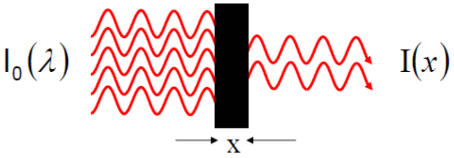
Interaction of gamma rays with matter

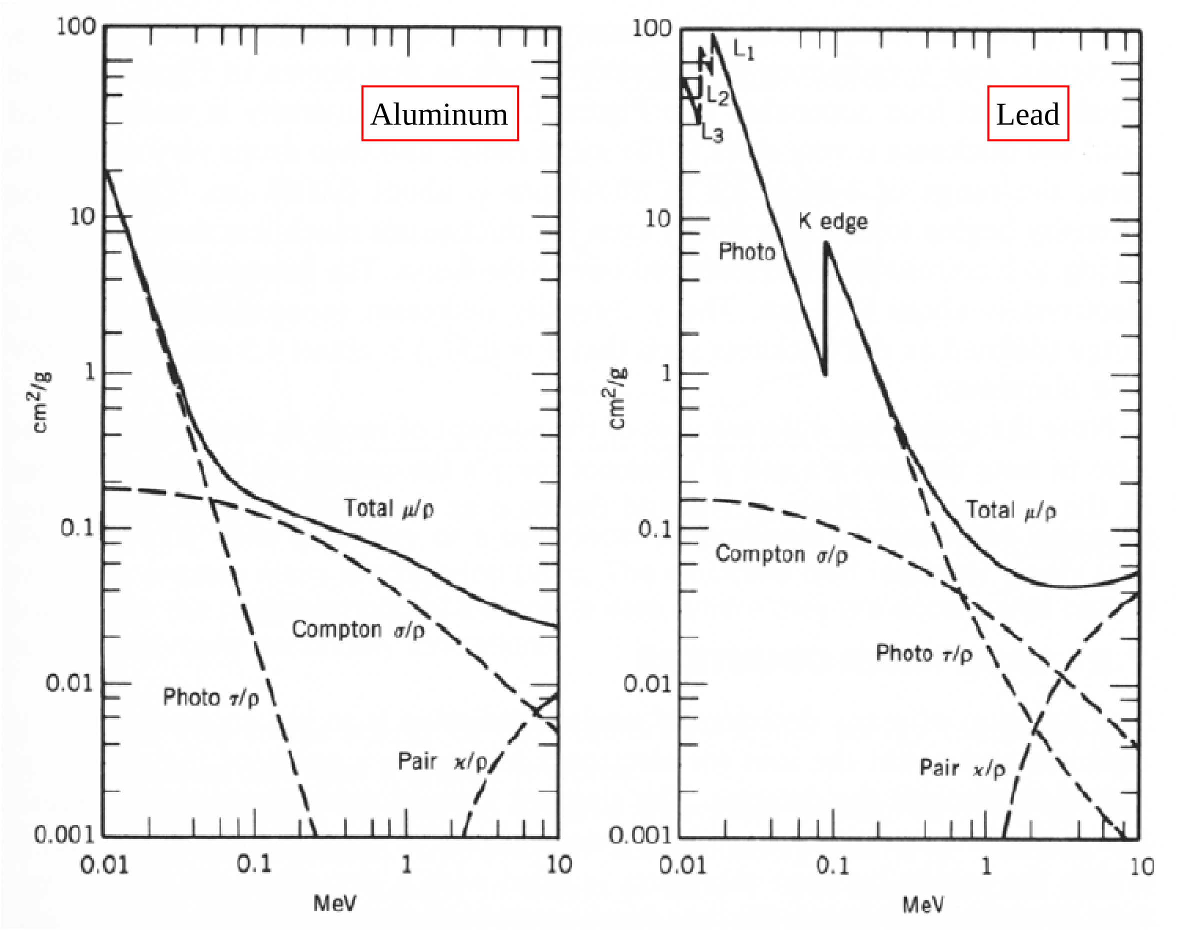


$$I(x) = I_0(\lambda) \cdot e^{-\frac{\mu(\lambda,Z)}{\rho}\rho\cdot x}$$

total absorption coefficient: **μ/ρ** [cm^2/g]

$$\frac{\mu_{total}}{\rho} = \sum_{i=1}^{3} \sigma_i$$

i=1 photoelectric effect
i=2 Compton scattering
i=3 pair production

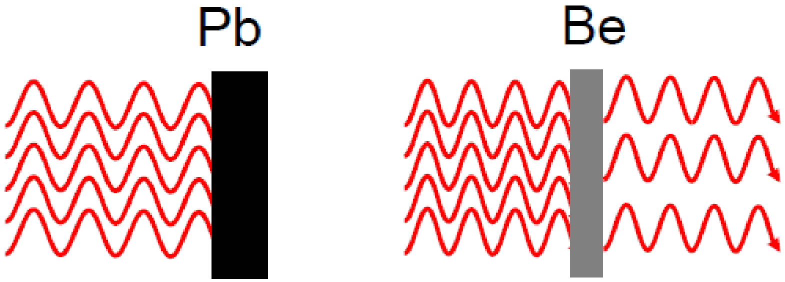


Mass dependence of X-ray absorption

For X-ray radiation the photoelectric effect is the most important interaction.

$$(\mu/\rho)_{Photo} \approx \lambda^3 \cdot Z^5$$

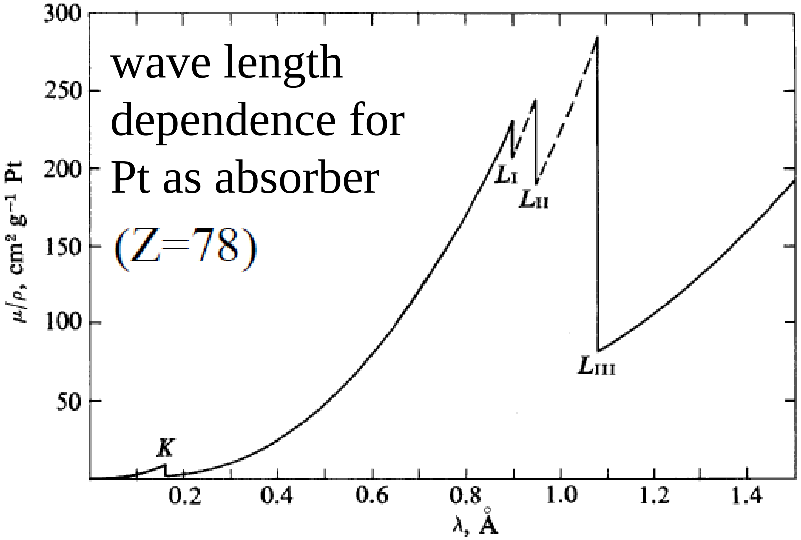
Lead absorbs more than Beryllium!



$_{82}$Pb serves as shielding for X-ray and γ-ray radiation; lead vests are used by medical staff people who are exposed to X-ray radiation. Co-sources are transported in thick lead container.

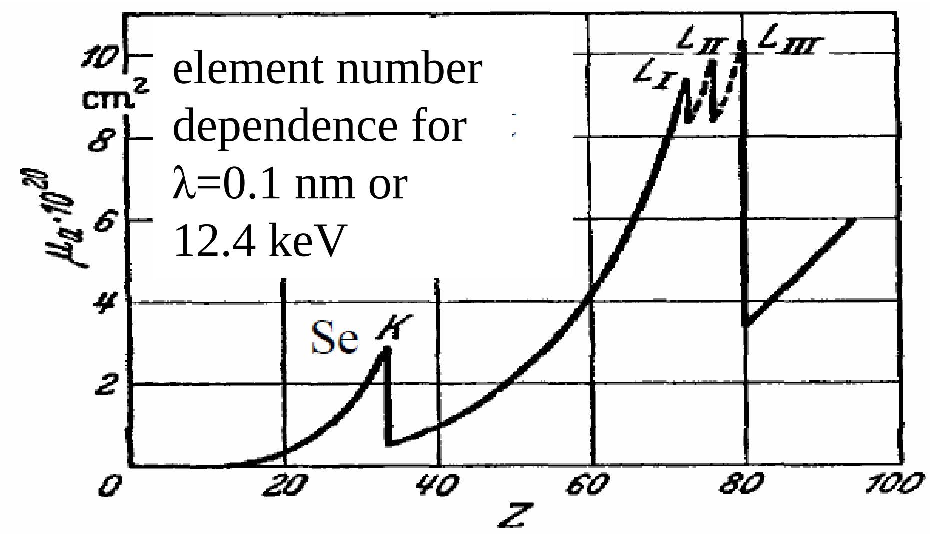
On the contrary:
$_4$Be is often used as windows in X-ray tubes to allow for almost undisturbed transmission of X-ray radiation.

Mass dependence μ/ρ of X-ray absorption



wave length dependence for Pt as absorber

(Z=78)

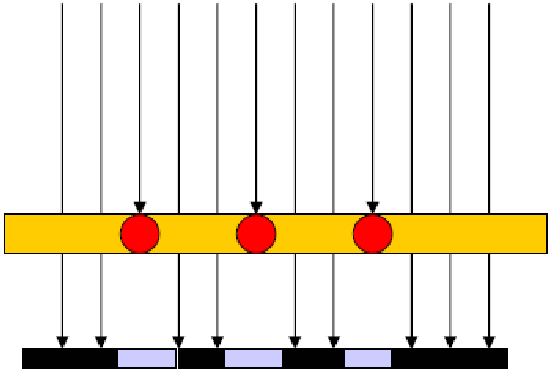


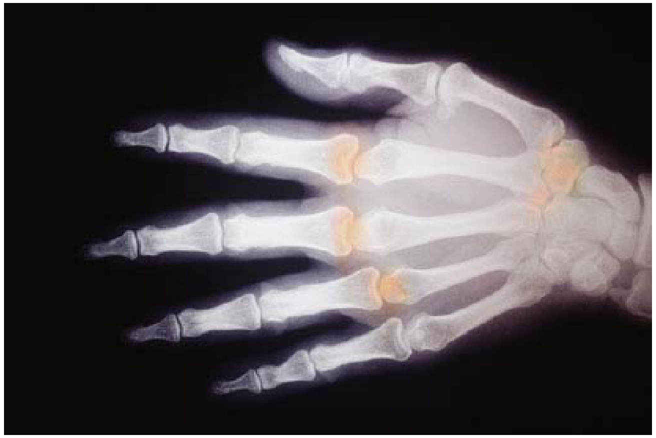
element number dependence for λ=0.1 nm or 12.4 keV

Se

X-ray image shows the effect of different absorptions

Bones absorb more radiation as tissues because of their higher $_{20}$Ca content

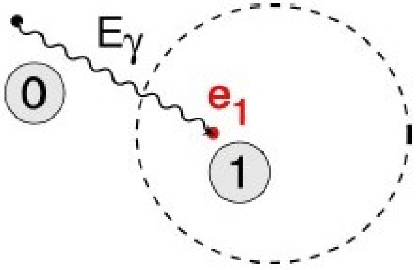




Interaction of gamma rays with matter

~ 100 keV ~1 MeV ~ 10 MeV γ-ray energy

Photoelectric

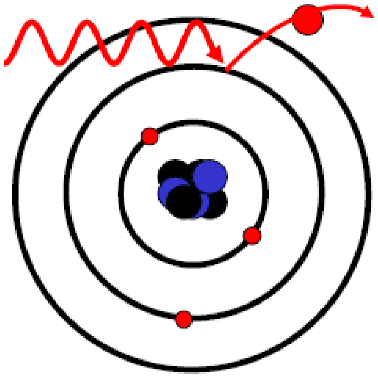


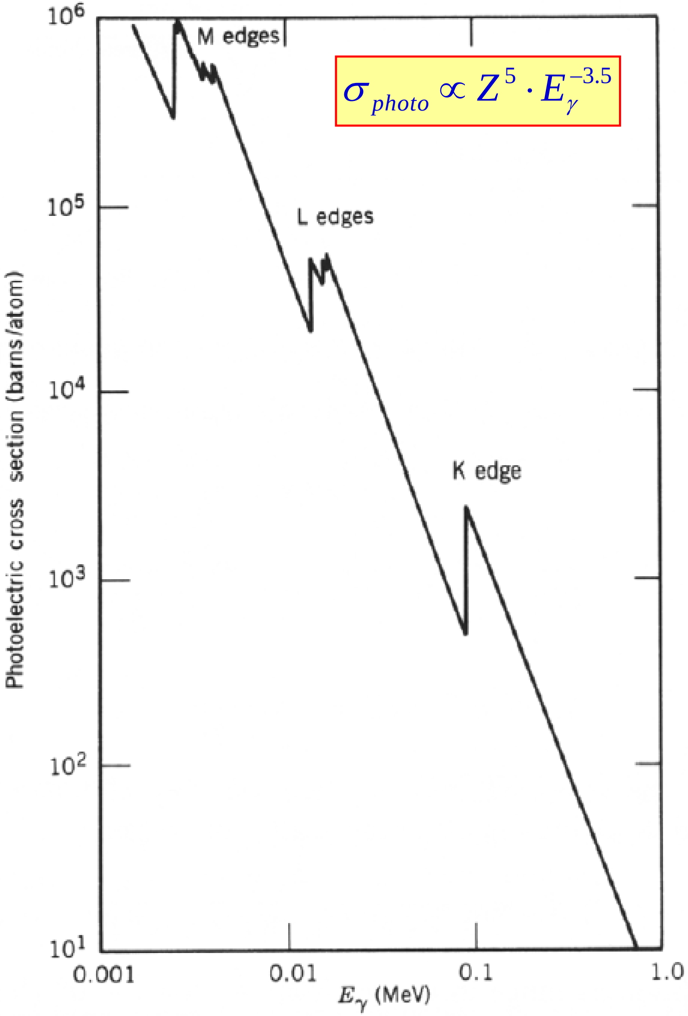
Isolated hits

Probability of interaction depth

Interaction of gamma rays with matter

$$E_{e,kin} = h \cdot \nu - E_{Bindung}$$





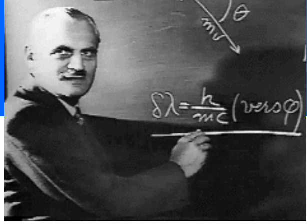
$$\sigma_{photo} \propto Z^5 \cdot E_\gamma^{-3.5}$$

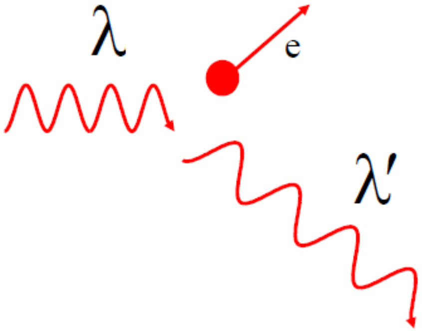
Photo effect:

Absorption of a photon by a bound electron and conversion of the γ-energy in potential and kinetical energy of the ejected electron.

(Nucleus preserves the momentum conservation.)

Interaction of gamma rays with matter





Compton scattering:
Elastic scattering of a γ-ray on a free electron. A fraction of the γ-ray energy is transferred to the Compton electron. The wave length of the scattered γ-ray is increased: λ‘ > λ.

relativistic $E^2 = (pc)^2 + (m_0c^2)^2$ $photons:\ m_0 = m_\gamma = 0$

$$\rightarrow E_\gamma = p_\gamma c$$

Momentum balance:

$$\overrightarrow{p_e} = \overrightarrow{p_\gamma} - \overrightarrow{p'_\gamma} \quad \rightarrow \quad |\overrightarrow{p_e}c|^2 = \left|\left(\overrightarrow{p_\gamma} - \overrightarrow{p'_\gamma}\right)c\right|^2$$

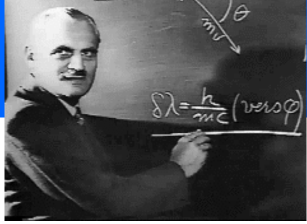
$$p_e^2c^2 = E_\gamma^2 + E_{\gamma'}^2 - 2E_\gamma E_{\gamma'} \cdot cos\theta$$

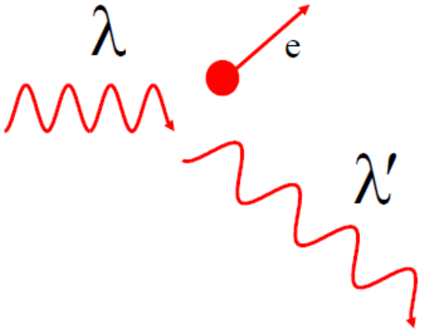
Energy balance:

$$E_\gamma + m_ec^2 = E_{\gamma'} + \sqrt{(p_ec)^2 + (m_ec^2)^2}$$

$$\boxed{E_{\gamma'} = \frac{E_\gamma}{1 + \left(E_\gamma/m_ec^2\right)(1 - cos\theta)}}$$

Interaction of gamma rays with matter





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Maximum energy of the scattered electron:

$$T\left(e^-\right)_{\max} = E_\gamma \cdot \frac{2\cdot E_\gamma}{m_e c^2 + 2\cdot E_\gamma}$$

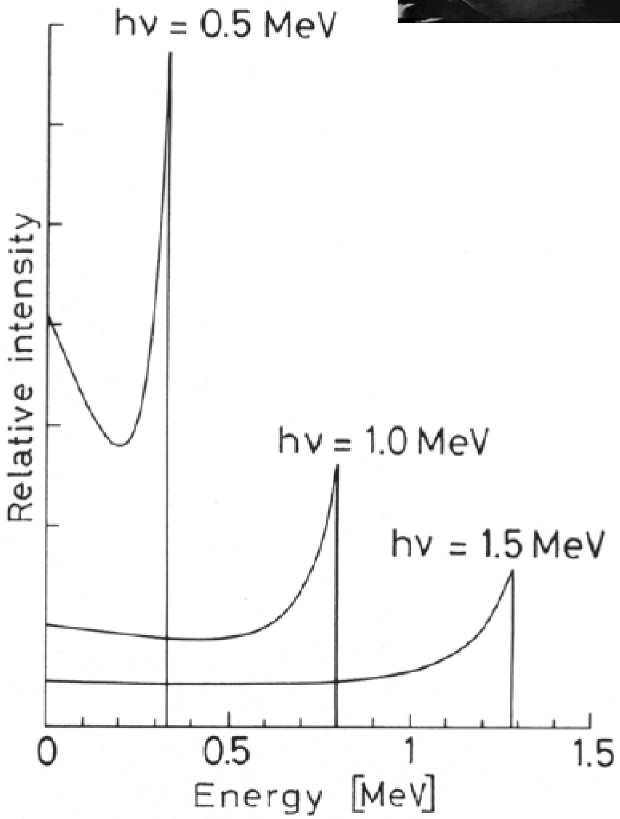
Energy of the scattered γ-photon:

$$E_\gamma' = \frac{E_\gamma \cdot m_e c^2}{m_e c^2 + E_\gamma \cdot (1-\cos\theta)}$$

$$\cos\theta = 1 + \frac{m_e c^2}{E_\gamma} - \frac{m_e c^2}{E_\gamma'}$$

Special case for $E >> m_e c^2$:
γ-ray energy after 180^0 scatter is approximately

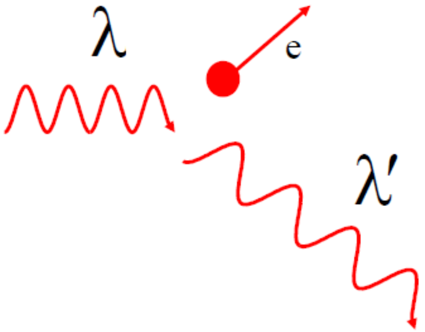
$$E_\gamma' = \frac{m_e c^2}{2} = 256\,keV$$

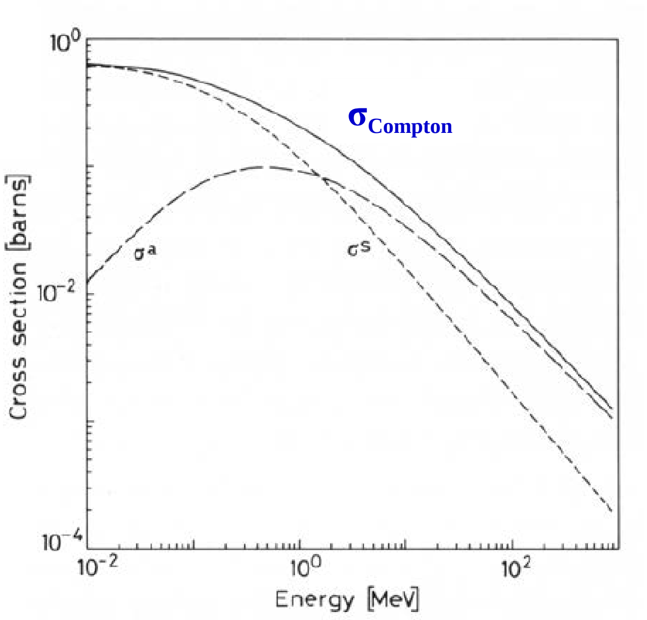


Gap between the incoming γ-ray and the maximum electron energy.

$$E_{kin}^{\max} = E_\gamma - E_\gamma' = E_\gamma \cdot \frac{2\cdot E_\gamma / m_e c^2}{1 + 2\cdot E_\gamma / m_e c^2}$$

Interaction of gamma rays with matter

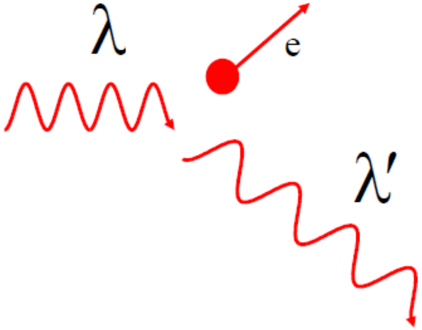




Compton scattering:

Elastic scattering of a γ-ray on a free electron. A fraction of the γ-ray energy is transferred to the Compton electron. The wave length of the scattered γ-ray is increased: λ‘ > λ.

Interaction of gamma rays with matter



Compton scattering:

Elastic scattering of a γ-ray on a free electron. The angle dependence is expressed by the

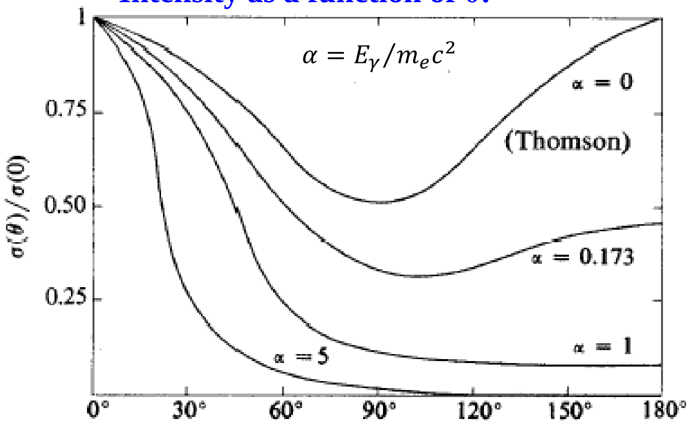
Klein-Nishina-Formula:

$$\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2}\left(\frac{E_{\gamma'}}{E_\gamma}\right)^2 \cdot \left\{\frac{E_\gamma}{E_{\gamma'}} + \frac{E_{\gamma'}}{E_\gamma} - 2sin^2\theta \cdot cos^2\phi\right\}$$

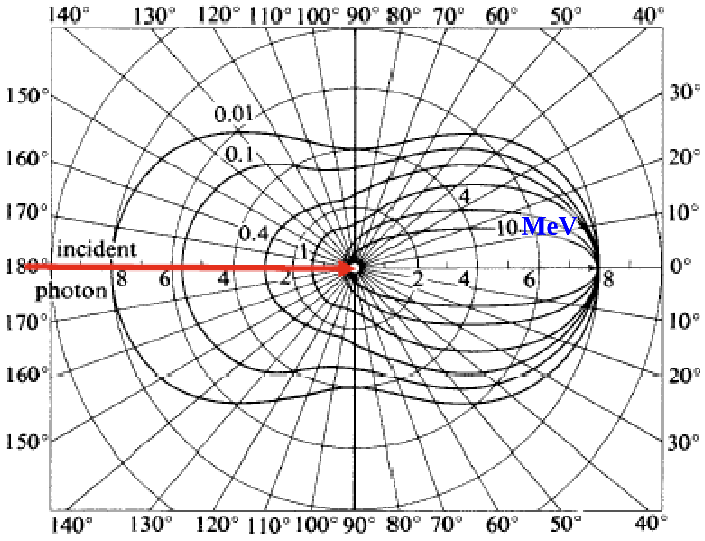
As shown in the plot **forward scattering** (θ small) is dominant for E_γ>100 keV.

r_0=2.818 fm (classical electron radius)

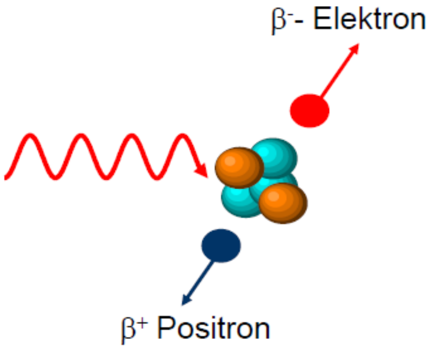
Intensity as a function of θ:

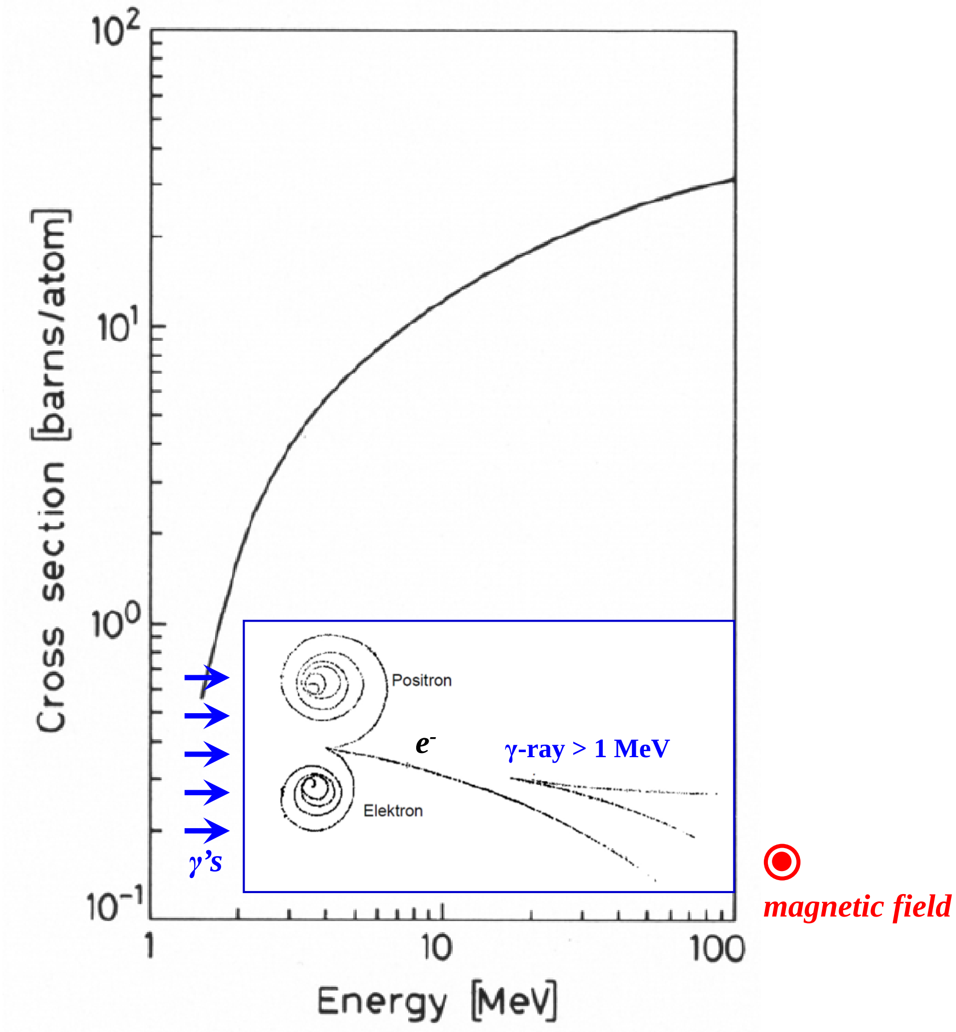


Angular distribution:



Interaction of gamma rays with matter





Pair production:
If γ-ray energy is >> $2m_0c^2$ (electron rest mass 511 keV), a positron-electron pair can be formed in the strong Coulomb field of a nucleus. This pair carries the γ-ray energy minus $2m_0c^2$.

Pair production for $E_\gamma > 2m_ec^2 = 1.022$MeV

picture of a bubble chamber

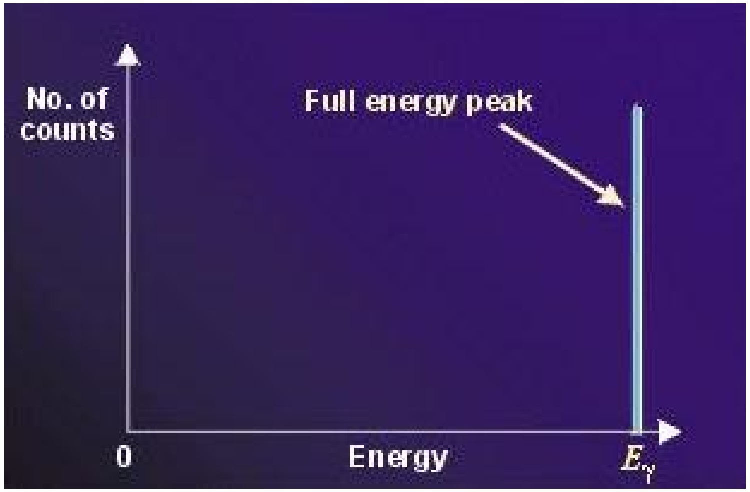
Interaction of gamma rays with matter

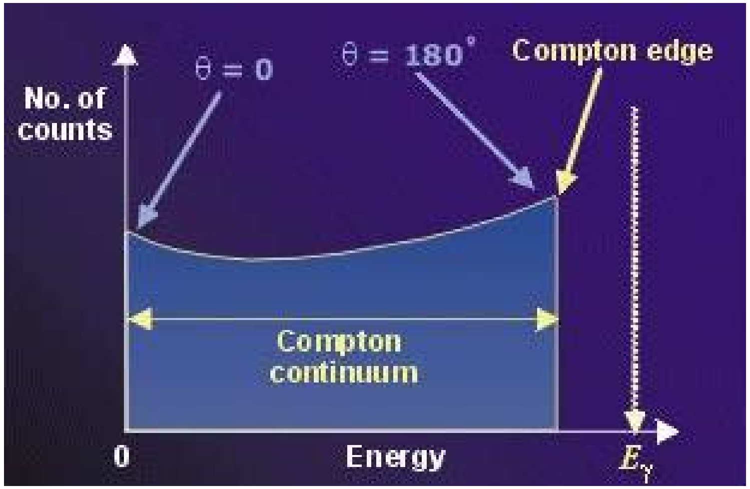
γ-rays interaction with matter via three main reaction mechanisms:

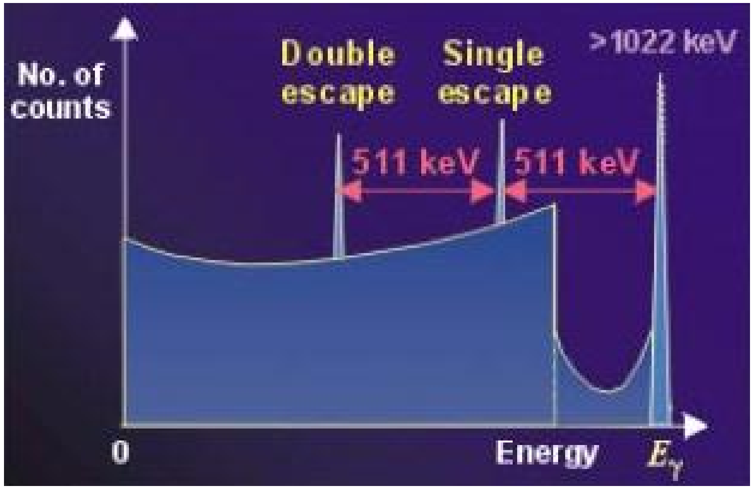
Photoelectric absorption

Compton scattering

Pair production



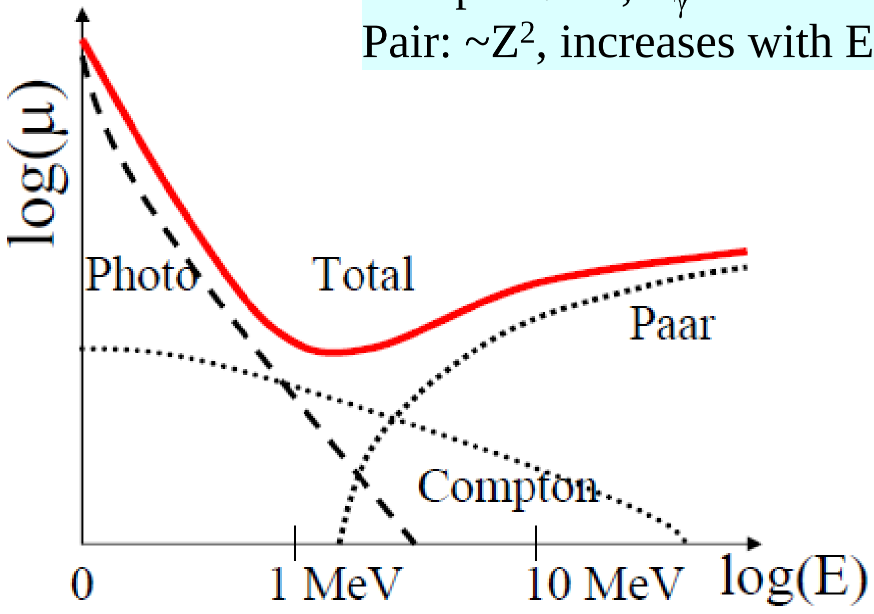


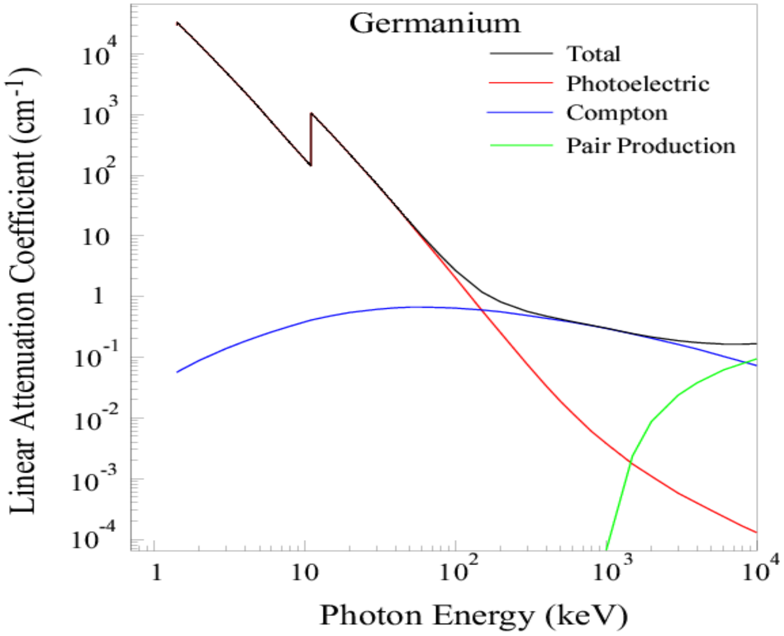


Gamma-ray interaction cross section

All three interaction (photo effect, Compton scattering and pair production) lead to an attenuation of the γ-ray or X-ray radiation when passing through matter. The particular contribution depends on the γ-ray energy:

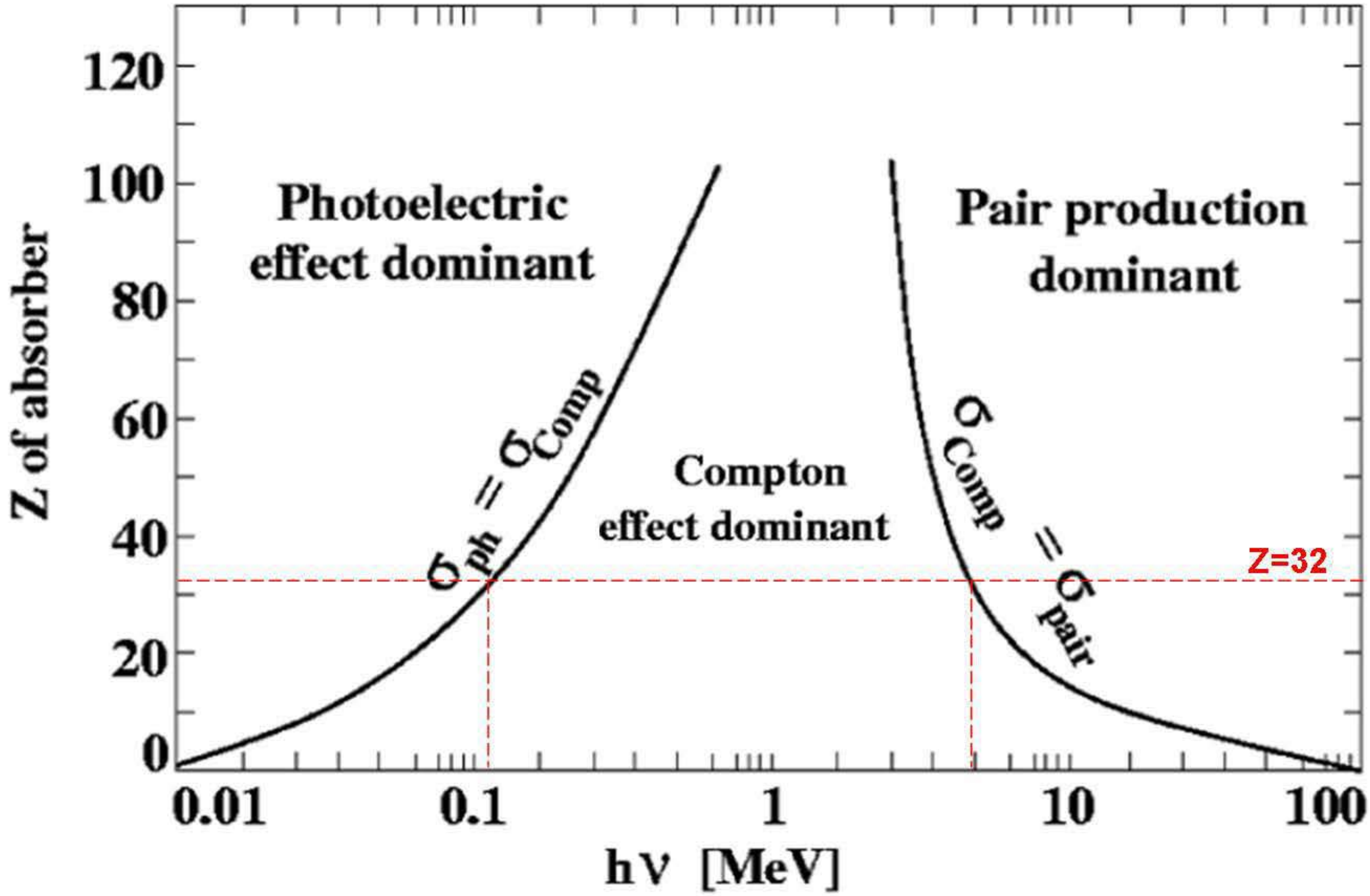
Photo effect: $\sim Z^{4-5}$, $E_\gamma^{-3.5}$
Compton: $\sim Z$, E_γ^{-1}
Pair: $\sim Z^2$, increases with E_γ





The absorption attenuates the intensity, but the energy and the frequency of the γ-ray and X-ray radiation is preserved!

Z dependence of interaction probabilities



Gamma-ray spectrum of a radioactive decay