Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

\[- \frac{dE}{dx} = 4 \cdot \pi \cdot r_e^2 \cdot N_a \cdot m_e c^2 \cdot \rho \cdot \frac{Z}{A} \cdot \frac{Z^2}{\beta^2} \cdot \left[ \frac{1}{2} \ln \left( \frac{2 \cdot m_e c^2 \cdot \gamma^2 \cdot \beta^2 \cdot T_{\text{max}}}{l^2} \right) \right] - \beta^2 - \delta - 2 \cdot \frac{C}{Z} \approx \frac{Z^2}{A} \cdot f(\beta, l)\]

\[T_{\text{max}} = 2 \cdot m_e c^2 \cdot \beta^2 \cdot \gamma^2\]

\[- \frac{dE}{dx} = 0.3071 \text{ MeV g}^{-1}\text{cm}^2\]

\[\text{dE/dx vs. E of protons in silicon}\]
Solid angle

Ω = solid angle between source and detector (sr)

For a point source:

\[ \frac{\Omega}{4\pi} = \frac{1}{2} \left( 1 - \frac{d}{\sqrt{d^2 + r^2}} \right) \]

1 Ci = 3.7\cdot10^{10} Bq

<table>
<thead>
<tr>
<th>d (cm)</th>
<th>r = 3 cm</th>
<th>(\Omega/4\pi) [%]</th>
<th>(\Omega/4\pi) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td>7.13</td>
<td>55</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>2.11</td>
<td>2.25</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>0.97</td>
<td>1</td>
</tr>
</tbody>
</table>
The area above the background represents the total counts between the vertical lines $P$ minus the trapezoidal area $B$ (red hatched). If the total counts are $(P+B)$ and the end-points of the horizontal line are $B_1$ and $B_2$ (width of $B_1 + B_2 = \text{width of } B$), then the net area is given by:

$$P = (P + B) - B$$

The *standard deviation of $\Delta P$* is given by:

$$\Delta P = \sqrt{P + 2 \cdot B}$$
Quality of Measurements: Resolution

Resolution generally defined as 1 standard deviation ($1\sigma$) for a Gaussian distribution, or the FWHM ($\Delta z = 2.355 \cdot \sigma_z$)

If the measurement is dominated by Poissonian fluctuations:

- **Fano factor $F$**: fluctuations on $N$ are reduced by correlation in the production of consecutive e-hole pairs. For Germanium detectors $F \sim 0.1$

$$\frac{\sigma_z}{\langle z \rangle} = \sqrt{\frac{F}{N}}$$

- lowest limit for the resolution apart from Fano factor correction
Luminosity

\[ L = N_{\text{projectiles}} \cdot N_{\text{target nuclei}} \]

accelerator current: 1 nA

What is the number of projectiles?

\[
\begin{align*}
1 \text{ particle / s} & \equiv 1.6 \cdot 10^{-19} \text{ C/s} \\
6.25 \cdot 10^9 \text{ particles / s} & \equiv 1 \text{ nA}
\end{align*}
\]

\(^{28}\text{Si} \) target thickness: 1 mg/cm\(^2\)

How many target nuclei?

\[
\begin{align*}
28 \text{ g/cm}^2 \text{ Silicon} & \equiv 6.02 \cdot 10^{23} \text{ atoms/cm}^2 \\
1 \text{ mg/cm}^2 \text{ Silicon} & \equiv 2.15 \cdot 10^{19} \text{ atoms/cm}^2
\end{align*}
\]

Luminosity = \(6.25 \cdot 10^9 \cdot 2.15 \cdot 10^{19} = 1.34 \cdot 10^{29} \text{ [s}^{-1} \text{ cm}^{-2}]\)

event rate [s\(^{-1}\)] = luminosity [s\(^{-1} \text{ cm}^{-2}\)] \cdot cross section [cm\(^2\)]

\[
= 1.34 \cdot 10^{29} \text{ [s}^{-1} \text{ cm}^{-2}] \cdot \text{cross section [\sim mb = 10}^{-27} \text{ cm}^2\] \approx 10^2 \text{ [s}^{-1}\]