Particle and Radiation Detectors: Advances & Applications

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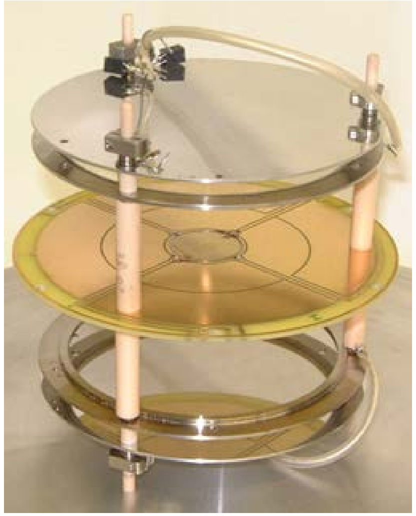
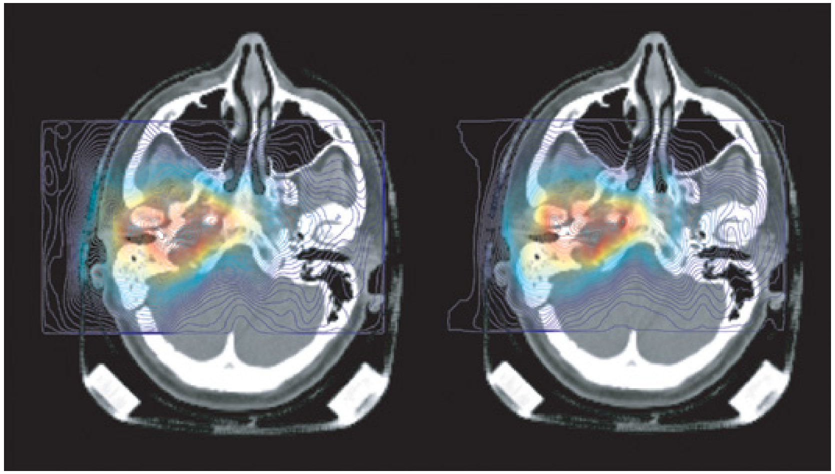
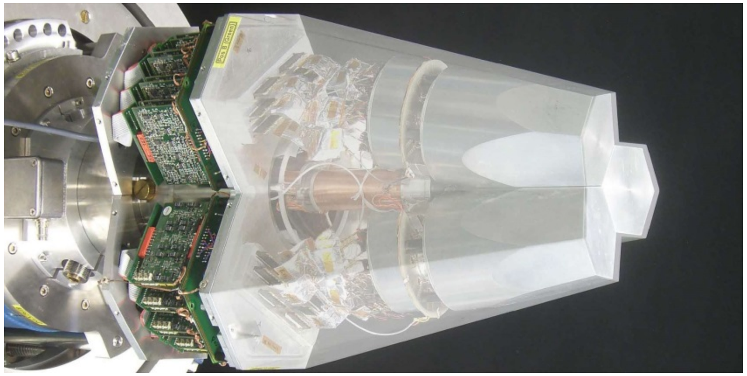
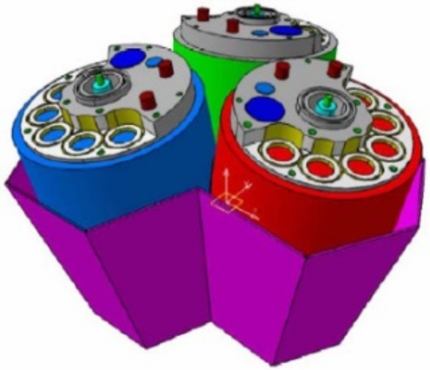


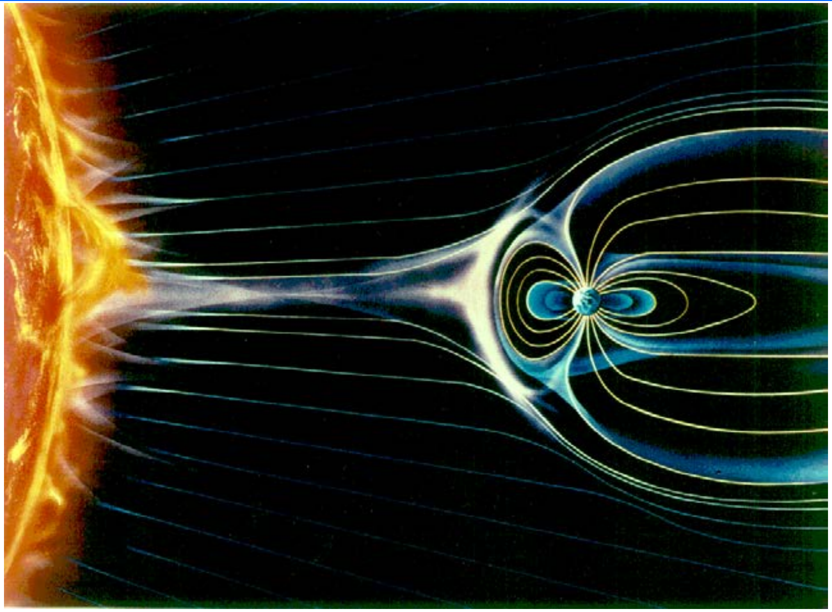






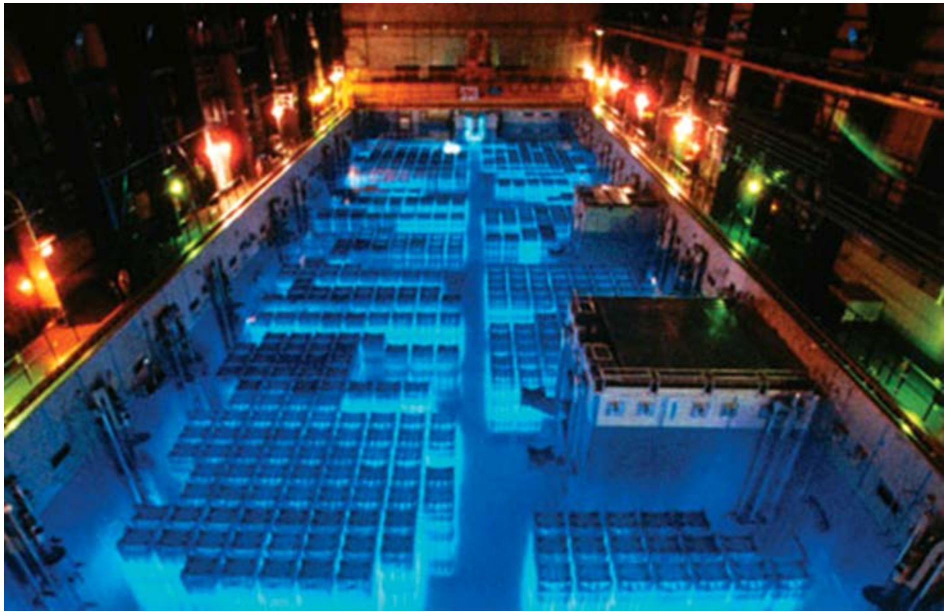
Particle Interaction with Matter

Aurora Borealis



Ionization

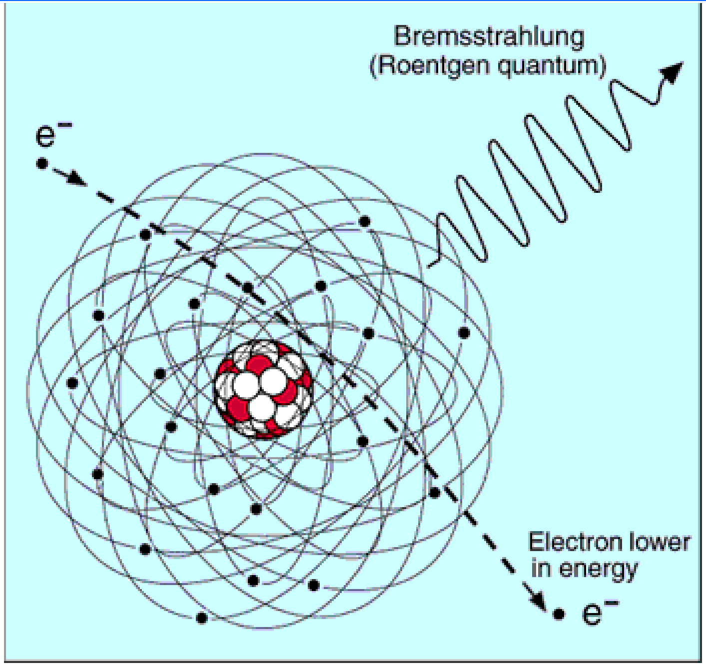
Particle Interaction with Matter



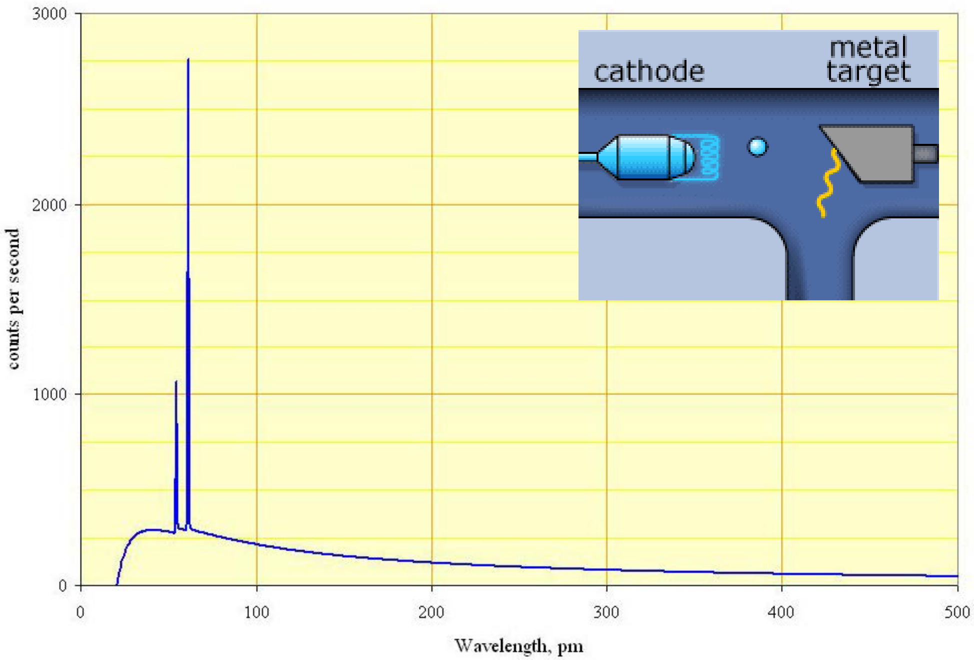
Characteristic glow from a reactor

Cherenkov radiation is an effect similar to sonic booms when the plane exceeds the velocity of sound

Particle Interaction with Matter



Bremsstrahlung or 'braking radiation'

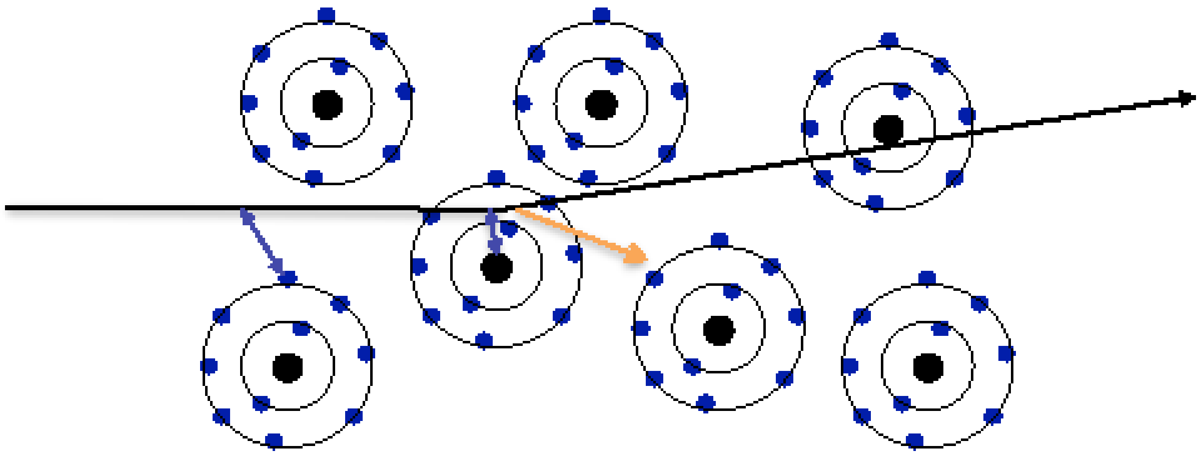


Spectrum of the X-rays emitted by an X-ray tube with a rhodium target, operated at 60 kV. The continuous curve is due to bremsstrahlung, and the spikes are characteristic K lines for rhodium.

Energetic charged particles in matter

Three types of electromagnetic interactions:

1. Ionization (of the atoms of the transversed material)
2. Emission of Cherenkov light
3. Emission of transition radiation



1) Interaction with the atomic electrons. The incoming particle looses energy and the atoms are **excited** or **ionized.**

2) Interaction with the atomic nucleus. The particle is deflected (scattered) causing **multiple scattering** of the particle in the material. During this scattering a **Bremsstrahlung** photon can be emitted.

3) In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as **Cherenkov Radiation**. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X-ray photon, called **Transition Radiation**.

Measurement Principles

A particle detector is an instrument to measure one or more properties of a particle …

Properties of a particle		Type of detection principle:
- position and direction	$x, \vec{x}$	position and tracking
- momentum	$\lvert\vec{p}\rvert$	tracking in a magnetic field
- energy	E	calorimetry
- mass	m	spectroscopy and PID
- velocity	β	Cherenkov radiation or time of flight
- transition radiation	γ	TRD

- spin, lifetime

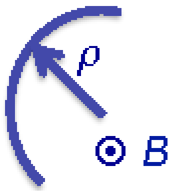
Detection and Identification of Particles

- **Detection** = particle counting (is there a particle?)
- **Identification** = measurement of mass and charge of the particle

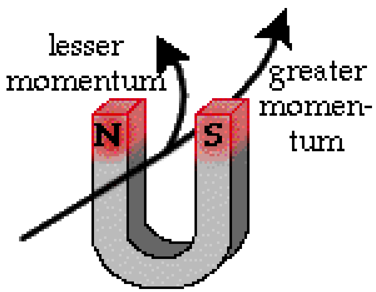
  (most elementary particle have $Ze = \pm 1$)

- **How:**

- charged particles are deflected by B fields such that:

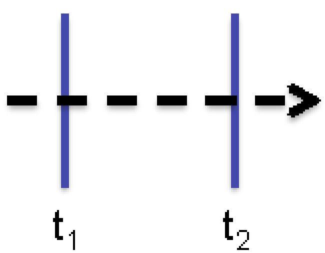


$$\rho = \frac{p}{ZeB} \propto \frac{p}{Z} = \frac{\gamma m_0 \beta c}{Z}$$



$p = particle\ momentum$
$m_0 = rest\ mass$
$\beta c = particle\ velocity$

- particle velocity measured with time-of-flight (ToF) method



$$\beta \propto \frac{1}{\Delta t}$$

- ToF for known distance
- Ionization $-\frac{dE}{dx} = f(\beta)$
- Cherenkov radiation
- Transition radiation

Detection and Identification of Particles

- Detection = particle counting (is there a particle?)
- Identification = measurement of mass and charge of the particle
  (most elementary particle have $Ze = \pm 1$)

- How:

- kinetic energy determined via a calorimetric measurement

$$E_{kin} = (\gamma - 1) m_0 c^2 \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- for Z=1 the mass is extracted from E_{kin} and p
- to determine Z (particle charge) a Z-sensitive variable is e.g. the ionization energy loss

$$\frac{dE}{dx} \propto \frac{z^2}{\beta^2} ln(a \cdot \beta^2 \gamma^2)$$

a = material-dependent constant

Relevant Formulae

The relevant formulae are calculated if A_1, Z_1 and A_2, Z_2 are the mass number (amu) and charge number of the projectile and target nucleus, respectively, and T_{lab} is the laboratory energy (MeV)

$$E = T_{lab} + m_0 \cdot c^2$$

$$m \cdot c^2 = T_{lab} + m_0 \cdot c^2$$

$$\frac{m_0 \cdot c^2}{\sqrt{1-\beta^2}} = T_{lab} + m_0 \cdot c^2$$

beam velocity:

$$\beta = \frac{\sqrt{T_{lab}{}^2 + 1863 \cdot A_1 \cdot T_{lab}}}{931.5 \cdot A_1 + T_{lab}}$$

Lorentz contraction factor:

$$\gamma = (1-\beta^2)^{-1/2}$$

$$\gamma = \frac{931.5 \cdot A_1 + T_{lab}}{931.5 \cdot A_1}$$

$$\beta \cdot \gamma = \frac{\sqrt{T_{lab}{}^2 + 1863 \cdot A_1 \cdot T_{lab}}}{931.5 \cdot A_1}$$

Some Nuclear Units

Nuclear energies are very high compared to atomic processes, and need larger units. The most commonly used unit is the MeV.

1 electron Volt = 1 eV = $1.6 \cdot 10^{-19}$ Joules

1 MeV = 10^6 eV; 1 GeV = 10^9 eV; 1 TeV = 10^{12} eV

However, the nuclear size are quite small and need smaller units:

Atomic sizes are on the order of 0.1 nm = 1 Angstrom = 10^{-10} m. Nuclear sizes are on the order of femtometers which in the nuclear context are usually called fermis:

1 fermi = 1 fm = 10^{-15} m

Atomic masses are measured in terms of atomic mass units with the carbon-12 atom defined as having a mass of exactly 12 amu. It is also common practice to quote the rest mass energy $E=m_0c^2$ as if it were the mass. The conversion to amu is:

1 u = $1.66054 \cdot 10^{-27}$ kg = 931.494 MeV/c^2

electron mass = 0.511 MeV/c^2; proton mass = 938.27 MeV/c^2; neutron mass = 939.56 MeV/c^2

AMDC Atomic Mass Data Center

Mass data: www-nds.iaea.org/amdc/ (mass16.txt)

Measurement Principles

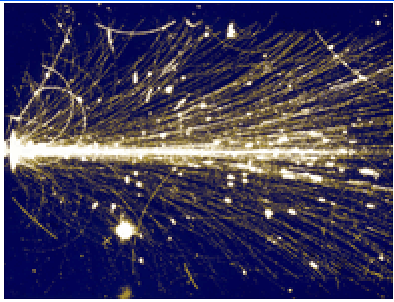
A measurement requires an interaction of the particle with the material of the detector. The interaction provokes two effects:

1st Creation of a detectable signal, e.g.
ionization → charges
excitation → scintillation
excitation of phonons → heat

2nd Alternation of the particles properties, e.g.
energy loss
change of trajectory due to scattering
absorption

unwanted side effects. They need to be as small as possible and well understood.

Energy loss – dE/dx



Particles interact differently with matter. Important for detectors is the energy loss per path length. The total energy loss per path length is the sum of all contributions.

$$-\left(\frac{dE}{dx}\right)_{tot} = -\left(\frac{dE}{dx}\right)_{coll} - \left(\frac{dE}{dx}\right)_{rad} - \left(\frac{dE}{dx}\right)_{photoeff} - \left(\frac{dE}{dx}\right)_{compton} - \left(\frac{dE}{dx}\right)_{pair} - \left(\frac{dE}{dx}\right)_{hadron} \cdots$$

Depending on the particle type, the particle energy and the material some processes dominate, other do not occur. For instance only charged particles will interact with electrons of atoms and produce ionization, etc.

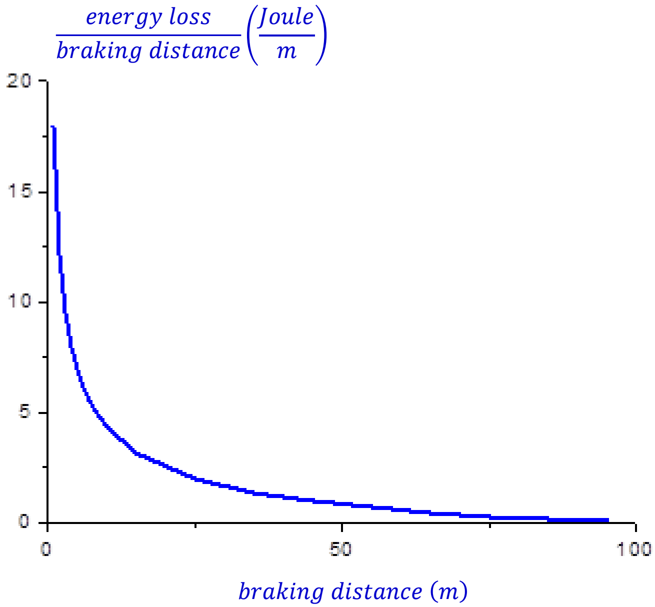
Interaction with matter

accelerated motion:

$$s = \frac{b}{2} t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2 \cdot s}{b}}$$

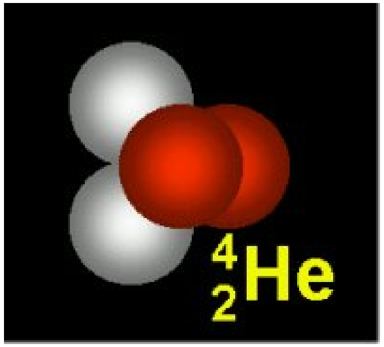
$$v = v_0 - b \cdot t = v_0 - \sqrt{2 \cdot s \cdot b}$$

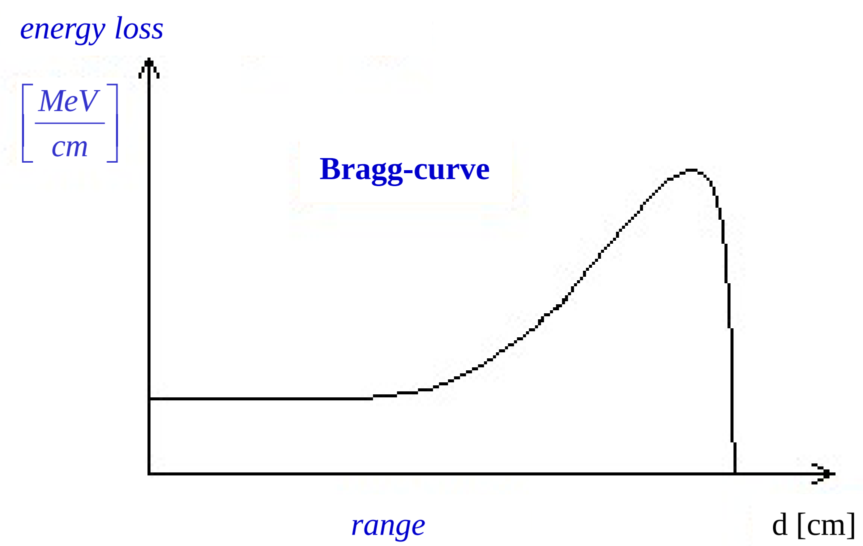
$$E = \frac{1}{2} m v^2 \quad \Rightarrow \quad \frac{dE}{ds} = m \cdot \left(\frac{v_0}{\sqrt{2 \cdot s \cdot b}} - 1 \right) \cdot b$$

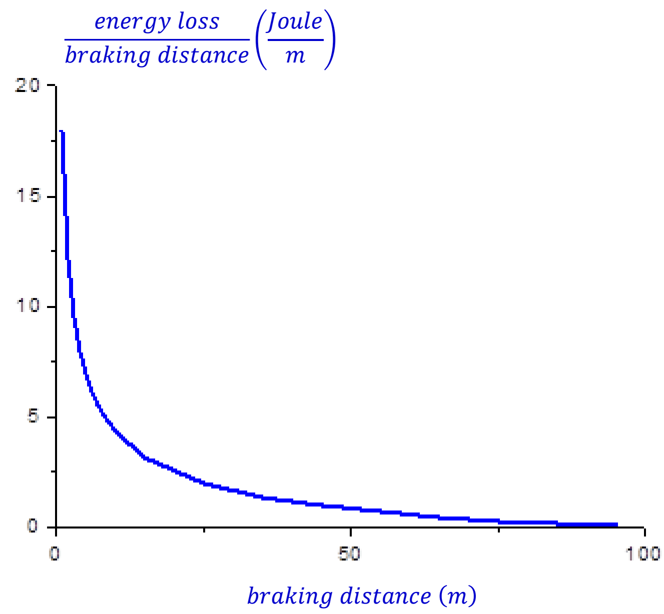


$$v_0 = 100 \frac{km}{h} = 27.78 \frac{m}{s} \qquad b = 3.86 \frac{m}{s^2}$$

Interaction with matter



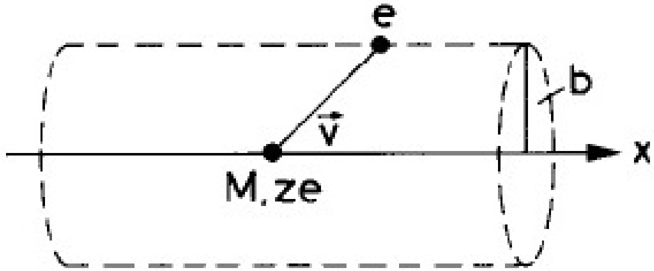




Bethe-Bloch – Classical Derivation (Bohr 1913)

Particle with charge ze and velocity v moves through a medium with electron density n.

Electrons considered free and initially at rest.



Interaction of a heavy charged particle with an electron of an atom inside medium.

Momentum transfer:

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v}$$

Symmetry!

$\Delta p_{||}$: averages to zero

Gauss law (infinite cylinder, electron in center):

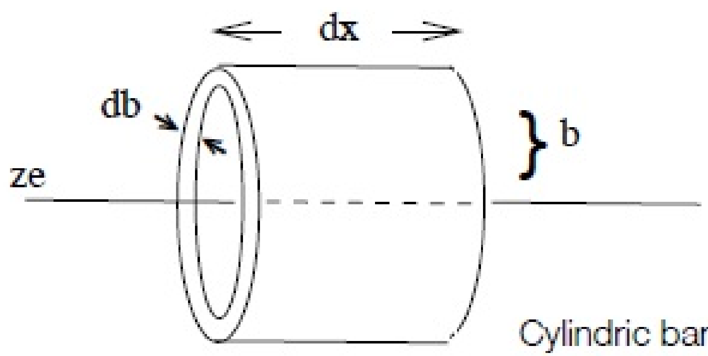
$$\int E_\perp (2\pi b) dx = 4\pi (ze) \rightarrow \int E_\perp dx = \frac{2ze}{b}$$

$$\Delta p_\perp = \int F_\perp \frac{dx}{v} = e \int E_\perp \frac{dx}{v} = \frac{2ze^2}{bv}$$

Bethe-Bloch – Classical Derivation (Bohr 1913)

Energy transfer onto single electron
for impact parameter b:

$$\Delta E(b) = \frac{\Delta p^2}{2m_e}$$



Cylindric barrel
with N_e electrons

Consider cylindric barrel → $N_e = n \cdot (2\pi b) \cdot db\, dx$

Bethe-Bloch – Classical Derivation (Bohr 1913)

Determination of relevant range [b_{min}, b_{max}]:
[Arguments: $b_{min} > \lambda_e$, i.e. de Broglie wavelength; $b_{max} < \infty$ due to screening ...]

$$b_{\text{min}} = \lambda_\text{e} = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_\text{e} v}$$

Use Heisenberg uncertainty principle or that electron is located within de Broglie wavelength ...

$$b_{\text{max}} = \frac{\gamma v}{\langle \nu_\text{e} \rangle} \; ; \quad \left[\gamma = \frac{1}{\sqrt{1-\beta^2}} \right]$$

Interaction time (b/v) must be much shorter than period of the electron (γ/ν_e) to guarantee relevant energy transfer ...

[adiabatic invariance]

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_\text{e} c^2 \beta^2} \, n \cdot \ln \frac{m_\text{e} c^2 \beta^2 \gamma^2}{2\pi\hbar \langle \nu_\text{e} \rangle}$$

Deviates by factor 2 from QM derivation

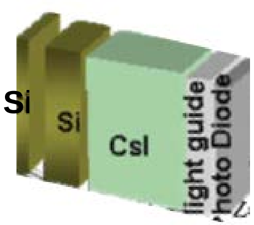
Electron density: $n = N_A \cdot \rho \cdot Z/A$!!

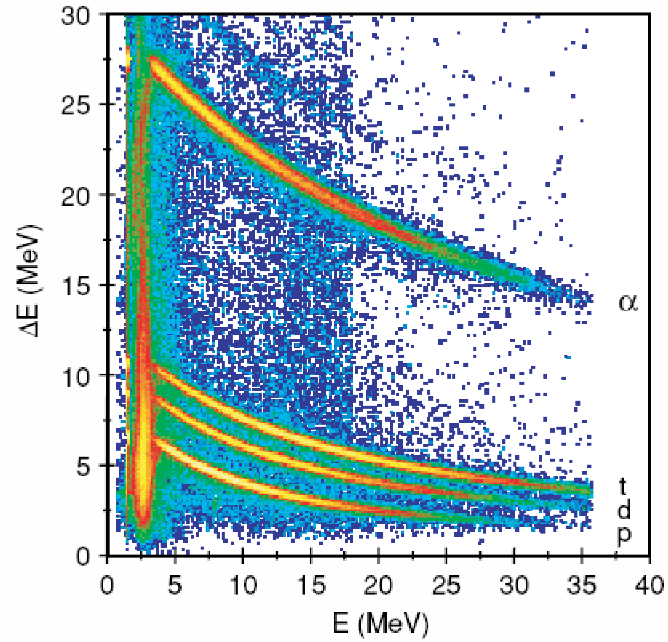
Effective Ionization potential: $I \sim h \langle \nu_e \rangle$

Energetic charged particles in matter

$$-\frac{dE}{dx} \propto \frac{mz^2}{E}$$

Charged particle identification with segmented or stacked detectors





$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_0 v^2} nZ \left[\ln \frac{2m_0 v^2}{I} - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2} \right]$$

Bethe-Block formula

z – projectile atomic number

v – projectile velocity

m_0 - electron mass

e – electron charge

n – target number density

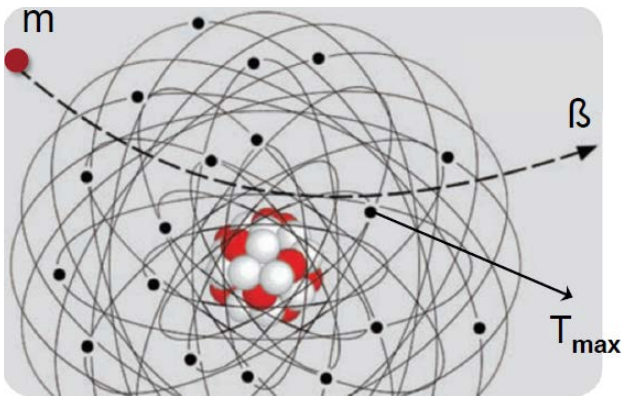
Z – target atomic number

nZ – target electron density

I – average excitation and ionization potential

Interaction of α-particles in matter

α-particles are highly ionized and lose their energy very fast by ionization and excitation when passing through matter.



- maximum energy transfer T_{max} of a projectile with mass **m** and velocity β on an electron $\mathbf{m_e}$ at rest

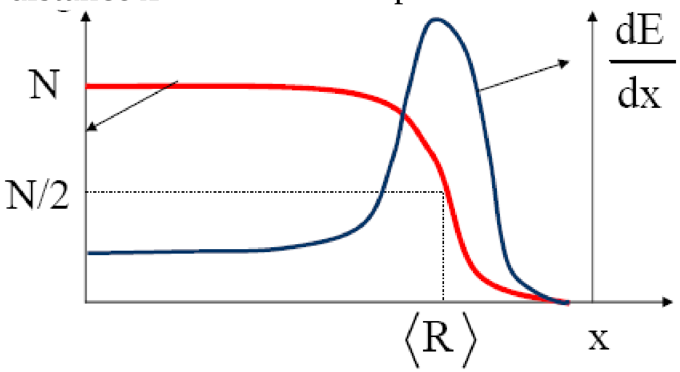
$$T_{max} = \frac{2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2 \cdot m^2}{m^2 + m_e{}^2 + 2 \cdot \gamma \cdot m \cdot m_e}$$

$$T_{max} = 2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2$$

for all heavy primary particles except electrons and positrons

number of α-particles as a function of distance **x**

energy loss of α-particles per distant unit



average range <R> of α-particles with 5 MeV
2,5cm in air, 2,3cm in Al, 4,3cm in tissue

Energy loss and range of charged particles

$$-\frac{dE}{d\varepsilon} = -\frac{1}{\rho}\cdot\frac{dE}{dx} = z^2\cdot\frac{Z}{A}\cdot f(\beta, I)$$

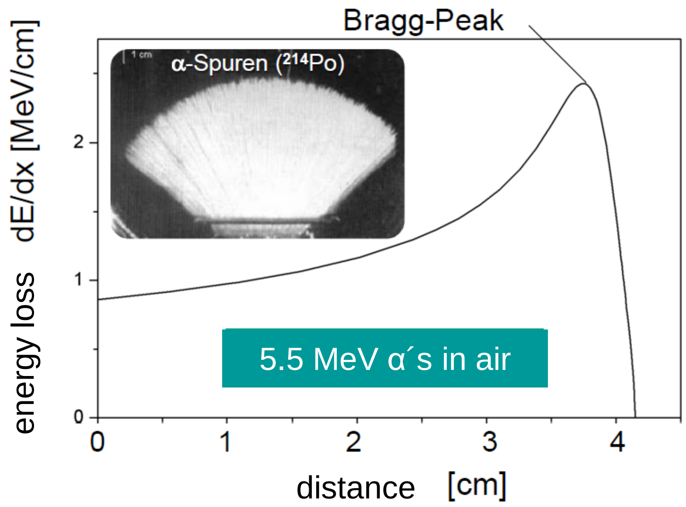
-dE/dε is independent of the material for equal particles

- the average range for particles with kin. energy T is obtained by integration

$$\bar{R} = \int_{E_0}^{0}\left(\frac{dE}{dx}\right)^{-1} dE$$

- 7.7 MeV α-particles in air: $\bar{R}/\rho \approx 7cm$

- range is not exact but there is range straggling,
  the number of interactions is a statistical process.

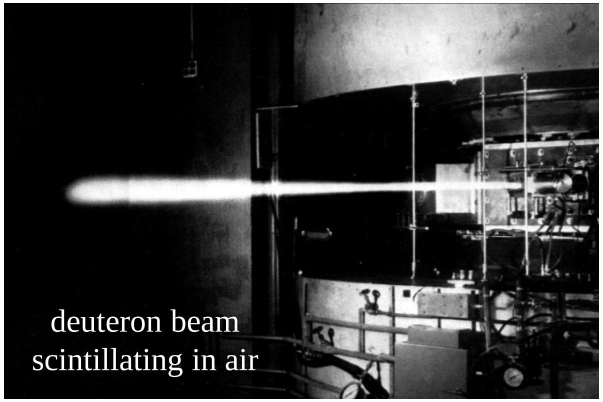


Several empirical and semi-empirical formulae have been proposed to compute range of α-particles in air.

$$R_\alpha^{air}[mm] = \begin{cases} e^{1.61\sqrt{E_\alpha}} & for\ E_\alpha < 4MeV \\ (0.05E_\alpha + 2.85)E_\alpha^{3/2} & for\ 4MeV \le E_\alpha \le 15MeV \end{cases}$$

$$R_\alpha^{air}[cm] = \begin{cases} 0.56E_\alpha & for\ E_\alpha < 4MeV \\ 1.24E_\alpha - 2.62 & for\ 4MeV \le E_\alpha < 8MeV \end{cases}$$

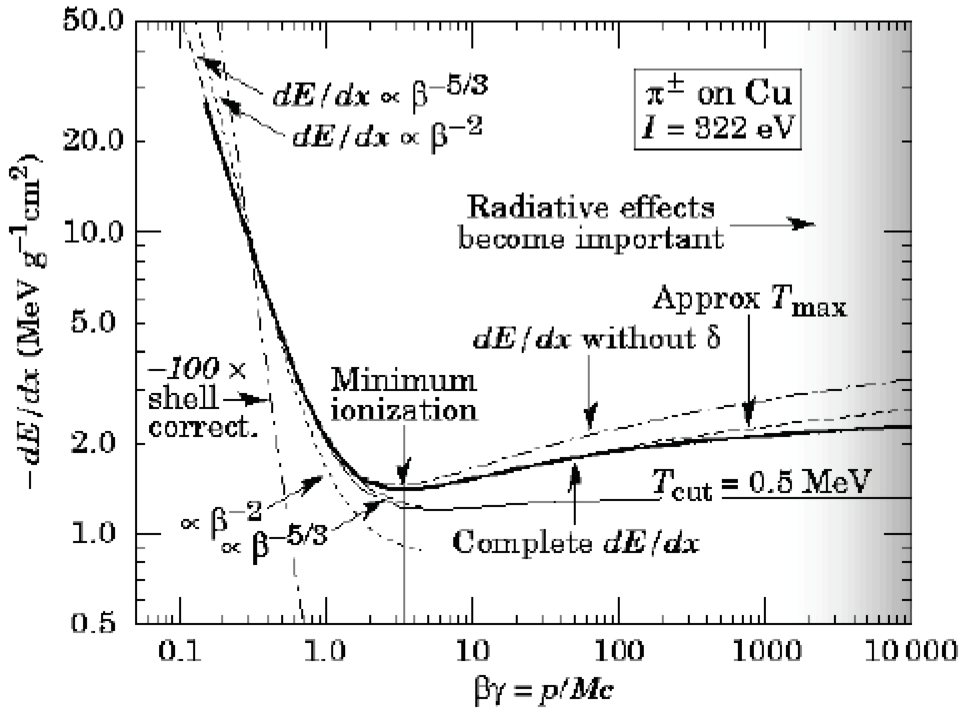
Scaling the range of other materials $R_\alpha^x = 3.37\cdot 10^{-4} R_\alpha^{air}\frac{\sqrt{A_x}}{\rho_x}$


deuteron beam
scintillating in air

Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

$$-\frac{dE}{dx} = \underbrace{4\cdot\pi\cdot r_e^2\cdot N_a\cdot m_e c^2}_{= 0.3071\ \text{MeV g}^{-1}\text{cm}^2}\cdot\rho\cdot\frac{Z}{A}\cdot\frac{z^2}{\beta^2}\cdot\left[\frac{1}{2}ln\left(\frac{2\cdot m_e c^2\cdot\gamma^2\cdot\beta^2\cdot T_{max}}{I^2}\right)-\beta^2-\delta-2\cdot\frac{C}{Z}\right] \approx z^2\cdot\frac{Z}{A}\cdot f(\beta,I)$$

N_a : Avogadro number $6.02\cdot10^{23}$ mol^{-1}
r_e : class. electron radius $2.81\cdot10^{-13}$ cm
m_e : electron mass
ρ : density of abs. matter
Z : element number of abs. matter
A : mass of abs. matter
z : charge number of incoming particle
W_{max} : max. energy transfer in a single collision
I : average ionization potential



- for small β the term $1/\beta^2$ is dominant
- dE/dx has a minimum at βγ~3-4 (minimum ionizing particle)
- for high momenta dE/dx reaches a saturation

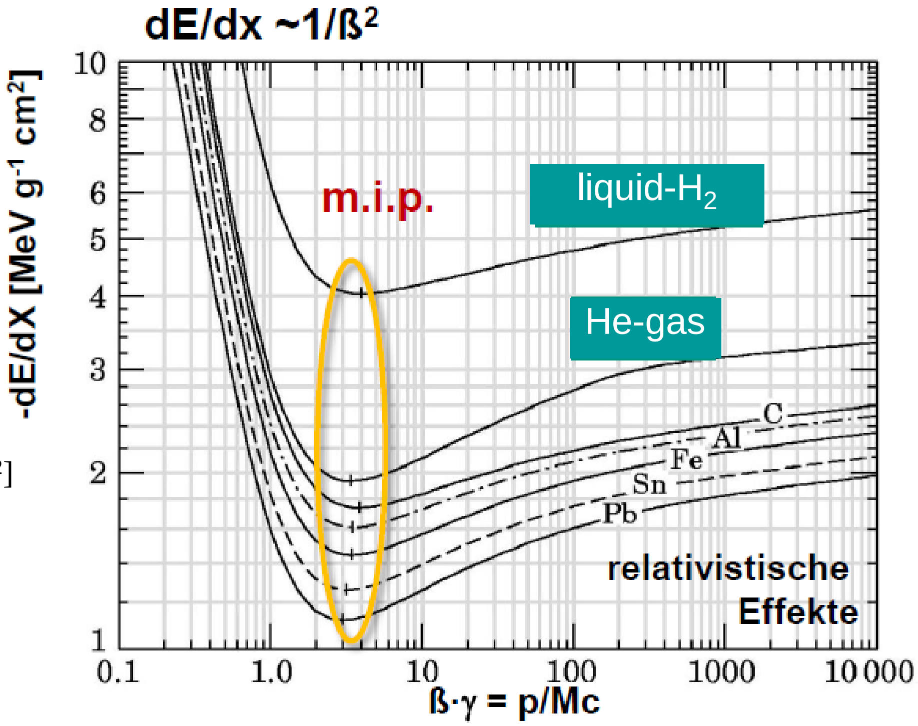
Interaction of charged particles in matter

Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

$$-\frac{dE}{dx} = \underbrace{4\cdot\pi\cdot r_e^2\cdot N_a\cdot m_e c^2}_{= 0.3071\ \text{MeV g}^{-1}\text{cm}^2}\cdot\rho\cdot\frac{Z}{A}\cdot\frac{z^2}{\beta^2}\cdot\left[\frac{1}{2}ln\left(\frac{2\cdot m_e c^2\cdot\gamma^2\cdot\beta^2\cdot T_{max}}{I^2}\right)-\beta^2-\delta-2\cdot\frac{C}{Z}\right] \approx z^2\cdot\frac{Z}{A}\cdot f(\beta,I)$$

- Specific energy loss rate $\frac{1}{\rho}\frac{dE}{dx}$
  for muons, pions and protons in different materials

- Dependence on mass A, charge Z of target nucleus
- Minimum ionization: 1-2 MeV/g cm^{-2} [H_2: 4 MeV/g cm^{-2}]

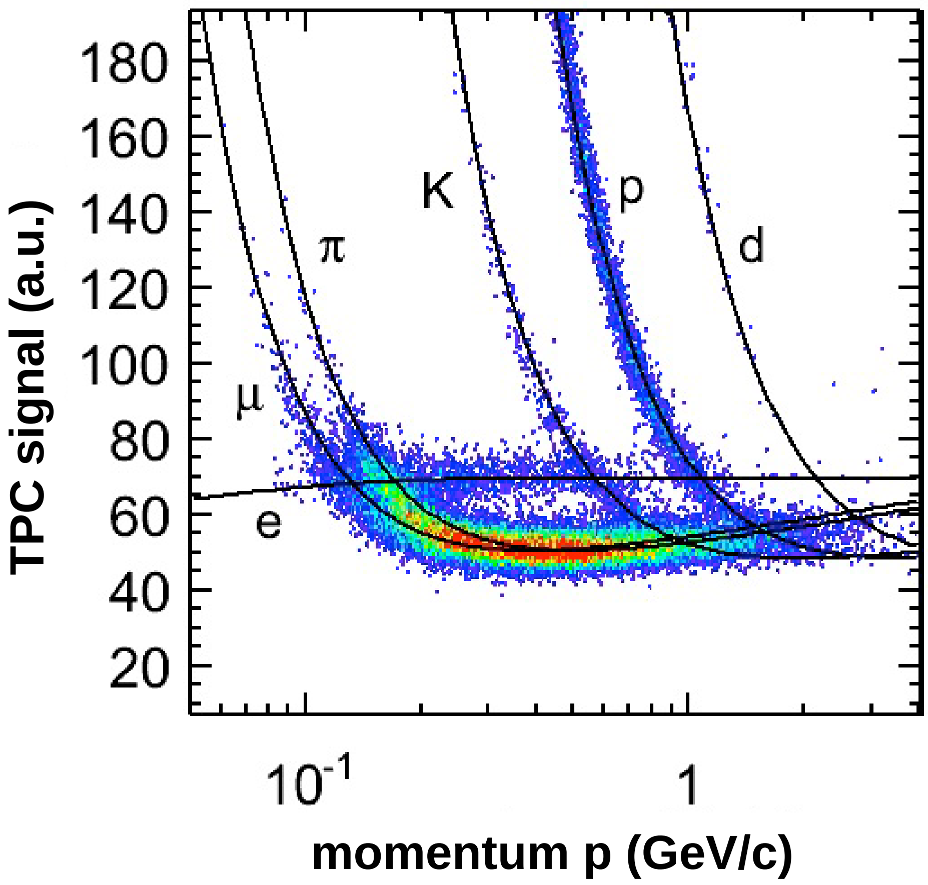


- for small β the term 1/β^2 is dominant
- dE/dx has a minimum at βγ~3-4 (minimum ionizing particle)
- for high momenta dE/dx reaches a saturation

Energy loss of charged particles – dE/dx for different particles

- dE/dx for heavy particles in this momentum regime is well described by Bethe-Bloch formula, i.e. the dominant energy loss is collisions with atoms

- dE/dx for electrons does not follow Bethe-Bloch formula. The dominant process is Bremsstrahlung



[ALICE TPC, 2009]

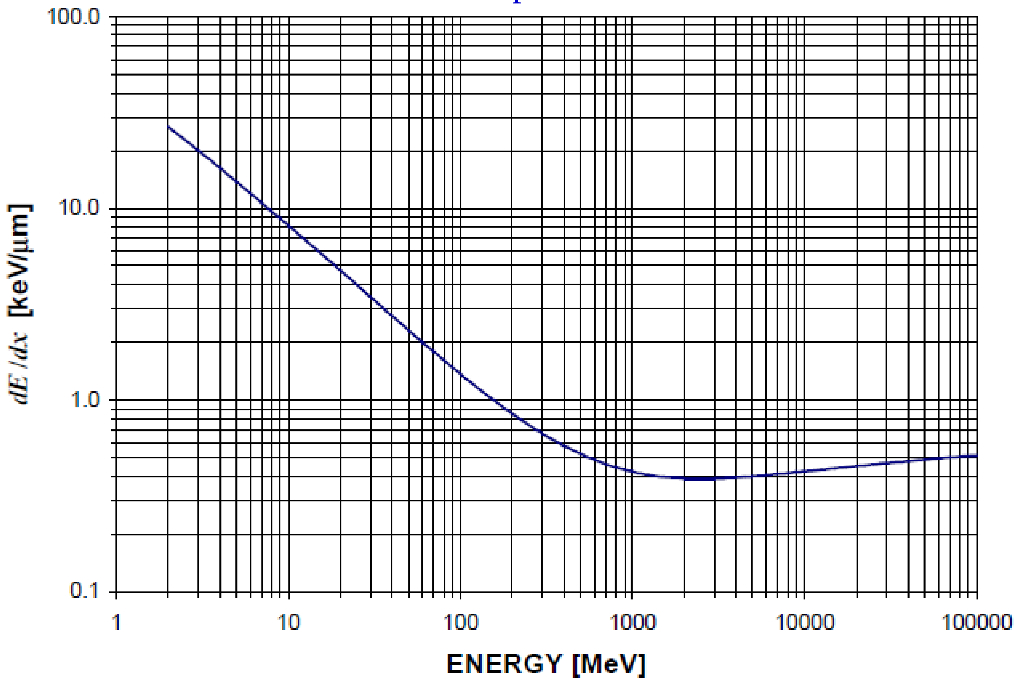
Example: Proton in Silicon

Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

$$-\frac{dE}{dx} = \underbrace{4 \cdot \pi \cdot r_e{}^2 \cdot N_a \cdot m_e c^2}_{= 0.3071\ \mathrm{MeV\ g^{-1}cm^2}} \cdot \rho \cdot \frac{Z}{A} \cdot \frac{z^2}{\beta^2} \cdot \left[\frac{1}{2} ln\left(\frac{2 \cdot m_e c^2 \cdot \gamma^2 \cdot \beta^2 \cdot T_{max}}{I^2}\right) - \beta^2 - \delta - 2 \cdot \frac{C}{Z}\right] \approx z^2 \cdot \frac{Z}{A} \cdot f(\beta, I)$$

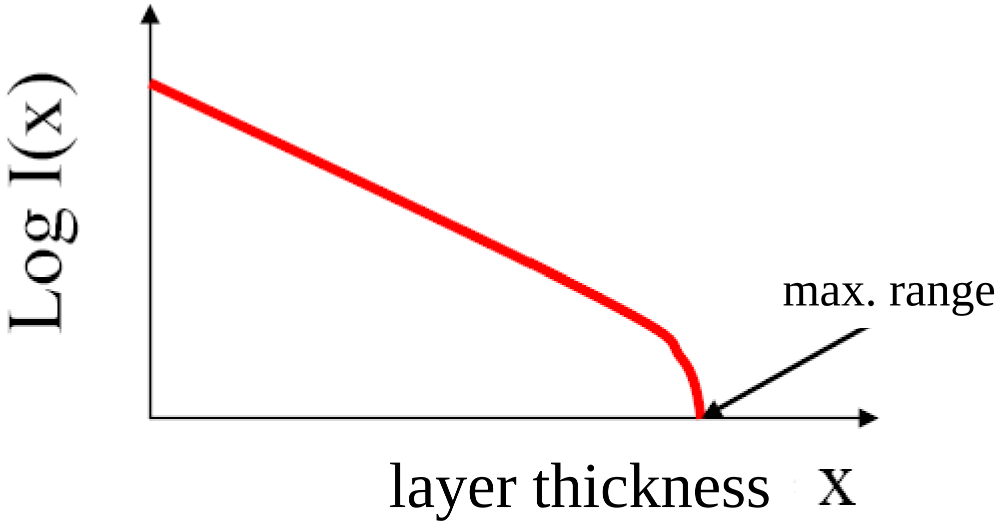
$$T_{max} = 2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2$$

dE/dx vs. E of protons in silicon



Interaction of β-particles with matter

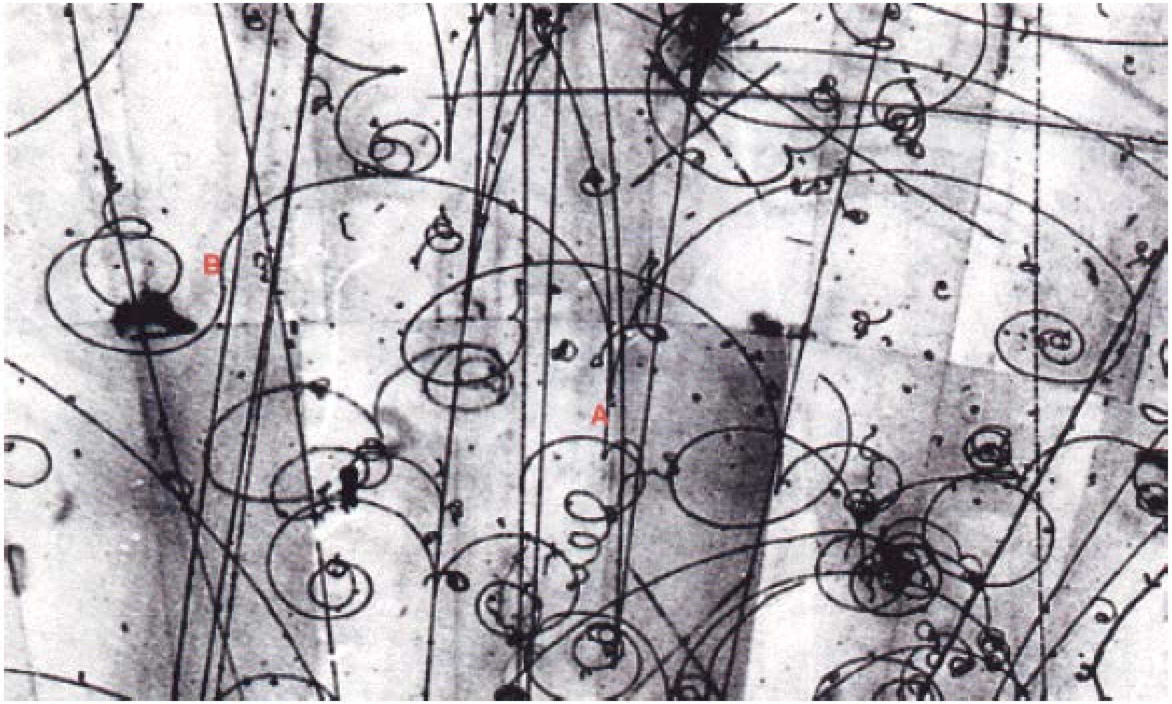
β-particles are also ionizing, similar to α-particles. Since the mass of the electrons and positrons are very small, the energy transfer per collision is small and the range large. Similar to the X-rays there is first only an attenuation, which finally leads to a maximum range for larger layer thicknesses.



Interaction of β-particles with matter

β^+ particles behave similarly as β^- particles; they are ionizing and attenuated on their way through matter.

But at the end of the their attenuation one observes a pair annihilation with an electron, which yields high energetic γ-emission. Positrons are hence more dangerous than electrons.

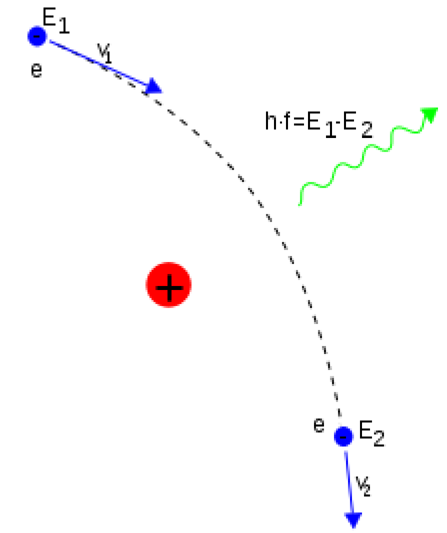


Bremsstrahlung

Bremsstrahlung (‘braking radiation’) is electromagnetic radiation produced by the deceleration of a charged particle when deflected by another charged particle, typically an electron by an atomic nucleus.

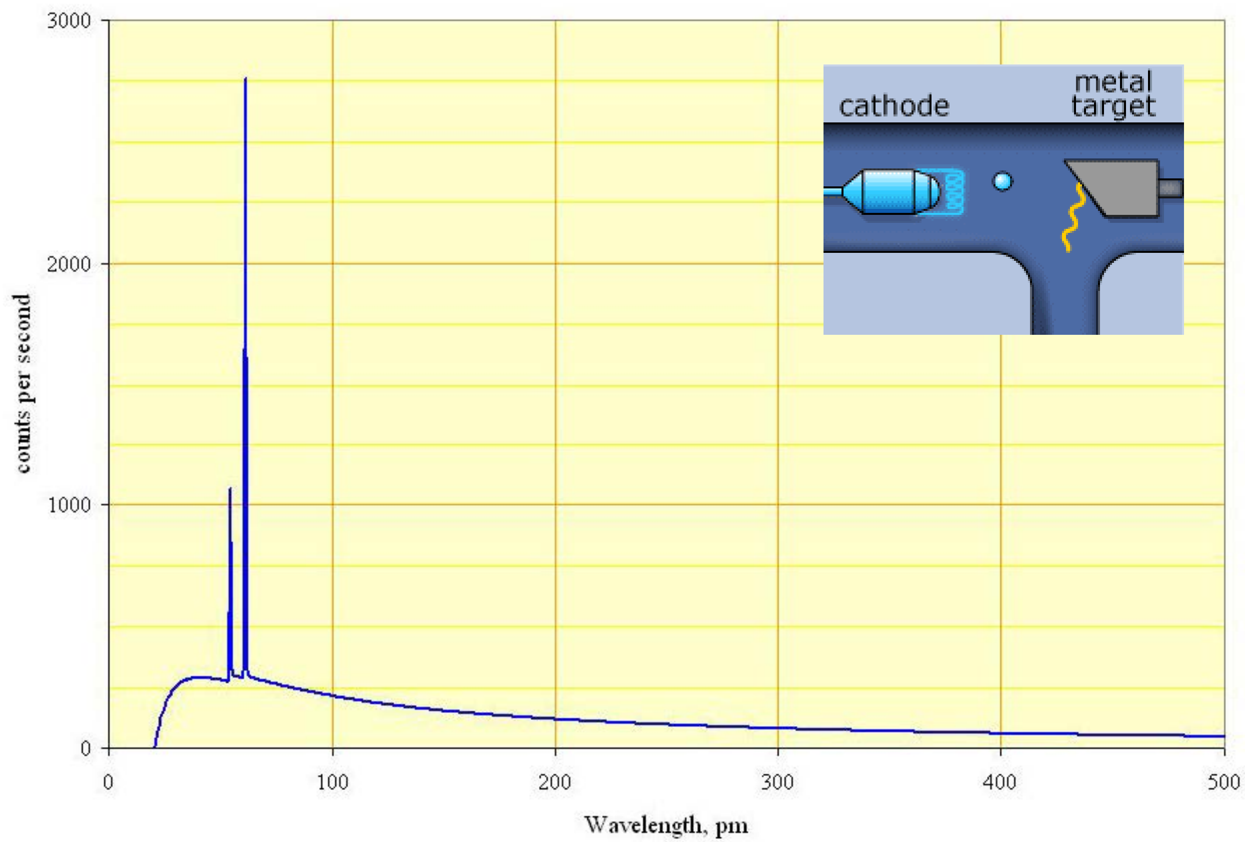
The moving particle loses kinetic energy, which is converted into a photon, thus satisfying the law of conservation of energy.

Bremsstrahlung has a continuous spectrum.



Bremsstrahlung

$$E_{photon} = h \cdot \nu = E_{kin} = e \cdot U$$



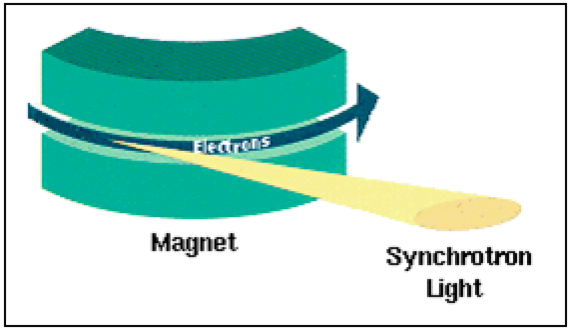
spectral distribution:

$$J(\lambda) = K \cdot I \cdot Z \cdot \left(\frac{\lambda}{\lambda_{min}} - 1\right) \cdot \frac{1}{\lambda^2}$$

K = Kramer constant
I = electron current
Z = element number of material

Spectrum of the X-rays emitted by an X-ray tube with a rhodium target, operated at 60 kV. The continuous curve is due to bremsstrahlung, and the spikes are characteristic K lines for rhodium.

The electromagnetic radiation emitted when charged particles are accelerated radially (**a** ⊥ **v**) is called **synchrotron radiation.** It is produced, for example, in synchrotrons using bending magnets.

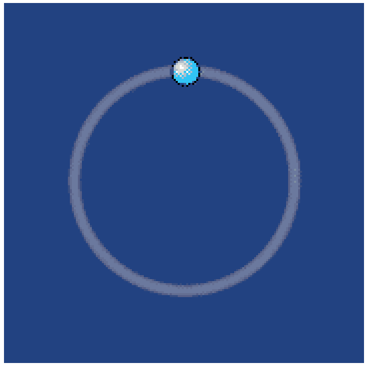


The energy loss of a charged particle (Z·e) due to radiation (during one cycle) is given by

$$\Delta E = \frac{(Ze)^2 \cdot \beta^3 \cdot \gamma^4}{\epsilon_0 \cdot 3R}$$

with Z = element number, ε_0 = electric field constant, R = radius of the storage ring, β=v/c and the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \equiv \frac{E}{m_0 c^2}$$



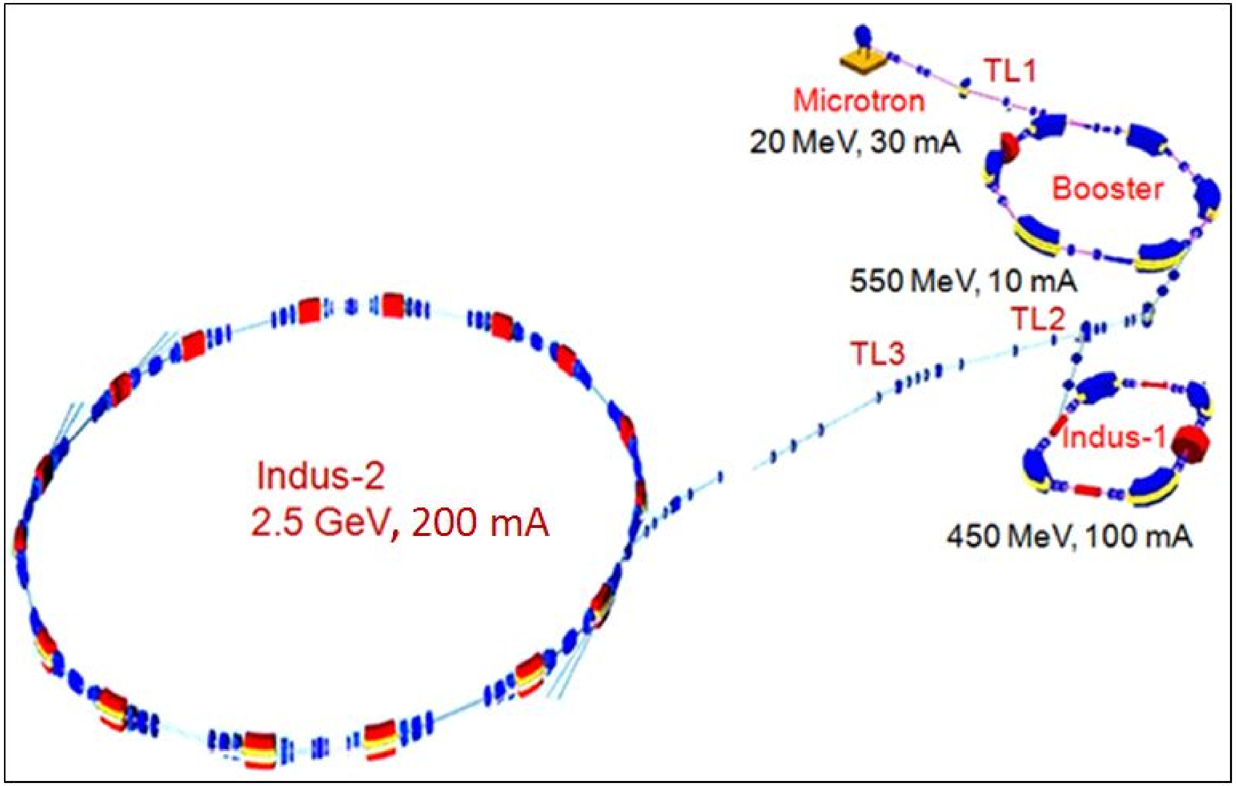
For relativistic velocities $\beta \approx 1$

$$\Delta E = \frac{(Ze)^2 \cdot E^4}{\epsilon_0 \cdot 3R \cdot (m_0 c^2)^4}$$

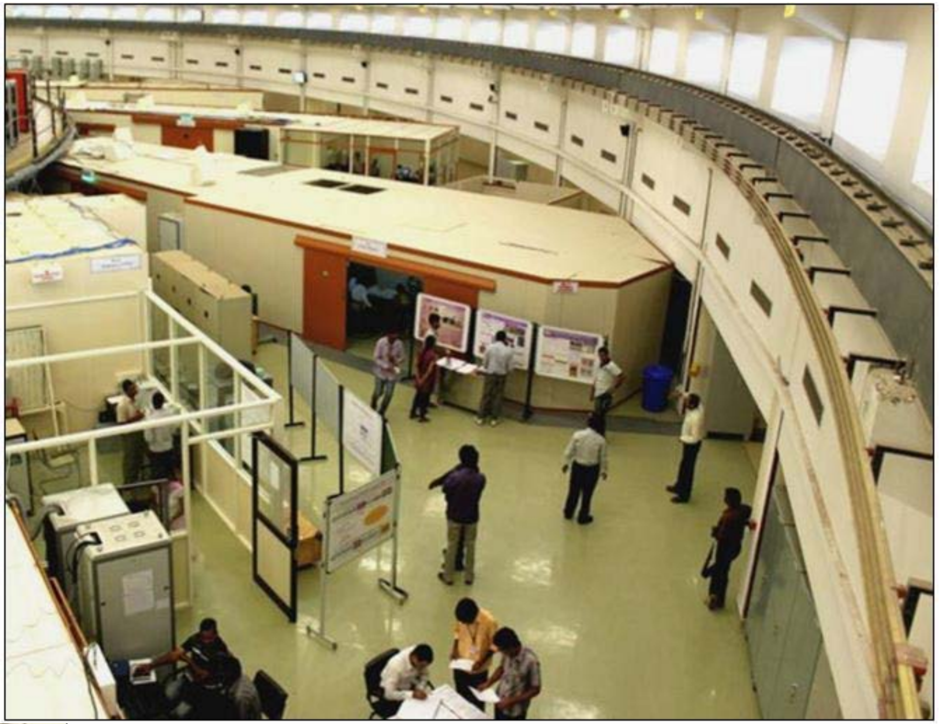
It is apparent that one uses light particles to create synchrotron radiation.

Raja Ramanna Centre for Advanced Technology

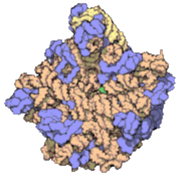
Indore



Raja Ramanna Centre for Advanced Technology - Synchrotron radiation



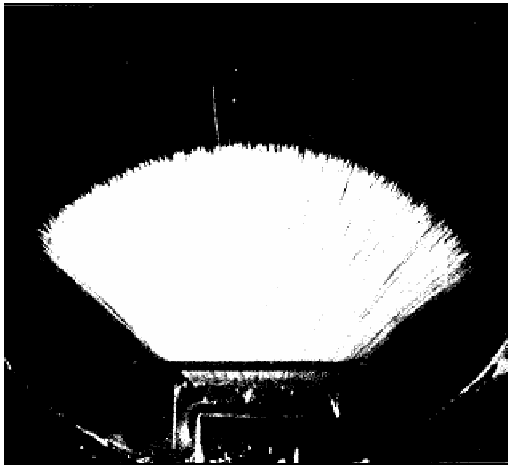
- Applications: condensed matter physics, material science, biology and medicine.



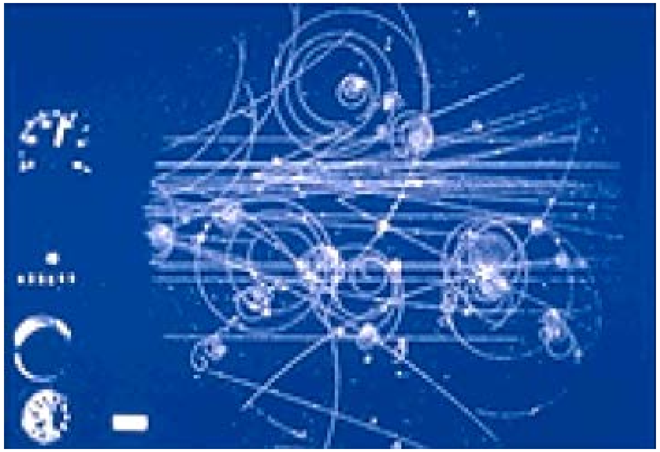
Structure of a ribosome (components of a cell)

Typical range of radioactive radiation in air

range of 5.5 MeV α-particles in air is ~ 4.2 cm



range of 1 MeV β-particles in air is ~ 4 m



range of X-rays, γ-rays and neutrons is very large.

shielding or large distances ($1/r^2$ law) are the solution

Energy loss for electrons and positrons

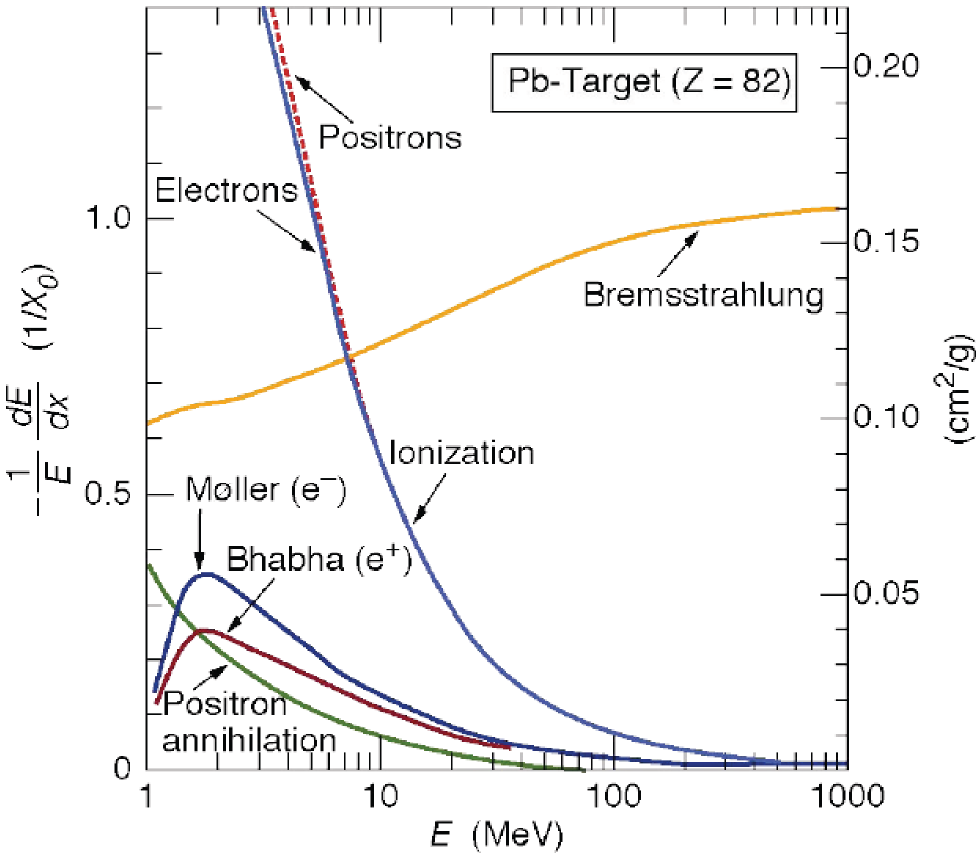
e$^{\pm}$ are exceptional cases due to their low mass.
They will be deflected significantly in each collision.
In addition to the energy loss due to **ionization**, the energy loss due to **Bremsstrahlung** is of importance.

$$-\left(\frac{dE}{dx}\right)_{tot} = -\left(\frac{dE}{dx}\right)_{coll} - \left(\frac{dE}{dx}\right)_{rad}$$

For high energies the energy loss due to Bremsstrahlung is given by

$$-\left(\frac{dE}{dx}\right)_{rad} \propto E$$ and $$-\left(\frac{dE}{dx}\right)_{rad} \propto \frac{1}{m^2}$$

Other particles like muons also radiate, especially at higher energies.



Bremsstrahlung

$$-\left(\frac{dE}{dx}\right)_{rad} = \frac{E}{X_0}$$

X_0 is the radiation length. It is the mean distance over which a high-energy electron loses all but 1/e of its energy by Bremsstrahlung

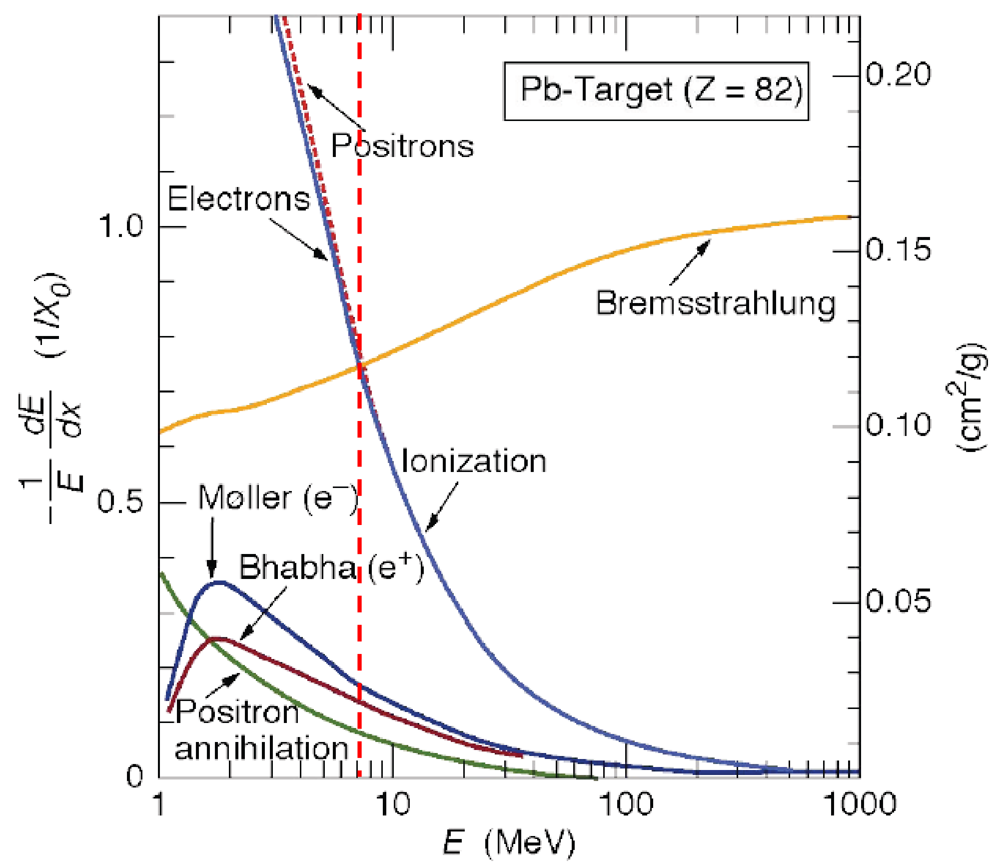
fit to data:

$$X_0 = \frac{716.4 \cdot A}{Z \cdot (Z+1) \cdot ln\left(287/\sqrt{Z}\right)}$$

Usual definition for the critical energy $\mathbf{E_c}$ (electron)

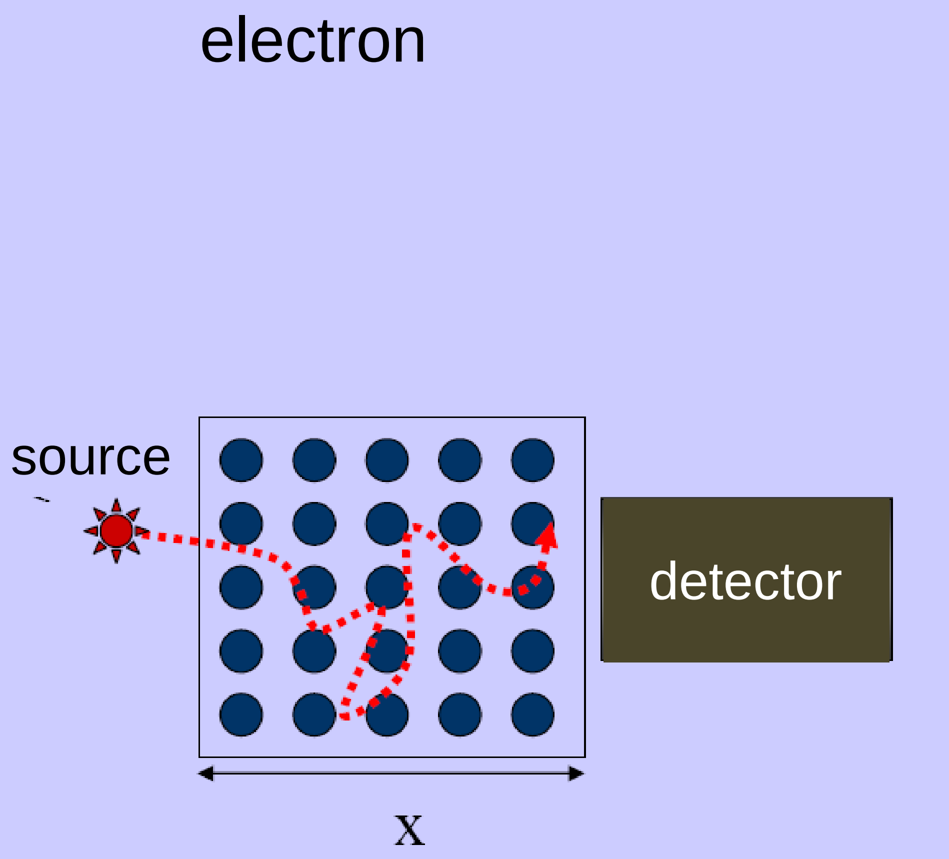
$$\left(\frac{dE}{dx}\right)_{ionization} = \left(\frac{dE}{dx}\right)_{bremsstrahlung}$$

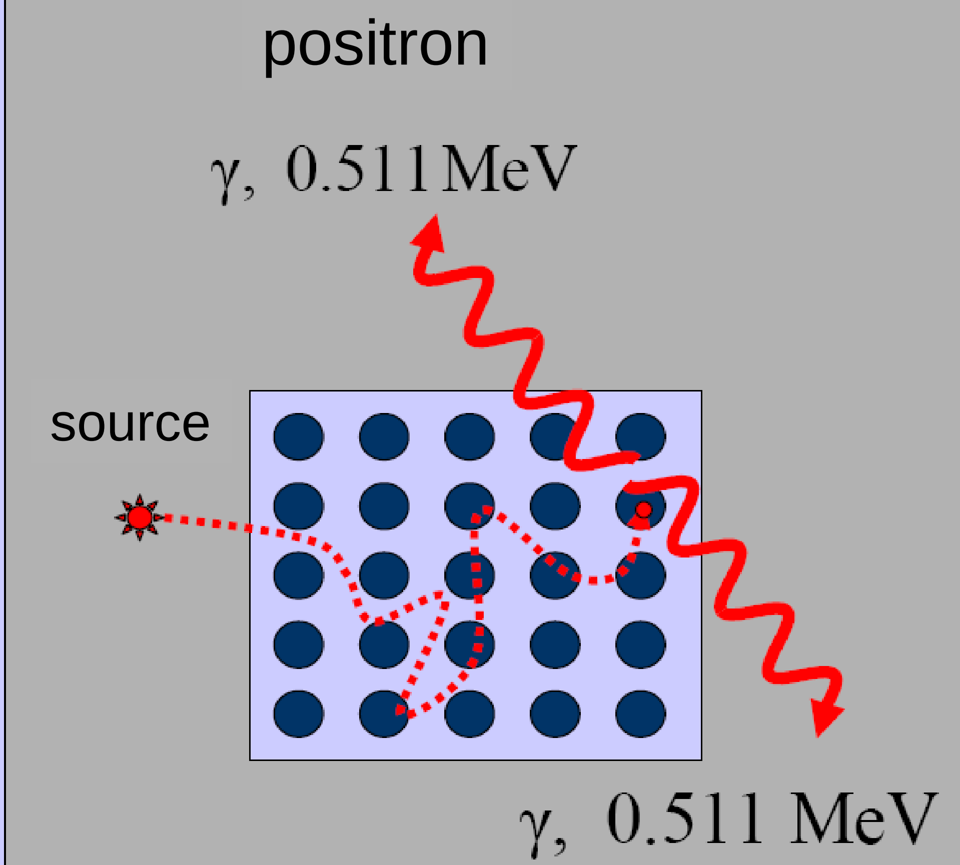
$$E_c\ (e^-) = \begin{cases} \dfrac{610\ MeV}{Z+1.24} & for\ solids\ and\ liquids \\ \dfrac{710\ MeV}{Z+0.92} & for\ gases \end{cases}$$



example: Pb (Z=82, ρ = 11.34 [g/cm³] → E_c = 7.34 MeV

Comparison between electrons (β^-) and positrons (β^+) on their way through matter





Cherenkov radiation

$$-\frac{dE}{dx} = \underbrace{4 \cdot \pi \cdot {r_e}^2 \cdot N_a \cdot m_e c^2}_{= 0.3071\ \text{MeV g}^{-1}\text{cm}^2} \cdot \rho \cdot \frac{Z}{A} \cdot \frac{z^2}{\beta^2} \cdot \left[\frac{1}{2} ln\left(\frac{2 \cdot m_e c^2 \cdot \gamma^2 \cdot \beta^2 \cdot T_{max}}{I^2}\right) - \beta^2 - \delta - 2 \cdot \frac{C}{Z}\right]$$

$\delta \equiv$ polarization of medium

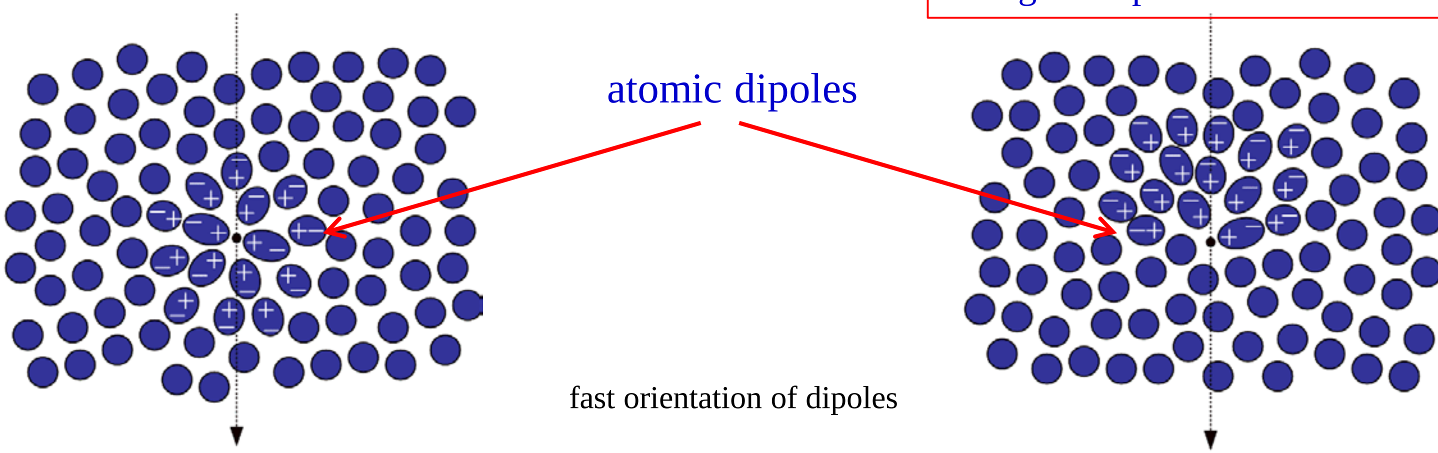
Cherenkov radiation is emitted if the particle velocity v is larger than the velocity of light in the medium.

$$v > \frac{c}{n}$$

$c \cdots$ *speed of light in vacuum*
$n \cdots$ *refraction index of medium*

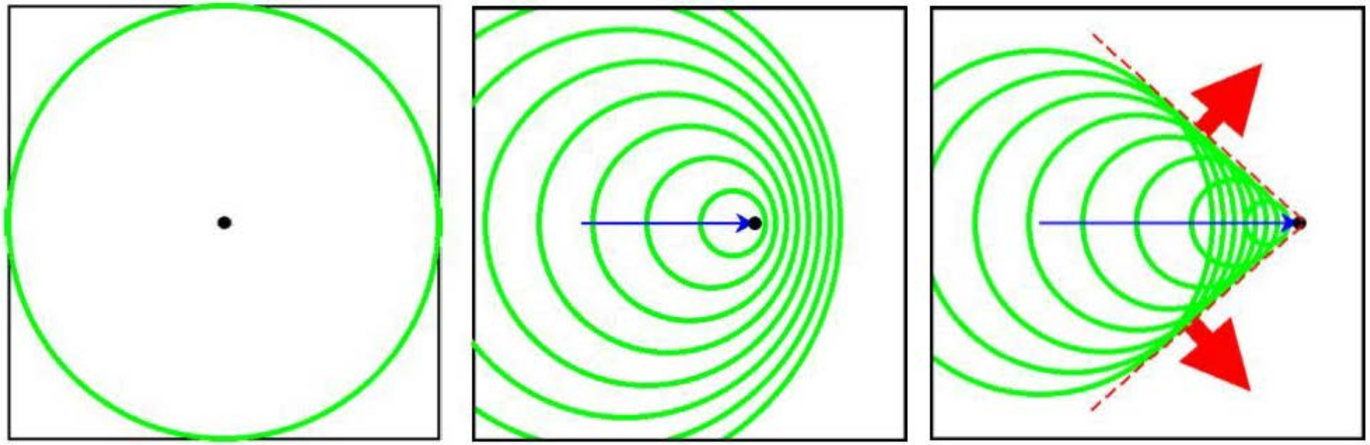
$v < c/n$ symmetric orientation
→ no resulting dipole field

$v > c/n$ asymmetric orientation
→ resulting dipole field
change of dipole field → **radiation**

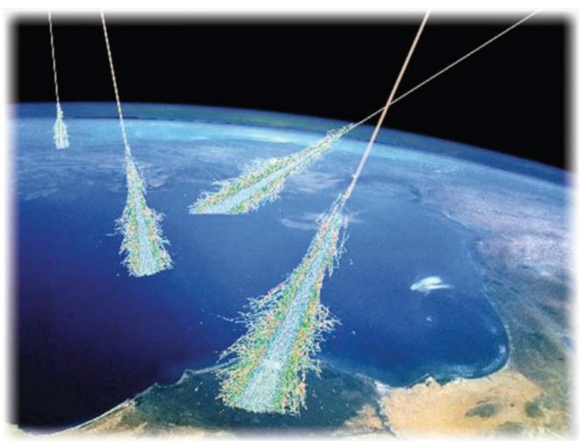


Cherenkov radiation

$\beta = 0$ $\quad$ $\beta \leq \frac{1}{n}$ $\quad$ $\beta \geq \frac{1}{n}$

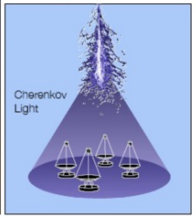


Pavel Alekseyevich Cherenkov

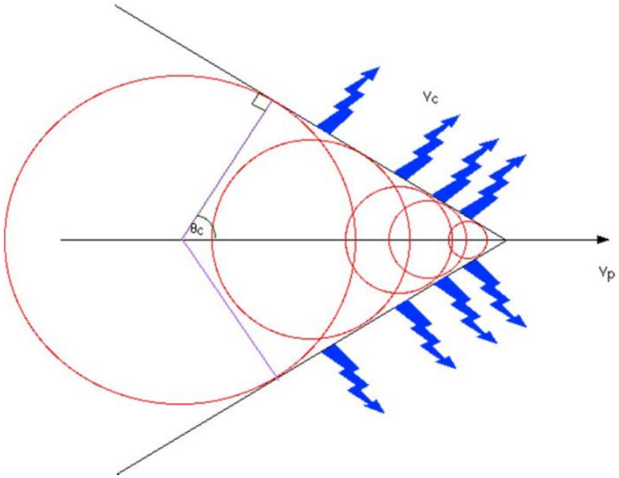


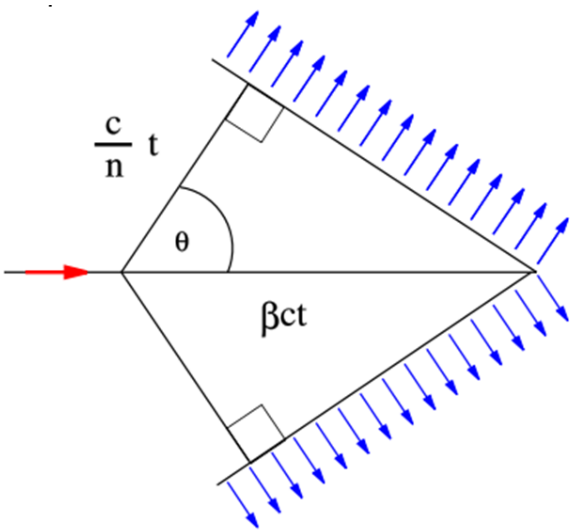
Hess Telescopes Namibia

γ-ray detection



Cherenkov radiation





threshold velocity: $\beta \geq \frac{1}{n}$

threshold angle: $\cos\theta_C = \frac{1}{\beta \cdot n}$

$$\gamma = (1-\beta^2)^{-1/2} \geq \frac{n}{\sqrt{n^2-1}}$$

Parameters of typical radiators

medium	n	β_{thr}	$\theta_{max}(\beta=1)$	$N_{ph}(eV^{-1}cm^{-1})$
air	1.000283	0.9997	1.36	0.208
isobutene	1.00127	0.9987	2.89	0.941
water	1.33	0.752	41.2	160.8
quartz	1.46	0.685	46.7	196.4

Note: Energy loss due to Cherenkov radiation very small compared to ionization (<1%)