Outline: Interacting Boson Model

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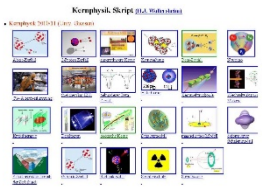
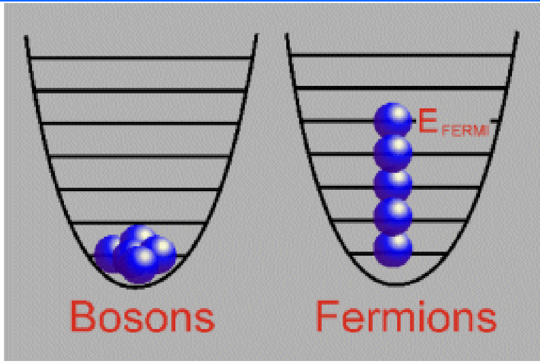


1. particle classification
2. parameters in IBM Hamiltonian
3. vibrational limit U(5)
4. rotational limit SU(3)
5. extension of IBM

Particle classification



➢ Fermions:

- Spin 1/2 particles
- Obey Pauli Exclusion Principle
  - No more than one fermion can occupy the same quantum state
- Antisymmetric wave functions
- Two classifications
  - Hadrons - interact via strong nuclear force; composed of quarks; neutrons, protons, etc.
  - Leptons - fundamental particles, ie no substructure; electron, muon, tau, neutrino.

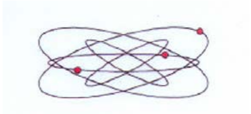
➢ Bosons:

- Integer spins
- Obey Bose-Einstein Statistics
  - Any number of bosons can share the same quantum state
- Symmetric wave functions
- Force mediators- Photons, gluons, etc.

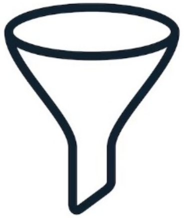
Modeling a nucleus

$^{162}_{66}Dy_{96}$ ➡ shell model ➡

~ 7·10^{19} 2$^+$ states



one needs a truncation



Interacting Boson Model

assumptions:

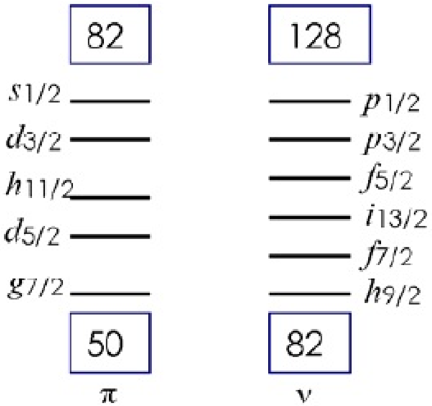
1. only valence nucleons
2. Fermions → Bosons

J = 0 (s bosons)

J = 2 (d bosons)

30 basic states 2$^+$ states

$^{132}_{50}Sn_{82}$ inert core



Dimensions

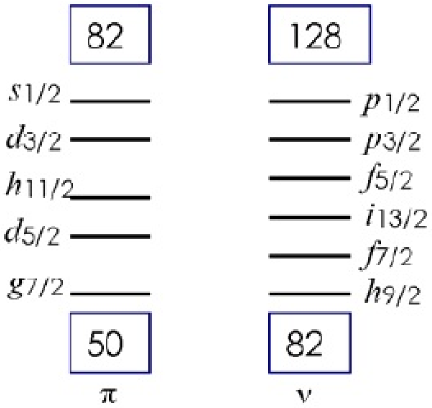
- Assume Ω available 1-fermion states. Number of *n*-fermion states is

$$\binom{\Omega}{n} = \frac{\Omega!}{n! \cdot (\Omega - n)!}$$

- Assume Ω available 1-boson states. Number of *n*-boson states is

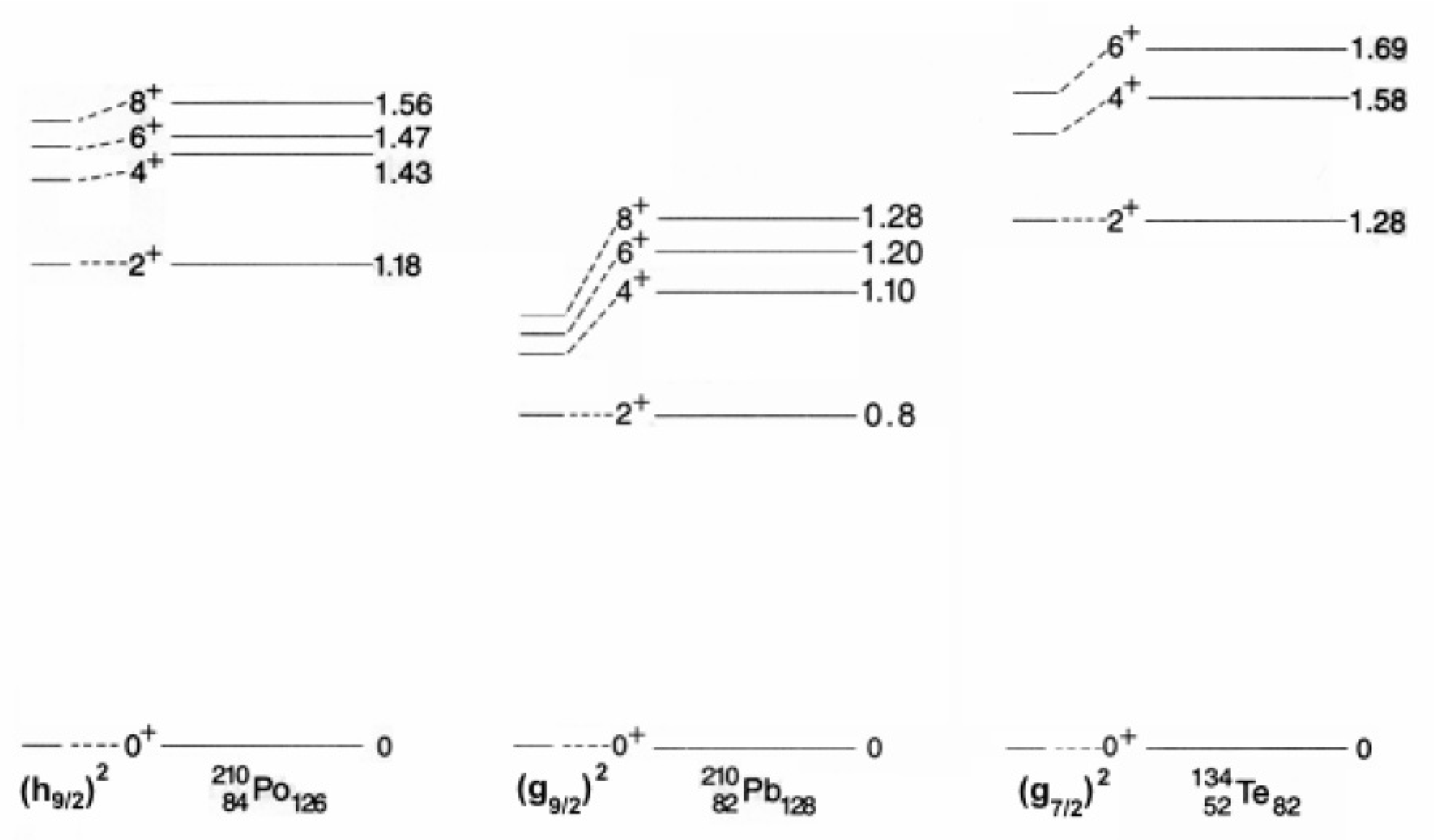
$$\binom{\Omega + n - 1}{n} = \frac{(\Omega + n - 1)!}{n! \cdot (\Omega - 1)!}$$

- Example: $^{162}_{66}Dy_{96}$ with 14 neutrons (Ω = 44) and 16 protons (Ω = 32) ($^{132}Sn_{82}$ inert core).
  - SM dimension: ~$7 \cdot 10^{19}$
  - IBM dimension: 15504



Why s- and d-bosons?

- s-boson lowest state of all even-even nuclei is 0^+.
  δ-force gives 0^+ ground state
- d-boson first excited state in non-magic even-even nuclei almost always 2^+
  δ-force gives 2^+ (occupation μ=-2,-1,0,1,2) next above 0^+



Interacting boson model

- Describe the nucleus as a system of *N* interacting *s* and *d* bosons.
  Hamiltonian:

$$\hat{H}_{\mathrm{IBM}} = \sum_{i=1}^{6} \varepsilon_i \hat{b}_i^+ \hat{b}_i + \sum_{i_1 i_2 i_3 i_4 = 1}^{6} \upsilon_{i_1 i_2 i_3 i_4} \hat{b}_{i_1}^+ \hat{b}_{i_2}^+ \hat{b}_{i_3} \hat{b}_{i_4} + \cdots$$

Since boson number is conserved for a given nucleus, H can only contain “bilinear” terms: 36 of them.

$s^+s, s^+d_m, d_m^+s, d_m^+d_{m'}$ ➡ group theoretical classification of Hamiltonian   U(6)

- Justification from
  - Shell model: *s* and *d* bosons are associated with *S* and *D* fermion (*Cooper*) pairs.
  - Geometric model: for large boson number the IBM reduces to a liquid-drop Hamiltonian.

- 36 Operators of IBM all conserve total boson number

$$N = s^{+}s + d^{+}\tilde{d}$$
$$= n_s + n_d$$

N is an example of a **Casimir** Operator – Operator that commutes with all the generators of a group (Lie algebra: $[a, b] = ab - ba$).

e.g:

$$[N, s^{+}\tilde{d}]\Psi = [N(s^{+}\tilde{d}) - s^{+}\tilde{d}N]\Psi$$
$$= Ns^{+}\tilde{d}\Psi - s^{+}\tilde{d}N\Psi$$
$$= Ns^{+}\tilde{d}\Psi - Ns^{+}\tilde{d}\Psi = 0$$

Set of states of the IBA with given *N* are an irreducible representation of the group U(6) spanned by these 36 operators

Concept of group theory

For IBA, the 36 operators s^+s, d^+s, s^+d, d^+d are generators of the group U(6)

$$[d^+s, s^+s]|n_d n_s > = (d^+ss^+s - s^+sd^+s)|n_d n_s >$$

$$= d^+sn_s|n_d n_s > - s^+sd^+s|n_d n_s >$$

$$= (n_s - s^+s)d^+s|n_d n_s >$$

Subsets of generators that commute among themselves.

e.g: d^+d 25 generators—span U(5)

They conserve n_d (# d bosons)

Set of states with same n_d are the representations of the group U(5)

General IBM Hamiltonian

- Most general rotationally invariant IBM Hamiltonian:

$$\hat{H}_{\mathrm{IBM}} = E_0 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \cdots$$

$$\hat{H}_1 = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d$$

$$\hat{H}_2 = \sum_{l_1 l_2 l_1' l_2', L} \tilde{\upsilon}^{L}_{l_1 l_2 l_1' l_2'} \left(b^+_{l_1} \times b^+_{l_2} \right)^{(L)} \cdot \left(\tilde{b}_{l_1'} \times \tilde{b}_{l_2'} \right)^{(L)}$$

$$\hat{H}_3 = \sum_{l_1 l_2 l_3 l_1' l_2' l_3', L} \tilde{\upsilon}^{L}_{l_1 l_2 l_3 l_1' l_2' l_3'} \left(b^+_{l_1} \times b^+_{l_2} \times b^+_{l_3} \right)^{(L)} \cdot \left(\tilde{b}_{l_1'} \times \tilde{b}_{l_2'} \times \tilde{b}_{l_3'} \right)^{(L)}$$

Parameters in the IBM Hamiltonian

- Spectrum of a single nucleus: 0+1+5+10 parameters.
- Overall binding energy: 1+1+2+7 parameters.
- ⇒ A total of 27 parameters if all interactions up to three-body are included (cfr. 63 2-body *sd*-shell model matrix elements).

Order	Number of interactions		
	total	type I[a]	type II[b]
$n = 0$	1	1	0
$n = 1$	2	1	1
$n = 2$	7	2	5
$n = 3$	17	7	10

[a] Interaction energy is constant for all states with the same N.
[b] Interaction energy varies from state to state.

Classical limit

- Coherent state:

$$|N;\beta,\gamma\rangle \propto \left[s^{+} + \beta\cos\gamma\, d_{0}^{+} + \sqrt{\tfrac{1}{2}}\beta\sin\gamma\left(d_{-2}^{+} + d_{+2}^{+}\right)\right]^{N}|o\rangle$$

- Generic form of the potential:

$$V(\beta,\gamma) \equiv \langle N;\beta,\gamma| E_{0} + \hat{H}_{1} + \hat{H}_{2} + \hat{H}_{3} + \cdots |N;\beta,\gamma\rangle$$

$$= E_{0} + \sum_{n\geq 1}\frac{N(N-1)\cdots(N-n+1)}{\left(1+\beta^{2}\right)^{n}}\sum_{kl} a_{kl}^{(n)}\beta^{2k+3l}\cos^{l}3\gamma$$

β is elongation of nuclear ellipsoidal shape – related to ratio of major and minor axes.

γ is the deviation from axial symmetry along the major axis. It is measured as an angle from 0 to 30 degrees.

Coefficients up to third order

$$a_{00}^{(1)} = \varepsilon_s, \quad a_{10}^{(1)} = \varepsilon_d,$$

$$a_{00}^{(2)} = \tfrac{1}{2} v_{ssss}^{0}, \quad a_{10}^{(2)} = \sqrt{\tfrac{1}{5}} v_{ssdd}^{0} + v_{sdsd}^{2}, \quad a_{01}^{(2)} = -\sqrt{\tfrac{2}{7}} v_{sddd}^{2},$$

$$a_{20}^{(2)} = \tfrac{1}{10} v_{dddd}^{0} + \tfrac{1}{7} v_{dddd}^{2} + \tfrac{9}{35} v_{dddd}^{4},$$

$$a_{00}^{(3)} = \tfrac{1}{6} v_{ssssss}^{0}, \quad a_{10}^{(3)} = \sqrt{\tfrac{1}{15}} v_{ssssdd}^{0} + \tfrac{1}{2} v_{ssdssd}^{2},$$

$$a_{01}^{(3)} = -\tfrac{1}{3}\sqrt{\tfrac{2}{35}} v_{sssddd}^{0} - \sqrt{\tfrac{2}{7}} v_{ssdsdd}^{2},$$

$$a_{20}^{(3)} = \tfrac{1}{10} v_{sddsdd}^{0} + \sqrt{\tfrac{1}{7}} v_{ssdddd}^{2} + \tfrac{1}{7} v_{sddsdd}^{2} + \tfrac{9}{35} v_{sddsdd}^{4},$$

$$a_{11}^{(3)} = -\tfrac{1}{5}\sqrt{\tfrac{2}{21}} v_{sddddd}^{0} - \tfrac{\sqrt{2}}{7} v_{sddddd}^{2} - \tfrac{18}{35}\sqrt{\tfrac{2}{11}} v_{sddddd}^{4},$$

$$a_{30}^{(3)} = \tfrac{1}{14} v_{dddddd}^{2} + \tfrac{1}{30} v_{dddddd}^{3} + \tfrac{3}{154} v_{dddddd}^{4} + \tfrac{7}{165} v_{dddddd}^{6},$$

$$a_{02}^{(3)} = \tfrac{1}{105} v_{dddddd}^{0} - \tfrac{1}{30} v_{dddddd}^{3} + \tfrac{3}{110} v_{dddddd}^{4} - \tfrac{4}{1155} v_{dddddd}^{6}.$$

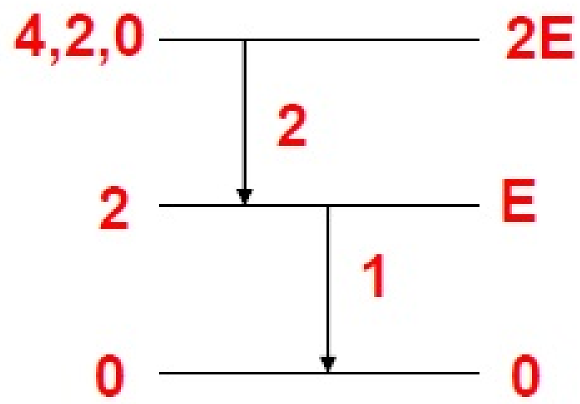
Review of phonon creation and annihilation operators

$$\mathbf{b}|n_b\rangle = \sqrt{n_b}\,|n_b - 1\rangle$$

$$\mathbf{b}^\dagger|n_b\rangle = \sqrt{(n_b + 1)}\,|n_b + 1\rangle$$

Why are creation and annihilation operators useful?

Example: Consider a vibrational nucleus with quadrupole phonon excitations, with angular momentum 2. These are vibrational excitations with energy E.



Therefore, the lowest energy state will have N=0 of these vibrations. The lowest excited state will have one of them and excitation energy 1 E.

$$B(E2;4 \rightarrow 2) = 2 \cdot B(E2;2 \rightarrow 0)$$

The solvable IBM Hamiltonians Dynamical symmetries

- U(5) Vibrator
- SU(3) Axial Rotator
- O(6) Axially asymmetric Rotor (“gamma-soft”)

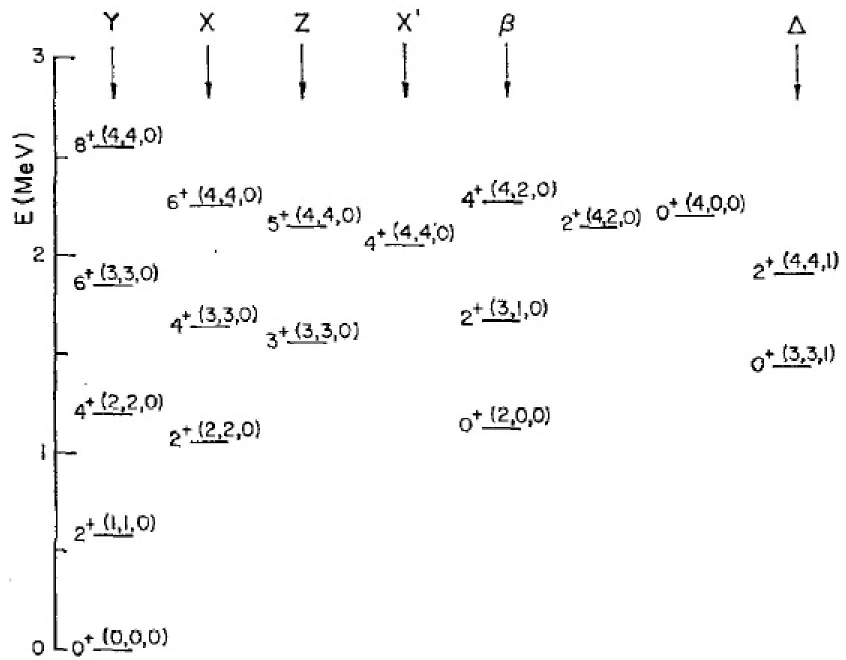
Application to U(5)-like nuclei 4 + 4 parameters

$$E(n_d, v, n_\Delta, L, M) = \epsilon \cdot n_d + \frac{\alpha}{2} \cdot n_d(n_d - 1) + \beta(n_d - v)(n_d + v + 3) + \gamma \cdot [L(L+1) - 6n_d]$$

eigenvalues do not depend on n_Δ and M.

The parameters α, β, γ can be related to c_0, c_2, c_4 by considering states wit $n_d = 2$.

$\alpha = (1/14)(6c_4 + 8c_2)$ $\beta = (3/70)c_4 - (1/7)c_2 + (1/10)c_0$ $\gamma = (1/14)(c_4 - c_2)$



A typical spectrum in the *d*-boson limit. The numbers in parenthesis are the *SU*(5) quantum numbers (n_d, v, n_Δ). The angular momentum quantum number is explicitly written to the left of each level. The parameters used are ϵ = 579 keV, c_4 = 39.4 keV, c_2 = −95.3 keV, c_0 = −27.4 keV. $v = n_d - 2n_\beta$ boson seniority number

F.Iachello, A. Arima ; *Ann. Phys. 99 (1976) 253*

Application to U(5)-like nuclei 4 + parameters

B(E2) branching ratios in the Xe-isotopes

	$^{132}_{54}Xe_{78}$	$^{130}_{54}Xe_{76}$	$^{128}_{54}Xe_{74}$	$^{126}_{54}Xe_{72}$	$^{124}_{54}Xe_{70}$	d-boson limit
$\frac{2_2^+ \to 0_1^+}{2_2^+ \to 2_1^+} (\times 10^{-2})$	0.17	0.57—0.64	1.01	1.48—1.99	1.86 — 3.89	0
$\frac{3_1^+ \to 2_1^+}{3_1^+ \to 2_2^+} (\times 10^{-2})$	1.03	1.41—1.54	—	1.99	2.99	0
$\frac{3_1^+ \to 4_1^+}{3_1^+ \to 2_2^+}$	0.51	0.24—0.25	—	0.46—0.72	0.16	$\frac{2}{5} = 0.400$
$\frac{4_2^+ \to 2_1^+}{4_2^+ \to 2_2^+} (\times 10^{-2})$	—	3.11—3.42	—	1.28	—	0
$\frac{4_2^+ \to 4_1^+}{4_2^+ \to 2_2^+}$	—	0.95—1.05	—	0.94	0.90	$\frac{10}{11} = 0.909$
$\frac{5_1^+ \to 4_1^+}{5_1^+ \to 3_1^+} (\times 10^{-2})$	—	3.38	—	4.97	3.90	0
$\frac{5_1^+ \to 4_2^+}{5_1^+ \to 3_1^+}$	—	0.46	—	1.26	0.98	$\frac{5}{11} = 0.454$
$\frac{5_1^+ \to 6_1^+}{5_1^+ \to 3_1^+}$	—	0.64	—	—	—	$\frac{104}{231} = 0.450$

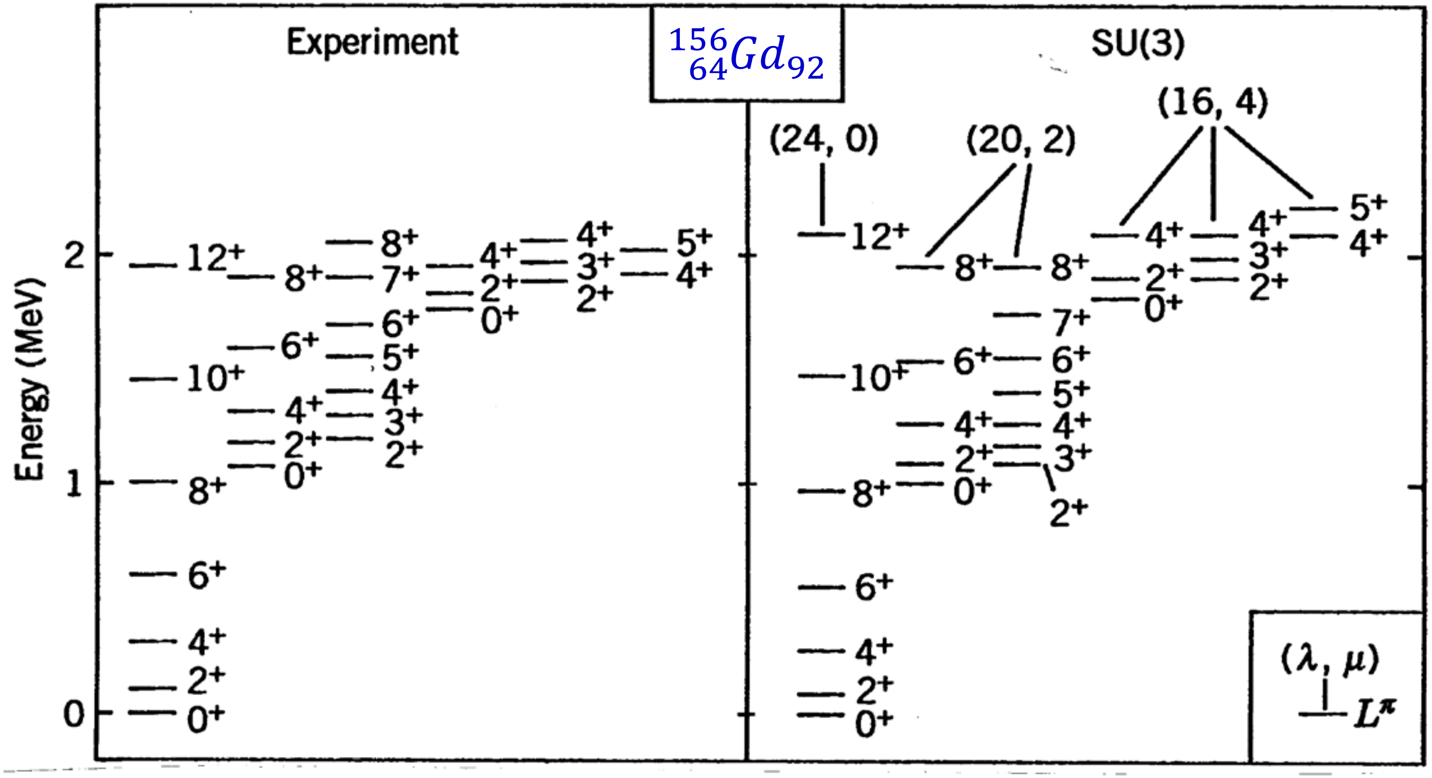
F.Iachello, A. Arima ; *Ann. Phys. 99 (1976) 253*

Application to SU(3)-like nuclei 2 + parameters

$$E([N](\lambda,\mu)KLM) = \alpha \cdot L(L+1) - \beta \cdot C(\lambda,\mu) \text{ with } C(\lambda,\mu) = \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu)$$

eigenvalues do not depend on K and M. C(λ,μ) is the quadratic Casimir operator.

The parameters α, β can be related to κ and κ´: $\alpha = \frac{3}{4}\kappa - \kappa'$ and $\beta = \kappa$



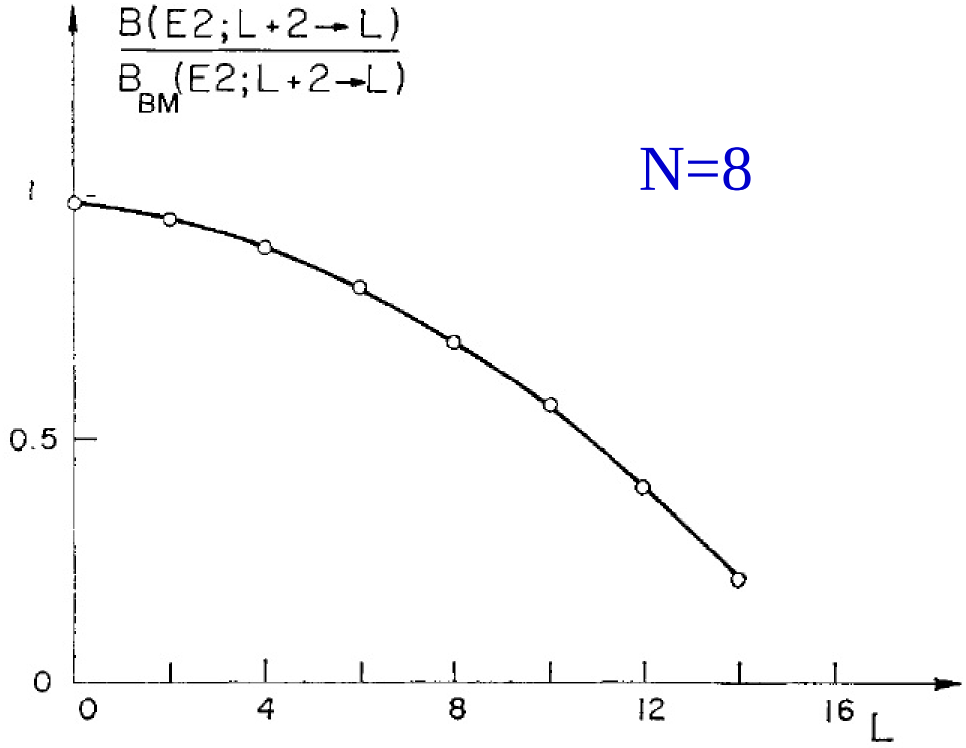
The energy difference between the lowest (2N,0) and the next lowest (2N-4,2) representation is in units of κ, 12N-6

F.Iachello, A. Arima ; *Ann. Phys. 111 (1978) 201*

Application to SU(3)-like nuclei 2 + 3 parameters

$$B(E2;\, L+2 \rightarrow L) = \alpha_2{}^2 \, \frac{3}{4} \, \frac{(L+2)(L+1)}{(2L+3)(2L+5)} \, (2N-L)(2N+L+3)$$

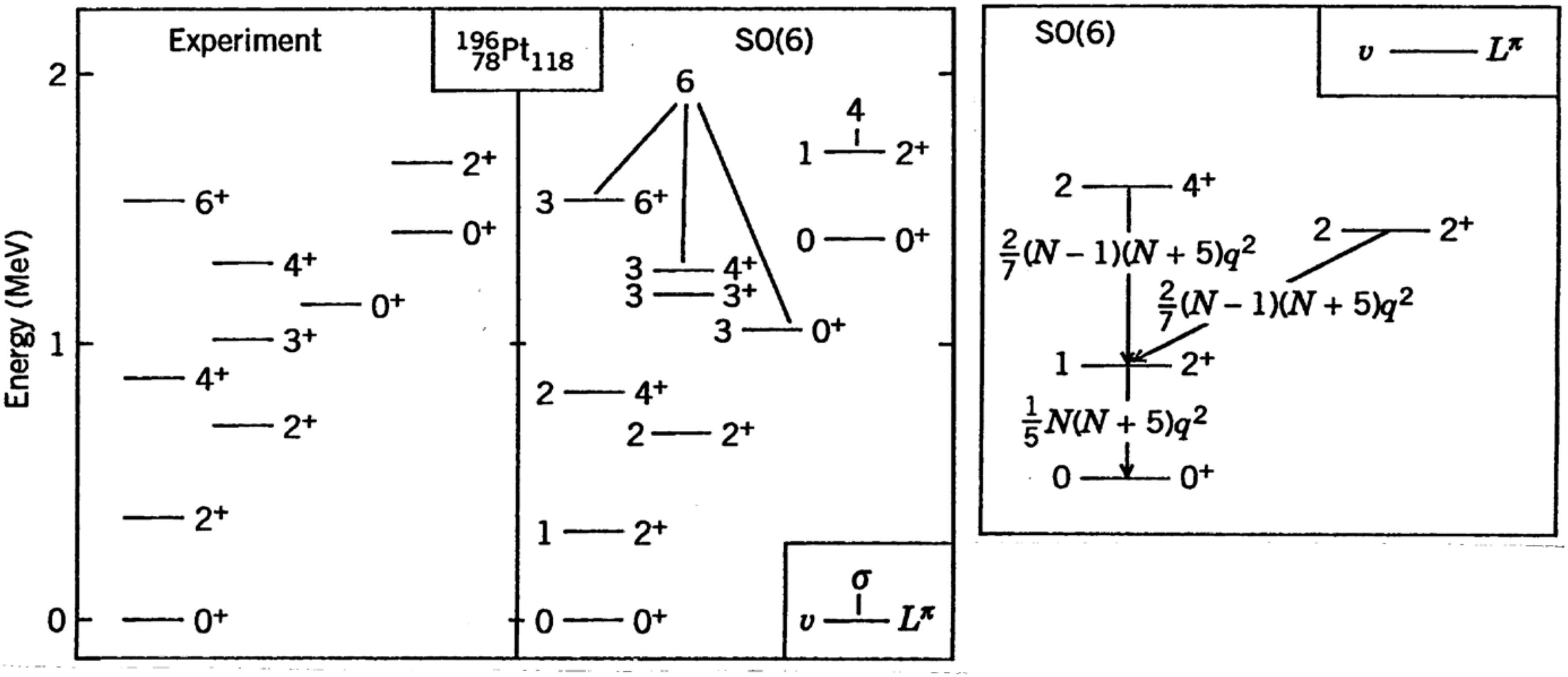
$$B(E2;\, L+2 \rightarrow L) = B_{BM}(E2;\, L+2 \rightarrow L) \, \frac{(2N-L)(2N+L+3)}{(2N+\frac{3}{2})^2}$$



F.Iachello, A. Arima ; *Ann. Phys. 111 (1978) 201*

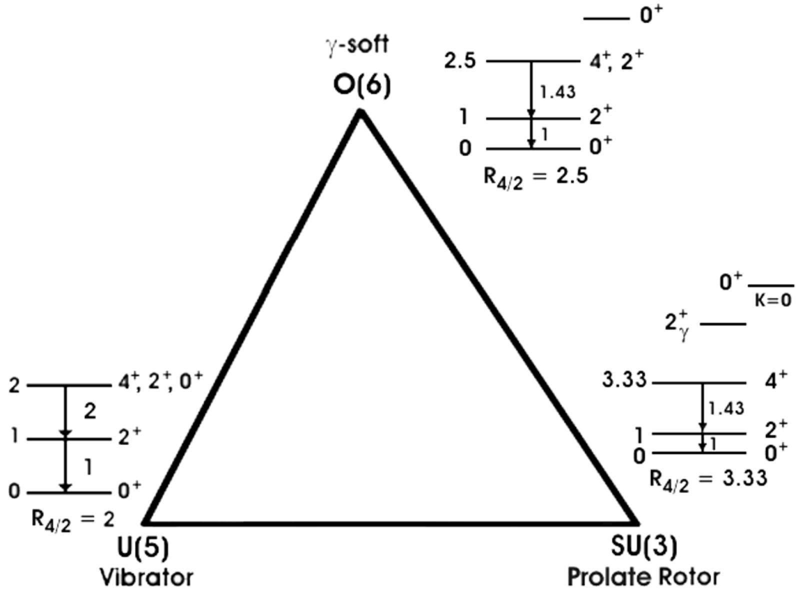
Application to SO(6) γ-unstable nuclei 2 + parameters

- Rotation-vibration spectrum of a γ-unstable body.
- Conserved quantum numbers: *σ, υ, L*.

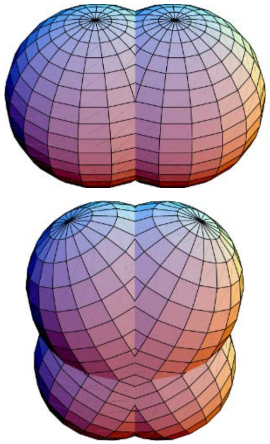


F.Iachello, A. Arima ; *Ann. Phys. 123 (1979) 468*

- Symmetry triangle of the IBM:
  - Three standard solutions: U(5), SU(3), SO(6).
  - SU(1,1) analytic solution for U(5) → SO(6).
  - Hidden symmetries (parameter transformations).
  - Deformed-spherical coexistent phase.
  - Partial dynamical symmetries.
  - Critical-point symmetries?

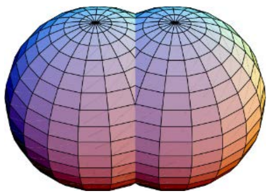


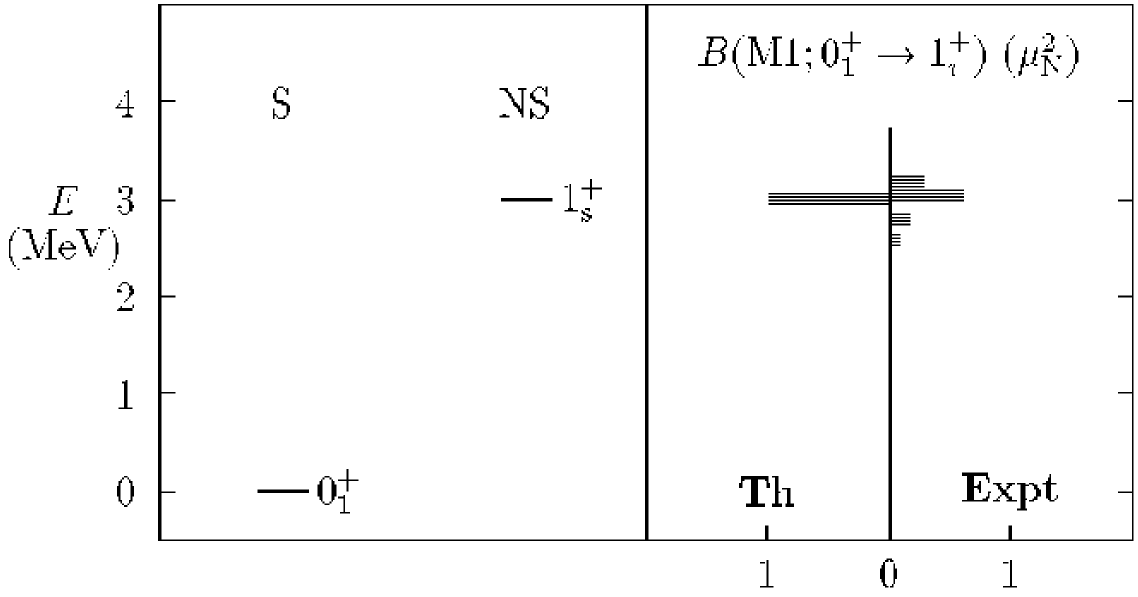
Extension of IBM



- Neutron and proton degrees freedom (IBM-2):
  - *F*-spin multiplets ($N_\nu + N_\pi$=constant)
  - Scissors excitations
- Fermion degrees of freedom (IBFM):
  - Odd-mass nuclei
  - Supersymmetry (doublets & quartets) $^{194}_{78}Pt_{116}$ $^{195}_{78}Pt_{117}$ $^{195}_{79}Au_{116}$ $^{196}_{79}Au_{117}$
- Other boson degrees of freedom:
  - Isospin *T*=0 & *T*=1 pairs (IBM-3 & IBM-4)
  - Higher multipole (g,…) pairs

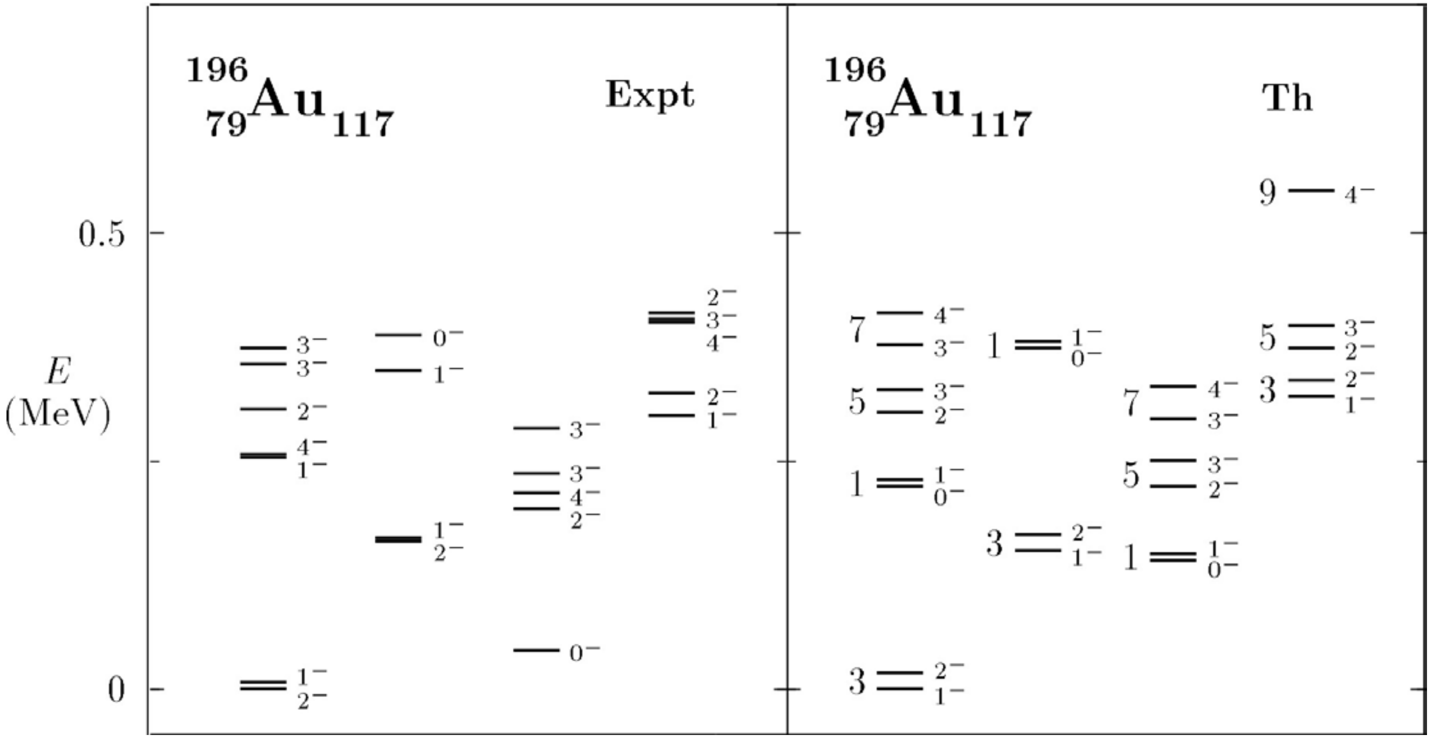
- Collective displacement modes between neutrons and protons:
  - *Linear* displacement (giant dipole resonance): $R_\nu - R_\pi \Rightarrow E1\ excitation.$
  - *Angular* displacement (scissor resonance): $L_\nu - L_\pi \Rightarrow M1\ excitation$





Supersymmetry

- A simultaneous description of even- and odd-mass nuclei (*doublets*) or of even-even, even-odd, odd-even and odd-odd nuclei (*quartets*).
- Example of ${}^{194}_{78}Pt_{116}$, ${}^{195}_{78}Pt_{117}$, ${}^{195}_{79}Au_{116}$ & ${}^{196}_{79}Au_{117}$



Comments on the use of NNDC

- Nuclear spectra:
  - Systematic use of band-plotting option in NuDat
    ⇒ important for identification of collective bands.
  - Occasional problems:
    - Gamma-band structure in ^{160}Dy and ^{170}Yb??
    - Gamma band cut in two in ^{180}W??
    - 2^+ @ 691 keV in ^{174}Os should be in beta band??
    - Beta band cut in two in ^{180}Os??
  - In general: confusion concerning beta band in deformed nuclei.

    What are the correct criteria to label a $K^\pi = 0^+$ band as collective beta band?