Interaction of gamma rays with matter

$I(x) = I_0(\lambda) \cdot e^{-\frac{\mu(\lambda, Z)}{\rho} \rho \cdot x}$

total absorption coefficient: $\mu/\rho$ [cm$^2$/g]

$\frac{\mu_{total}}{\rho} = \sum_{i=1}^{3} \sigma_i$

i=1  photoelectric effect
i=2  Compton scattering
i=3  pair production
For X-ray radiation the **photoelectric effect** is the most important interaction.

\[(\mu / \rho)_{\text{photo}} \approx \lambda^3 \cdot Z^5\]

**Lead absorbs more than Beryllium!**

\[{}^{82}\text{Pb}\text{ serves as shielding for X-ray and } \gamma \text{-ray radiation; lead vests are used by medical staff people who are exposed to X-ray radiation. Co-sources are transported in thick lead container.}\]

On the contrary:

\[{}^{4}\text{Be}\text{ is often used as windows in X-ray tubes to allow for almost undisturbed transmission of X-ray radiation.}\]
Mass dependence $\mu/\rho$ of X-ray absorption

Wave length dependence for Pt as absorber

$(Z=78)$

Element number dependence for

$\lambda=0.1 \text{ nm or } 12.4 \text{ keV}$
X-ray image shows the effect of different absorptions

Bones absorb more radiation as tissues because of their higher $^{20}\text{Ca}$ content
Interaction of gamma rays with matter

- Photoelectric
- Isolated hits

<table>
<thead>
<tr>
<th>~ 100 keV</th>
<th>~1 MeV</th>
<th>~ 10 MeV</th>
<th>γ-ray energy</th>
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Probability of interaction depth
Interaction of gamma rays with matter

Photo effect:
Absorption of a photon by a bound electron and conversion of the $\gamma$-energy in potential and kinetical energy of the ejected electron. (Nucleus preserves the momentum conservation.)

\[ E_{e,\text{kin}} = h \cdot \nu - E_{\text{Bindung}} \]
**Compton scattering:**
Elastic scattering of a $\gamma$-ray on a free electron. A fraction of the $\gamma$-ray energy is transferred to the Compton electron. The wavelength of the scattered $\gamma$-ray is increased: $\lambda' > \lambda$.

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\[
\begin{align*}
\text{relativistic} & \quad E^2 = (pc)^2 + (m_0c^2)^2 \\
\text{photons:} & \quad m_0 = m_\gamma = 0 \\
\rightarrow E_\gamma & = p_\gamma c \\
\text{Momentum balance:} & \quad \overrightarrow{p_e} = \overrightarrow{p_\gamma} - \overrightarrow{p'_\gamma} \\
& \quad \rightarrow |p_e c|^2 = \left|\left(\overrightarrow{p_\gamma} - \overrightarrow{p'_\gamma}\right)c\right|^2 \\
& \quad p_e^2 c^2 = E_\gamma^2 + E_{\gamma'}^2 - 2E_\gamma E_{\gamma'} \cdot \cos \theta \\
\text{Energy balance:} & \quad E_\gamma + m_e c^2 = E_{\gamma'} + \sqrt{(p_e c)^2 + (m_e c^2)^2} \\
E_{\gamma'} & = \frac{E_\gamma}{1 + (E_\gamma / m_e c^2)(1 - \cos \theta)}
\end{align*}
\]
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Maximum energy of the scattered electron:

\[
T(e^-)_{\text{max}} = E_\gamma \cdot \frac{2 \cdot E_\gamma}{m_e c^2 + 2 \cdot E_\gamma}
\]

Energy of the scattered γ-photon:

\[
E'_\gamma = \frac{E_\gamma \cdot m_e c^2}{m_e c^2 + E_\gamma \cdot (1 - \cos \theta)}
\]

\[
\cos \theta = 1 + \frac{m_e c^2}{E_\gamma} - \frac{m_e c^2}{E_\gamma}
\]

Special case for \( E >> m_e c^2 \):
γ-ray energy after 180° scatter is approximately

\[
E'_\gamma = \frac{m_e c^2}{2} = 256 \text{ keV}
\]

Gap between the incoming γ-ray and the maximum electron energy.

\[
E_{\text{kin}}^{\text{max}} = E_\gamma - E'_\gamma = E_\gamma \cdot \frac{2 \cdot E_\gamma / m_e c^2}{1 + 2 \cdot E_\gamma / m_e c^2}
\]
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Compton scattering:
Elastic scattering of a $\gamma$-ray on a free electron. The angle dependence is expressed by the **Klein-Nishina-Formula**:

$$\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2} \left( \frac{E_{\gamma'}}{E_\gamma} \right)^2 \cdot \left\{ \frac{E_\gamma}{E_{\gamma'}} + \frac{E_{\gamma'}}{E_\gamma} - 2 \sin^2 \theta \cdot \cos^2 \phi \right\}$$

As shown in the plot **forward scattering** ($\theta$ small) is dominant for $E_\gamma > 100$ keV.

$r_0 = 2.818$ fm (classical electron radius)
**Pair production:**
If γ-ray energy is >> 2m₀c² (electron rest mass 511 keV), a positron-electron pair can be formed in the strong Coulomb field of a nucleus. This pair carries the γ-ray energy minus 2m₀c².

Pair production for E_γ>2mₑc²=1.022MeV

picture of a bubble chamber
γ-rays interaction with matter via three main reaction mechanisms:

- Photoelectric absorption
- Compton scattering
- Pair production
All three interaction (photo effect, Compton scattering and pair production) lead to an attenuation of the $\gamma$-ray or X-ray radiation when passing through matter. The particular contribution depends on the $\gamma$-ray energy:

- **Photo effect:** $\sim Z^{4-5}, E_\gamma^{-3.5}$
- **Compton:** $\sim Z, E_\gamma^{-1}$
- **Pair:** $\sim Z^2$, increases with $E_\gamma$

The absorption attenuates the intensity, but the energy and the frequency of the $\gamma$-ray and X-ray radiation is preserved!
Z dependence of interaction probabilities

- Photoelectric effect dominant
- Compton effect dominant
- Pair production dominant

hν [MeV]

Z of absorber
Gamma-ray spectrum of a radioactive decay

\[ \gamma_1, 0.30 \text{ MeV} \quad \gamma_2, 2.1 \text{ MeV} \]

Stable Daughter, \[^{A(Z-1)}\]

\[ \beta^+, 0.83 \text{ MeV} \]

\[ 511 \text{ keV} \]

\[ 511 \text{ keV} \quad \gamma_1 + \gamma_2 \]

\[ DE\gamma_2, SE\gamma_2, CE\gamma_2 \]

\[ Pb X-ray \]

\[ BSc \]

\[ Pb-box \]