# Outline: Rotational Spectroscopy


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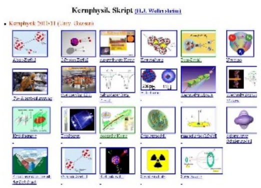
web-page: https://web-docs.gsi.de/~wolle/ and click on

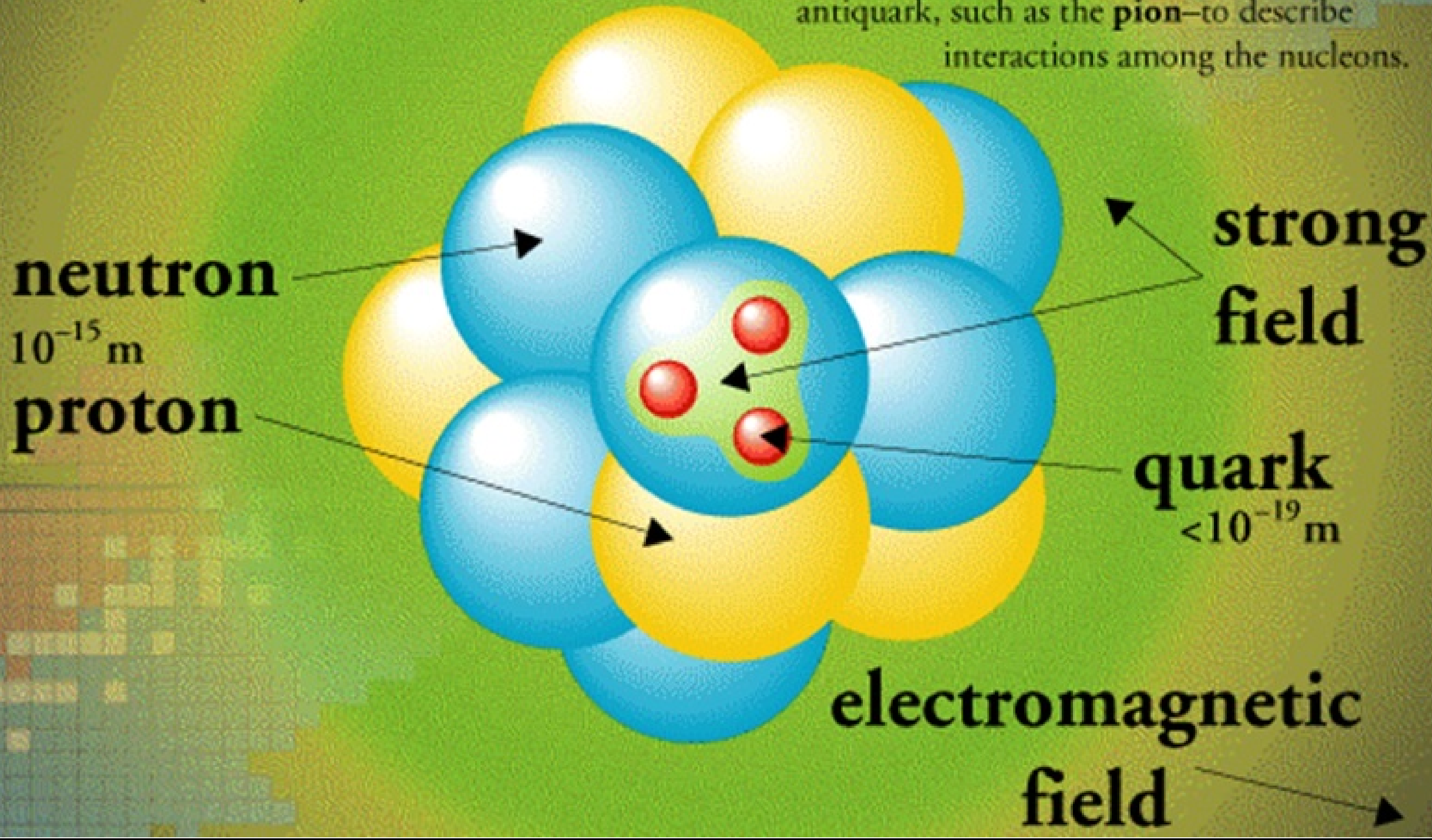


1. rotational bands
2. band crossings
3. backbending
4. rotational alignment

# *The* Nucleus

(1-10) × $10^{-15}$ m

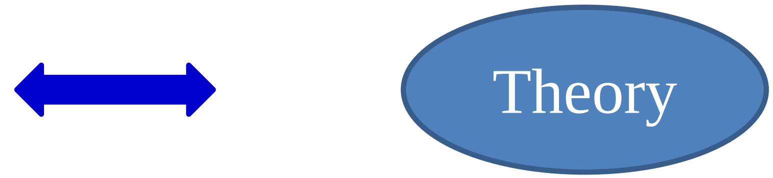
*A*t the center of the atom is a nucleus formed from **nucleons**–protons and neutrons. Each nucleon is made from three **quarks** held together by their strong interactions, which are mediated by gluons. In turn, the nucleus is held together by the **strong** interactions between the gluon and quark constituents of neighboring nucleons. Nuclear physicists often use the exchange of mesons–particles which consist of a quark and an antiquark, such as the **pion**–to describe interactions among the nucleons.



# Why study nuclear structure?

Studying exotic nuclei extends the range over which theories can be tested

Goal: Establish the physical properties of exotic nuclei and their interactions (reactions) to constrain theory and improve predictive power



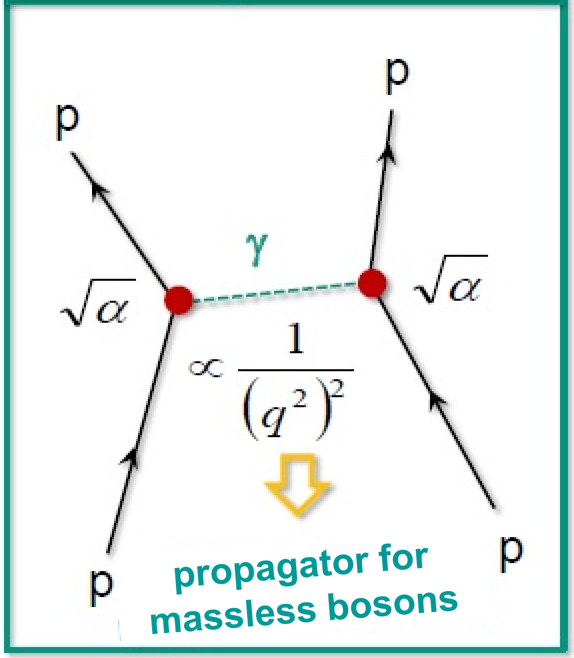

Physical properties

Mass, decay lifetime, production cross-section, electric and magnetic moments, excited state properties (moments, energies, lifetimes), etc…

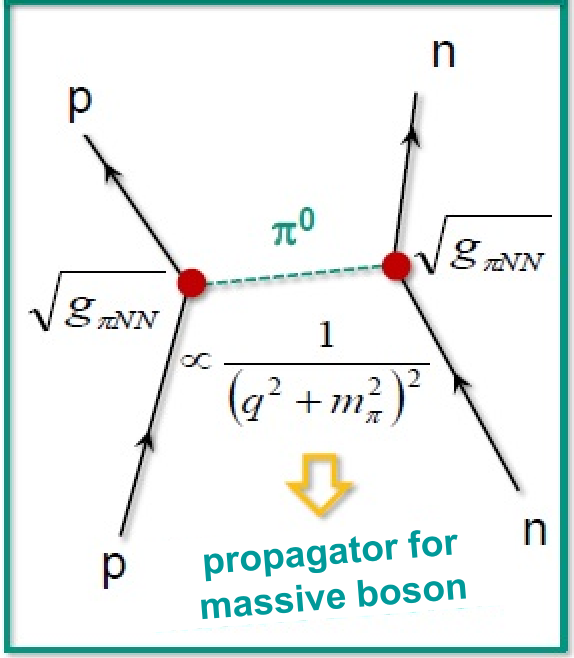
Note → the observables in an experiment may or may not require interpretation to relate to physical properties

# Feynman diagrams

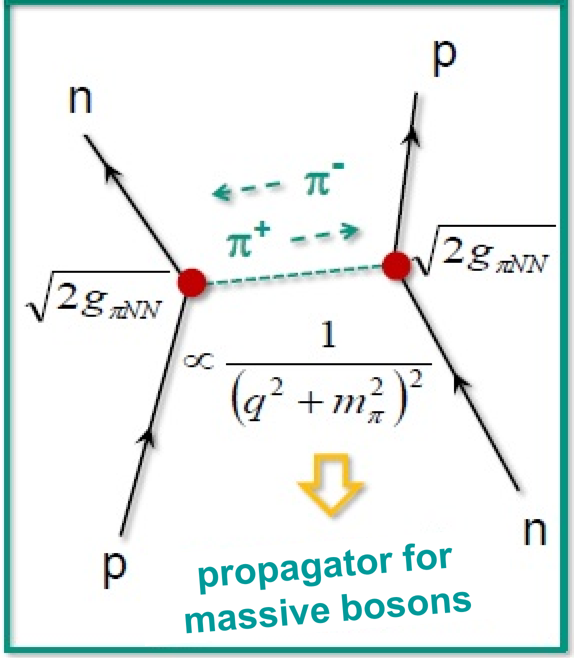
electromagnetic repulsion of protons



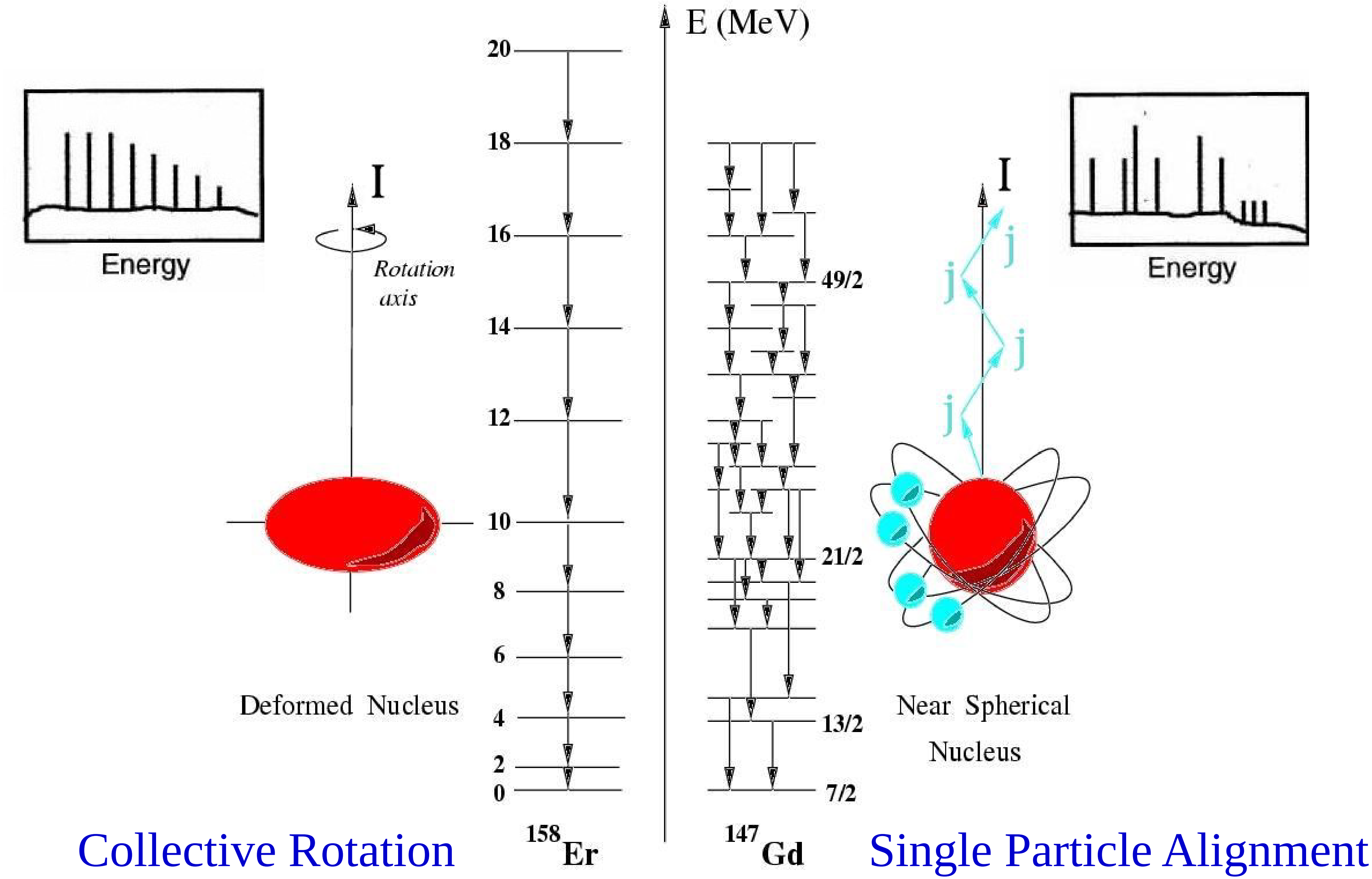

strong interaction of a proton and a neutron



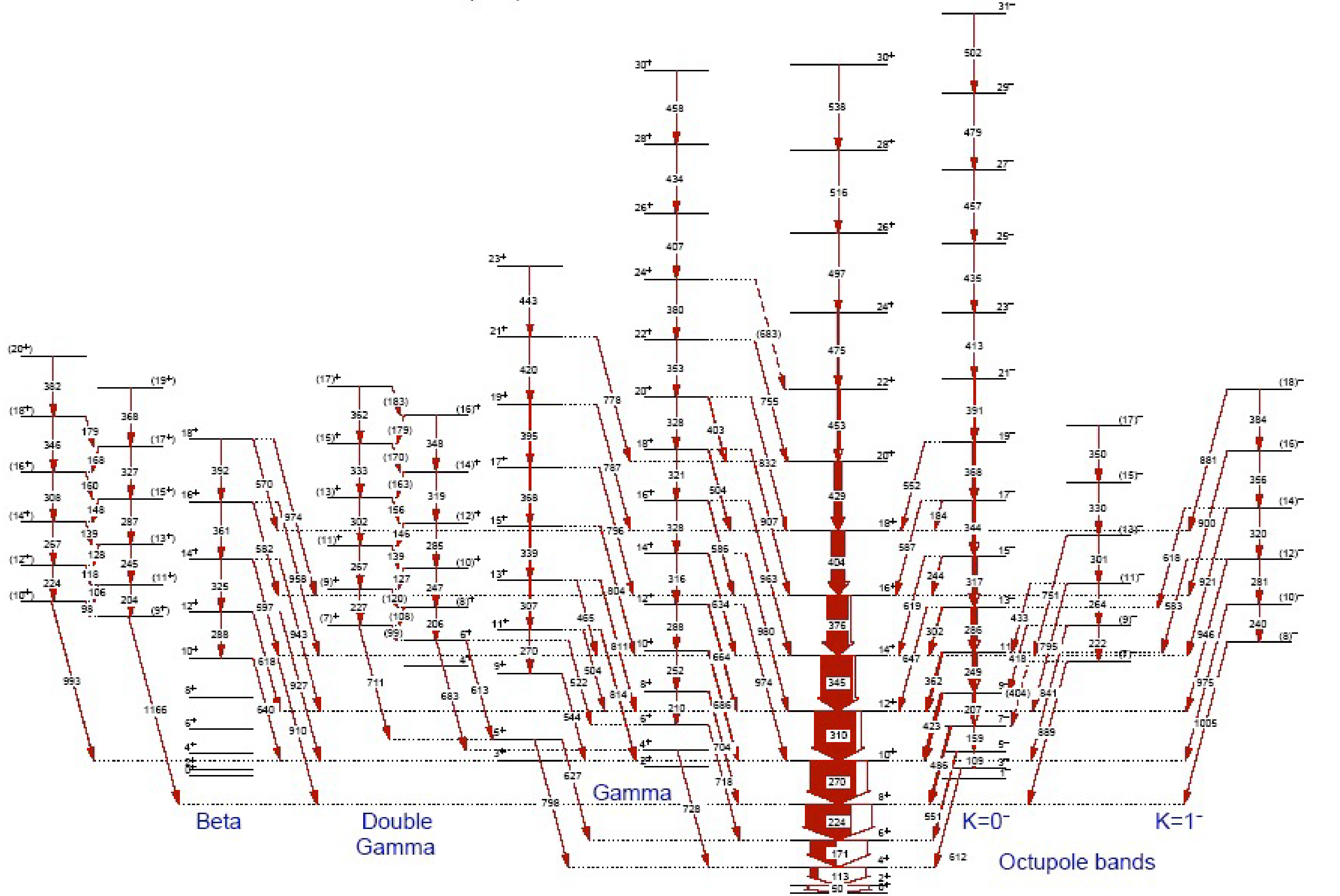

strong interaction of a proton and a neutron



# Level schemes – collective versus single particle

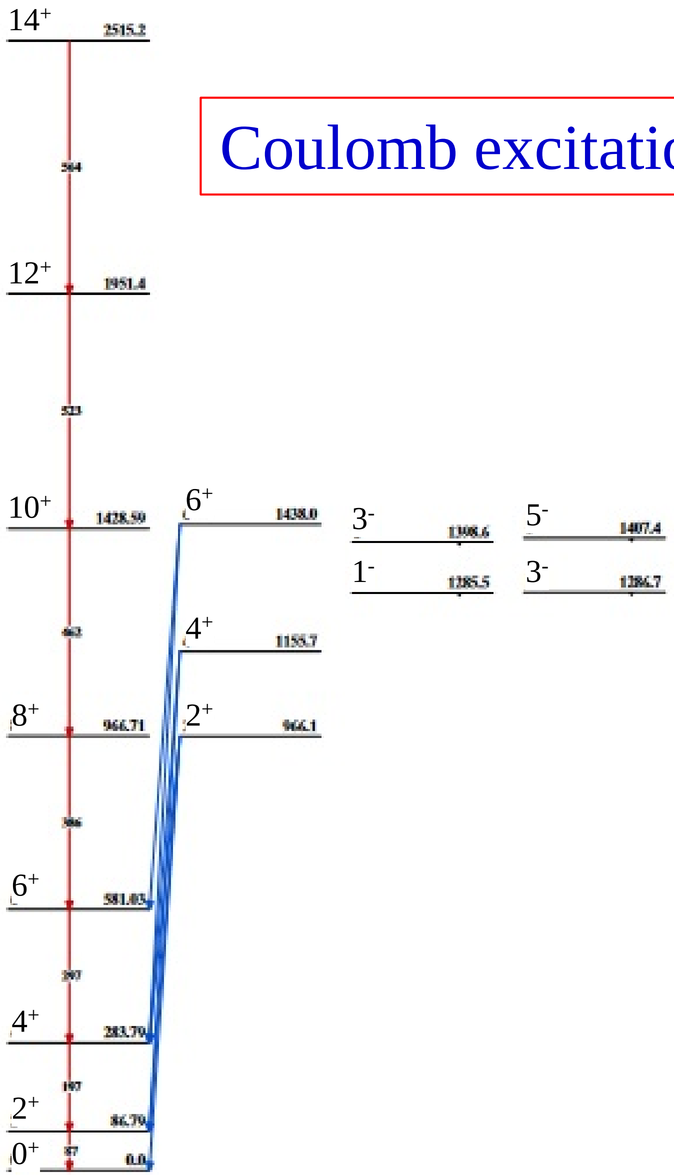


# Rotational bands: sequences of excited states

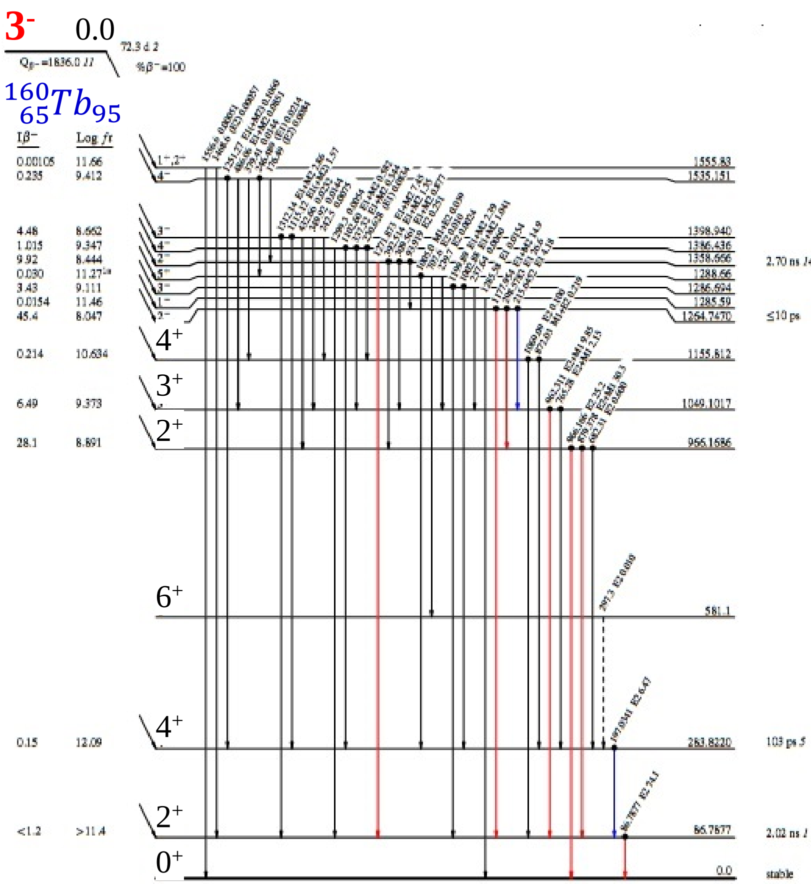


# Rotational bands in $^{160}$Dy: sequences of excited states

Coulomb excitation




β- decay



# Rotation of deformed nuclei

rotational axis ⊥ symmetry axis

$$E(I) = \frac{\hbar^2}{2\mathfrak{J}}\left[I(I+1) - K^2\right]$$

kinematic moment of inertia

$$\mathfrak{J}^{(1)} = I \cdot \left(\frac{\partial E}{\partial I}\right)^{-1} = \frac{I}{\hbar\omega} \approx \frac{\Delta I(I)}{E_\gamma}$$

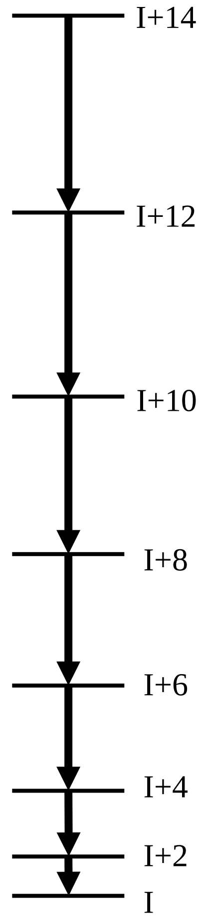
dynamic moment of inertia

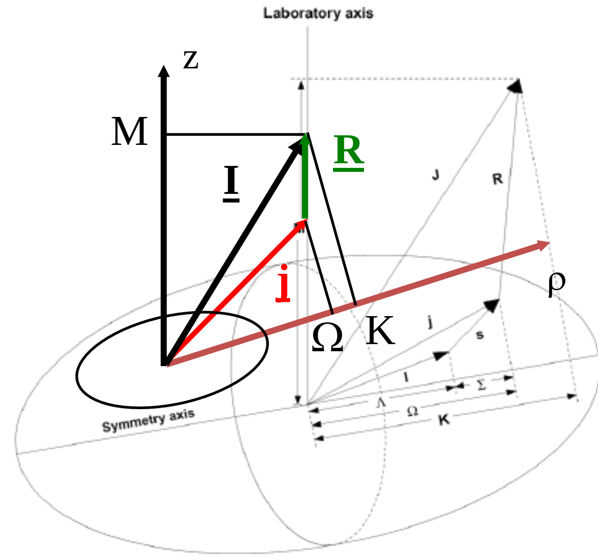
$$\mathfrak{J}^{(2)} = \left(\frac{\partial^2 E}{\partial I^2}\right)^{-1} = \frac{\partial I}{\hbar\partial\omega} = \frac{(\Delta I)^2}{\Delta E_\gamma}$$

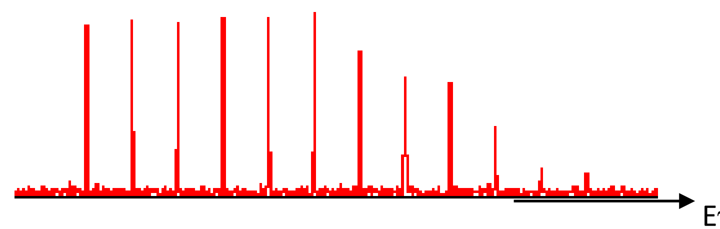
rigid rotor: $\mathfrak{J}^{(2)} = \mathfrak{J}^{(1)}$

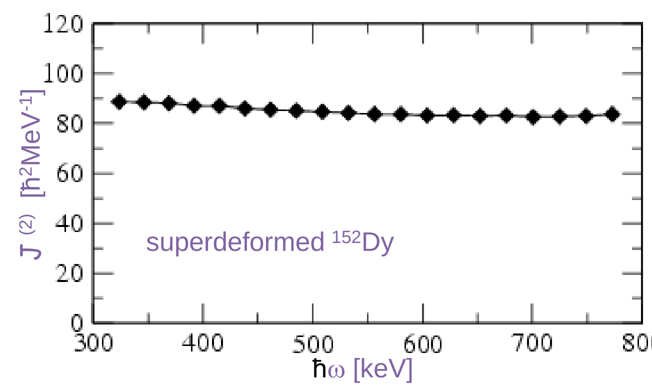
rotational frequency

$$\hbar\omega = \frac{\partial E}{\partial I} \approx \frac{E}{\Delta I}$$

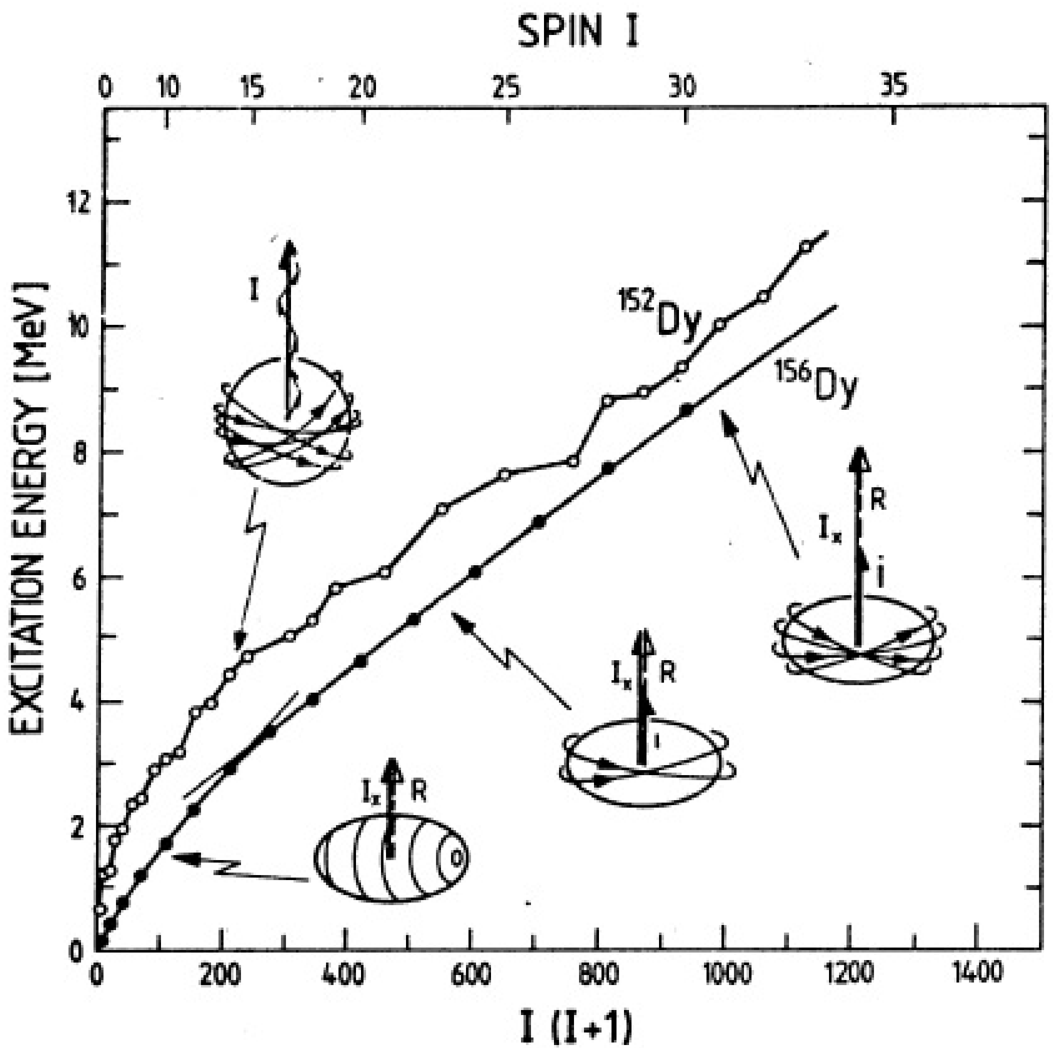


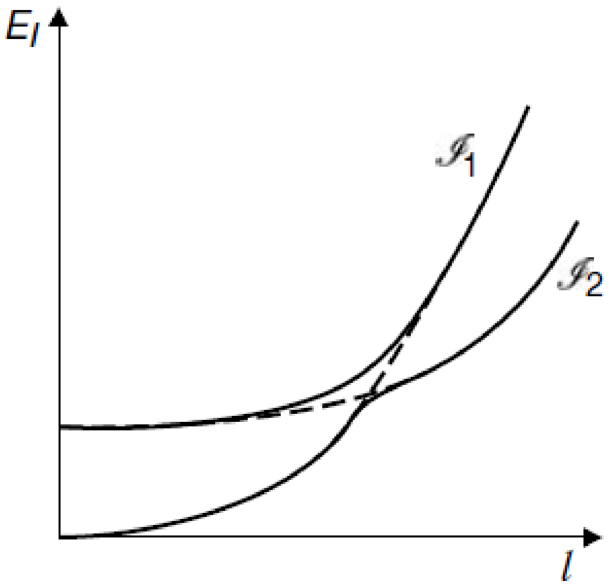
# Rotational bands: sequences of excited states

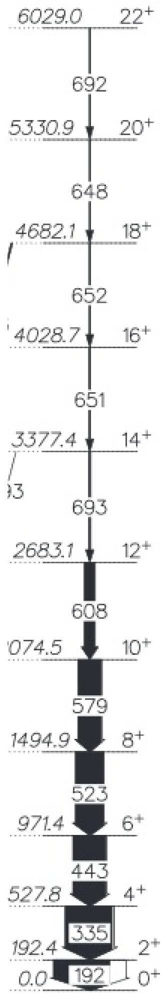


# Rotational bands

$^{158}$Er

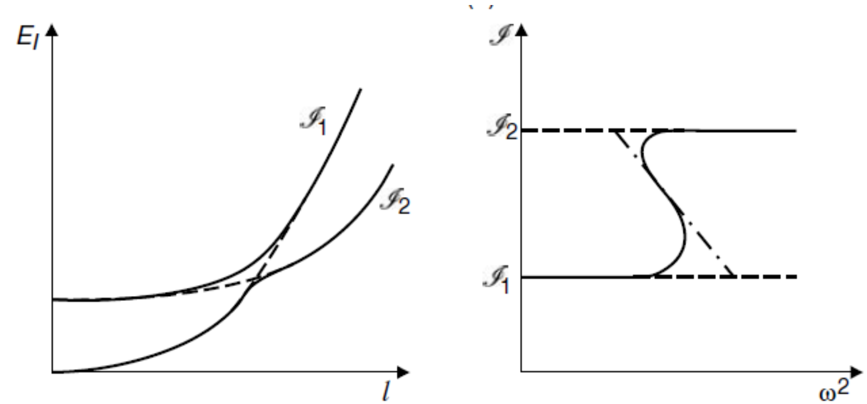
- Rotation can exist on top of other excitations
- As such, a nucleus can have several different rotational bands and the moment of inertia $\mathfrak{I}$ is often different for different bands
- The different $\mathfrak{I}$ lead to different energy spacings for the different bands

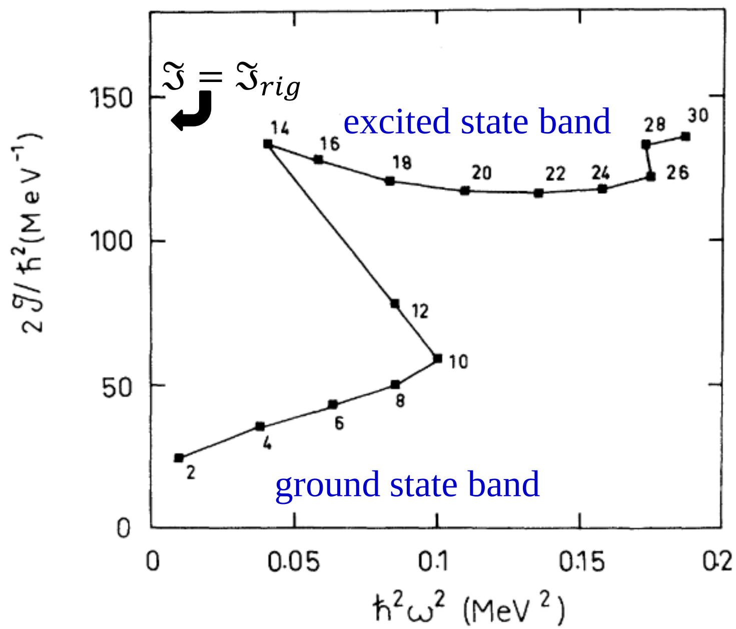


# Rotational bands: Backbending

- The different $\Im$ for different rotational bands creates the so-called "backbend"
- This is when we follow the lowest-energy state for a given spin-parity ("yrast" state) belonging to a given rotational band and plot the moment of inertia and square of the rotational frequency




at high spin → break up of J=0 pair, reduction of the moment of inertia

$$\frac{2\Im}{\hbar^2} = \frac{4I-2}{E_\gamma}$$

$$\hbar\omega \cong \frac{E_\gamma}{2}$$

$$E_\gamma = E(I) - E(I-2) \qquad E_{rot} = \frac{\hbar^2}{2\Im} I(I+1) = \frac{1}{2}\Im\omega^2$$

# Rotational alignment: Coriolis coupling

at high spin → break up of J=0 pair, reduction of the moment of inertia

$$H_{rot} = \frac{\hbar^2}{2\Im} \cdot \vec{R}^2 = \frac{\hbar^2}{2\Im} \left(\vec{I} - \vec{j}\right)^2$$

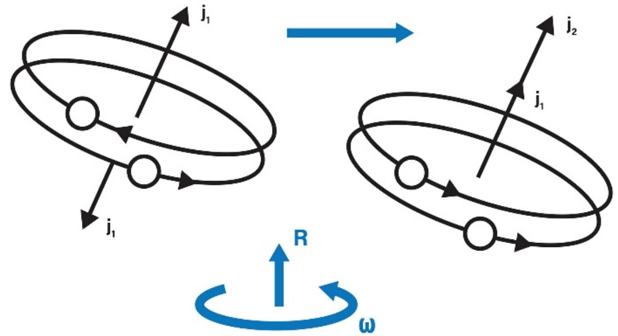



Illustration of the “rotational alignment” effect where a pair of high-**j** quasi-particles in time-reversed orbits at a critical rotational frequency are broken and align themselves with the collective rotation, **R**, of the nucleus as a whole leading to the “backbending” effect in nuclei

$$\vec{R}^2 = \left(\vec{I} - \vec{j}\right)^2 = \vec{I}^2 - 2 \cdot \overrightarrow{I \cdot j} + \vec{j}^2$$

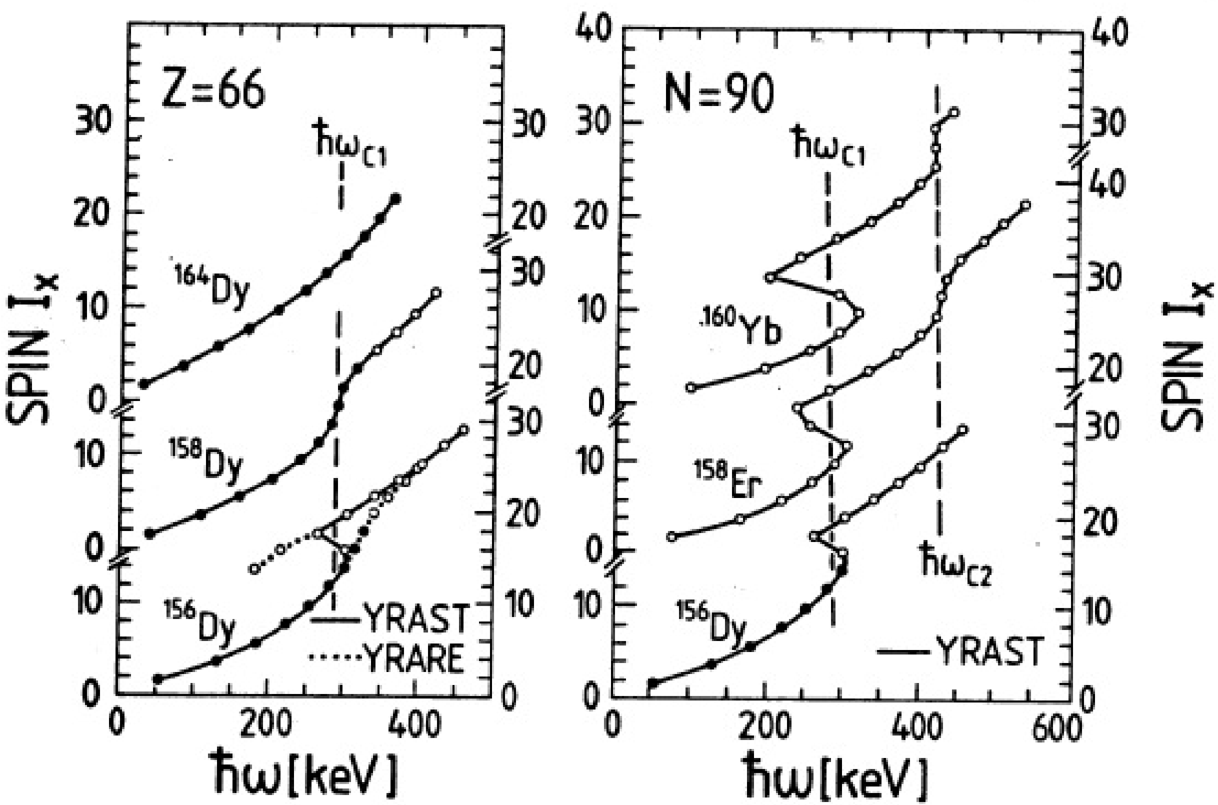
$$= I^2 + j^2 - 2K^2 - (I_+ j_- + I_- j_+)$$

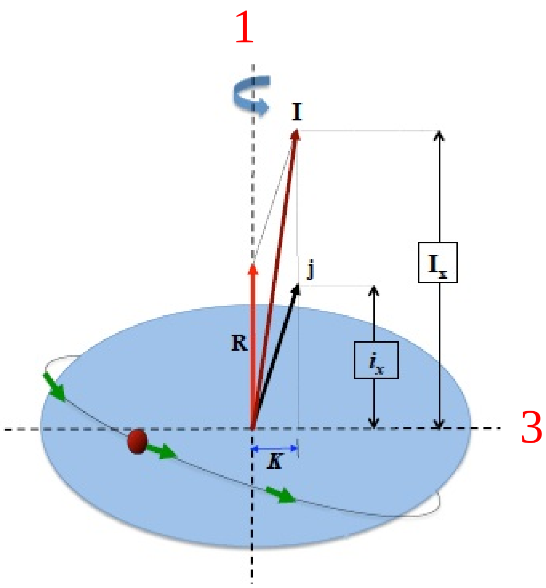
where $I_\pm = I_x \pm i \cdot I_y$ , $j_\pm = j_x \pm i \cdot j_y$ and $j_z = I_Z = \pm K$

The quantity ***K*** is the projection of ***I*** along the rotational axis.

The coupling term $(I_+ j_- + I_- j_+)$ corresponds to the Coriolis force and couples $\vec{j}$ to $\vec{R}$

# Rotational alignment



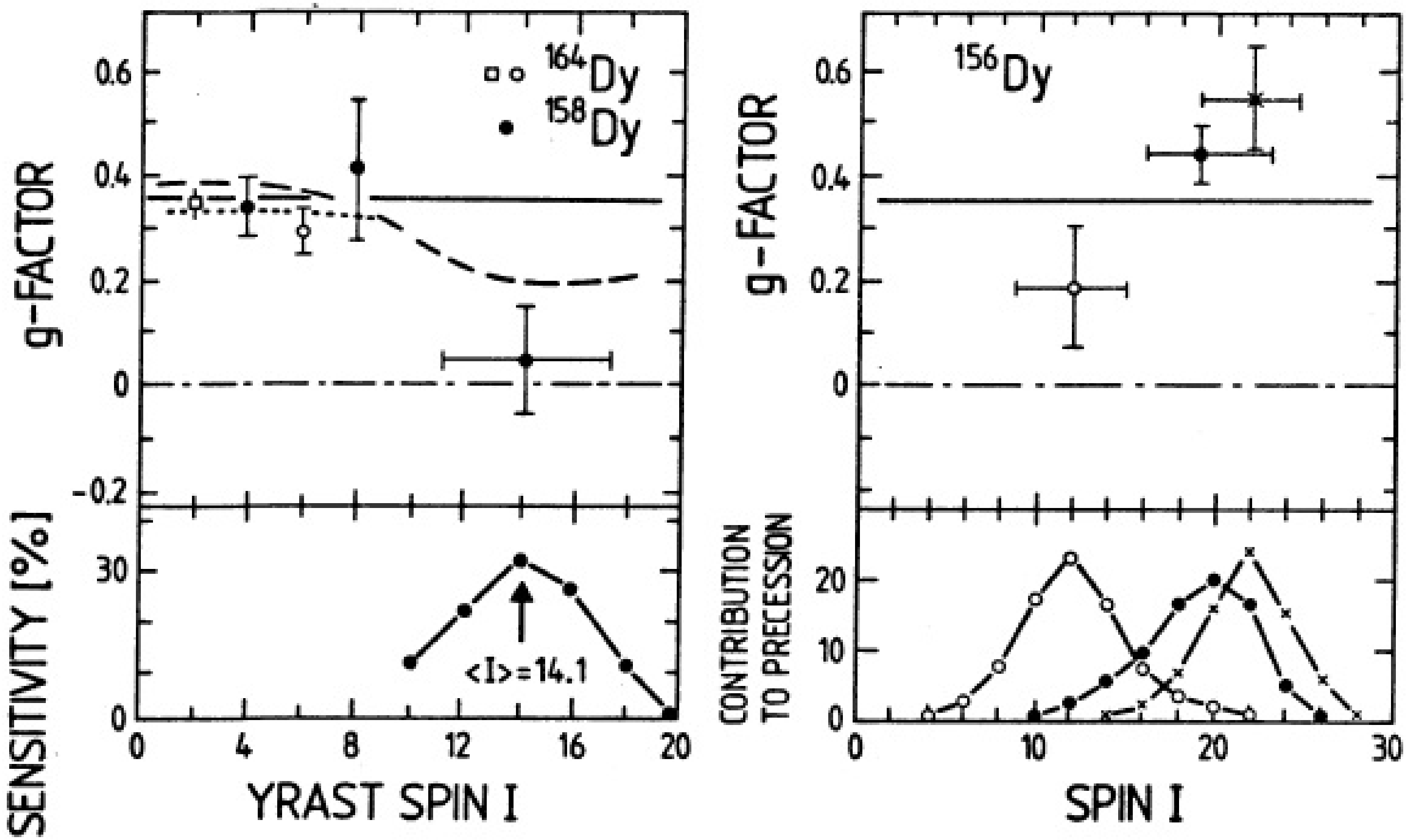




$$I_x(I) = \sqrt{I(I+1)} \approx I + 0.5$$

$$I_x(I) = R(\omega(I)) + i \; = \Im \cdot \omega + i$$

$$\hbar\omega(I_x(I-1)) = \frac{\Delta E(I \to I-2)}{I_x(I) - I_x(I-2)} \approx \frac{1}{2}\Delta E(I \to I-2)$$

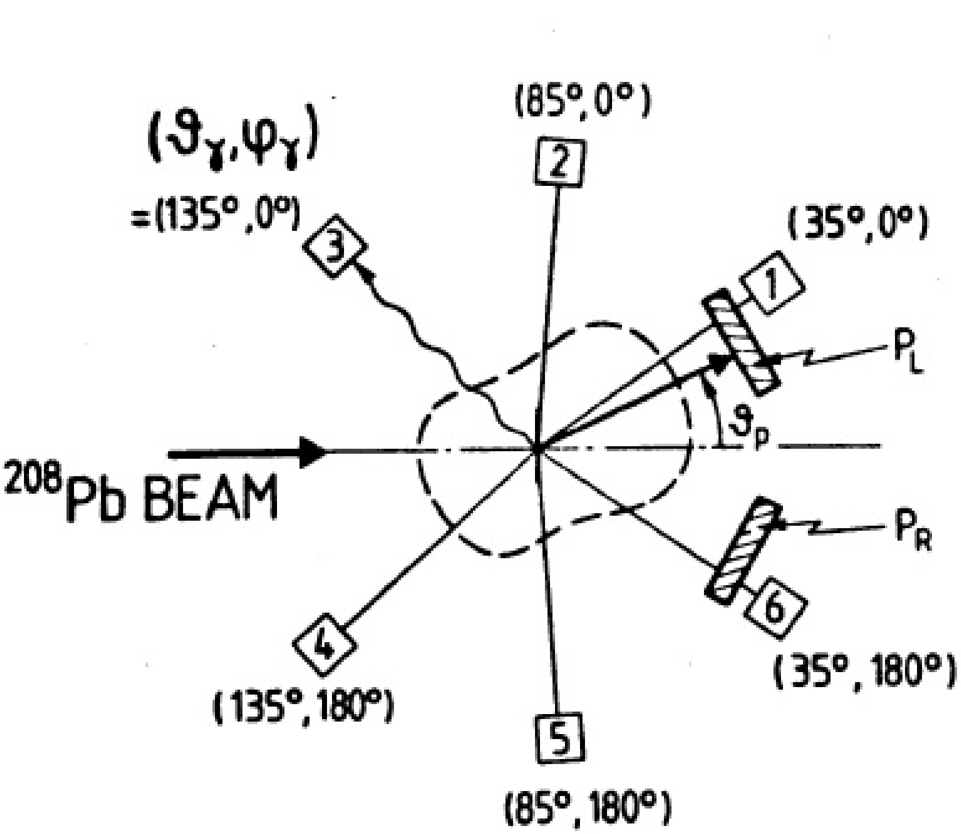
# Rotational alignment: g-factor

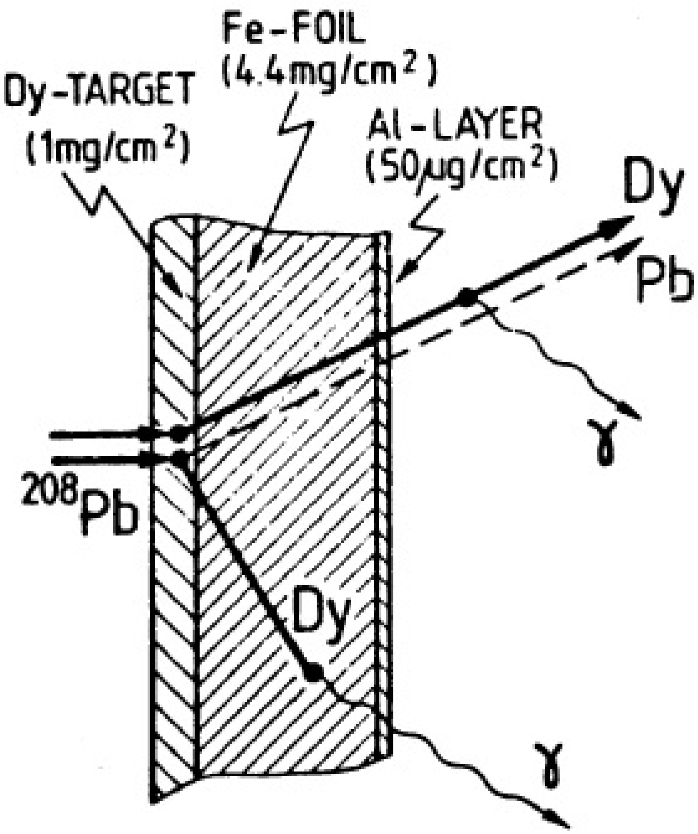



$$\boldsymbol{g(I) = \frac{\mu(I)}{\mu_N \cdot I} = g_R + \sum_{\mu} (g_\mu - g_R) \frac{i_\mu}{I}}$$

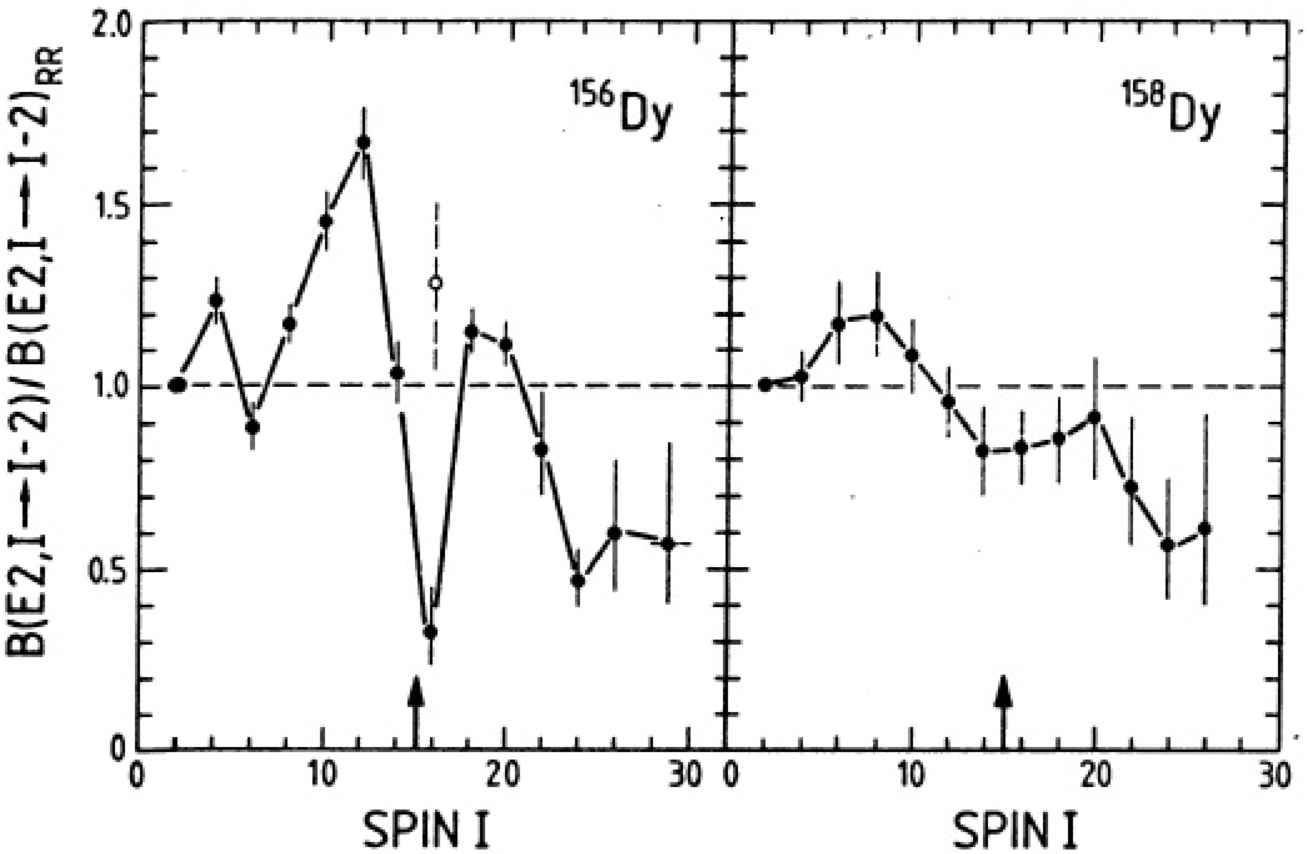
$g_R = \frac{Z}{A}$ $\quad g_\mu \approx -0.15 \quad g_\mu \approx +1.2 \quad i_\mu = 10$

# Rotational alignment: g-factor



# Rotational alignment: B(E2)-values




$$B(E2, I \to I-2) = \frac{5}{16\pi}\frac{3I(I-1)}{2(4I^2-1)}Q_t^2$$

$$Q(I) = -\frac{I}{2I+3}Q_d$$

$$Q_t = Q_0\frac{2}{\sqrt{3}}cos(30^0-\gamma)$$

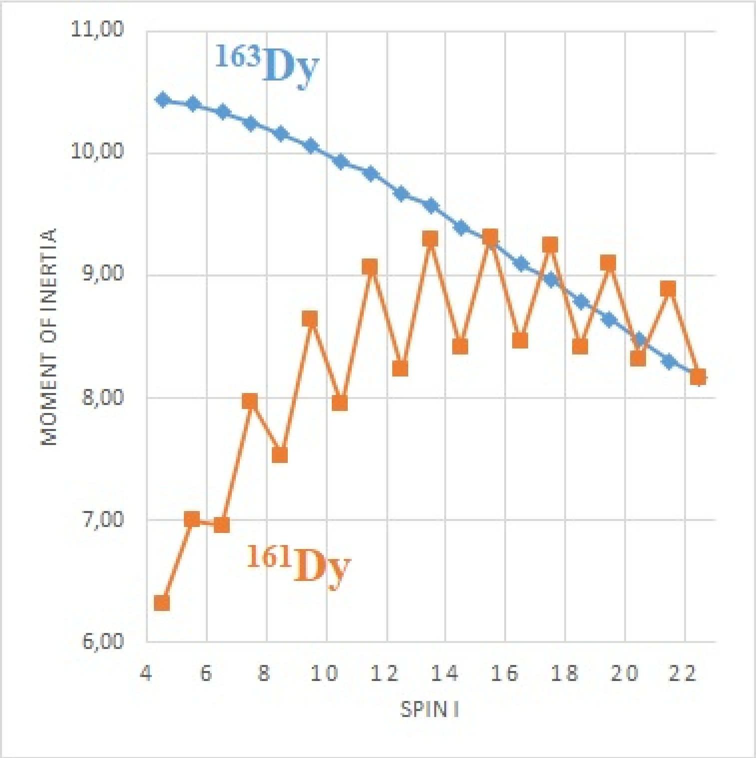
$$Q_d = Q_0 \cdot 2 \cdot sin(30^0-\gamma)$$

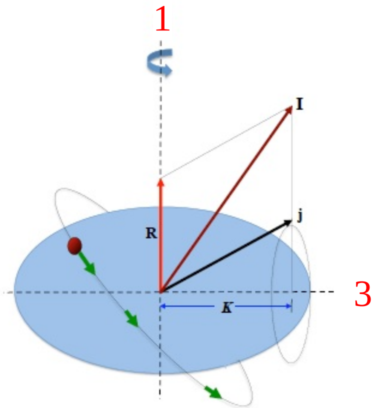
$$Q_0 = \frac{3}{\sqrt{5\pi}}ZeR_0^2\beta$$

# Rotational alignment: Coriolis coupling

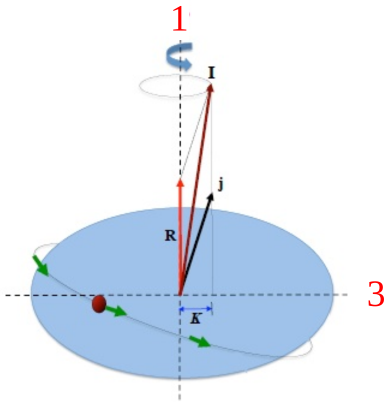
$^{161}$Dy: **5/2**[642]   ($i_{13/2}$)

$^{163}$Dy: **5/2**[523]   ($f_{7/2}$)




strong coupling




weak coupling

rotational alignment

# Nilsson diagram