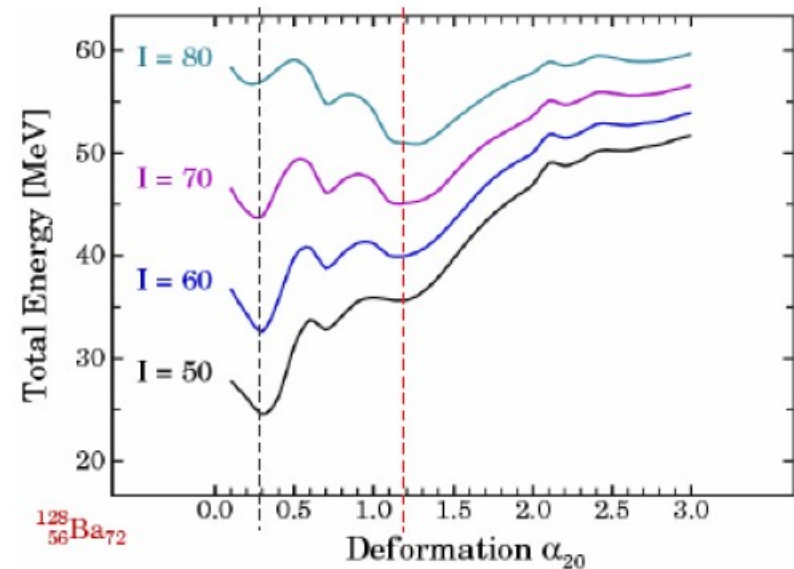
- (N+1) intruder
⇒ normal deformed, e.g. ^{235}U
- (N+2) super-intruder
⇒ super deformation, e.g. ^{152}Dy , ^{80}Zr
- (N+3) hyper-intruder
⇒ hyper deformation in ^{108}Cd , ?

Nuclear deformation



For large spins: interaction between

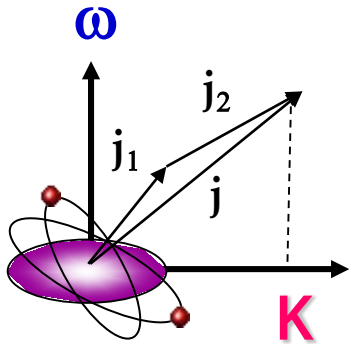
- macroscopic effects: liquid drop
- microscopic effects: shell structure



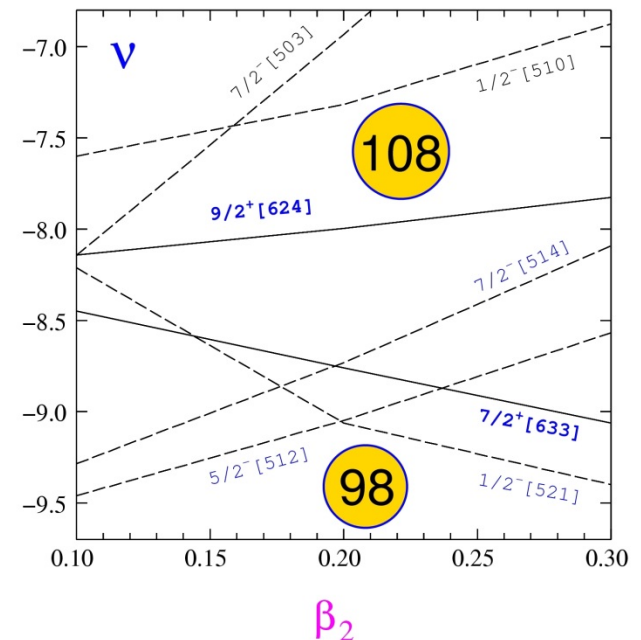
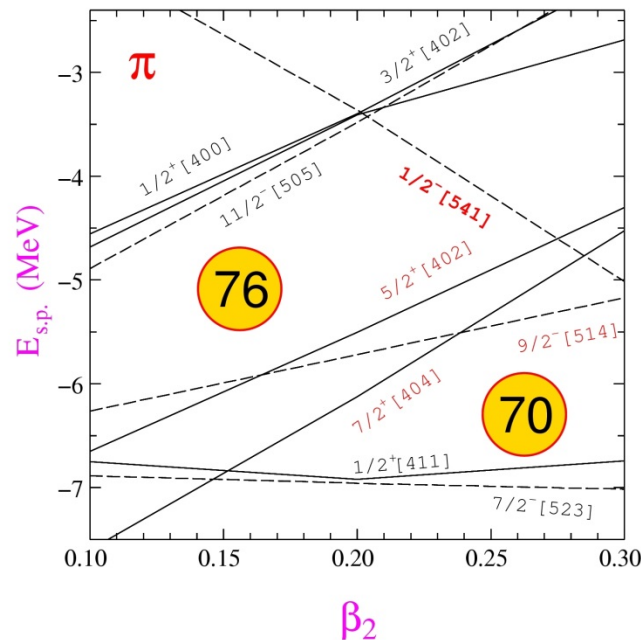
The different slopes of the single-particle states create different minima for the same number of nucleons

K-Isomers

- Deformed nuclei with axially-symmetric shape



Mass 180 region : Yb (Z=70) - Ir (Z=77)



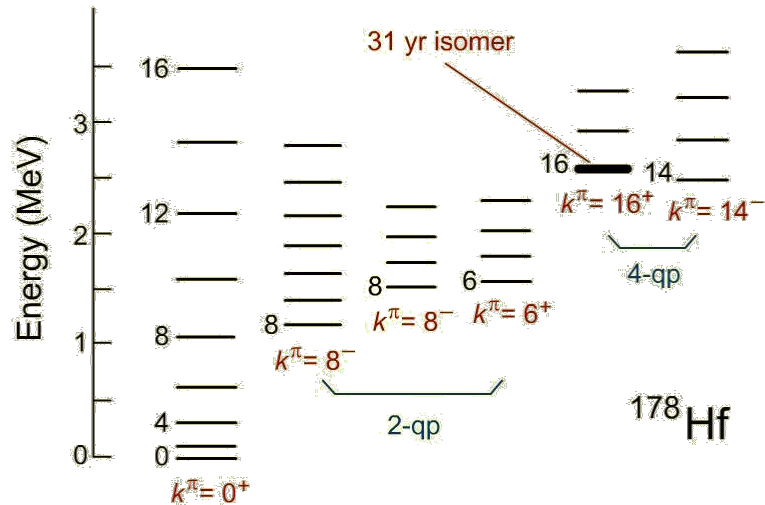
- High-K orbitals near the Fermi surface

π : 7/2[404], 9/2[514], 5/2[402]

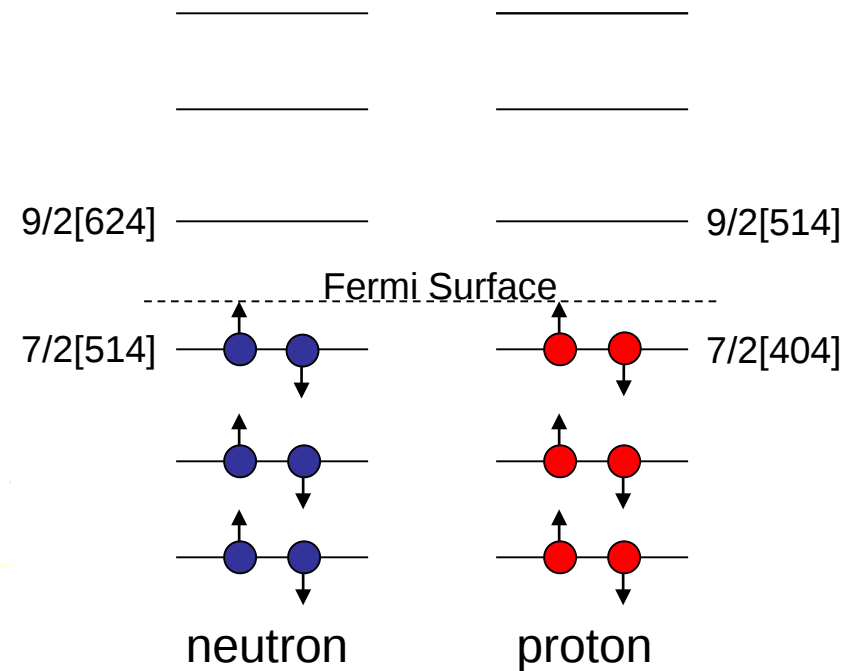
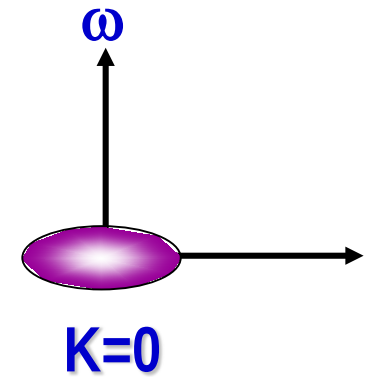
ν : 7/2[514], 9/2[624], 5/2[512], 7/2[633]

- A well-known example:

High-K isomers in ^{178}Hf

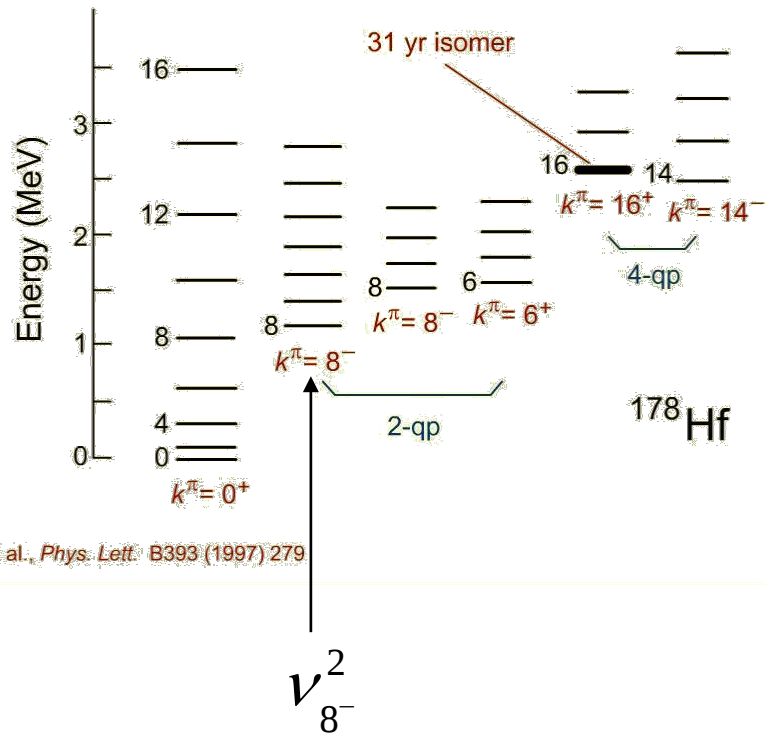


Mullins et al., *Phys. Lett. B* 393 (1997) 279

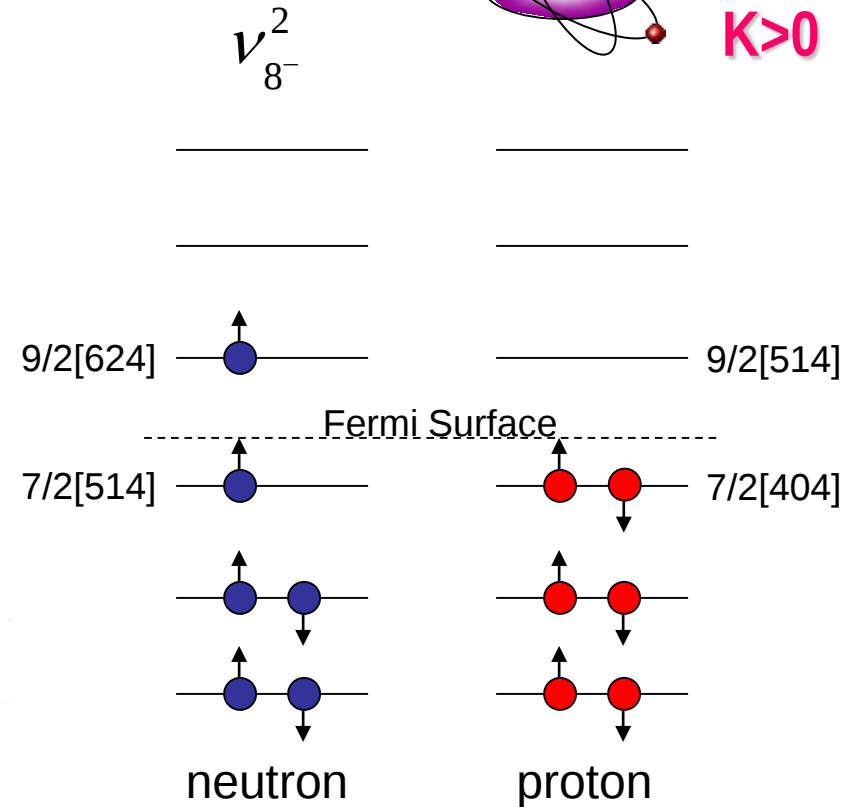
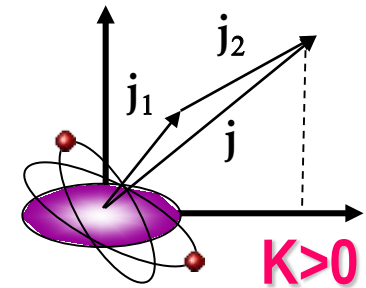


- A well-known example:

High-K isomers in ^{178}Hf

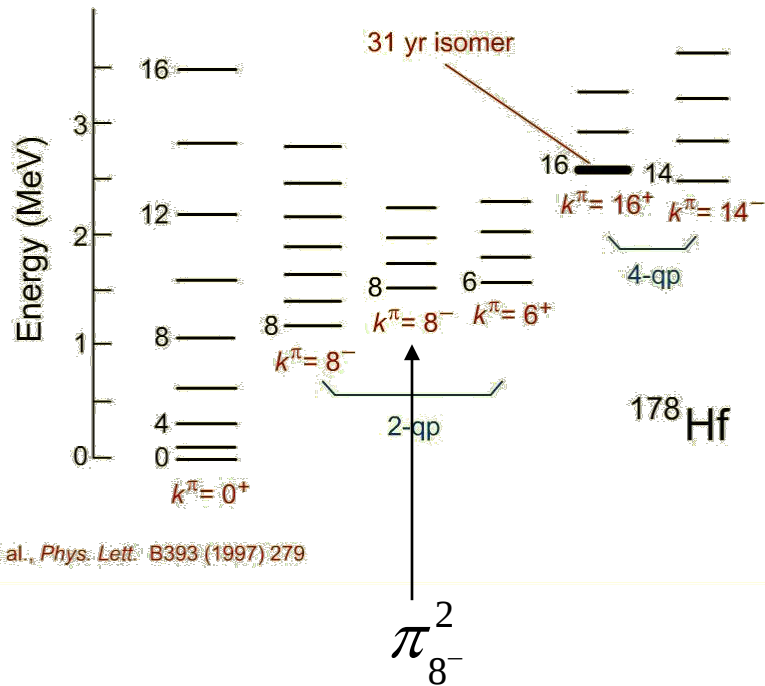


Mullins et al., *Phys. Lett. B* 393 (1997) 279

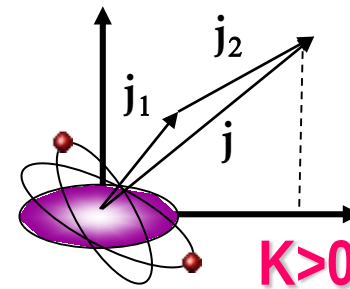


- A well-known example:

High-K isomers in ^{178}Hf



Mullins et al., *Phys. Lett. B* 393 (1997) 279

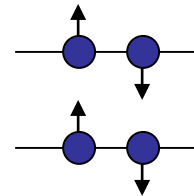


$\pi_{8^-}^2$

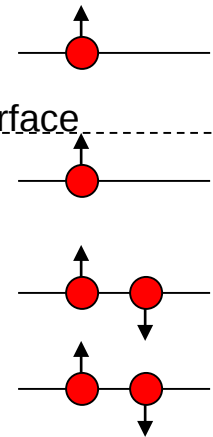
9/2[624]

7/2[514]

Fermi Surface



neutron



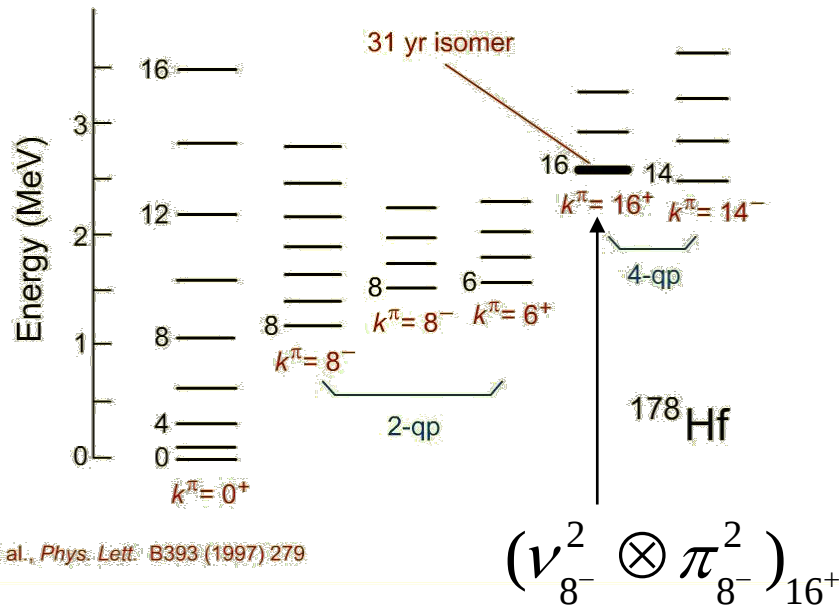
proton

9/2[514]

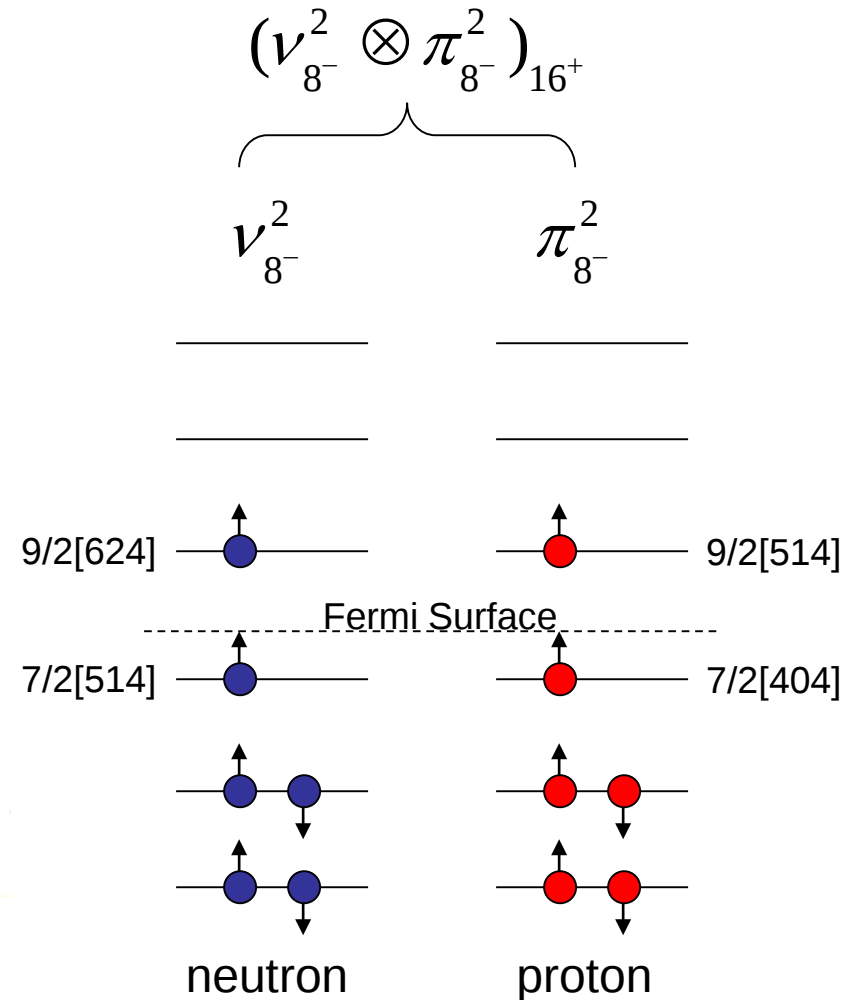
7/2[404]

- A well-known example:

High-K isomers in ^{178}Hf



Mullins et al., *Phys. Lett. B* 393 (1997) 279



Magnetic Moments in ^{178}Hf

$$g(j) = \begin{cases} \frac{2 \cdot \ell \cdot g_\ell + g_s}{2 \cdot \ell + 1} & \text{for } j = \ell + 1/2 \\ \frac{2 \cdot (\ell + 1) \cdot g_\ell - g_s}{2 \cdot \ell + 1} & \text{for } j = \ell - 1/2 \end{cases}$$

proton $g_\ell = 1$ $g_s = 5.59$
neutron $g_\ell = 0$ $g_s = -3.83$

$$g(\mathbf{h}_{11/2}) = 1.42 \quad g(\mathbf{g}_{7/2}) = 0.49 \quad g(\mathbf{f}_{7/2}) = -0.55 \quad g(\mathbf{i}_{13/2}) = -0.29$$

$$g(j_1 \times j_2; J) = \frac{1}{2} \cdot (g_1 + g_2) + \frac{j_1 \cdot (j_1 + 1) - j_2 \cdot (j_2 + 1)}{2 \cdot J \cdot (J + 1)} \cdot (g_1 - g_2)$$

$$g(\mathbf{h}_{11/2} \times \mathbf{g}_{7/2}; 8^-) = 1.08 \quad g(\mathbf{f}_{7/2} \times \mathbf{i}_{13/2}) = -0.36$$

$$g(8^- \times 8^-; 16^+) = 0.36 \rightarrow \mu = g \cdot I = 5.76 \text{ nm}$$

$$7.26 \pm 0.16 \text{ nm}$$

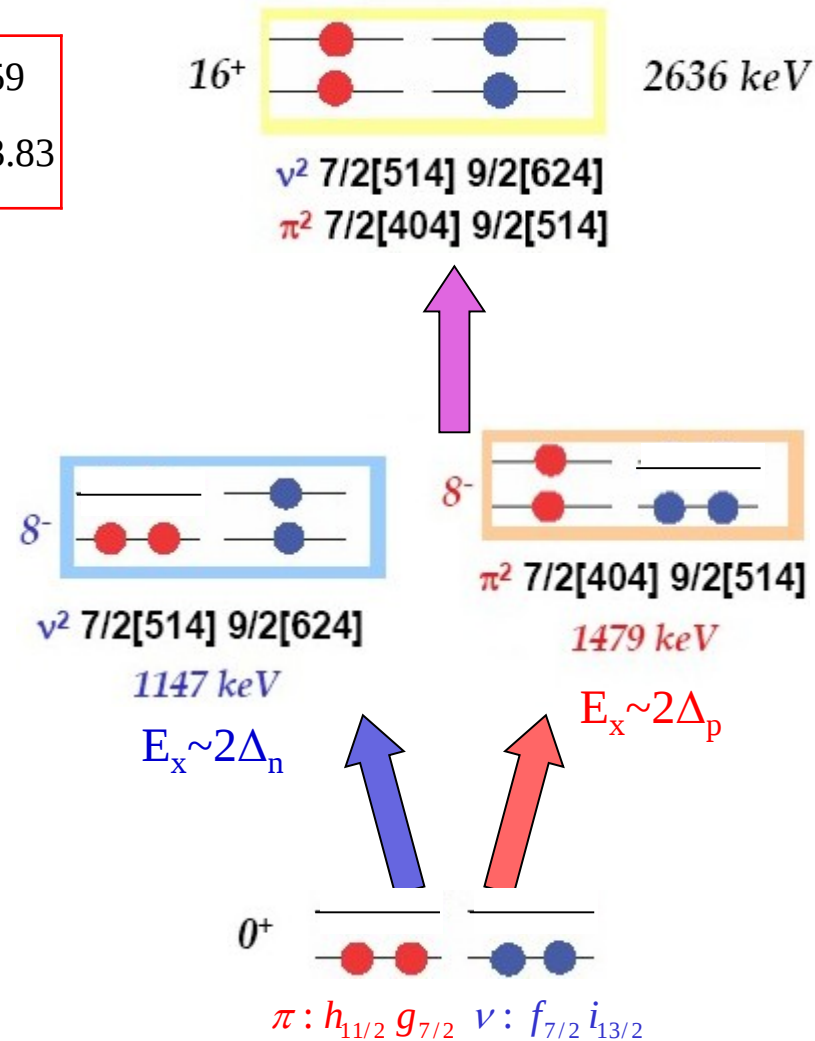
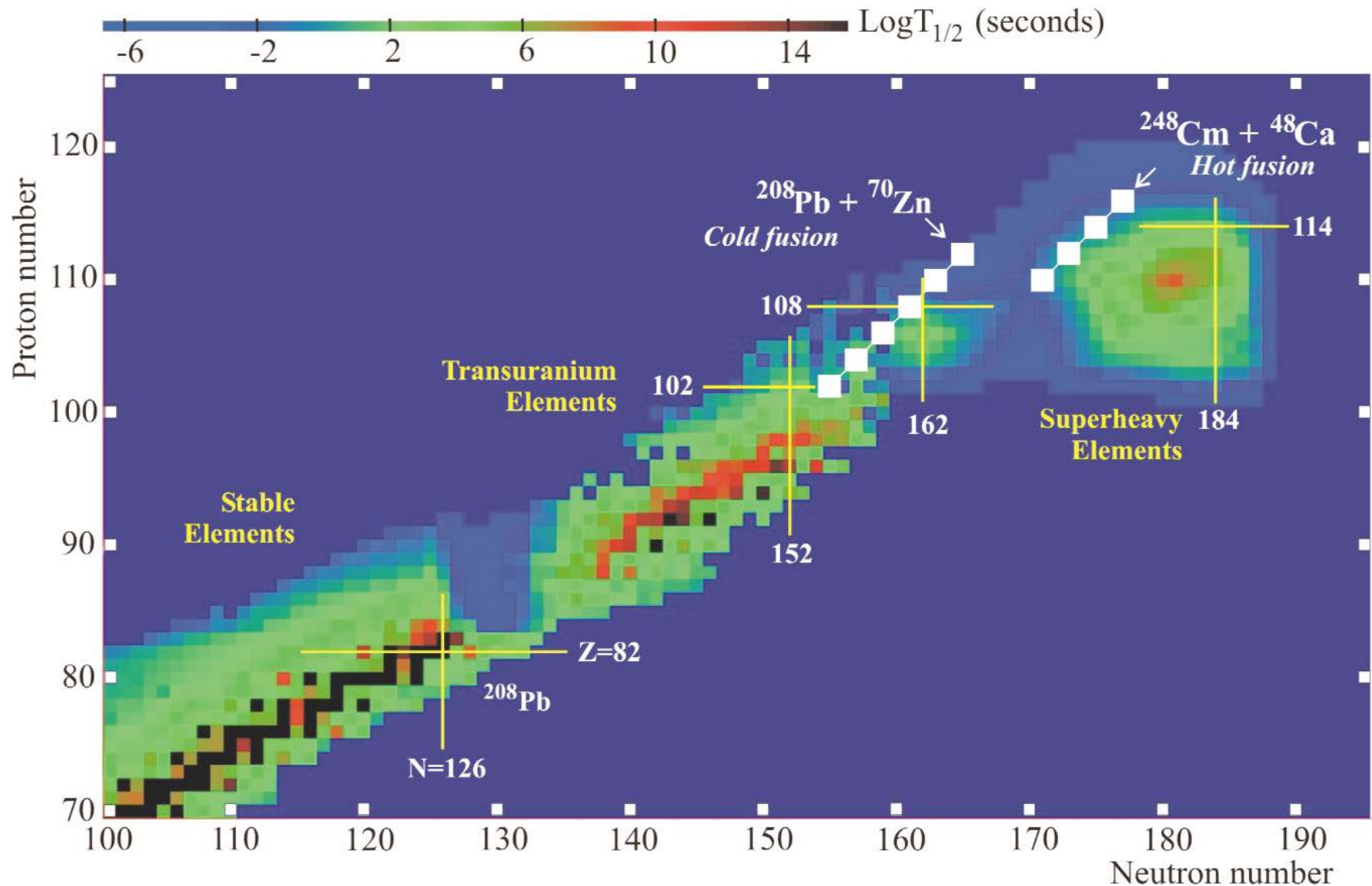
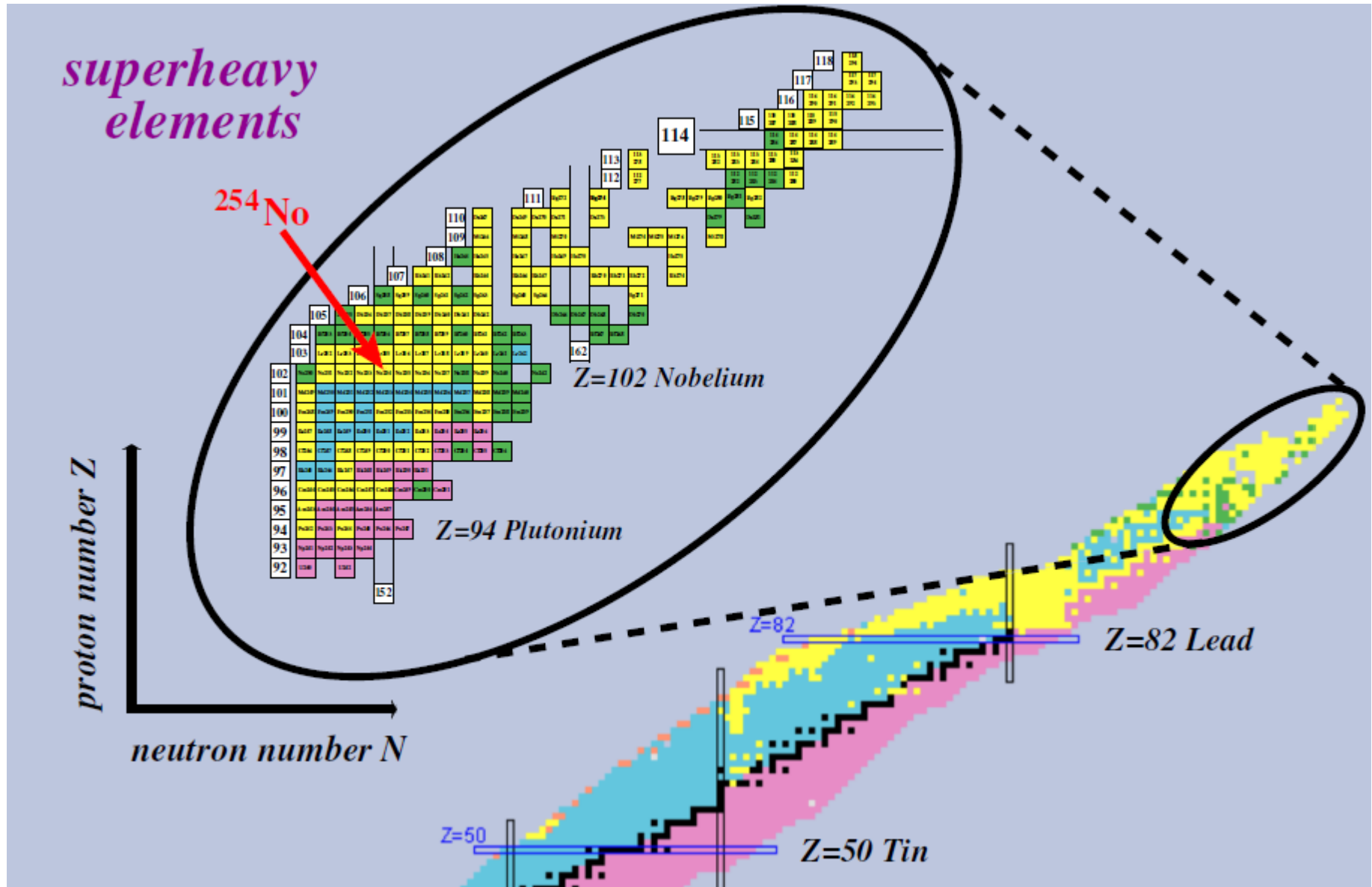


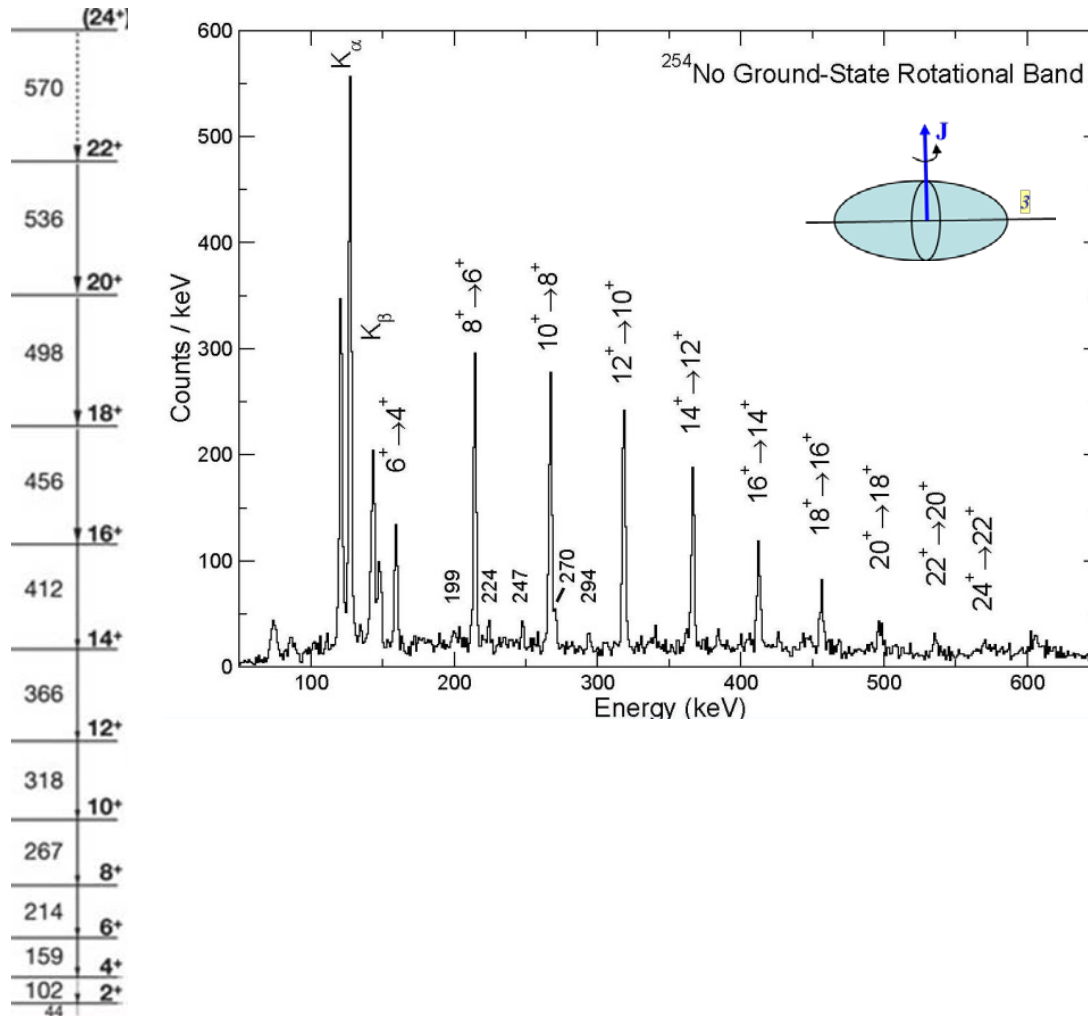
Chart of nuclides: the domain of heavy and superheavy elements



What is the structure of SHE?



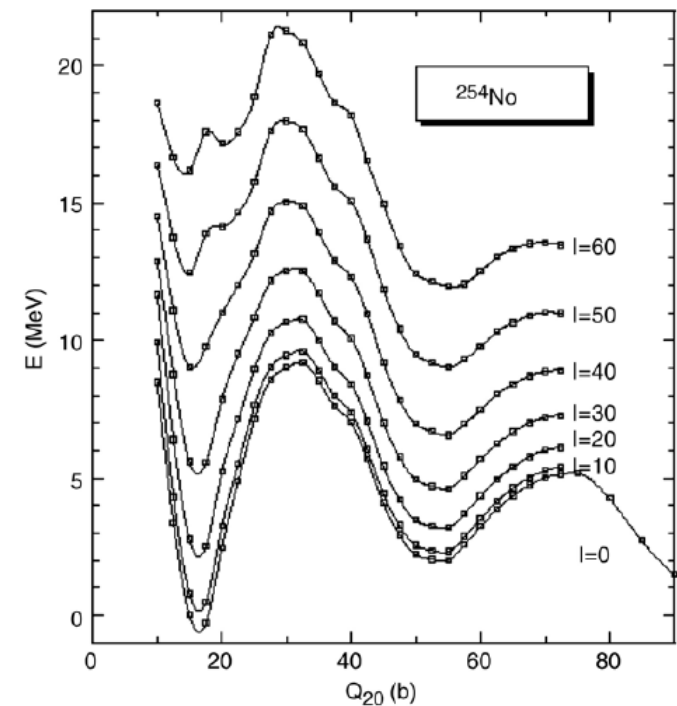
Spinning the heaviest elements



$$\text{rotational energy: } E_I = \frac{\hbar^2}{2\mathcal{I}} \cdot I(I+1)$$

$$\gamma\text{-ray energy: } E_I - E_{I-2} = \frac{\hbar^2}{2\mathcal{I}} \cdot (4I-2)$$

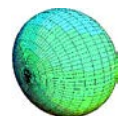
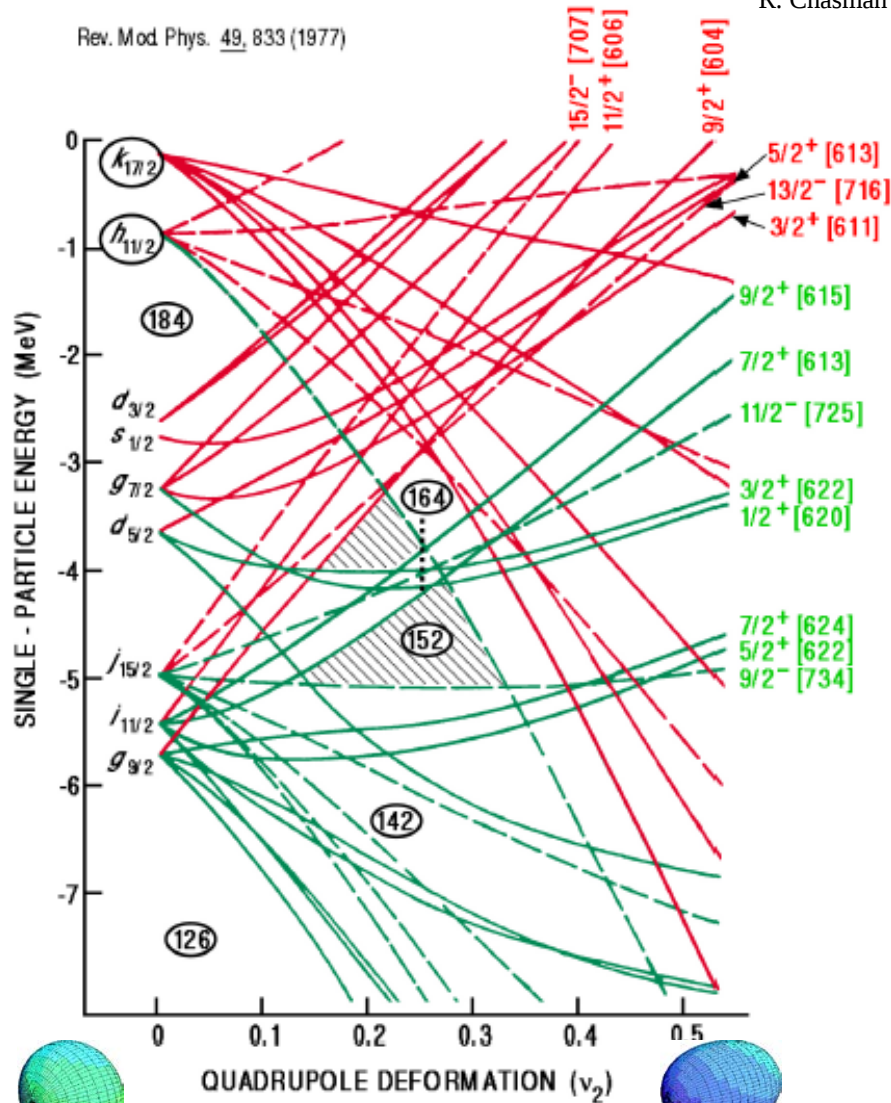
dependence of the fission barrier on spin I



Single particle orbitals

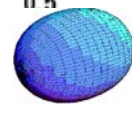
R. Chasman et al. Rev. Mod. Phys. 49 (1977), 833

Rev. Mod. Phys. 49, 833 (1977)



Oblate

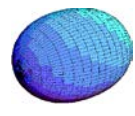
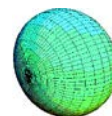
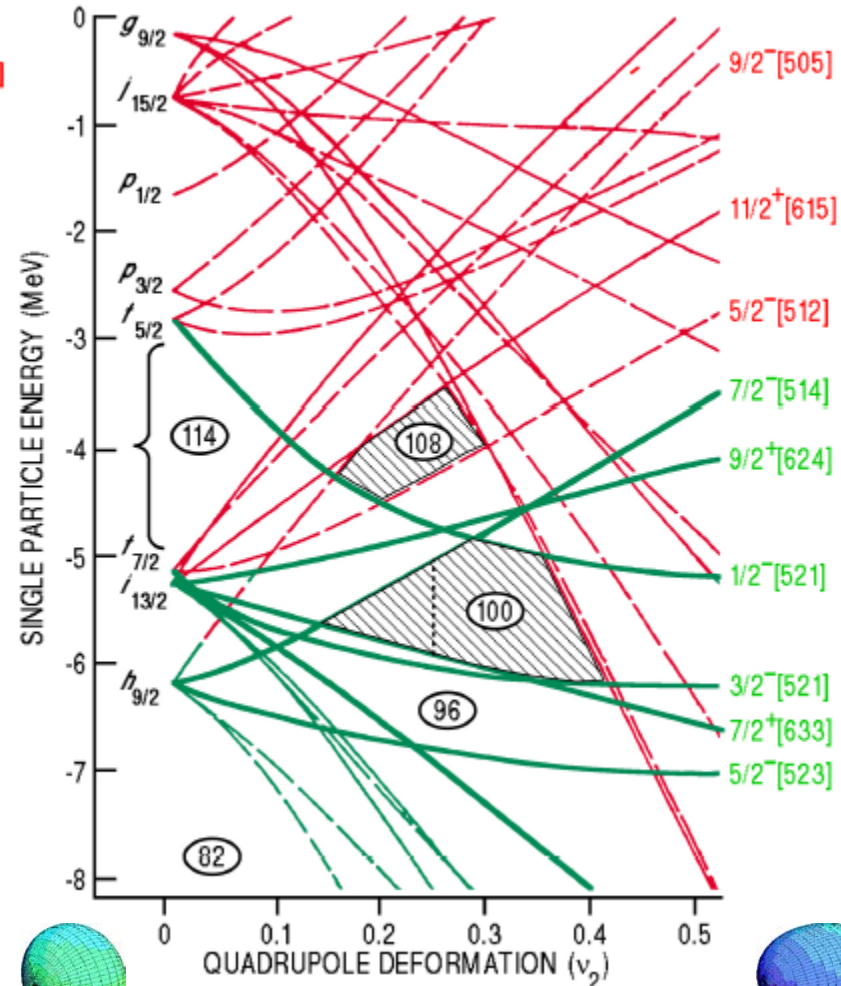
$^{254}_{102}\text{No}_{152}$ $\beta_2 \sim 0.28$



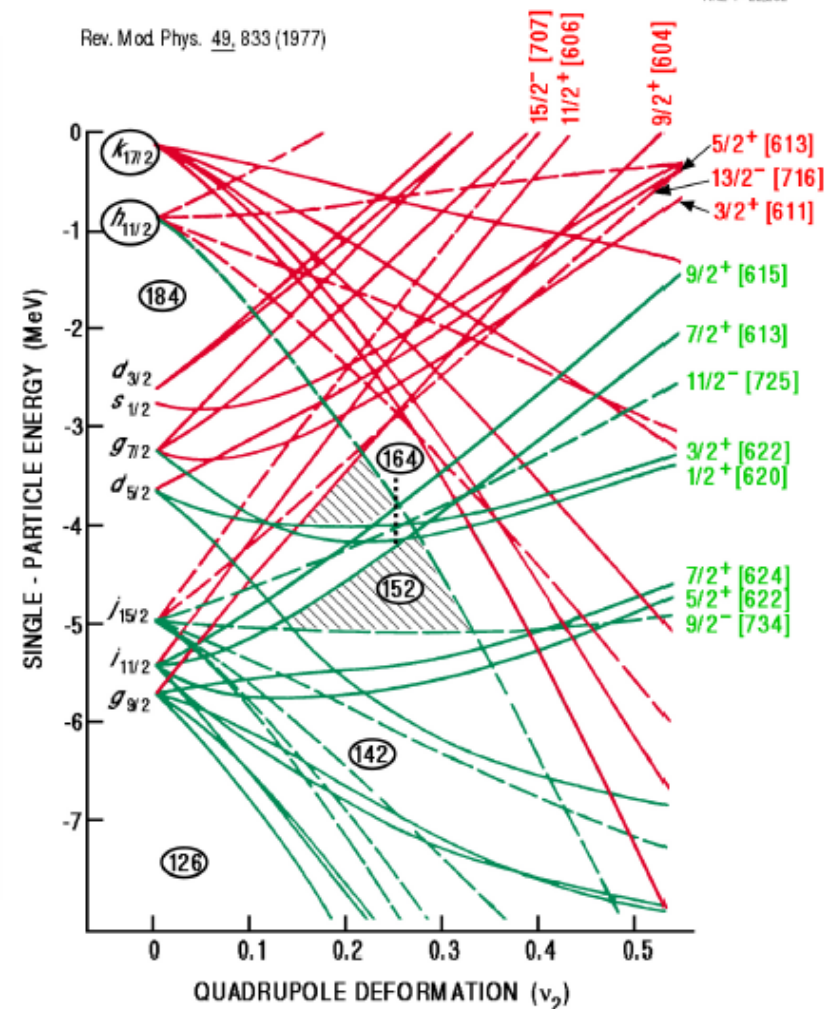
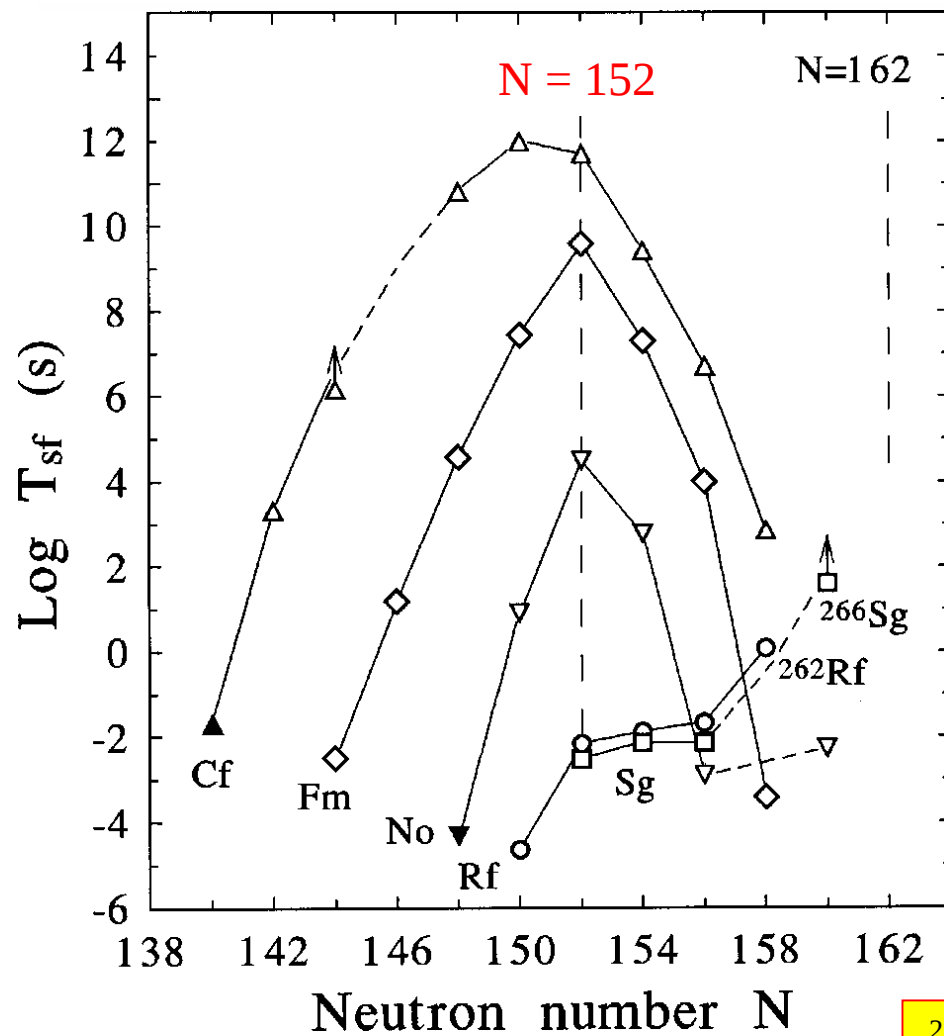
Prolate

Rev. Mod. Phys. 49, 833 (1977)

ANL-P-22,033



Stability of heavy elements – Nilsson level energy



^{254}No ($Z=102$), ^{252}Fm ($Z=100$) and ^{250}Cf ($Z=98$)
with $N=152$
 seem to be more stable than their neighbors