Gross Properties of Nuclei

Nuclear Masses and Binding Energies
In 1905 Albert Einstein following his derivation of the Special Theory of Relativity identifies relation between mass and energy of an object at rest:

\[ E = mc^2 \]

The corresponding relation for moving object is

\[ E = \frac{m_0 c^2}{\sqrt{1 + \left(\frac{v}{c}\right)^2}} \]

This discovery explains the energy powering nuclear decay. The question of energy release in nuclear decay was a major scientific puzzle from the time of the discovery of natural radioactivity by Henry Becquerel (1896) until Einstein’s postulate of mass-energy equivalence.
Heat is evolved in the chemical reaction in which hydrogen and oxygen are combined to be water and generates 3.0 eV energy emission:

\[ H_2 + \frac{1}{2} O_2 = H_2O + 3.0 \text{ eV} \]

Such chemical reaction in which heat is evolved is called exothermic reaction.

Another example is where one mol of carbon is oxidized to be carbon dioxide with producing 4.1 eV energy:

\[ C + O_2 = CO_2 + 4.1 \text{ eV} \]

The nuclear reaction in which two deuterons bind with each other is an example of nuclear fusion. This exoergic reaction is written as

\[ ^2H + ^2H \rightarrow ^3He + n + 3.27 \text{ MeV} \]

If a neutron is absorbed in the uranium-235 nucleus, it would split into two fragments of almost equal masses and evolves some number of neutrons and energy. One of the equations for the processes is written

\[ ^{235}_{92}U + n \rightarrow ^{137}_{56}Ba + ^{97}_{36}Kr + 2n + 215 \text{ MeV} \]
### Nuclear Masses

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<thead>
<tr>
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<th>mass (u)</th>
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<tbody>
<tr>
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Nuclear and atomic masses are expressed in **ATOMIC MASS UNITS (u)**

**definition:** 1/12 of mass of neutral $^{12}$C → $M(^{12}$C) = 12 u

1 u = $1.6605 \cdot 10^{-27}$ kg or 931.494 MeV/c² \( (E = mc^2) \)

\[
\Delta m \cdot c^2 = BE
\]

**Expect:**

\[
M\left(^{A}_{Z}X_{N}\right) = Z \cdot m_p + N \cdot m_n + Z \cdot m_e
\]

**Find:**

\[
M\left(^{A}_{Z}X_{N}\right) < Z \cdot m_p + N \cdot m_n + Z \cdot m_e
\]

We typically use **ATOMIC** and not **NUCLEAR** masses → mass of electrons also included
Nuclear Masses and Binding Energy

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1 u = 1.6605 · 10⁻²⁷ kg or 931.494 MeV/c²

A bound system has a lower potential energy than its constituents!

Binding energy:

$$BE = \left\{ [Z \cdot m_H + N \cdot m_n] - M\left(\frac{A}{Z}X_N\right) \right\} \cdot c^2 > 0$$

$$BE \left( ^4 He \right) = [2 \cdot 1.007825 + 2 \cdot 1.008665] - 4.002603 = 0.030377 \text{ u} \quad (= 28.296 \text{ MeV})$$
Binding Energy and Mass Excess

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1 u = 1.6605·10⁻²⁷ kg or 931.494 MeV/c²

Binding energy: \[ BE = \left\{ [Z \cdot m_H + N \cdot m_n] - M\left(\frac{A}{2}X_N\right) \right\} \cdot c^2 \]

Mass excess: \[ \Delta M = M(A,Z) - A \cdot M(u) \]

Example:

mass of ⁵⁶Fe: \[ M(56,26) = 55.934942 \; [u] \]

mass excess: \[ \Delta M = M(A,Z) - A \cdot M(u) = 55.934942 - 56 = -0.06506 \; [u] \]
\[ = -0.06506 \cdot 931.494 = -60.601 \; [\text{MeV/c}^2] \]

binding energy: \[ BE(Z,A) = [Z \cdot M(¹H) + N \cdot M_n - M(Z,A)] \cdot c^2 \]
\[ = [26 \cdot 1.007825 + 30 \cdot 1.008665 - 55.934942] \cdot 931.494 = 492.25 \; [\text{MeV}] \]
## Binding Energy

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$1 \text{ u} = 1.6605 \cdot 10^{-27} \text{ kg}$ or $931.494 \text{ MeV/c}^2$  

$$BE = \left\{ [Z \cdot m_H + N \cdot m_n] - M\left(\frac{A}{Z}X_N\right) \right\} \cdot c^2$$

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<td>$^2$H</td>
<td>2.01594</td>
<td>2.01355</td>
<td>2.23</td>
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<tr>
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<td>4.03188</td>
<td>4.00151</td>
<td>28.29</td>
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<td>$^7$Li</td>
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<td>$^9$Be</td>
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<td>$^{56}$Fe</td>
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<td>55.92069</td>
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<td>$^{127}$I</td>
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<td>239.93448</td>
<td>238.00037</td>
<td>1801.63</td>
<td>7.57</td>
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Characteristics of Nuclear Binding

Binding energy per nucleon (BE/A)

1. Most nuclei have almost exactly the same B/A, which is roughly 8 MeV/A. This means the nuclear force saturates such that only each nucleon can interact with a few of its neighbors.

2. The most bound nuclei are in the region of \( A \sim 56 \sim 62 \).

3. Nuclei on the left of the peak can release energy by joining together (nuclear fusion)

4. Nuclei on the right of the peak can release energy by breaking apart (nuclear fission)

5. Some structure in this curve also exists (particularly for \( 4\text{He} \)) that results from quantum effects of the nucleus.
Characteristics of Nuclear Binding

Binding energy per nucleon (BE/A)

![Graph showing the binding energy per nucleon (BE/A) for various elements.](graph)

- **Fusion**: Processes where two or more atomic nuclei combine to form a single nucleus, releasing energy.
- **Fission**: Processes where a large atomic nucleus splits into two or more smaller nuclei, releasing energy.

Elements shown include:
- $^4_2He$
- $^{14}_7N$
- $^{18}_8F$
- $^{235}_{92}U$
- $^{238}_{92}U$
- $^{144}_{56}Ba$
- $^{36}_{89}K$

The graph illustrates the trend of binding energy per nucleon with the number of nucleons in the nucleus.
Long vs short range interaction

Long range force: \( B \propto \frac{A(A-1)}{2} \quad \implies \quad B/A \propto A \)

Short range force: saturation
Mass spectroscopy

Mass of nuclei are measured by passing ion beams through magnetic and electric field (2 steps)

1) select a constant velocity with a velocity (Wien) filter
   $E$ and $B_1$ are at right angles
   \[ q \cdot E = q \cdot v \cdot B_1 \]
   \[ v = \frac{E}{B_1} \]

2) Measure a trajectory of ions in a uniform magnetic $B_2$ field

\[ \frac{m \cdot v^2}{r} = q \cdot v \cdot B_2 \]

If $B_1 = B_2 = B$

\[ \frac{q}{m} = \frac{v}{B \cdot r} = \frac{E}{B^2 \cdot r} \]
The most precise measurements come from storage devices that confine ions in three dimensions by means of well-controlled electromagnetic fields.

Examples: storage rings, Penning trap, Paul trap

In a Penning trap,

1) Ions are confined in 2D with a strong magnetic field \( B \)
2) A weak electrostatic field is used to trap ions in 3D (along the Z-axis)

1) Cyclotron oscillations \( (\omega_c) \)

\[
\frac{m \cdot v^2}{r} = q \cdot v \cdot B \\
\frac{v}{q} = \frac{B \cdot r}{m} \\
t = \frac{2\pi \cdot r}{v} = \frac{2\pi \cdot m}{q \cdot B} \\
\omega_c = 2\pi \cdot f = \frac{2\pi}{t} = \frac{qB}{m}
\]
2) A weak axially symmetric electrostatic potential is superimposed to produce a saddle point at the center.

This requires the **quadrupole potential**:

\[ \Phi(z, r) = \frac{U}{4d^2} \cdot (2z^2 - r^2) \]

\[ d = \frac{1}{2} (2z_0^2 - r_0^2)^{1/2} \]

(so that \( U \) is the potential difference between the endcap and the ring electrodes)
Penning trap measurement

Solving the equations of motions result in three independent motional modes with frequencies:

\[ \omega_z = \sqrt{\frac{q \cdot U}{m \cdot d^2}} \quad \text{(axial motion)} \]

\[ \omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \quad \text{(cyclotron motion)} \]

\[ \omega_- = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \quad \text{(magnetron motion)} \]

The condition on the magnetic field,

\[ \frac{\omega_c^2}{4} - \frac{\omega_z^2}{2} > 0 \quad B^2 > 2 \cdot m \cdot U/q \cdot d^2 \]

\[ \omega_\pm = \frac{\omega_c}{2} \cdot \left[ 1 \pm \left( 1 - \frac{\omega_z^2}{\omega_c^2} \right)^{1/2} \right] \]

\[ = \frac{\omega_c}{2} \cdot \left[ 1 \pm \left( 1 - \frac{\omega_z^2}{\omega_c^2} \right) \right] \]

\[ \omega_- = \frac{\omega_z^2}{2\omega_c} = \frac{U}{2} \cdot B \cdot d^2 \]

\[ \omega_+ = \omega_c - \frac{U}{2} \cdot B \cdot d^2 \]

\[ \rightarrow \omega_c = \omega_+ + \omega_- \quad \omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2 \quad (\omega_+ \cdot \omega_- = \omega_z^2/2) \]

In the experiment one needs to measure the sum

\[ \omega_+ + \omega_- \]

(not cyclotron frequency from \( \omega_+ \) only)
Schottky-Mass-Spectroscopy

4 particles with different m/q
Schottky-Mass-Spectroscopy

\[ \sin(\omega_1) \]  
\[ \sin(\omega_2) \]  
\[ \sin(\omega_3) \]  
\[ \sin(\omega_4) \]

Fast Fourier Transform

frequency

\[ \omega_4 \]  
\[ \omega_3 \]  
\[ \omega_2 \]  
\[ \omega_1 \]
Small-band Schottky frequency spectra

\[ ^{143m}_{62}\text{Sm} \quad \text{754 keV} \quad ^{143g}_{62}\text{Sm} \]

\[ \frac{m}{\Delta m} \approx 700000 \]