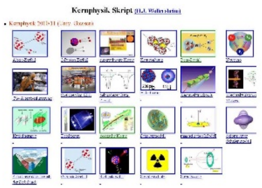
Outline: 2-band mixing

Lecturer: Hans-Jürgen Wollersheim

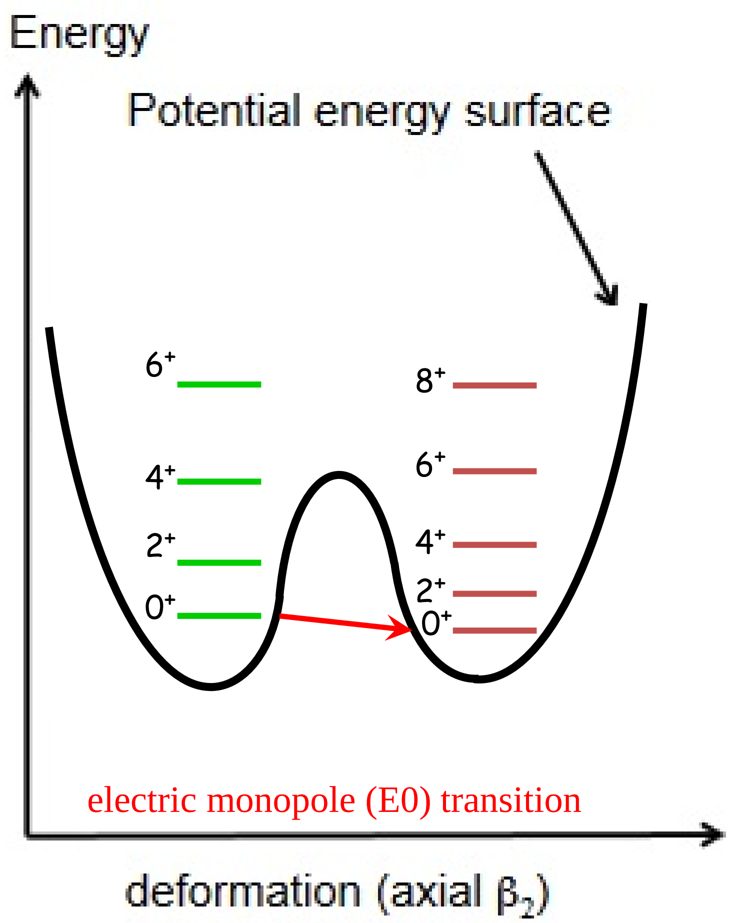
e-mail: h.j.wollersheim@gsi.de

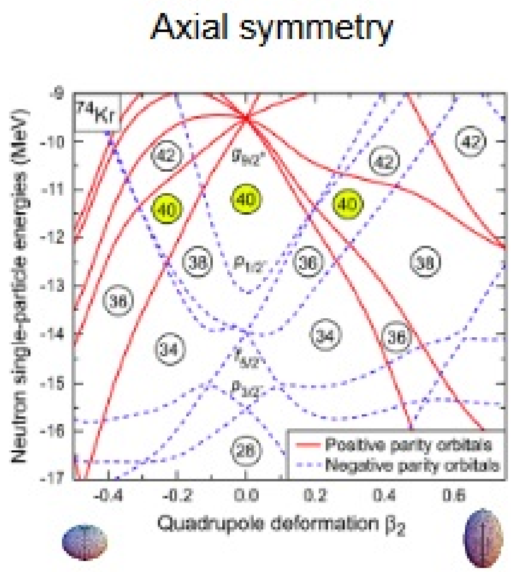
web-page: https://web-docs.gsi.de/~wolle/ and click on

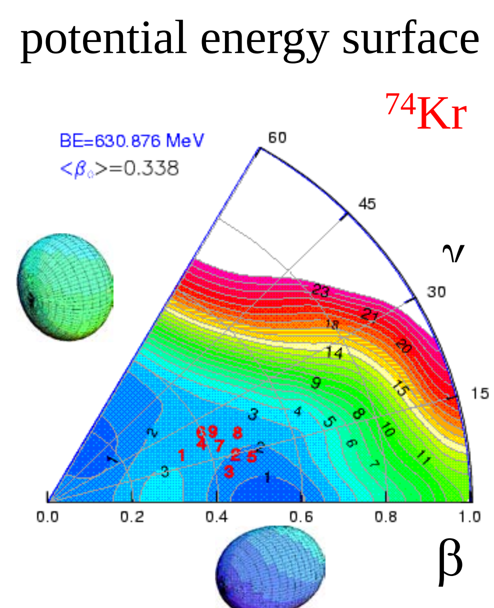


1. shape coexistence of low-lying 0^+ states in Kr isotopes
2. band mixing between the ground and γ-band in ^{164}Dy
3. Coulomb excitation of the 8^- isomer in ^{178}Hf

1. Shape coexistence in light Kr isotopes





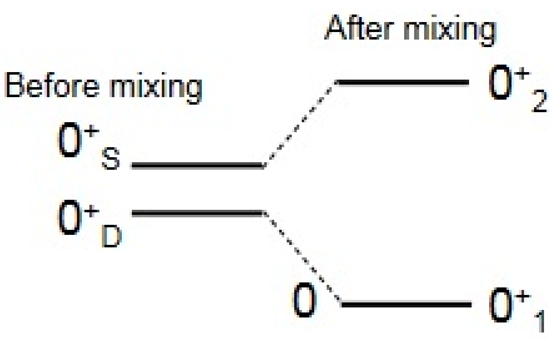


M. Girod

2-level mixing model

$$|0_1^+\rangle = \cos\theta_0|0_D^+\rangle + \sin\theta_0|0_S^+\rangle$$
$$|0_2^+\rangle = -\sin\theta_0|0_D^+\rangle + \cos\theta_0|0_S^+\rangle$$

$$\cos^2\theta_0 + \sin^2\theta_0 = 1$$



Maximum mixing $\cos^2\theta = \sin^2\theta = 0.5$

Weak mixing $\cos^2\theta \to 1 \quad \sin^2\theta \to 0$

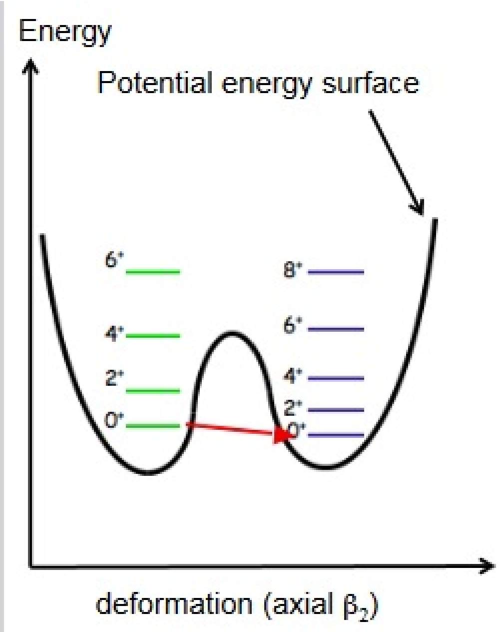
$$|2_1^+\rangle = \cos\theta_2|2_D^+\rangle + \sin\theta_2|2_S^+\rangle$$
$$|2_2^+\rangle = -\sin\theta_2|2_D^+\rangle + \cos\theta_2|2_S^+\rangle$$

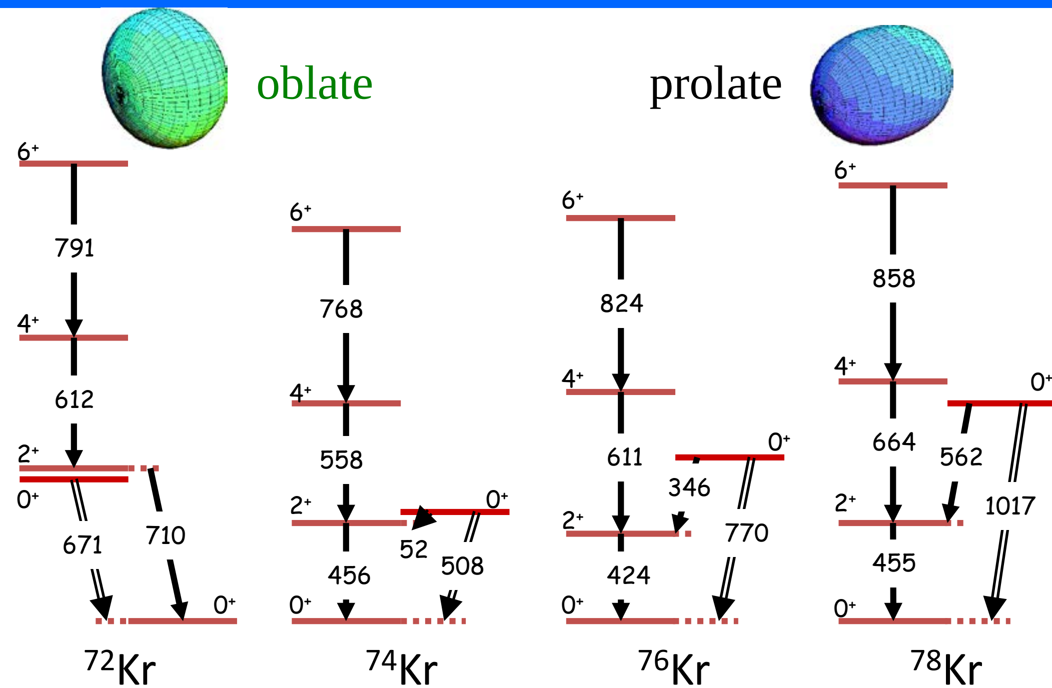
$$B(E2;0_1^+ \to 2_1^+) = \left|\cos\theta_0\cos\theta_2\langle 0_D^+|M(E2)|2_D^+\rangle + \sin\theta_0\sin\theta_2\langle 0_S^+|M(E2)|2_S^+\rangle\right|^2$$

$$B(E2;0_2^+ \to 2_1^+) = \left|-\sin\theta_0\cos\theta_2\langle 0_D^+|M(E2)|2_D^+\rangle + \cos\theta_0\sin\theta_2\langle 0_S^+|M(E2)|2_S^+\rangle\right|^2$$

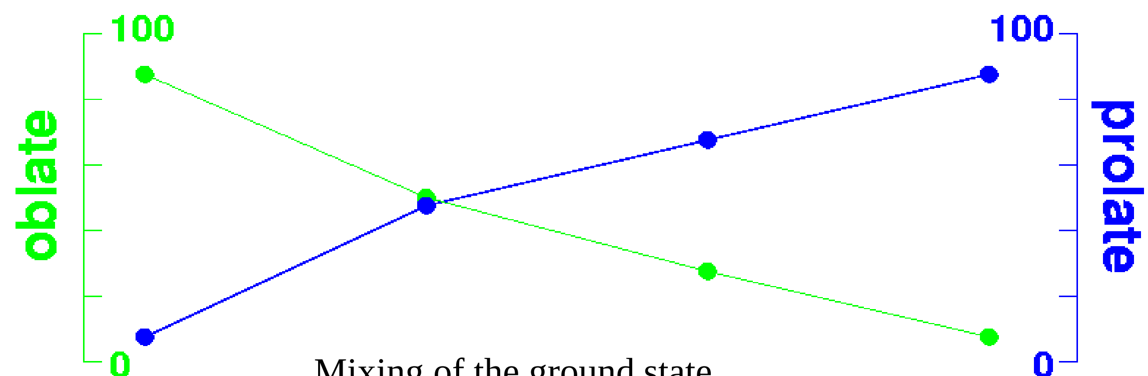
$$\langle 0_2^+\|M(E0)\|0_1^+\rangle \propto sin\theta_0 cos\theta_0\left(\beta_{pro}^2 - \beta_{obl}^2\right)$$

Systematics of the light Kr isotopes





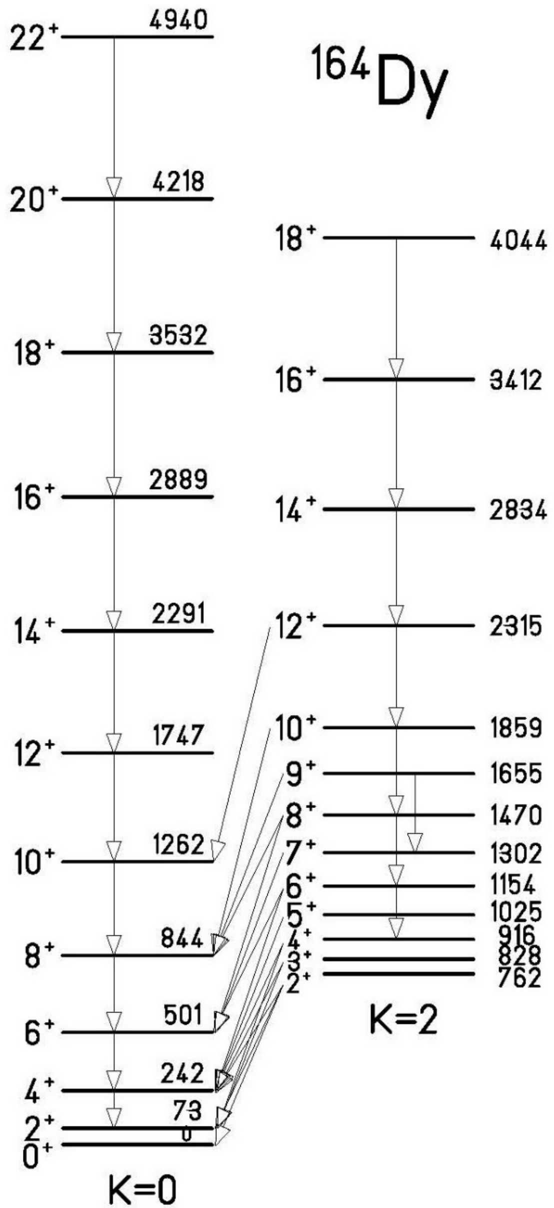
$\rho^2(E0)$ $72 \cdot 10^{-3}$ $85 \cdot 10^{-3}$ $79 \cdot 10^{-3}$ $47 \cdot 10^{-3}$



Mixing of the ground state
(two-level mixing extrapolated
from distortion of rotational bands)

E. Bouchez et al., Phys. Rev. Lett. 90, 082502 (2003)

2. Band mixing between the ground and γ-band in ^{164}Dy



$$\left|IMKn_2 n_0\right\rangle = \sqrt{\frac{2I+1}{16\pi^2}\,\frac{1}{1+\delta_{K,0}}}\left(D^I_{MK} + (-)^I D^I_{M-K}\right)\left|Kn_2 n_0\right\rangle$$

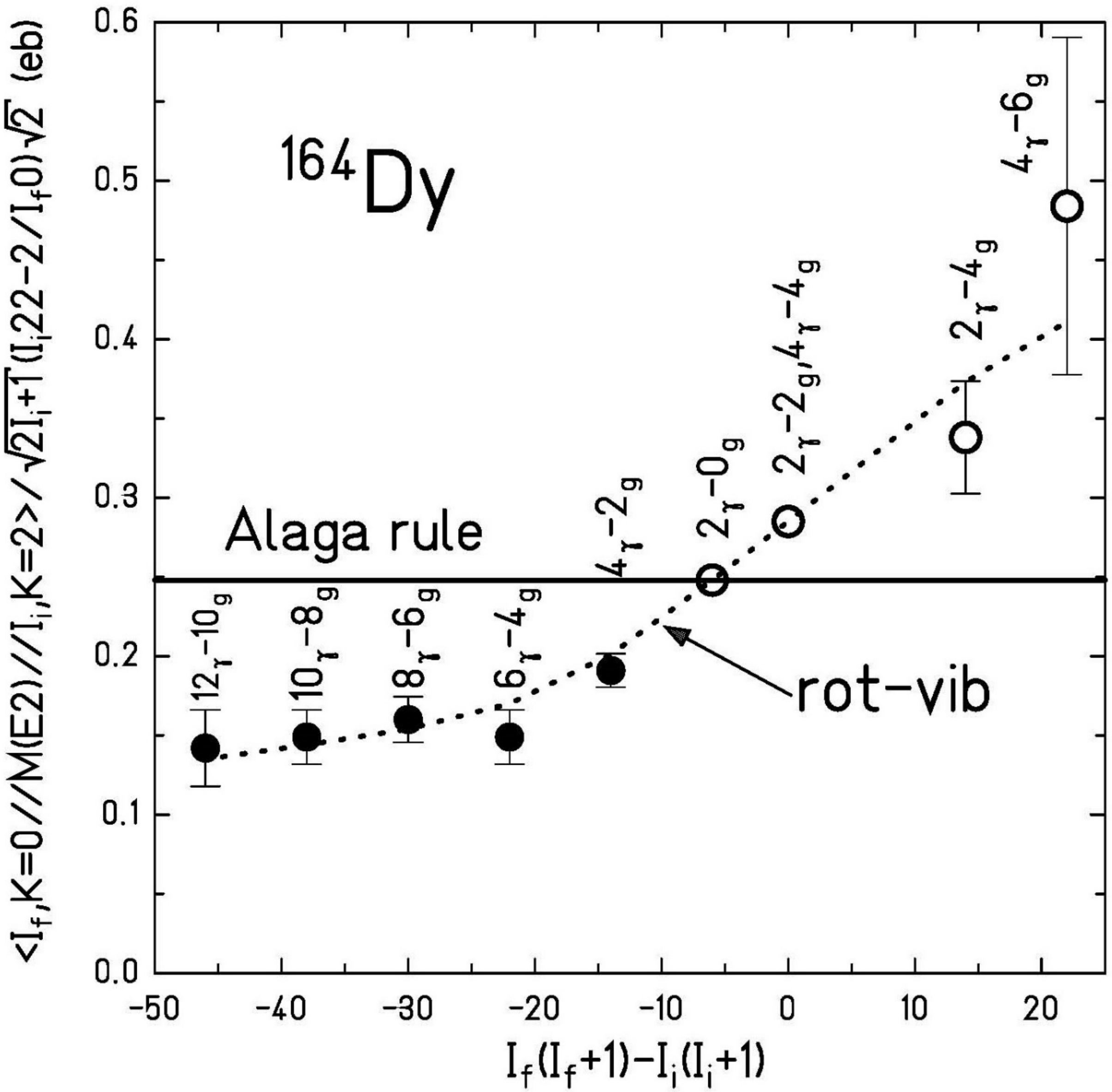
Band mixing between the ground and γ-band in ^{164}Dy

$$|IMKn_2n_0\rangle = \sqrt{\frac{2I+1}{16\pi^2}\,\frac{1}{1+\delta_{K,0}}}\left(D^I_{MK} + (-)^I\, D^I_{M-K}\right)|Kn_2n_0\rangle$$

$$|\widetilde{IMKn_2n_0}\rangle = A_1(I)\,|IM000\rangle + A_2(I)\,|IM200\rangle + A_3(I)\,|IM001\rangle$$

$$\begin{aligned}
\langle f\|M(E2)\|i\rangle = \sqrt{2I_i+1}\,\{ & A_1(I_f)\;A_1(I_i)\;(I_i200/I_f0)\,\langle 000/M(E2,0)/000\rangle \\
+ & A_1(I_f)\;A_2(I_i)\;(I_i22{-}2/I_f0)\sqrt{2}\;\langle 000/M(E2,-2)/200\rangle \\
+ & A_1(I_f)\;A_3(I_i)\;(I_i200/I_f0)\,\langle 000/M(E2,0)/001\rangle \\
+ & A_2(I_f)\;A_1(I_i)\;(I_i202/I_f2)\sqrt{2}\;\langle 200/M(E2,2)/000\rangle \\
+ & A_2(I_f)\;A_2(I_i)\;(I_i220/I_f2)\,\langle 200/M(E2,0)/200\rangle \\
+ & A_3(I_f)\;A_1(I_i)\;(I_i200/I_f0)\,\langle 001/M(E2,0)/000\rangle \\
+ & A_3(I_f)\;A_3(I_i)\;(I_i200/I_f0)\,\langle 001/M(E2,0)/001\rangle \}
\end{aligned}$$

Band mixing between the ground and γ-band in ^{164}Dy



Rotation Vibration Model

$$A_1(I) = 1$$

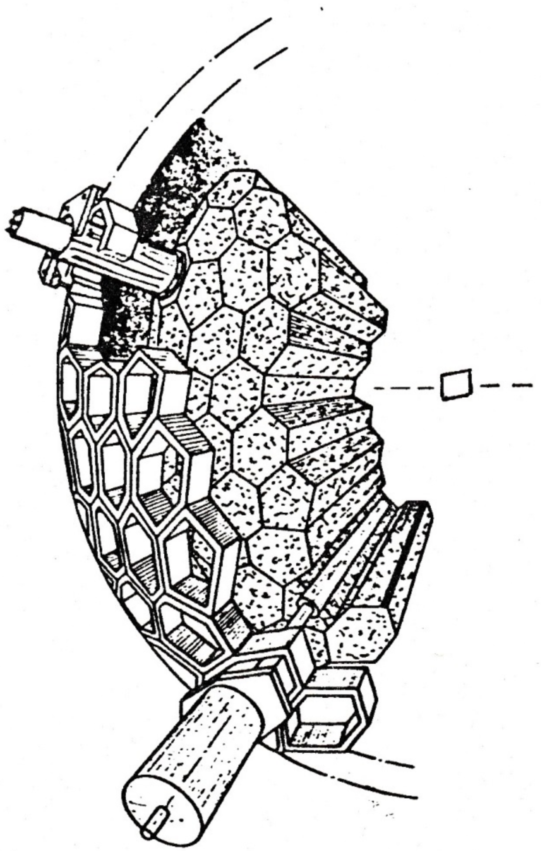
$$A_2(I) = \frac{\varepsilon}{E_\gamma - 2\varepsilon}\sqrt{\frac{\varepsilon}{E_\gamma}}\,(1/2\sqrt{2})\,\sqrt{(I+2)(I+1)I(I-1)}$$

$$A_3(I) = \frac{\varepsilon}{E_\beta}\sqrt{\frac{3\varepsilon}{2E_\beta}}\; I(I+1)$$

$$\alpha_0 = \frac{\varepsilon}{E_\beta}\sqrt{\frac{6\varepsilon}{E_\beta}}\sqrt{\frac{B(E2;0_g\rightarrow 2_\beta)}{B(E2;0_g\rightarrow 2_g)}} \qquad \alpha_2 = \frac{\varepsilon}{E_\gamma - 2\varepsilon}\sqrt{\frac{\varepsilon}{3E_\gamma}}\sqrt{\frac{B(E2;0_g\rightarrow 2_\gamma)}{B(E2;0_g\rightarrow 2_g)}}$$

	Rot-Vib-Modell			Experiment		
	α_0	α_2	$\alpha = \alpha_0 + \alpha_2$	α_0	α_2	$\alpha = \alpha_0 + \alpha_2$
^{150}Nd	5.7	0.6	6.3	–	–	5.7
^{152}Sm	2.1	0.5	2.6	2.2±0.6	0.3	2.5±0.6 2.2±0.7
^{154}Sm	0.5	0.2	0.7	–	–	0.6±0.6
^{154}Gd	2.1	0.8	2.9	2.7±0.7	0.6	3.3±0.7 2.6±1.0
^{156}Gd	0.4	0.3	0.7	0.1±0.1	–	0.1±0.1 1.5±1.6 0.6±0.6

3. Coulomb excitation of the K = 8 isomer in ^{178}Hf



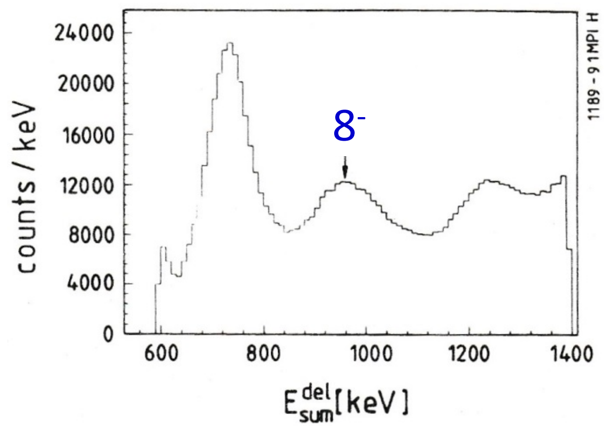
162 **NaI** detectors
6 **Ge** detector

parameters and resolutions FWHM

E_i	5.5% at 1332 keV
t_i	2.8 ns
W(θ)	14^0
E_{total}	18-22% for M_γ=20
M_γ	25-30% for M_γ=20

The two basic observables which can be measured for the resulting γ-ray shower are the **total energy** emitted as γ-radiation and the **number of γ-rays**.

- ^{178}Hf + ^{130}Te at 560, 590, 620 MeV
  particle detection at ~180^0, Pb catcher (0.5 mm thickness) was positioned 1cm downstream to stop the recoiling ^{178}Hf ions.

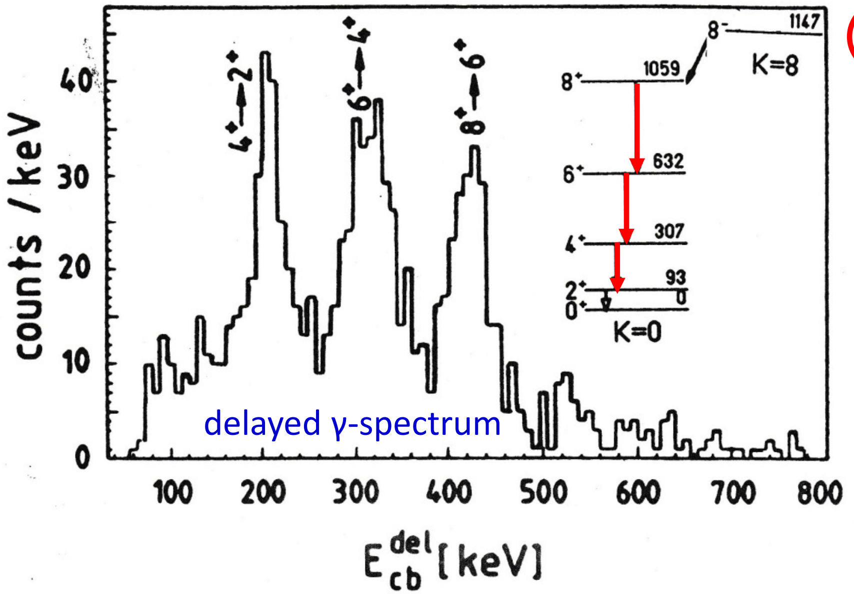


Delayed sum-energy spectrum taken at 590 MeV. (delayed time window 20-65ns with respect to beam pulse) A peak associated to the $K^\pi=8^-$ shows up.
The other peaks correspond to isomers of fusion products from target contaminants and β-decay.

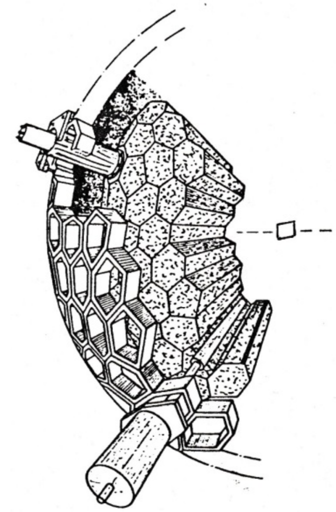
H. Xie et al.; Phys. Rev. C48 (1993), 2517

Coulomb excitation of the K = 8 isomer in ^{178}Hf

- ^{178}Hf + ^{130}Te at 560, 590, 620 MeV



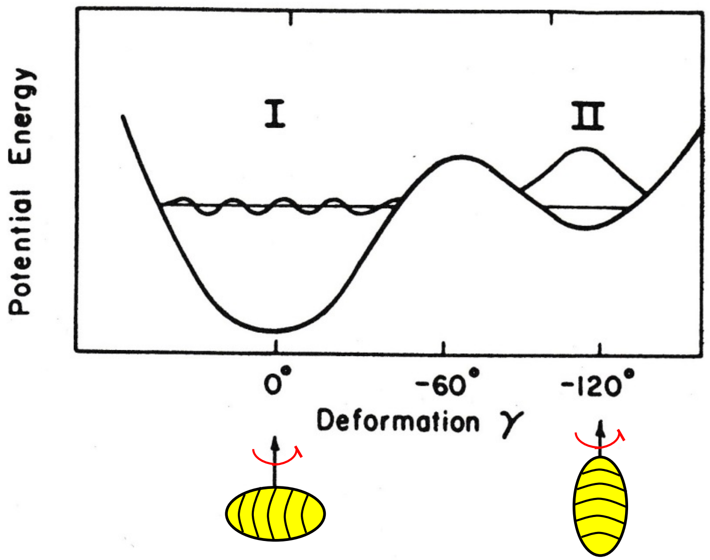
4 s $\rightarrow M_{30}\ (E3) \leq 0.01$



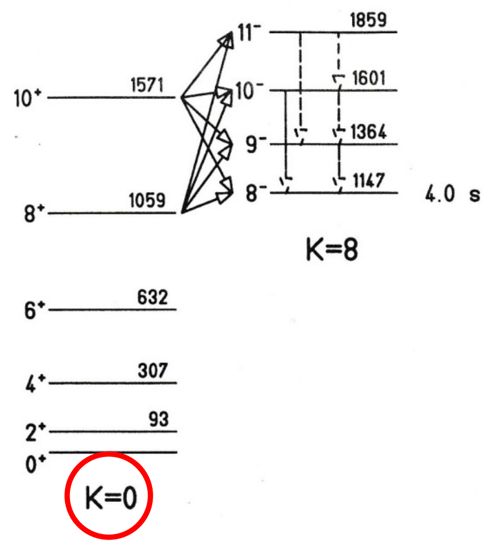
Delayed γ-ray spectrum of the Crystal Ball with $850keV \leq E_{sum}^{del} \leq 1100keV$ and $3 \leq N_{det} \leq 6$. In addition at least one of the delayed γ-rays must have been detected in one of the Ge-detectors.

H. Xie et al.; Phys. Rev. C48 (1993), 2517

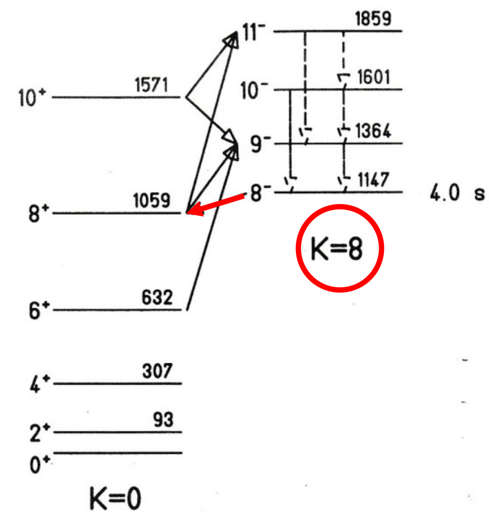
Decay of the isomer by barrier penetration



+ small K=8 admixture



+ small K=0 admixture



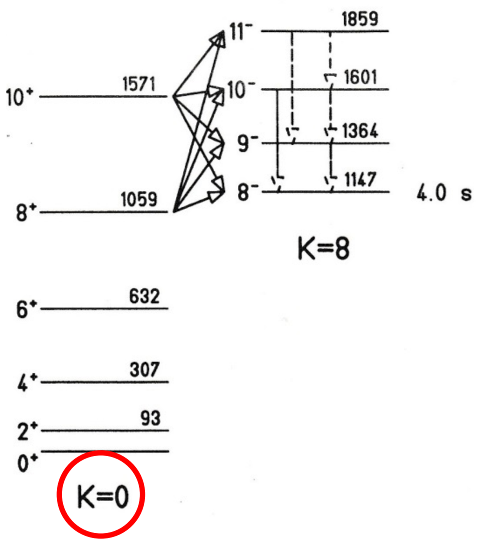
rigid rotor model:

$$\left\langle I_f \,\|M(E2)\|I_i\right\rangle = \sqrt{2I_i+1}\cdot\left(I_i 3K0|I_f K\right)\cdot M_{30}$$

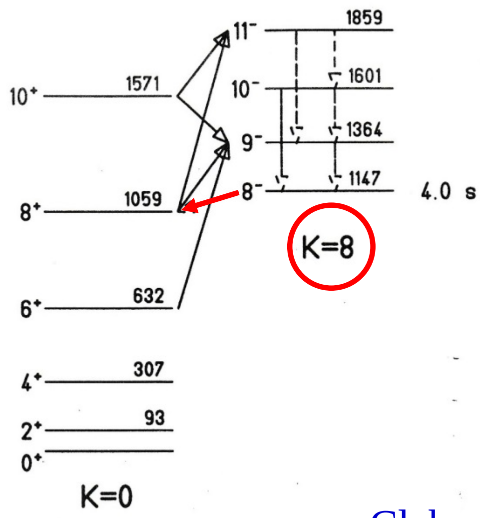
8^- lifetime is independent from excitation

2-band K-mixing model

+ small K=8 admixture



+ small K=0 admixture



8^- lifetime is independent from excitation

Clebsch Gordan coefficient:

$$< I3K0|(I-3)K > \; = \; -\sqrt{\frac{5(I+K-2)(I+K-1)(I+K)(I-K-2)(I-K-1)(I-K)}{2(I-2)(I-1)I(2I-3)(2I-1)(2I+1)}}$$

$$< I3K0|(I-2)K > \; = \; \sqrt{\frac{15(I+K-1)(I+K)(I-K-1)(I-K)}{(I-2)(I-1)I(2I-1)(2I+1)(2I+2)}} * K$$

$$< I3K0|(I-1)K > \; = \; -\sqrt{\frac{3(I+K)(I-K)}{(I-1)I(2I-3)(2I+1)(2I+2)(2I+3)}} * (5K^2 - I^2 + 1)$$

$$< I3K0|IK > \; = \; \frac{5K^2 - 3I^2 - 3I + 1}{\sqrt{(I-1)I(I+1)(I+2)(2I-1)(2I+3)}} * K$$

$$< I3K0|(I+1)K > \; = \; \sqrt{\frac{3(I+K+1)(I-K+1)}{I(I+1)(2I-1)(2I+1)(2I+4)(2I+5)}} * (5K^2 - I^2 - 2I)$$

$$< I3K0|(I+2)K > \; = \; \sqrt{\frac{15(I+K+1)(I+K+2)(I-K+1)(I-K+2)}{I(I+1)(I+2)(2I+1)(2I+3)(2I+6)}} * K$$

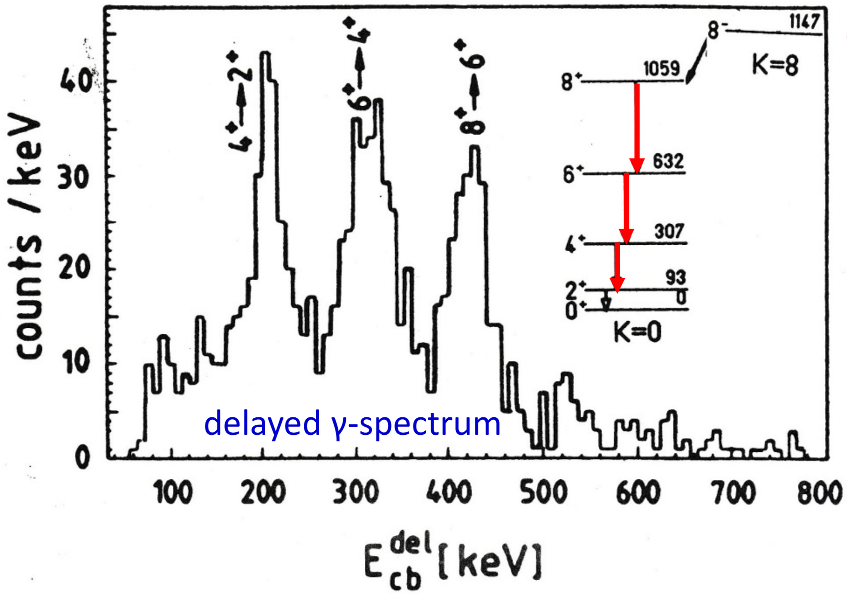
$$< I3K0|(I+3)K > \; = \; \sqrt{\frac{5(I+K+1)(I+K+2)(I+K+3)(I-K+1)(I-K+2)(I-K+3)}{2(I+1)(I+2)(I+3)(2I+1)(2I+3)(2I+5)}}$$

rigid rotor model:

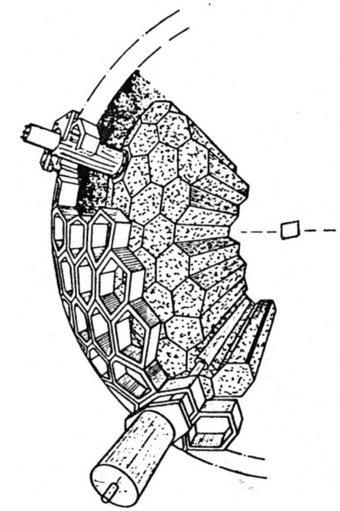
$$\left\langle I_f \| M(E3) \| I_i \right\rangle = \sqrt{2I_i + 1} \cdot \left(I_i 3K0 | I_f K \right) \cdot M_{30}$$

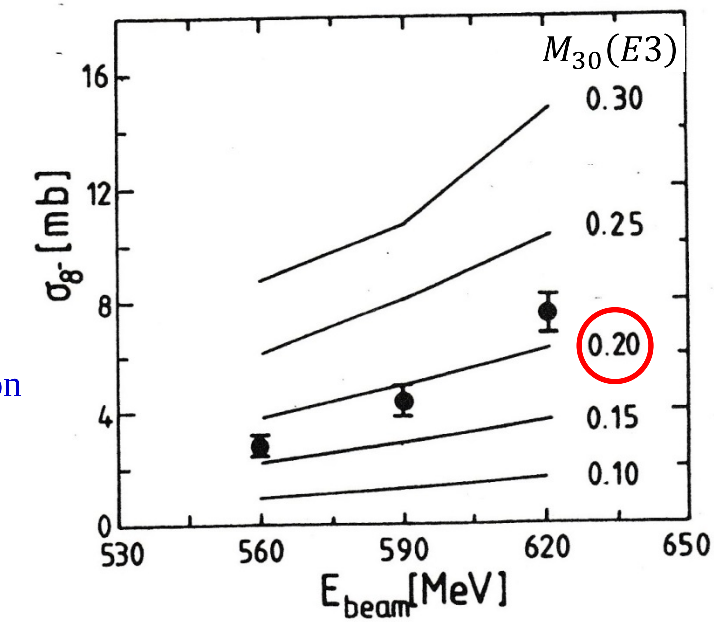
Coulomb excitation of the K = 8 isomer in ^{178}Hf

- ^{178}Hf + ^{130}Te at 560, 590, 620 MeV



4 s → $M_{30}\ (E3) \le 0.01$





coupling between rotational motion
and single particle excitation

$$\psi(8^-) \cong |K = 8\rangle + \alpha |K = 0\rangle$$

H. Xie et al.; Phys. Rev. C48 (1993), 2517