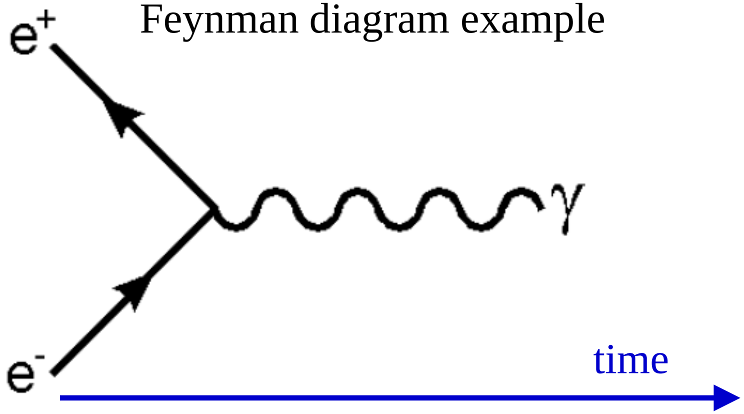
Feynman diagrams

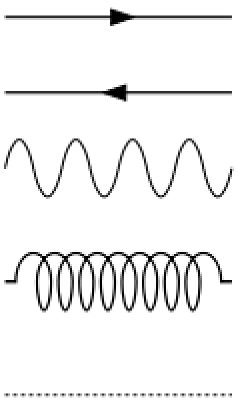
Richard Feynman

In 1940s, R. Feynman developed a diagram technique to describe particle interactions in space-time.

Feynman diagram example



❖ Particles are represented by lines



Fermion f

Antifermion $\bar{f}$

γ, Z, W

Gluon

Boson

- Particles go forward in time
- Antiparticles go backwards in time

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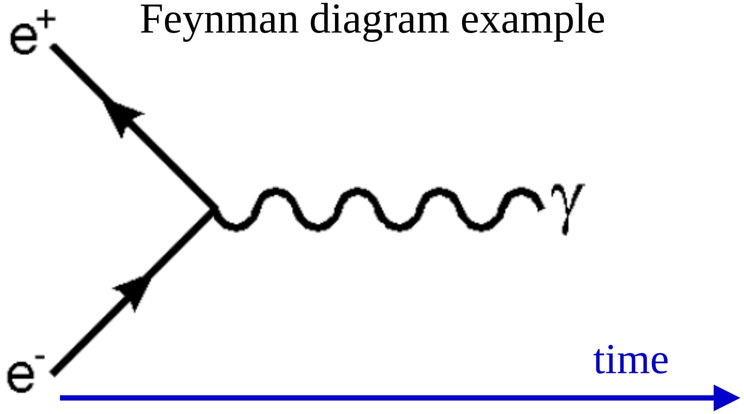
Feynman diagram example



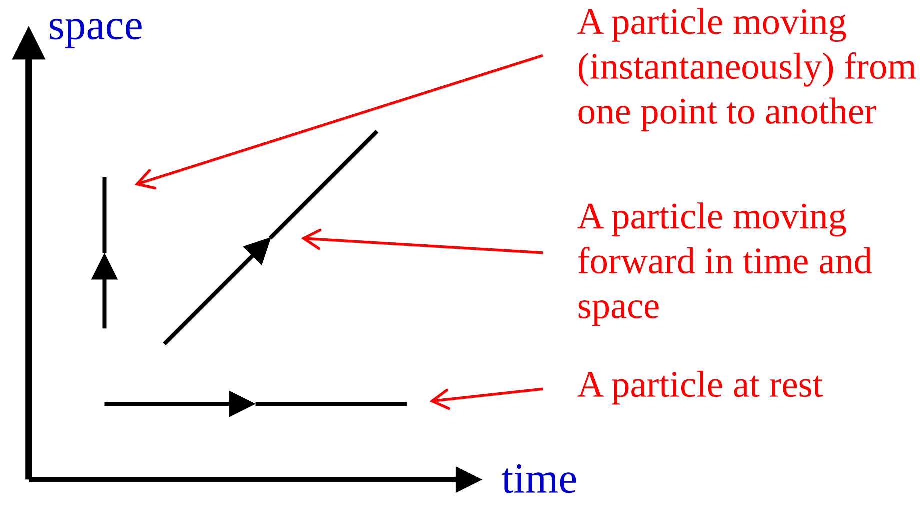
Main assumptions and requirements:

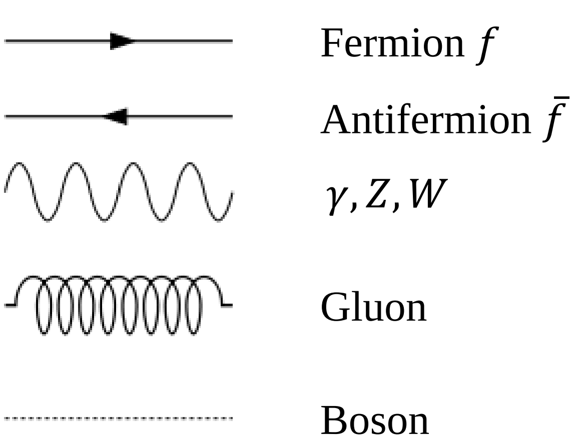
- Time runs from left to right (convention)
- Particles are usually denoted with **solid lines**, and gauge bosons - with **helices** or **dashed lines**
- **Arrow** directed towards the right indicates a particle, otherwise – antiparticle
- *Points at which 3 or more particles meet are called* ***vertices***
- *At any vertex, momentum, angular momentum and charge are conserved (but not energy)*

Feynman diagrams

Feynman diagrams are like circuit diagrams – they show what is connected to, but length and angle of momentum vectors are not relevant.

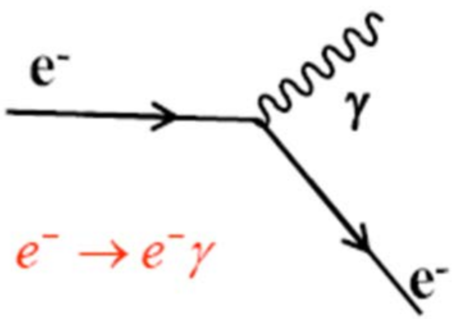
Richard Feynman

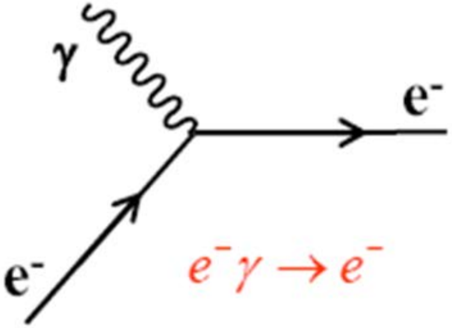




Vertices

- Lines connect into vertices, which are the building blocks of Feynman diagrams

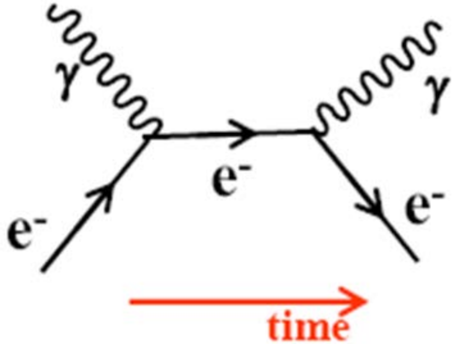




- Charge, lepton number and baryon number as well as momentum are always conserved at a vertex.

Compton scattering

A photon scatters from an electron producing a photon and an electron in the final state



Lowest order diagram has two vertices

Feynman Diagrams

Each Feynman diagram represents an Amplitude (**M**)

In lowest order perturbation theory **M** is the Fourier transformation of the potential. "Born Approximation"

Fermi's Golden Rule:

$$transition\ rate = \frac{2\pi}{\hbar}|M|^2 \times (phase\ space)$$

Differential cross section for two body scattering (e.g. pp → pp) in the CM system:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\frac{q_f^2}{v_i v_f}|M|^2$$

q_f = final state momentum
v_f = speed of final state particle
v_i = speed of initial state particle

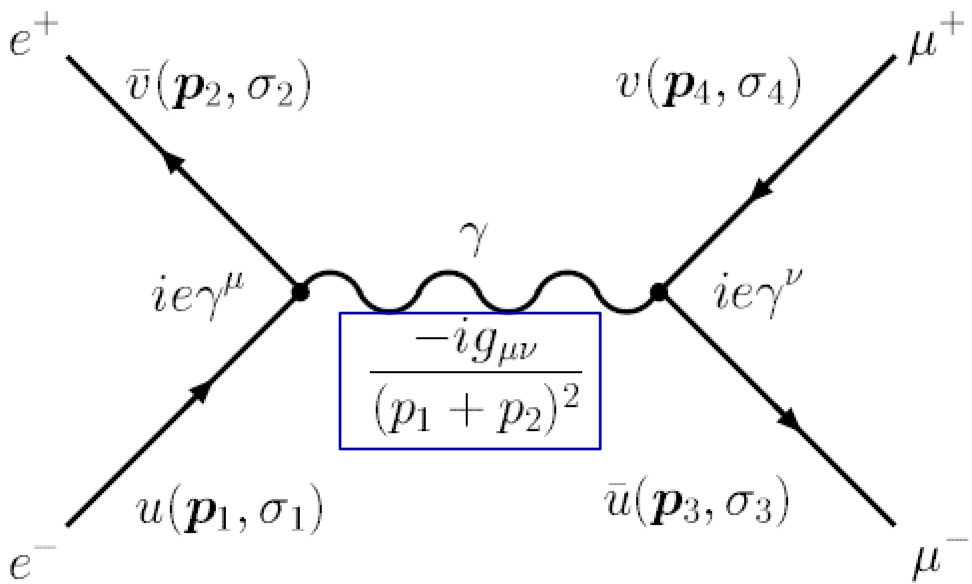
The decay rate (Γ) for a two body decay (e.g. $K^0 \rightarrow \pi^+\pi^-$) in the CM system:

$$\Gamma = \frac{S \cdot |\vec{p}|}{8\pi\hbar m^2 c}|M|^2$$

m = mass of parent
p = momentum of decay particle
S = statistical factor (fermions/bosons)

In most cases $|\boldsymbol{M}|^2$ cannot be calculated exactly.
Often ***M*** is expanded in a power series.
Feynman diagrams represent terms in the series expansion of ***M***.

Feynman Diagrams



$$-iM = [\bar{u}(p_3,\sigma_3)(ie\gamma^\nu)v(p_4,\sigma_4)]\,\frac{-ig_{\mu\nu}}{(p_1+p_2)^2}\,[\bar{v}(p_2,\sigma_2)(ie\gamma^\mu)u(p_1,\sigma_1)]$$

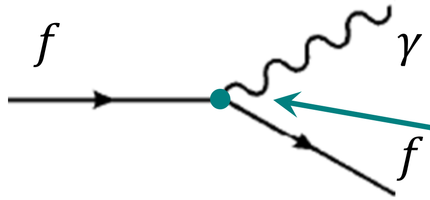
for massive particle: $\frac{-i\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{m^2}\right)}{p^2-m^2}$

Feynman Diagrams

- A coupling constant (multiplication factor) is associated with each vertex.
- Value of coupling constant depends on type of interaction

$Q_f = fermion\ charge$

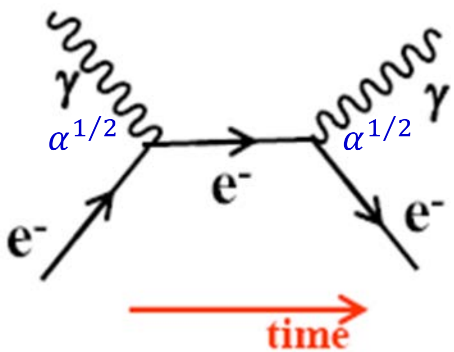
(in units of electron charge)



Electromagnetic $f \rightarrow f\gamma$

$$Q_f \cdot \sqrt{\alpha} = Q_f \cdot \sqrt{\frac{e^2}{4\pi}}$$

- Example: Compton scattering of an electron

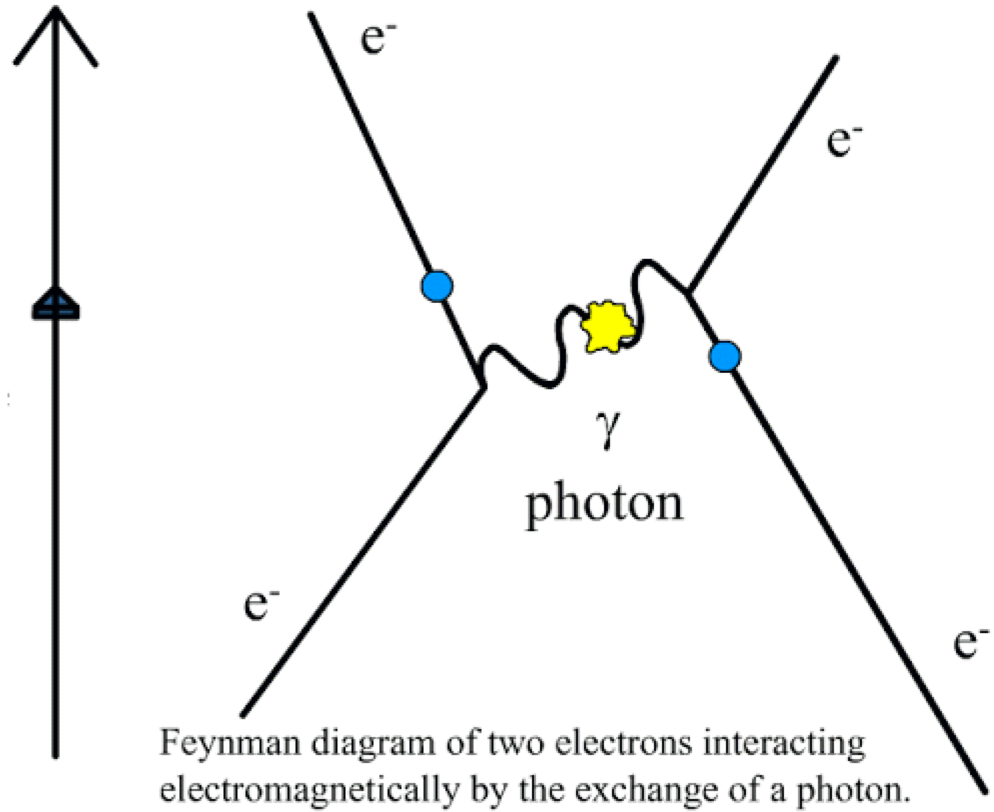


$$Diagram\ \propto (coupling)^2 \propto \alpha$$

$$\sigma \propto |Diagram|^2 \propto \alpha^2 \propto e^4$$

$= {}^{1}/_{137}$

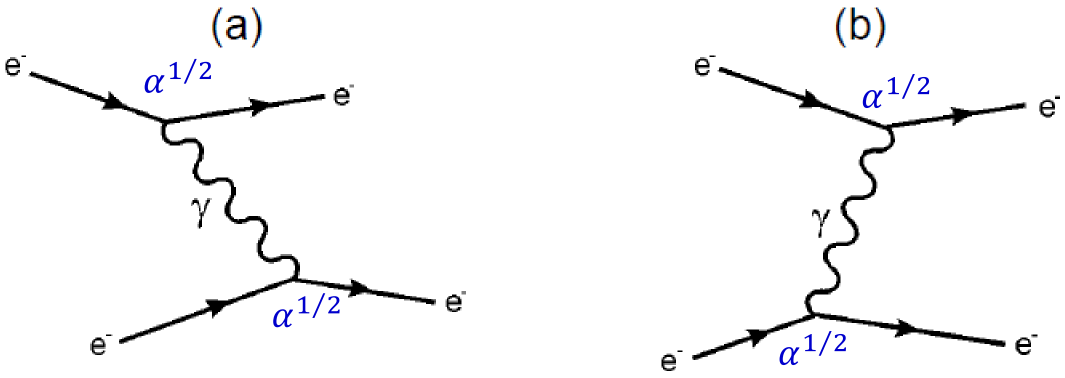
- Total four-momenta conserved at a vertex
- Can move particle from initial to final state by replacing it into its antiparticle $f \rightarrow f\gamma$ *becomes* $f\bar{f} \rightarrow \gamma$



Feynman diagram of two electrons interacting electromagnetically by the exchange of a photon.

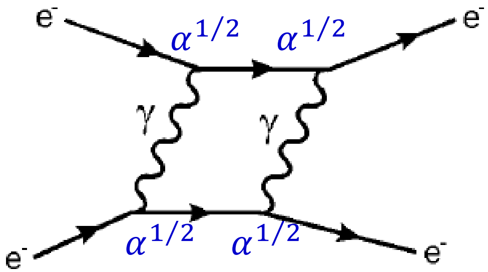
Real processes

For a real process there must be energy conversation → it has to be a combination of virtual processes



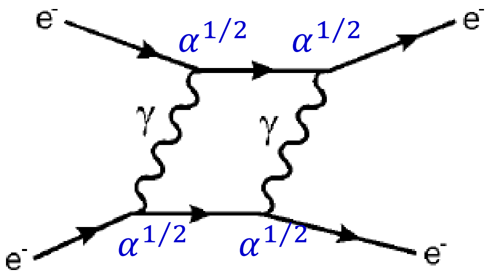
Electron-electron scattering, single photon exchange

Any real process receives contributions from all possible virtual processes



Two-photon exchange contribution

Real processes



Two-photon exchange contribution

- Number of vertices in a diagram is called its ***order***
- Each vertex has an associated probability proportional to a ***coupling constant***, usually denoted as "α". In the electromagnetic processes this constant is

$$\alpha_{em} = \frac{e^2}{4\pi\varepsilon_0} \approx \frac{1}{137}$$

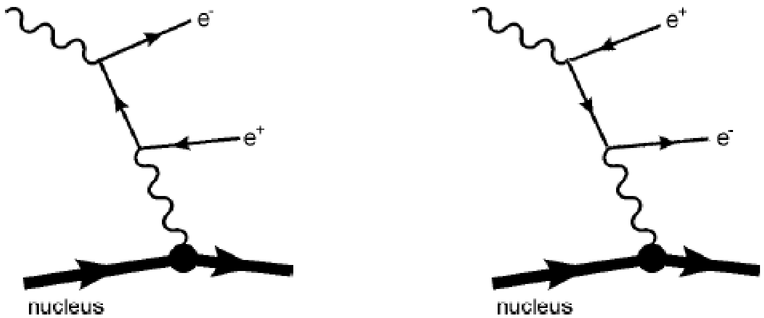
- For the real processes, a diagram of the ***order n*** gives a contribution of order $\boldsymbol{\alpha^n}$

- Provided that α is small enough, higher order contributions to many real processes can be neglected.

- From the order of diagrams one can estimate the ratio of appearance rates of processes:

$$R \equiv \frac{Rate(e^+e^- \rightarrow \gamma\gamma\gamma)}{Rate(e^+e^- \rightarrow \gamma\gamma)} = O(\alpha)$$

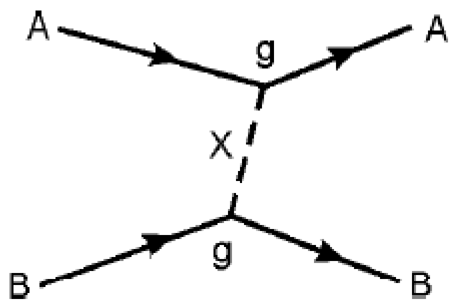
This ratio can be measured experimentally; it appears to be R = $0.9\cdot10^{-3}$, which is smaller than $\alpha_{em} = 7\cdot10^{-3}$, but the equation above is only a first order prediction.



Diagrams are not related by time ordering

For nucleus, the coupling is proportional to $Z^2\alpha$, hence the rate of this process is of the order of $Z^2\alpha^3$.

Exchange of a massive boson



Exchange of a massive particle X

In the rest frame of particle A:

$$A(E_0, \overrightarrow{p_0}) \rightarrow A(E_A, \vec{p}) + X(E_X, -\vec{p})$$

where $E_0 = M_A, \quad \overrightarrow{p_0} = (0,0,0)$

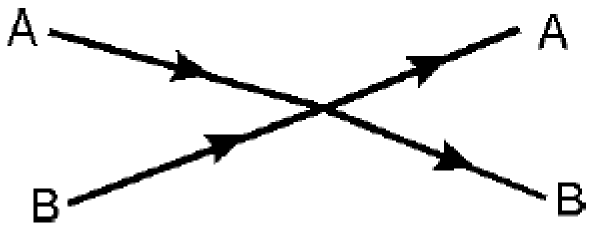
$$E_A = \sqrt{p^2 + {M_A}^2}, \qquad E_X = \sqrt{p^2 + {M_X}^2}$$

From this one can estimate the maximum distance over which X can propagate before being absorbed: $\Delta E = E_X + E_A - M_A \geq M_X$
This energy violation can exist only for $\Delta t \approx \hbar / \Delta E$, the *interaction range* is

$$r \approx R \equiv \hbar c / M_X$$

Exchange of a massive boson

- For a massless exchanged particle, the interaction has an infinite range (e.g. electromagnetic)
- In case of a very heavy exchanged particle (e.g. a W boson in weak interaction), the interaction can be approximated by a *zero-range*, or *point interaction*



Point interaction as a result of $M_X \rightarrow \infty$

$$R_W = \hbar c / M_W = \hbar c / 80.4\ GeV/c^2 = \frac{197.3 \cdot 10^{-18}}{80.4} \approx 2 \cdot 10^{-18} m$$

Considering particle X as an electrostatic potential V(r), the Klein-Gordon equation for it will look like

$$\nabla^2 V(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = M_X^2 \cdot V(r)$$

Yukawa potential (1935)

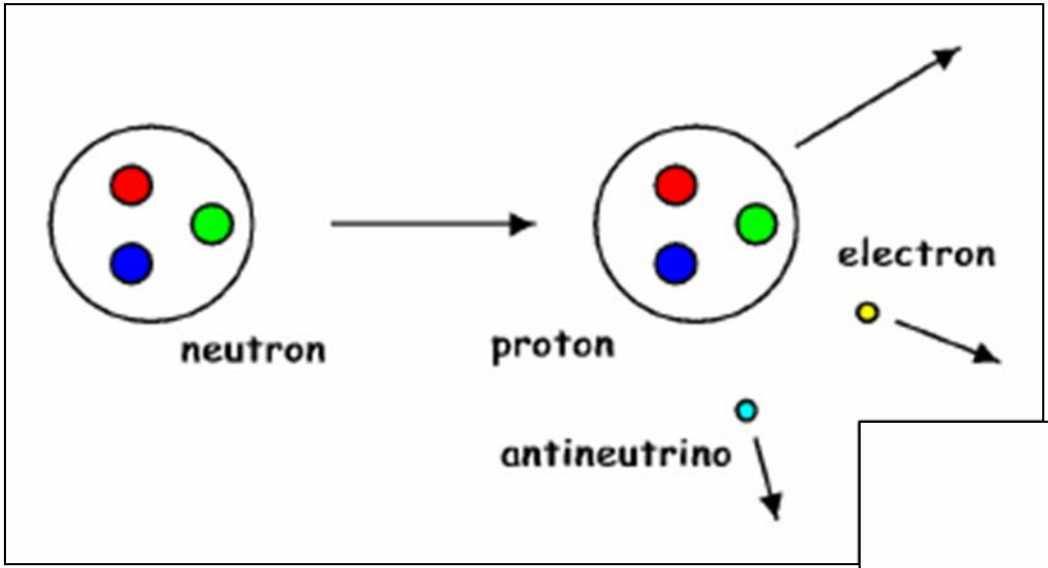
For nuclear forces with a range $R \sim 10^{-15} m$, Yukawa hypothesis predicted a spinless quantum of mass:

$$Mc^2 = \hbar c/R \approx 100\ MeV$$

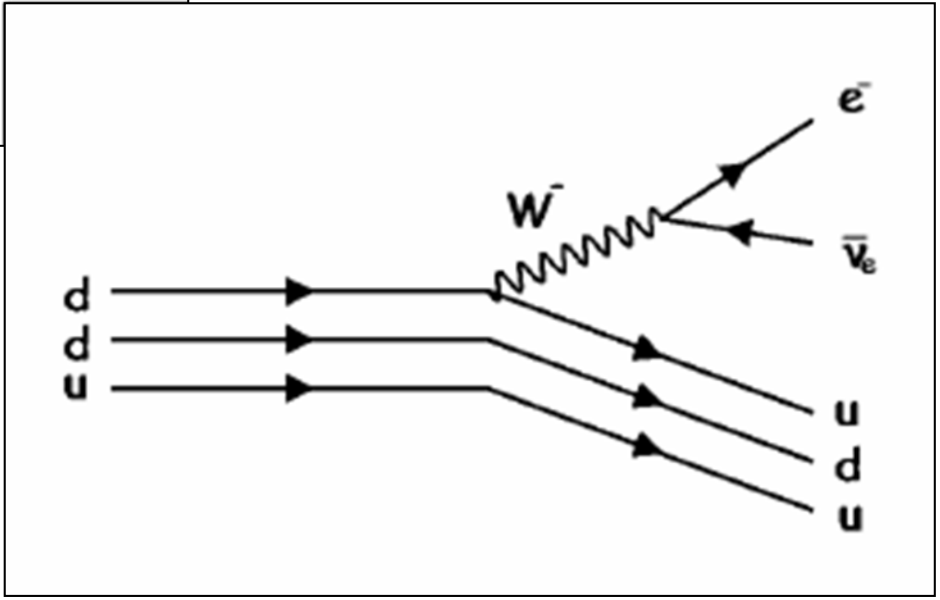
The pion observed in 1947 had $M = 140[MeV/c^2]$, $spin = 0$ and strong nuclear interactions.

Nowadays: pion exchange still accounted for the longer-range part of nuclear potential.

However, full details of interaction are more complicated.

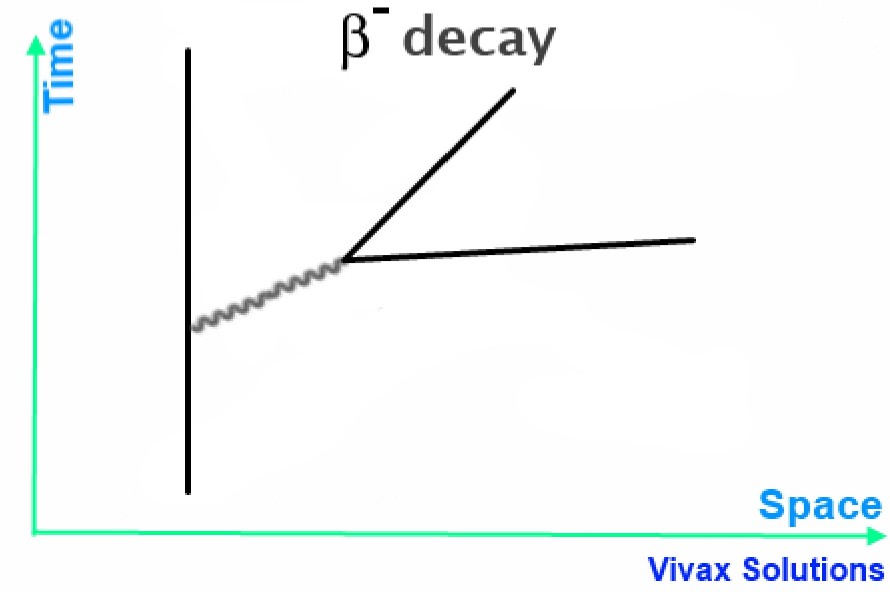


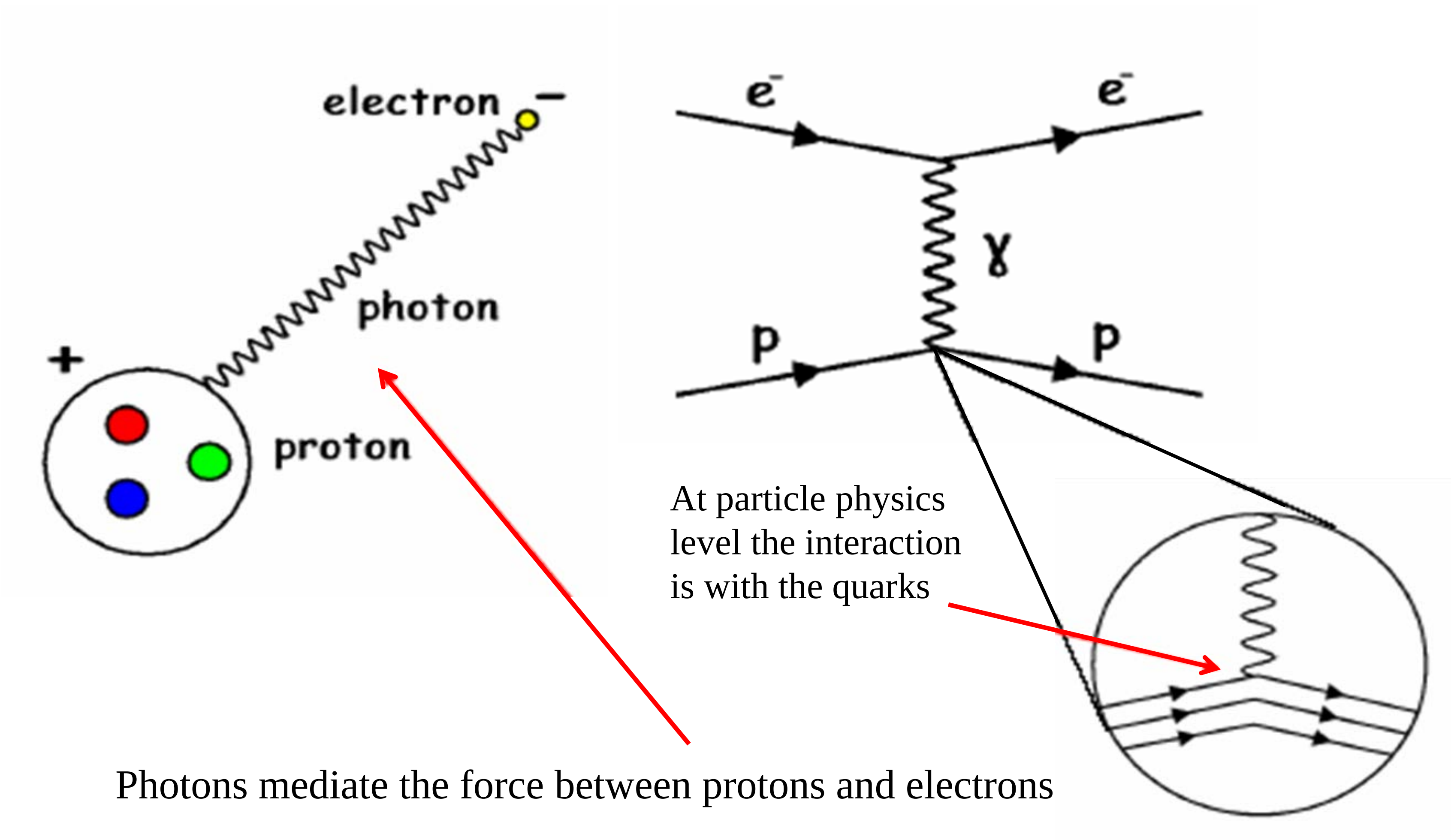
Beta decay: $n \rightarrow p + e^- + \bar{\nu}_e$



Mediated by charged W exchange:

The charge that goes into the vertex ***must*** equal the charge that comes out of it.

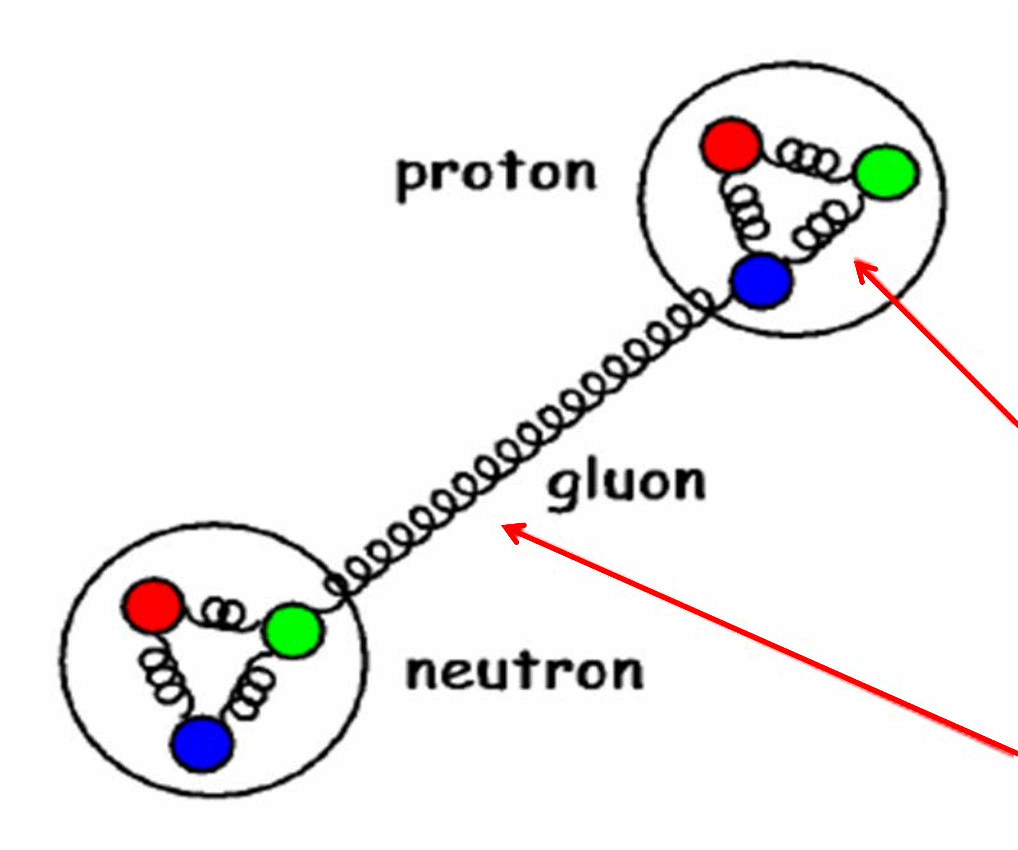

β⁻ decay
Time
Space
Vivax Solutions

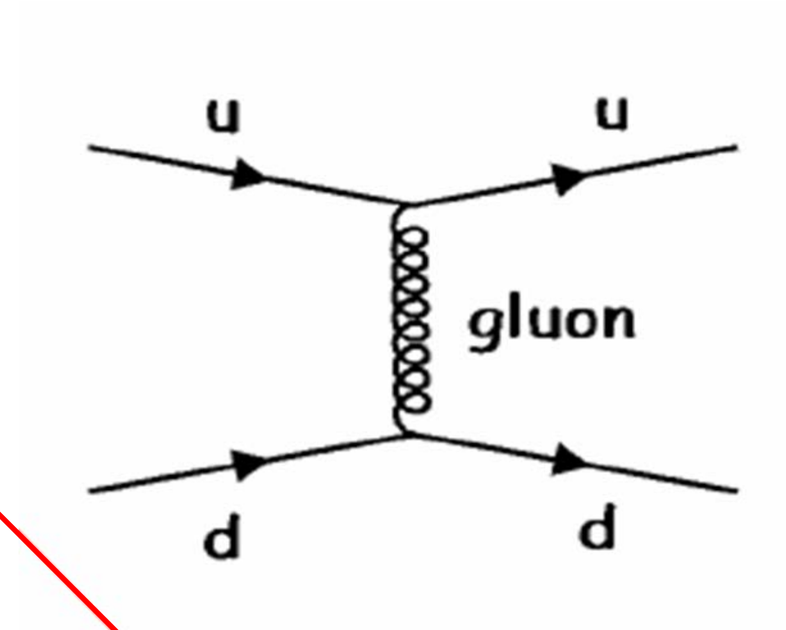


At particle physics level the interaction is with the quarks

Photons mediate the force between protons and electrons

Strong Interaction



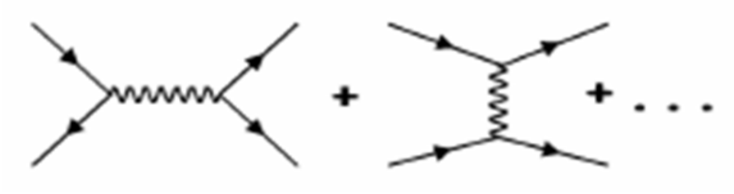


Gluons hold protons and neutron together and are responsible for the Strong force between them

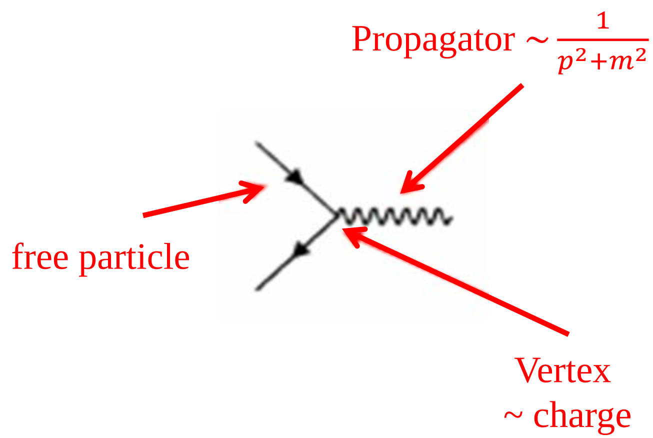
Use of Feynman Diagrams

Although they are used pictorially to show what is going on, Feynman Diagrams are used more seriously to calculate cross sections or decay rates.

- Draw all possible Feynman Diagrams for the process:



- Assign values to each part of the diagram:



- Calculate the amplitude by multiplying together.

- Add the amplitudes for each diagram (including interference).

- Square the amplitude to get the intensity/probability (cross section or decay rate).