Outline: Octupole collectivity

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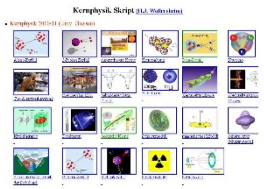
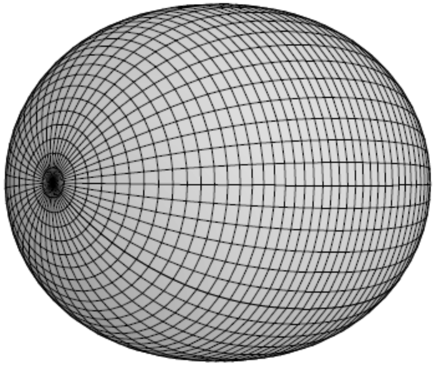


1. the double oscillator
2. Coulomb excitation of ^{226}Ra
3. signature of an octupole deformed nucleus
4. electric moments of ^{226}Ra

Shape parameterization

$$R(\theta,\phi) = R_0 \cdot \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} \cdot Y_{\lambda\mu}(\theta,\phi) \right]$$

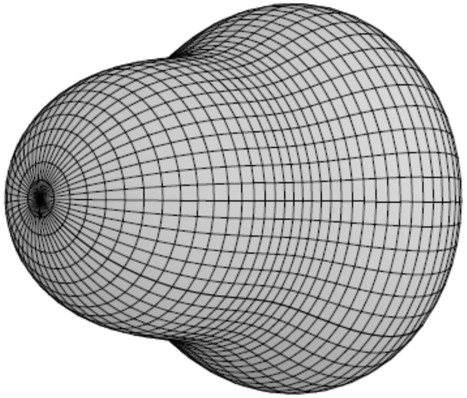
axially symmetric quadrupole



λ=2

$\alpha_{20} \neq 0$, $\alpha_{2\pm1} = \alpha_{2\pm2} = 0$

axially symmetric octupole

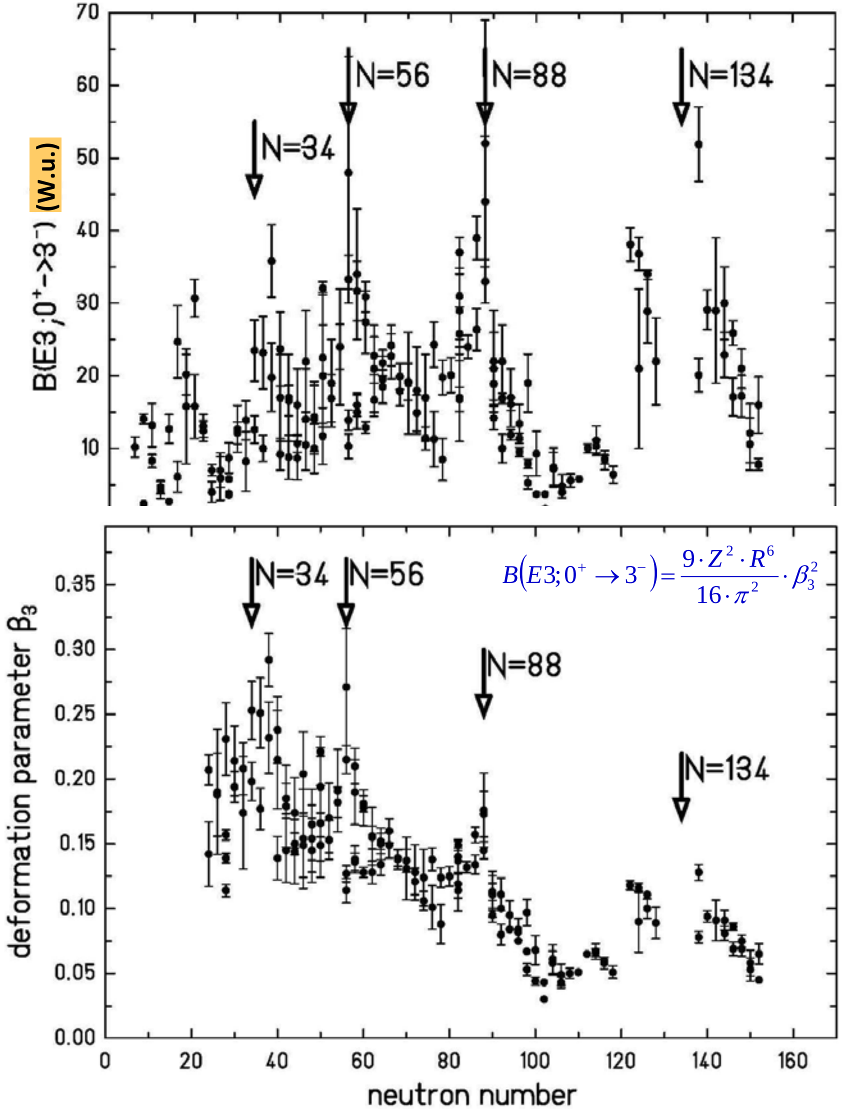


λ=3

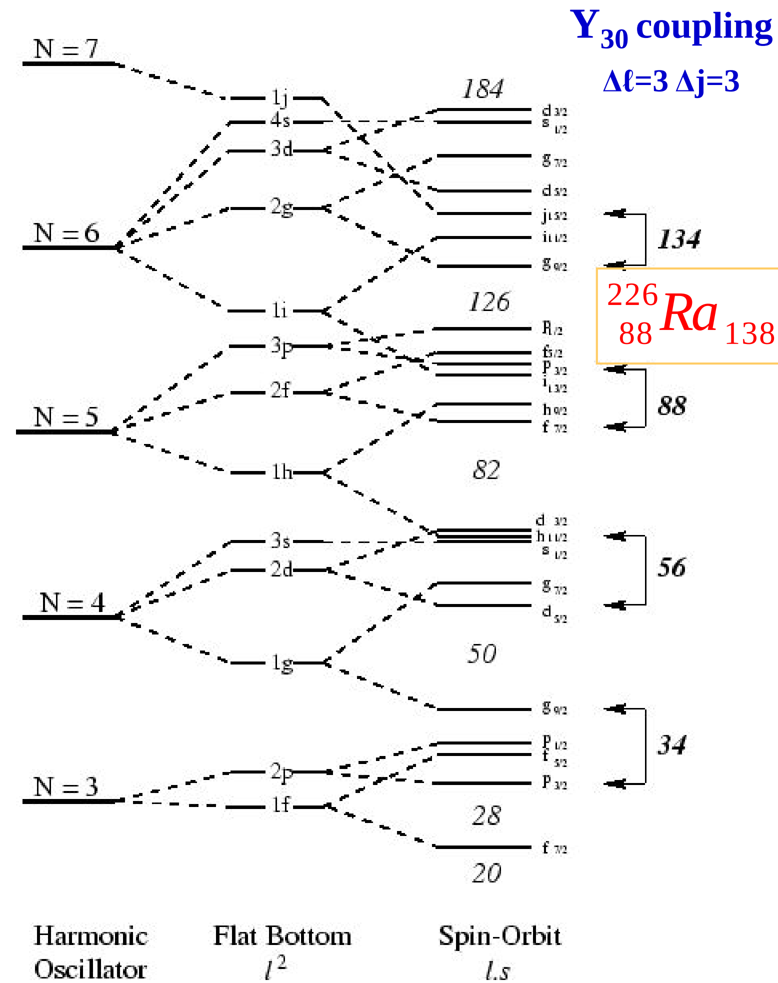
$\alpha_{30} \neq 0$, $\alpha_{3\pm1,2,3} = 0$

$\alpha_{20} \neq 0$, $\alpha_{2\pm1,2} = 0$

Octupole collectivity



R.H. Spear At. Data and Nucl. Data Tables 42 (1989), 55



some subshells interact via the r^3Y_{30} operator

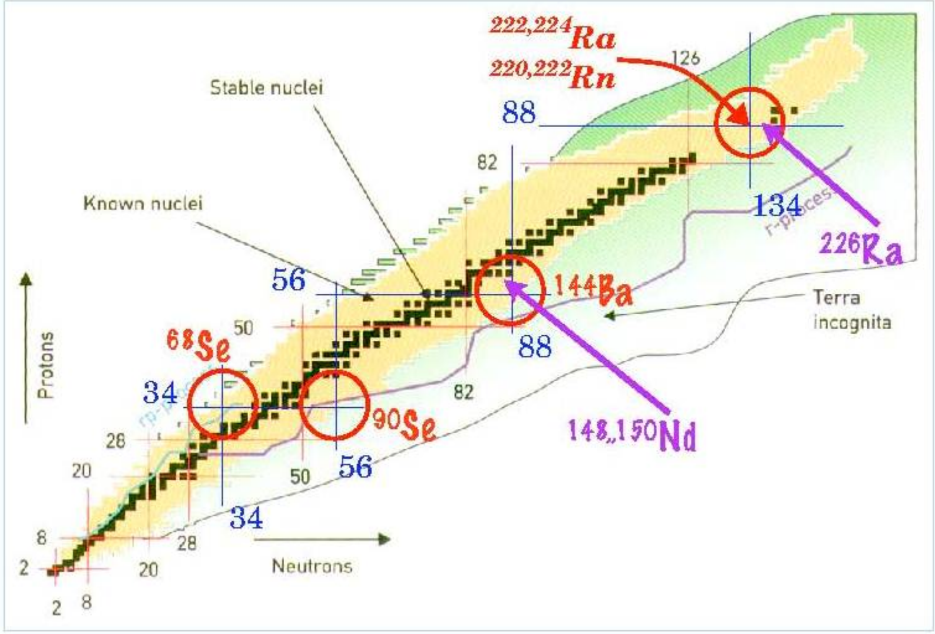
e.g. in light actinide nuclei one has an interaction between
$j_{15/2}$ and $g_{9/2}$ neutron orbitals
$i_{13/2}$ and $f_{7/2}$ proton orbitals

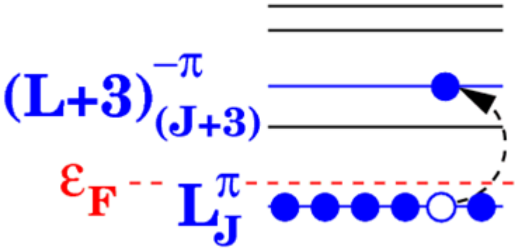
Octupole collectivity

Octupole correlations enhanced at the magic numbers: **34, 56, 88, 134**

Microscopically …

Intruder orbitals of opposite parity and **ΔJ, ΔL = 3** close to Fermi level

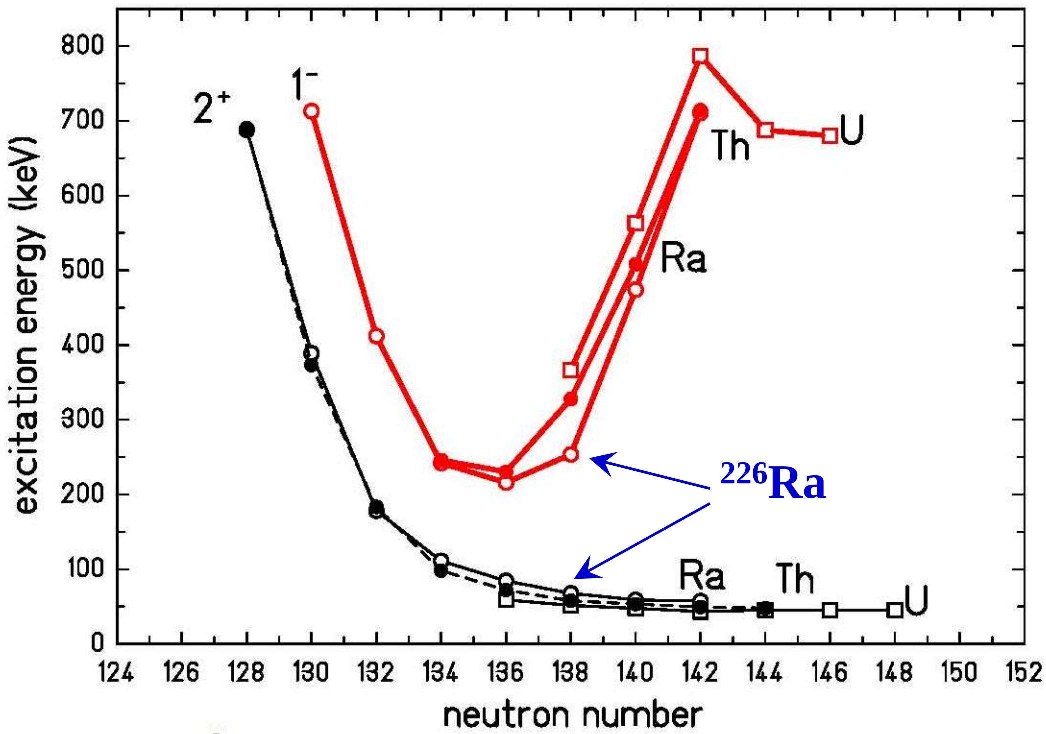


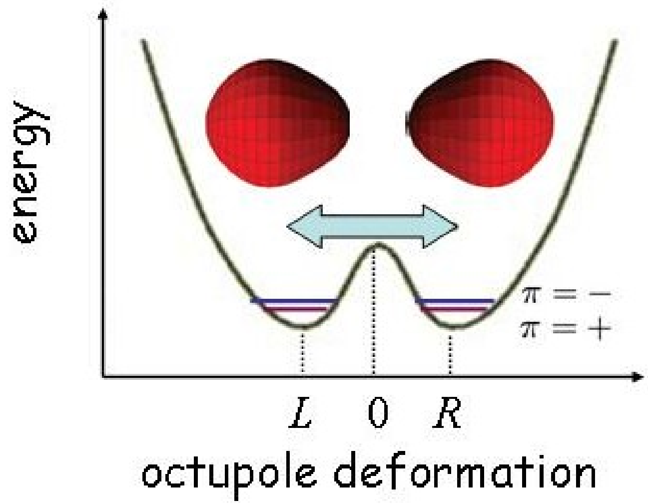


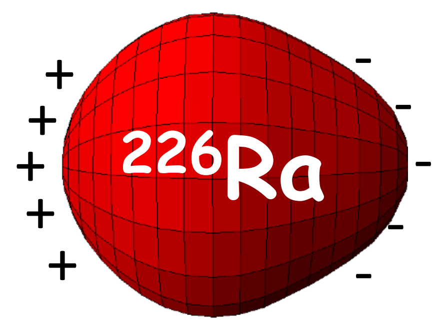
^{226}**Ra** close to Z=88 N=134

$$\pi\left(f_{7/2}\rightarrow i_{13/2}\right)\quad \nu\left(g_{9/2}\rightarrow j_{15/2}\right)$$

Octupole collectivity

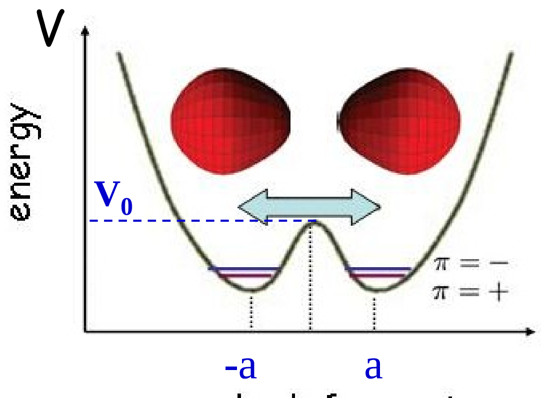






In an **octupole** deformed nucleus the center of mass and center of charge tend to separate, creating a non-zero **electric dipole moment**.

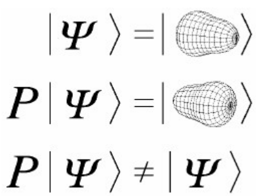
The double oscillator

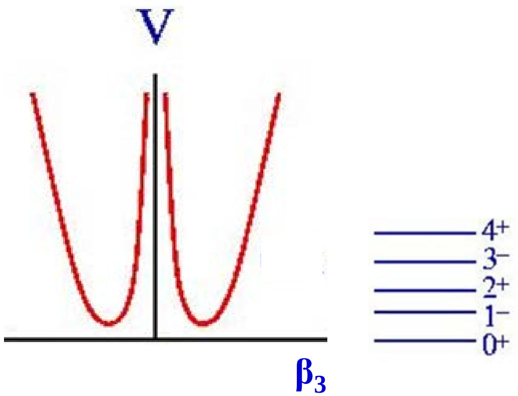


$$H \cdot \Psi = -\frac{\hbar^2}{2 \cdot B}\frac{\partial^2 \psi}{\partial \beta_3^2} + \frac{V_0}{a^2} \cdot (|\beta_3| - a)^2 \cdot \Psi = E \cdot \Psi$$

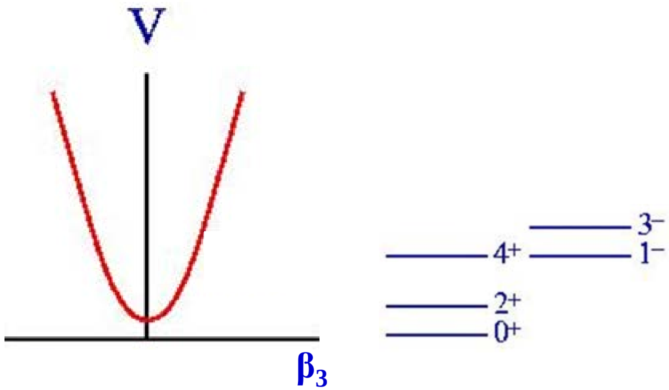
$$E_{even} = \hbar\omega \cdot \left(\nu_{even} + \frac{1}{2}\right) = \hbar\omega \cdot \left(\frac{1}{2} - \sqrt{\frac{2 \cdot V_0}{\hbar\omega \cdot \pi}} \cdot e^{-\frac{2V_0}{\hbar\omega}}\right)$$

$$E_{odd} = \hbar\omega \cdot \left(\nu_{odd} + \frac{1}{2}\right) = \hbar\omega \cdot \left(\frac{1}{2} + \sqrt{\frac{2 \cdot V_0}{\hbar\omega \cdot \pi}} \cdot e^{-\frac{2V_0}{\hbar\omega}}\right)$$





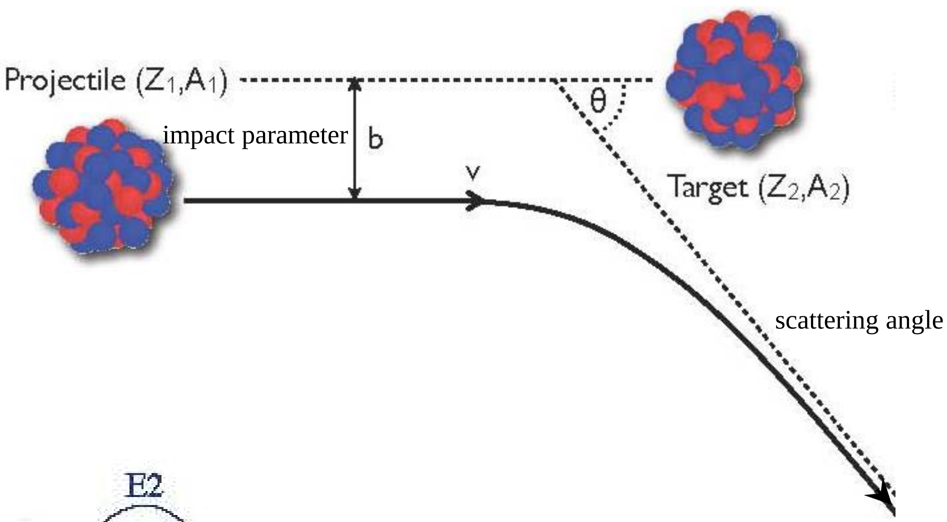
octupole deformed

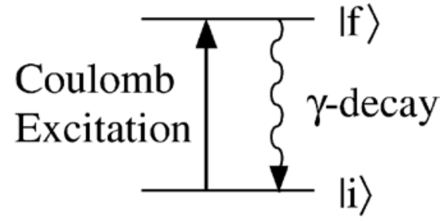


octupole vibrational

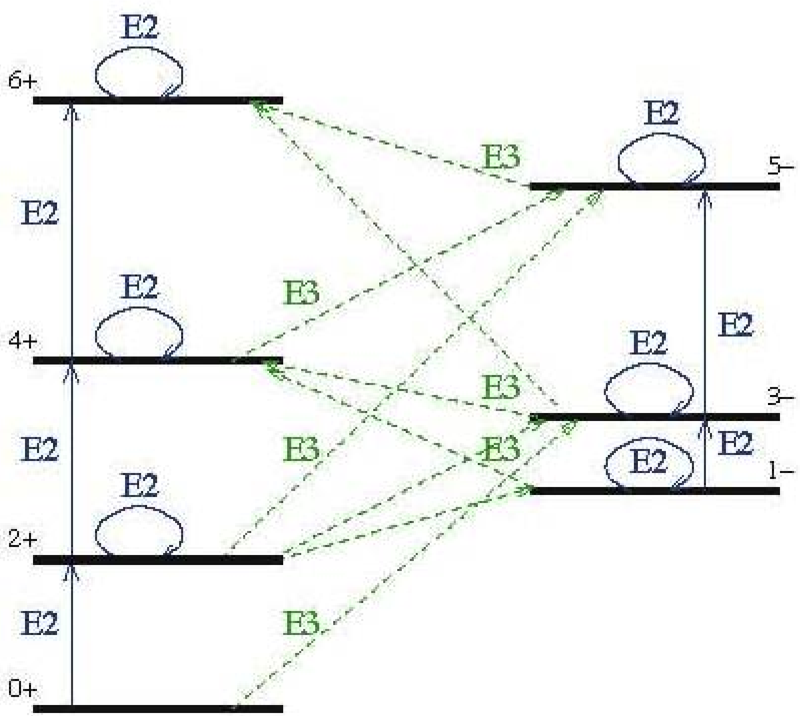
Merzbacher 'Quantum Mechanics'

Coulomb excitation



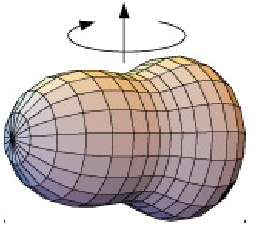


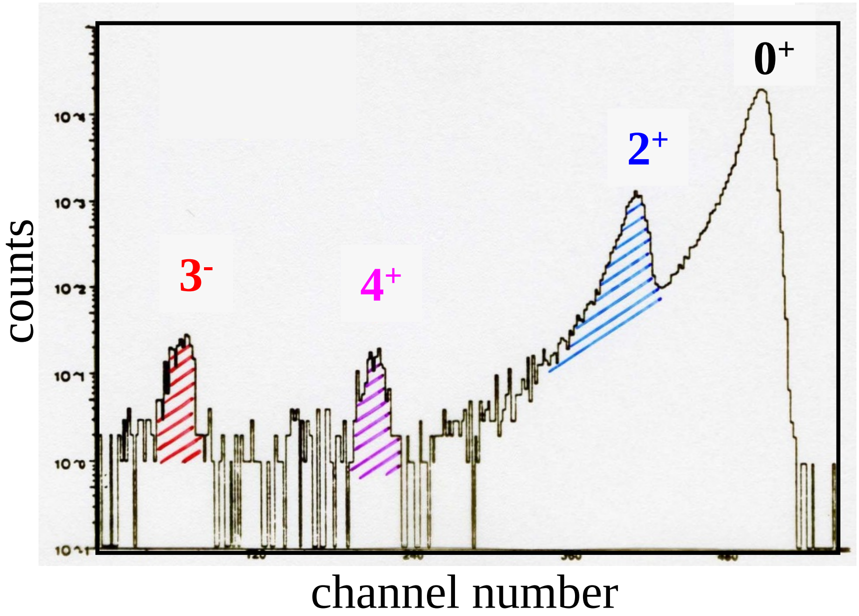
$$\frac{d\sigma_{i\to f}}{d\Omega_{cm}} = P_{i\to f}\cdot\frac{d\sigma_{Ruth}}{d\Omega_{cm}}$$



$$d\sigma_{E2} \cong 4.819\cdot\left(1+\frac{A_1}{A_2}\right)^{-2}\cdot\frac{A_1}{Z_2^2}\cdot E_{MeV}\cdot B(E2;I_i\to I_f)\cdot df_{E2}(\eta,\xi)\;[b]$$

Scattered α-spectrum of ^{226}Ra

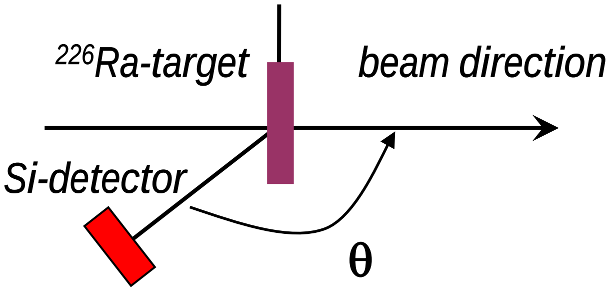




^{4}He → ^{226}Ra

E_α = 16 MeV

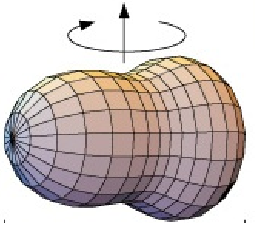
θ_{lab} = 145^0

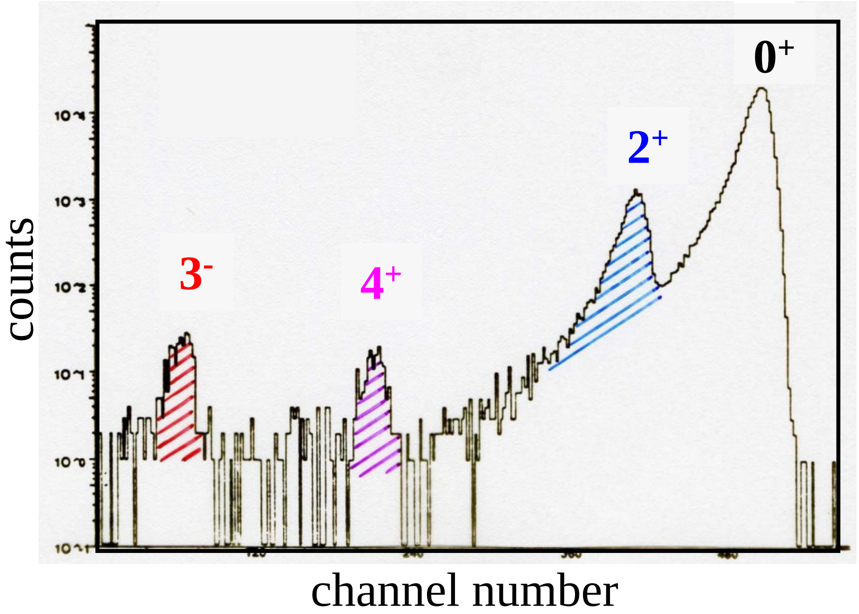


$$P_{i\to f} = \frac{d\sigma_{i\to f}}{d\sigma_{el}} \cong \frac{d\sigma_{i\to f}}{d\sigma_{Ruth}}$$

λ	$\langle\lambda\|M(E\lambda)\|0\rangle$ $[eb^{\lambda/2}]$	β_λ (exp)	β_λ (theo)
2	2.27 (3)	0.165 (2)	0.164
3	1.05 (5)	0.104 (5)	0.112
4	1.04 (7)	0.123 (8)	0.096

Scattered α-spectrum of ^{226}Ra





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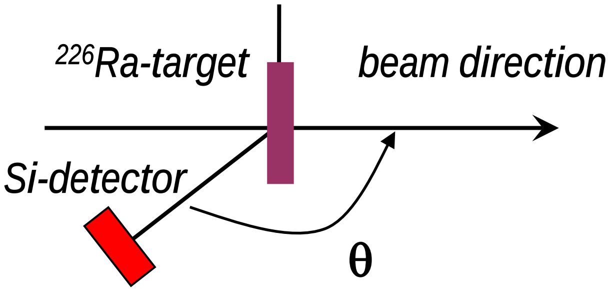

λ	$\langle \lambda \| M(E\lambda) \| 0 \rangle \; [eb^{\lambda/2}]$	β_{λ} (exp)	β_{λ} (theo)
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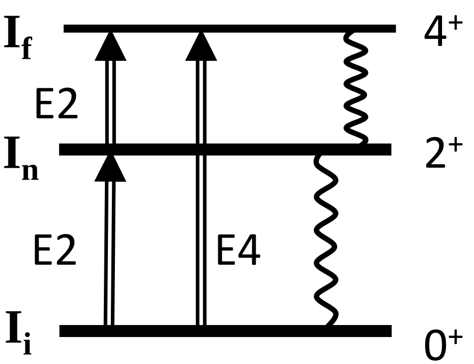
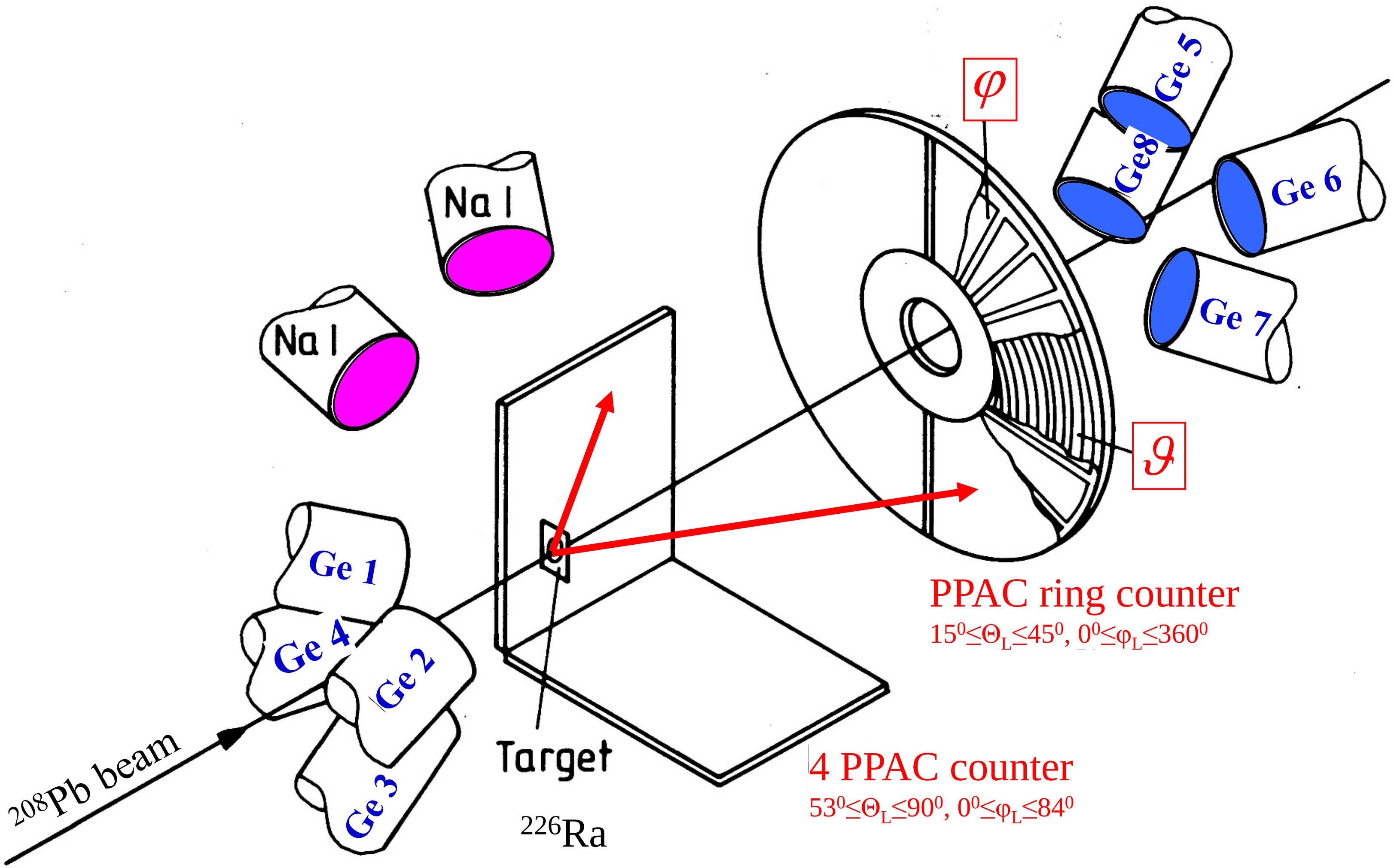
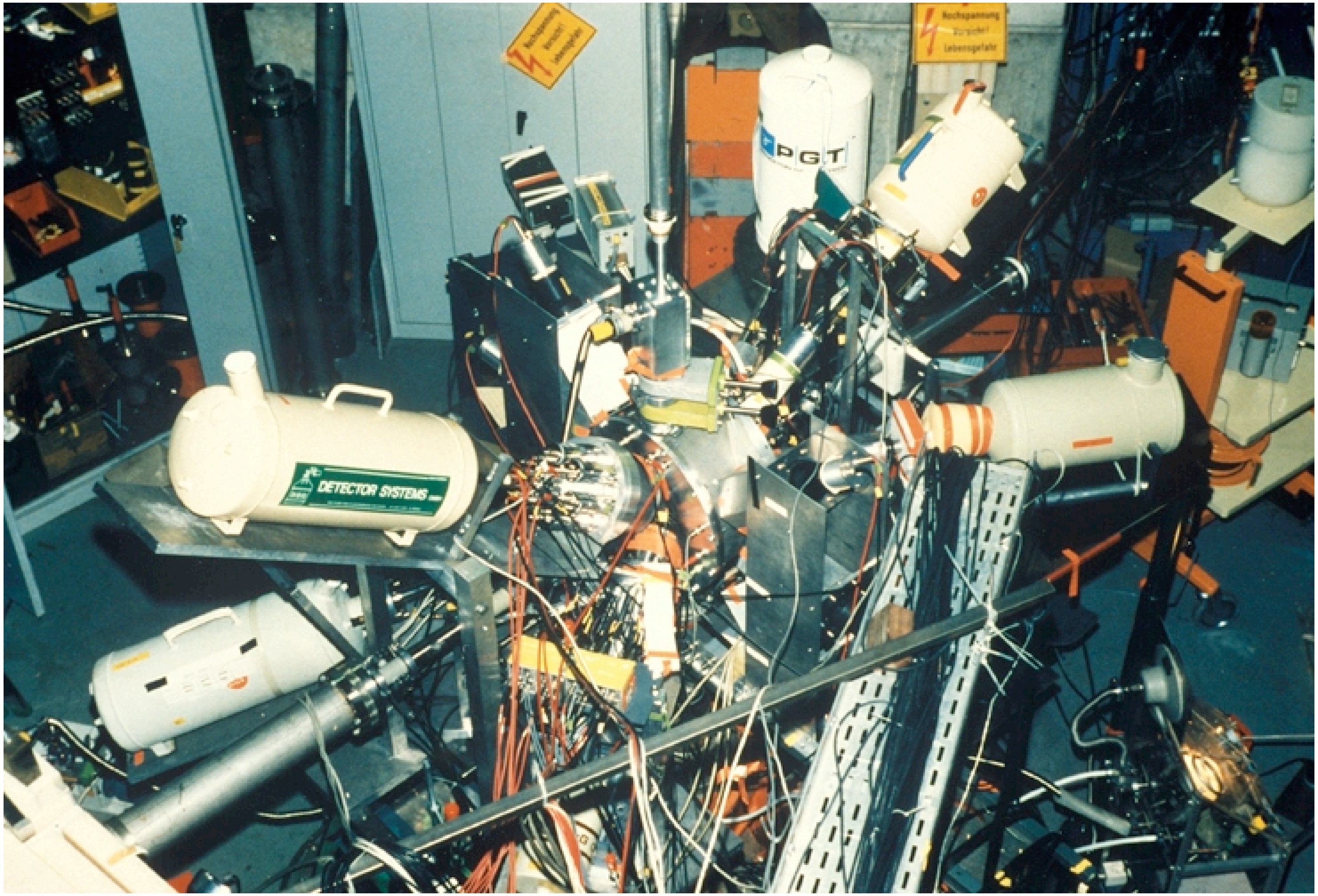


Experimental set-up

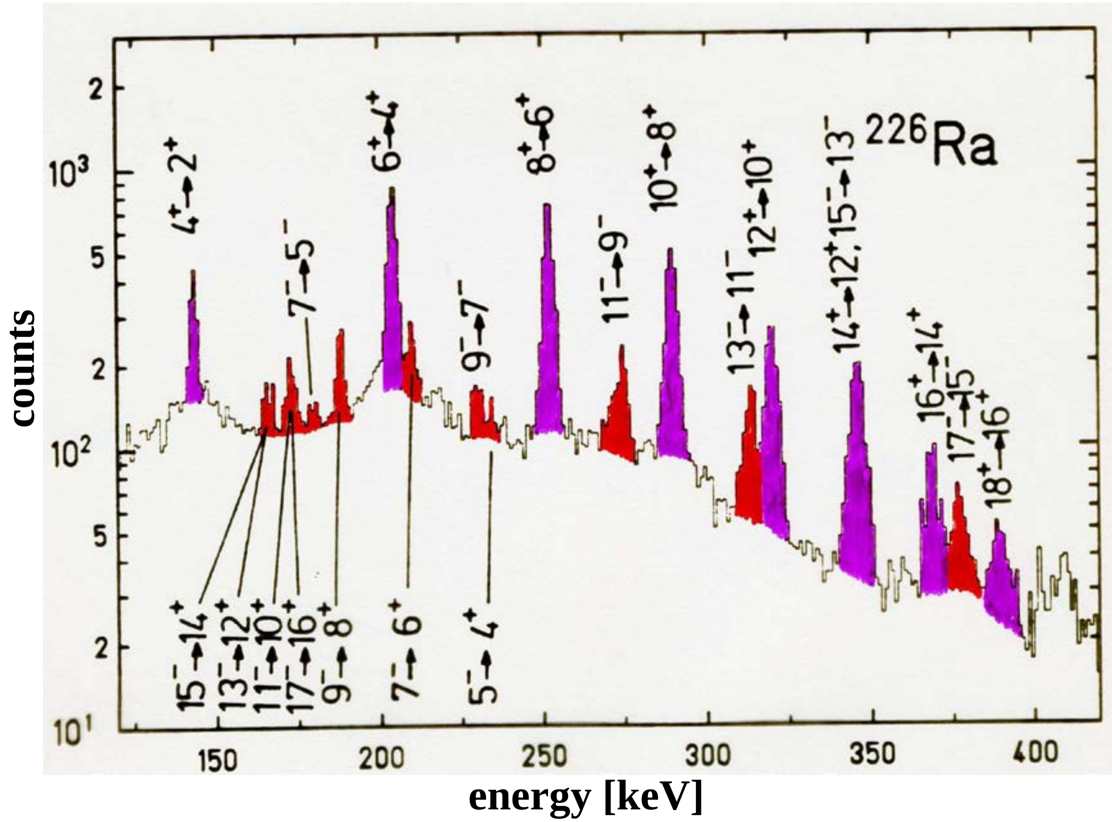


$^{226}RaBr_2$ (400 μg/cm^2) on C-backing (50 μg/cm^2) and covered by Be (40 μg/cm^2)

Coulomb excitation of ^{226}Ra



γ-ray spectrum of ^{226}Ra



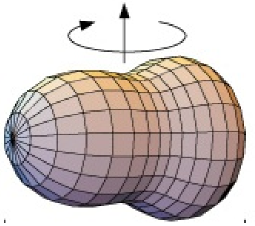
^{208}**Pb** $\rightarrow$ ^{226}**Ra**

$E_{lab} = 4.7$ AMeV

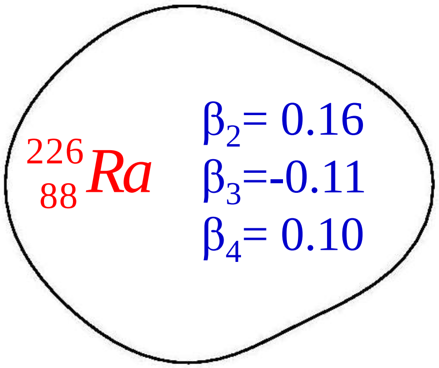
$15^0 \leq \theta_{lab} \leq 45^0$

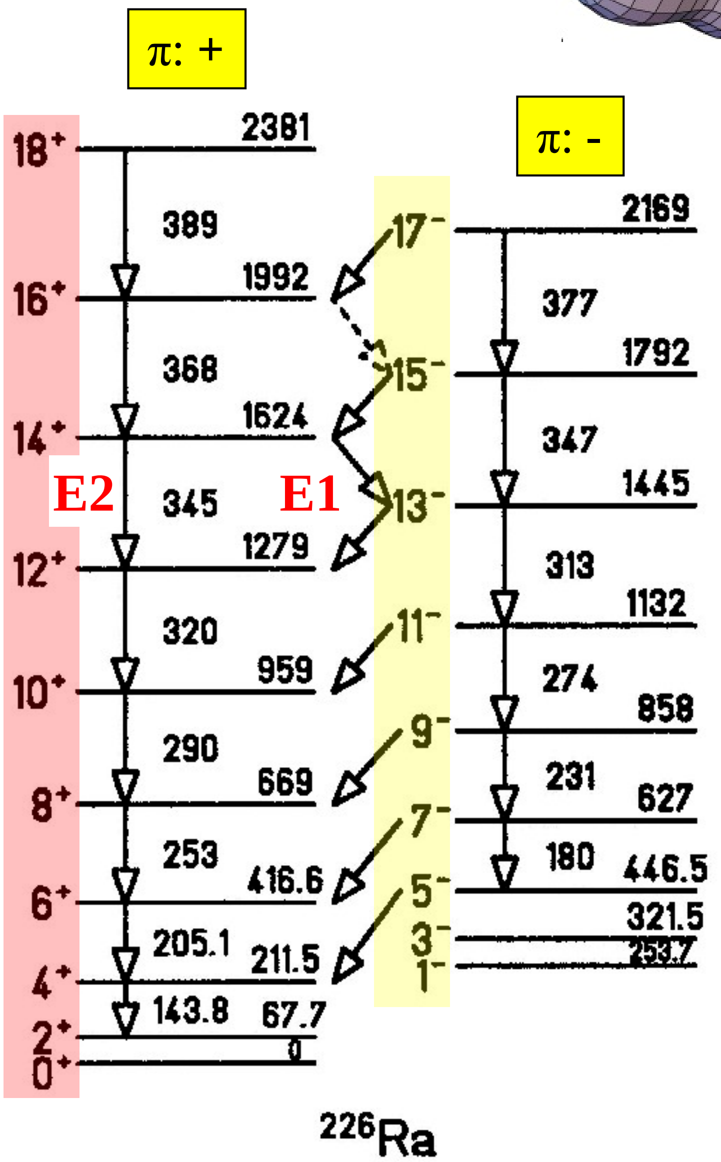
$0^0 \leq \varphi_{lab} \leq 360^0$

Signature of an octupole deformed nucleus



$$R(\theta) = R_0 \cdot \left[1 + \beta_2 \cdot Y_{20}(\theta) + \beta_3 \cdot Y_{30}(\theta) + \beta_4 \cdot Y_{40}(\theta)\right]$$



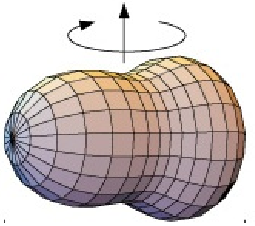


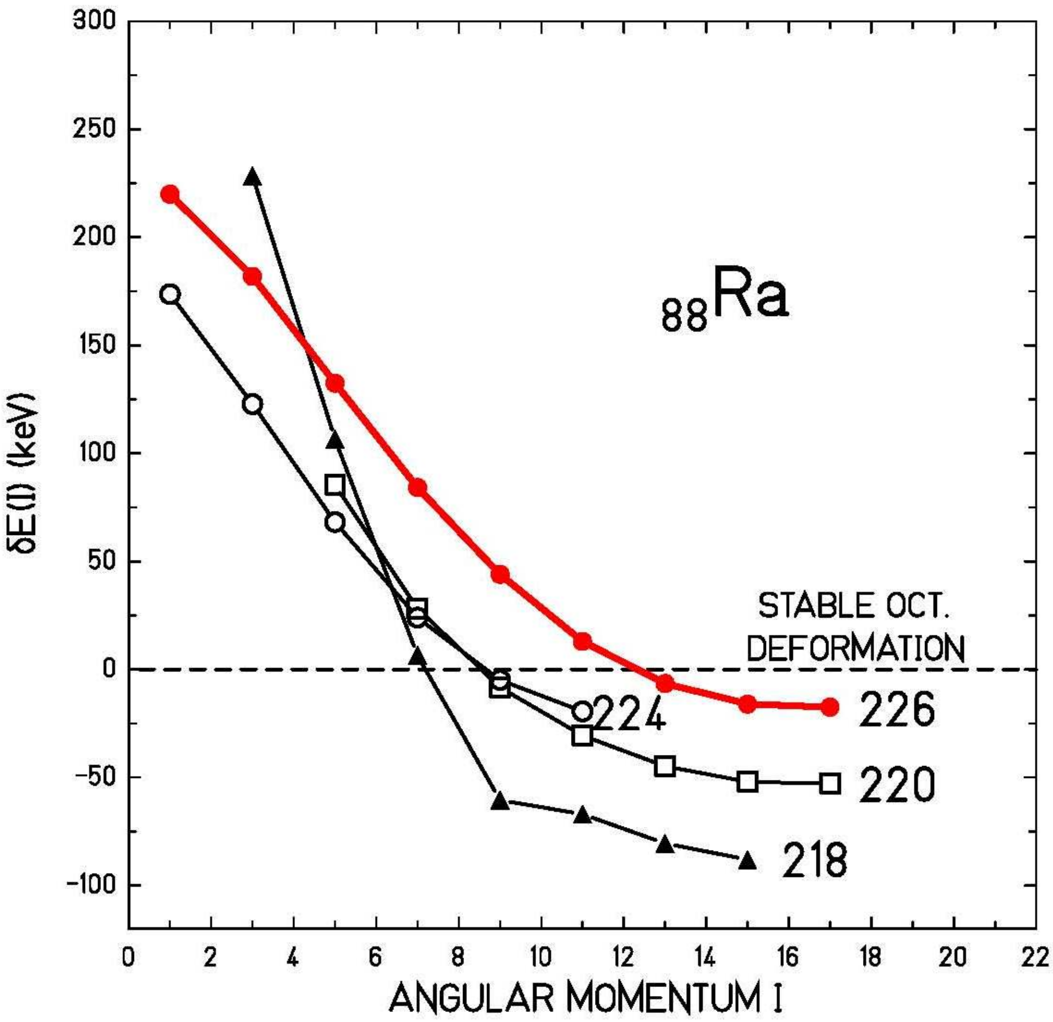
Single rotational band with spin sequence:
$I = 0^+, 1^-, 2^+, 3^-, \ldots$
excitation energy **$E \sim I \cdot (I+1)$**

competition between intraband **E2** and interband **E1** transitions

E1 transition strength **10^{-2} W.u.**

Signature of an octupole deformed nucleus





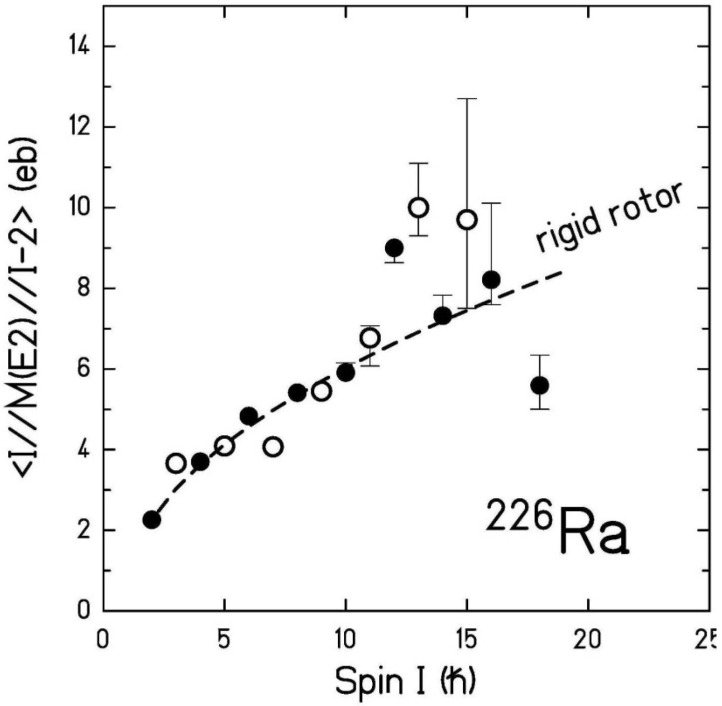
Energy displacement **δE** between the positive- and negative-parity states if they form a single rotational band

$$\delta E(I) = E(I)^{-} - \frac{E(I+1)^{+} + E(I-1)^{+}}{2}$$

$$= -\frac{\hbar^2}{2\Im} \quad \textit{for rigid rotor}$$

W. Nazarewicz et al.; Nucl. Phys. A441 (1985) 420

Electric transition quadrupole moments in ^{226}Ra



○ negative parity states
● positive parity states

rigid rotor model:

$$\langle I-2\|M(E2)\|I\rangle = \sqrt{\frac{15}{32\cdot\pi}}\cdot\sqrt{\frac{I\cdot(I-1)}{2I-1}}\cdot Q_2\cdot e$$

liquid drop:

$$Q_2 = \frac{3\cdot Z\cdot R_0^2}{\sqrt{5\cdot\pi}}\cdot(\beta_2 + 0.360\beta_2^2 + 0.336\beta_3^2 + 0.328\beta_4^2 + 0.967\beta_2\beta_4)\ [fm^2]$$

Q_2(exp) = 750 fm^2

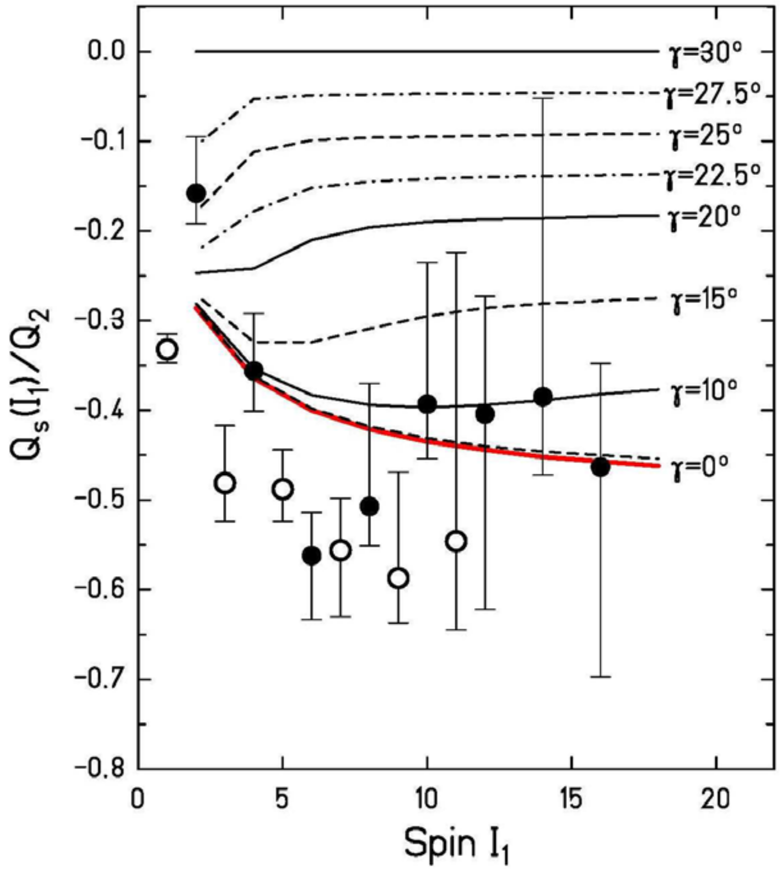
Q_2(theo) = 680 fm^2

$\beta_2 = 0.21$

H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261

W. Nazarewicz et al.; Nucl. Phys. A467 (1987) 437

Static quadrupole moments in ^{226}Ra



○ negative parity states
● positive parity states

rigid rotor model:

$$\frac{Q_S(I)}{Q_0} = \sqrt{\frac{I\cdot(2I-1)}{(I+1)\cdot(2I+1)\cdot(2I+3)}}\cdot\frac{\langle I\|M(E2)\|I\rangle}{\langle 2_1\|M(E2)\|0_1\rangle}$$

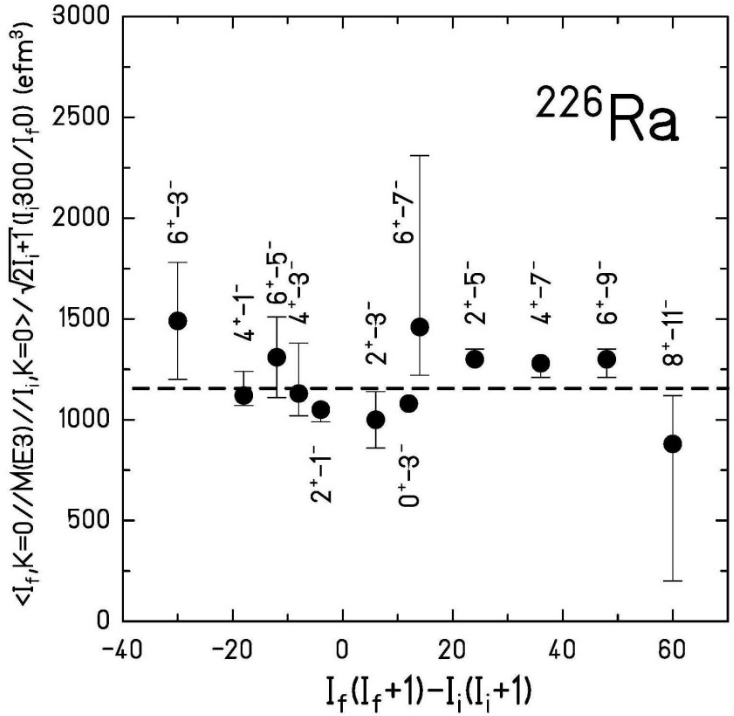
rigid triaxial rotor model:

$$\frac{Q_s(2_1)}{Q_0} = -\frac{6\cdot\cos(3\gamma)}{7\cdot\sqrt{9-8\cdot\sin^2(3\gamma)}}$$

H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261

Davydov and Filippov, Nucl. Phys. 8, 237 (1958)

Electric transition octupole moments in ^{226}Ra



β_3

0.18

0.12

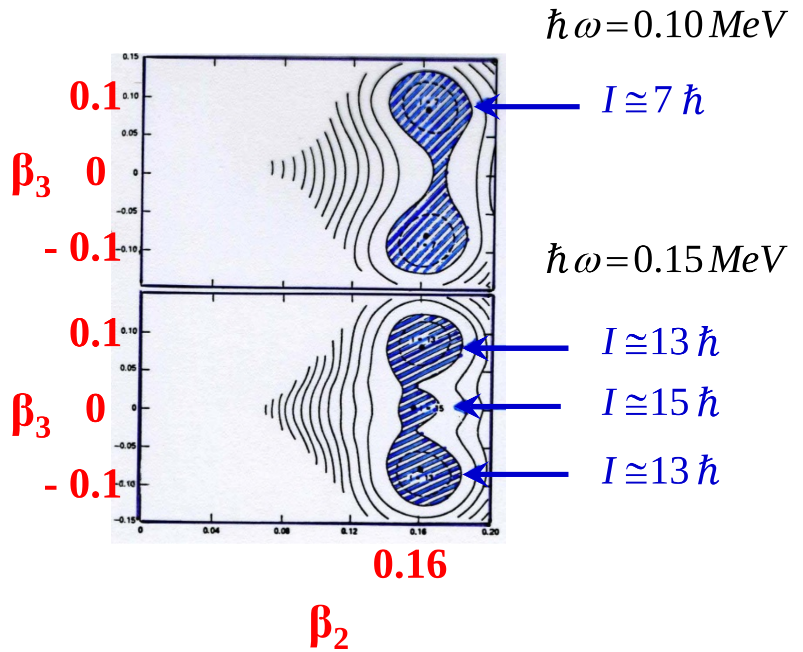
0.06

liquid drop:

$$Q_3 = \frac{3 \cdot Z \cdot R_0^3}{\sqrt{7 \cdot \pi}} \cdot (\beta_3 + 0.841\beta_2\beta_3 + 0.769\beta_3\beta_4)\ [fm^3]$$

$$\langle I-3 \| M(E3) \| I \rangle = -\sqrt{\frac{35}{32\pi}} \cdot \sqrt{\frac{I \cdot (I-1) \cdot (I-2)}{(2I-3) \cdot (2I+3)}} \cdot Q_3 \cdot e$$

$$\langle I-1 \| M(E3) \| I \rangle = \sqrt{\frac{21}{32\pi}} \cdot \sqrt{\frac{(I-1) \cdot I \cdot (I+1)}{(2I-3) \cdot (2I+3)}} \cdot Q_3 \cdot e$$



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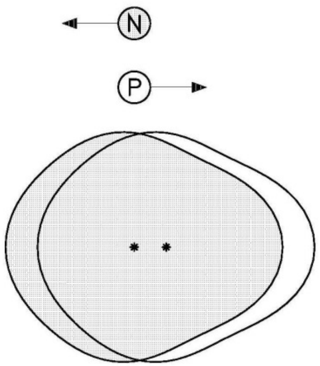
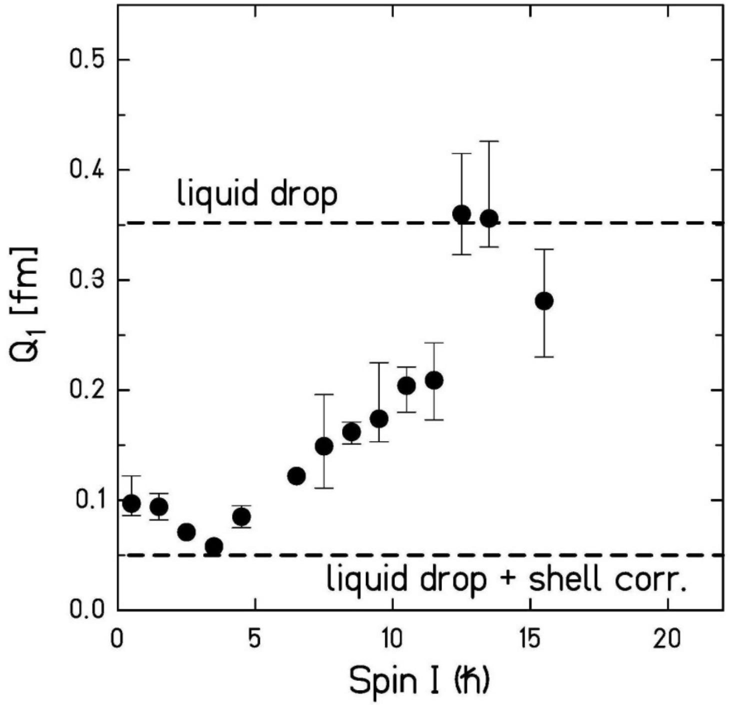
W. Nazarewicz et al.; Nucl. Phys. A467 (1987) 437

Intrinsic electric dipole moments in ^{226}Ra





liquid-drop contribution:

$$Q_1^{LD} = C_{LD} \cdot A \cdot Z \cdot \left(\beta_2\beta_3 + 1.458 \cdot \beta_3\beta_4\right)$$

$$with \;\; C_{LD} = 5.2 \cdot 10^{-4} \;\; [fm]$$

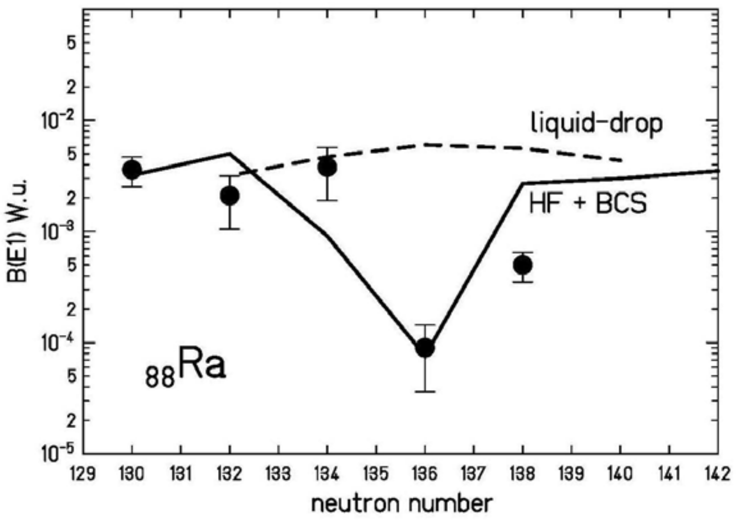
rigid rotor model:

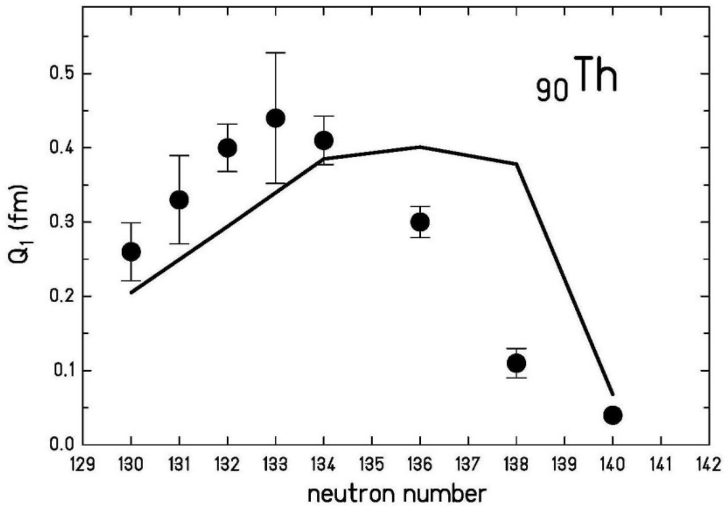
$$\left\langle I-1 \| M(E1) \| I \right\rangle = -\sqrt{\frac{3}{4\pi}} \cdot \sqrt{I} \cdot Q_1 \cdot e$$

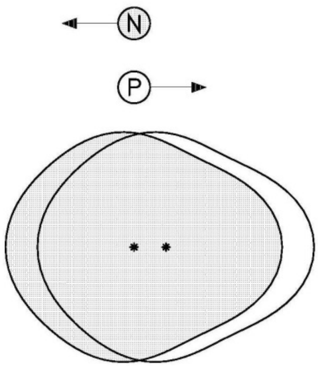
H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261

G. Leander et al.; Nucl. Phys. A453 (1986) 58

Intrinsic electric dipole moments in Ra / Th







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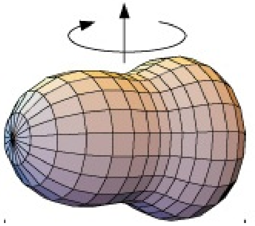
G. Leander et al.; Nucl. Phys. A453 (1986) 58

Summary

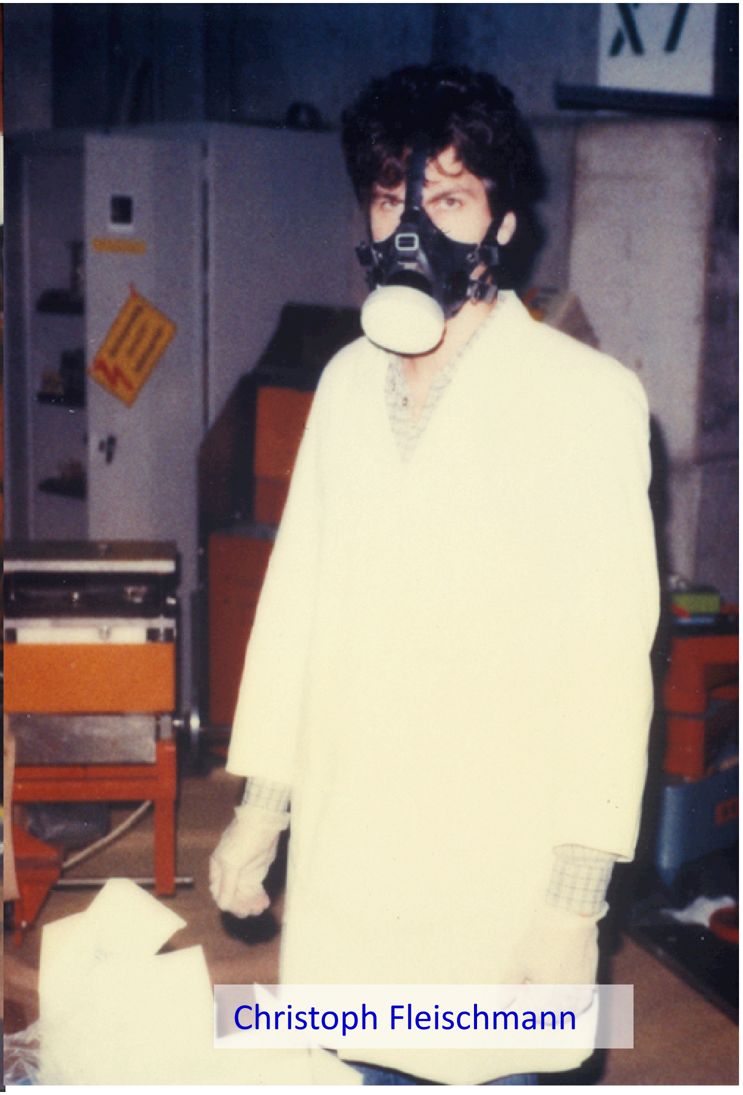


- single rotational band for $I > 10\,\hbar$
- no backbending observed
- β_2, β_3, β_4 deformation parameters are in excellent agreement with calculated values
- octupole deformation is three times larger than in octupole-vibrational nuclei
- equal transition quadrupole moments for positive- and negative-parity states
- static quadrupole moments are in excellent agreement with an axially symmetric shape
- electric dipole moments are close to liquid-drop value ($I > 10\,\hbar$)

- octupole deformation seems to be stabilized with increasing rotational frequency

Coulomb excitation of ^{226}Ra

^{226}Ra target broken after 8 hours



Christoph Fleischmann

Appendix: Center of mass conservation

$$0=\begin{Bmatrix}\int\rho_0\cdot x\cdot d\tau\\ \int\rho_o\cdot y\cdot d\tau\\ \int\rho_{0\cdot z\cdot d\tau}\end{Bmatrix}=\int\rho_0\cdot\vec{r}\cdot d\tau$$

The coordinates (x, y, z) can be expressed by

$$r\cdot Y_{1m}(\theta,\phi)=\begin{Bmatrix}\frac{1}{2}\cdot\sqrt{\frac{3}{\pi}}\cdot z & m=0\\ \mp\sqrt{\frac{3}{2\pi}}\cdot(x\pm iy) & m=\pm 1\end{Bmatrix}$$

$$0=\sqrt{\frac{4\pi}{3}}\cdot\int\rho_0\cdot r\cdot Y_{10}(\theta,\phi)\cdot d\tau=\iint r^3\cdot dr\cdot Y_{10}\cdot d\Omega$$

$$0=\frac{R_0^4}{4}\cdot\int\left\{1+4\sum_{\lambda_1 m_1}\alpha^*_{\lambda_1 m_1}Y_{\lambda_1 m_1}+6\sum_{\lambda_1 m_1\lambda_2 m_2}\alpha^*_{\lambda_1 m_1}\alpha^*_{\lambda_2 m_2}Y_{\lambda_1 m_1}Y_{\lambda_2 m_2}+\cdots\right\}\cdot Y_{10}\cdot d\Omega$$

$$0=4\cdot\alpha^*_{10}+6\sum_{\lambda_1 m_1\lambda_2 m_2}\alpha^*_{\lambda_1 m_1}\alpha^*_{\lambda_2 m_2}\cdot\left[\frac{(2\lambda_1+1)\cdot(2\lambda_2+1)\cdot 3}{4\pi}\right]^{1/2}\cdot\begin{pmatrix}\lambda_1 & \lambda_2 & 1\\ 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\lambda_1 & \lambda_2 & 1\\ m_1 & m_2 & 0\end{pmatrix}$$

$$\alpha^*_{10}=-\frac{3}{2}\sum_{\lambda_1 m_1\lambda_2 m_2}\alpha^*_{\lambda_1 m_1}\alpha^*_{\lambda_2 m_2}\cdot\left[\frac{(2\lambda_1+1)\cdot(2\lambda_2+1)\cdot 3}{4\pi}\right]^{1/2}\cdot\begin{pmatrix}\lambda_1 & \lambda_2 & 1\\ 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\lambda_1 & \lambda_2 & 1\\ m_1 & m_2 & 0\end{pmatrix}$$

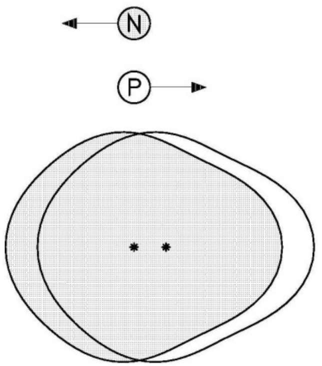
The dipole coordinate is not an independent quantity. It is non-zero for nuclear shapes with both quadrupole and octupole degrees of freedom.

$$\beta_1=-\sqrt{\frac{3}{4\pi}}\cdot\frac{9}{\sqrt{35}}\cdot\beta_2\cdot\beta_3$$

$$\beta_1=-\frac{3}{2}\cdot\left[\frac{5\cdot 7\cdot 3}{4\pi}\right]^{1/2}\cdot\begin{pmatrix}2 & 3 & 1\\ 0 & 0 & 0\end{pmatrix}^2\cdot\{\beta_2\cdot\beta_3+\beta_3\cdot\beta_2\}$$

$$\begin{pmatrix}2 & 3 & 1\\ 0 & 0 & 0\end{pmatrix}=-\sqrt{\frac{3}{35}}$$

Appendix: Intrinsic electric dipole moment



$$Q_1^{LD} = e \cdot \int z \cdot \rho_{proton} \cdot d\tau = \sqrt{\frac{4\pi}{3}} \cdot e \cdot \iint \rho_p \cdot r^3 \cdot dr \cdot Y_{10}(\theta,\phi) \cdot d\Omega$$

The local volume polarization of electric charge can be derived from the requirement of a minimum in the energy functional. (Myers Ann. of Phys. (1971))

$$\frac{\rho_{proton} - \rho_{neutron}}{\rho_{proton} + \rho_{neutron}} = -\frac{1}{4C_{LD}} \cdot e \cdot V_C(r) \qquad V_C(r) = \left\{ \frac{3}{2} - \frac{1}{2} \cdot \left(\frac{r}{R_0}\right)^2 + \sum_{l=1}^{\infty} \frac{3}{2l+1} \left(\frac{r}{R_0}\right)^l \cdot \beta_l \cdot Y_{l0} \right\} \cdot \frac{Ze}{R_0}$$

where $\boldsymbol{\rho_p}$ and $\boldsymbol{\rho_n}$ are the proton and neutron densities, $\mathbf{C_{LD}}$ is the volume symmetry energy coefficient of the liquid drop model and $\mathbf{V_C}$ is the Coulomb potential generated by $\boldsymbol{\rho_p}$ inside the nucleus ($r < R_0$)

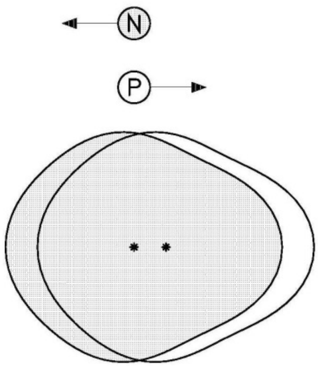
$$\rho_p = -\frac{\rho_0}{4 \cdot C_{LD}} \cdot e \cdot V_C(r) + \rho_n \qquad \rho_0 = \rho_p + \rho_n \qquad \rho_p = \frac{\rho_0}{2} \cdot \left[1 - \frac{1}{4 \cdot C_{LD}} \cdot e \cdot V_C(r)\right]$$

$$V_C(r) = \left\{ \frac{3}{2} - \frac{1}{2} \cdot \left(\frac{r}{R_0}\right)^2 + \left(\frac{r}{R_0}\right) \cdot \beta_1 \cdot Y_{10} + \frac{3}{5} \cdot \left(\frac{r}{R_0}\right)^2 \cdot \beta_2 \cdot Y_{20} + \frac{3}{7} \cdot \left(\frac{r}{R_0}\right)^3 \cdot \beta_3 \cdot Y_{30} \right\} \cdot \frac{Ze}{R_0}$$

Keeping the center of gravity fixed, the integral $0 = \iint \rho_0 \cdot r^3 \cdot dr \cdot Y_{10} \cdot d\Omega$

$$\beta_1 = -\sqrt{\frac{3}{4\pi}} \cdot \frac{9}{\sqrt{35}} \cdot \beta_2 \cdot \beta_3$$

Appendix: Intrinsic electric dipole moment



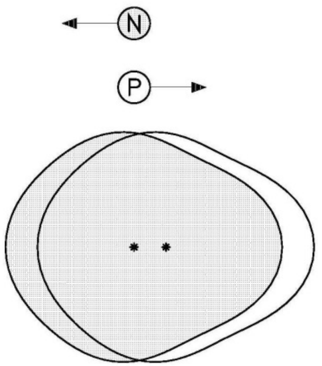
$$Q_1^{LD} = \sqrt{\frac{4\pi}{3}} \cdot e \cdot \iint \frac{\rho_0}{2} \cdot \left[1 - \frac{1}{4C_{LD}} \cdot e \cdot V_C(r)\right] \cdot r^3 \cdot dr \cdot Y_{10}(\theta,\phi) \cdot d\Omega$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \frac{3\cdot A}{8\cdot\pi\cdot R_0^3} \cdot \frac{1}{4C_{LD}} \frac{Z}{R_0} \cdot \iint \left\{\frac{3}{2} - \frac{1}{2}\cdot\left(\frac{r}{R_0}\right)^2 + \left(\frac{r}{R_0}\right)\cdot\beta_1\cdot Y_{10} + \frac{3}{5}\cdot\left(\frac{r}{R_0}\right)^2\cdot\beta_2\cdot Y_{20} + \frac{3}{7}\cdot\left(\frac{r}{R_0}\right)^3\cdot\beta_3\cdot Y_{30}\right\} \cdot r^3 \cdot dr \cdot Y_{10}\cdot d\Omega$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \frac{3\cdot A}{8\cdot\pi\cdot R_0^3} \cdot \frac{1}{4C_{LD}} \cdot \frac{Z}{R_0} \cdot \left\{ \begin{array}{l} \frac{3}{2}\frac{R_0^4}{4}\int\left[1 + 4\cdot(\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30}) + 6\cdot(\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30})^2 + \cdots\right]\cdot Y_{10}\cdot d\Omega \\ -\frac{1}{2}\frac{R_0^4}{6}\int\left[1 + 6\cdot(\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30}) + 15\cdot(\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30})^2 + \cdots\right]\cdot Y_{10}\cdot d\Omega \\ +\frac{R_0^4}{5}\int\left[\beta_1 Y_{10} + 5\cdot(\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30})\cdot\beta_1 Y_{10} + \cdots\right]\cdot Y_{10}\cdot d\Omega \\ +\frac{3}{5}\frac{R_0^4}{6}\int\left[\beta_2 Y_{20} + 6\cdot(\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30})\cdot\beta_2 Y_{20} + \cdots\right]\cdot Y_{10}\cdot d\Omega \\ +\frac{3}{7}\frac{R_0^4}{7}\int\left[\beta_3 Y_{30} + 7\cdot(\beta_1 Y_{10} + \beta_2 Y_{20} + \beta_3 Y_{30})\cdot\beta_3 Y_{30} + \cdots\right]\cdot Y_{10}\cdot d\Omega \end{array} \right\}$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \frac{3\cdot A}{8\cdot\pi\cdot R_0^3} \cdot \frac{1}{4C_{LD}} \cdot \frac{Z}{R_0} \cdot \left\{ \begin{array}{l} \frac{3}{2}\frac{R_0^4}{4}\left[4\beta_1 + 6\cdot\int(2\beta_1\beta_2 Y_{10}Y_{20} + 2\beta_2\beta_3 Y_{20}Y_{30})\cdot Y_{10}\cdot d\Omega\right] \\ -\frac{1}{2}\frac{R_0^4}{6}\left[6\beta_1 + 15\cdot\int(2\beta_1\beta_2 Y_{10}Y_{20} + 2\beta_2\beta_3 Y_{20}Y_{30})\cdot Y_{10}\cdot d\Omega\right] \\ +\frac{R_0^4}{5}\left[\beta_1 + 5\cdot\int\beta_1\beta_2 Y_{10}Y_{20}\cdot Y_{10}\cdot d\Omega\right] \\ +\frac{3}{5}\frac{R_0^4}{6}\left[6\cdot\int(\beta_1\beta_2 Y_{10}Y_{20} + \beta_2\beta_3 Y_{20}Y_{30})\cdot Y_{10}\cdot d\Omega\right] \\ +\frac{3}{7}\frac{R_0^4}{7}\left[7\cdot\int\beta_2\beta_3 Y_{20}Y_{30}\cdot Y_{10}\cdot d\Omega\right] \end{array} \right\}$$

Appendix: Intrinsic electric dipole moment



$$\int Y_{20} Y_{30} Y_{10} \cdot d\Omega = \sqrt{\frac{5\cdot 7\cdot 3}{4\pi}} \begin{pmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \end{pmatrix}^2 = \sqrt{\frac{3}{4\pi}} \frac{3}{\sqrt{35}}$$

$$Q_1^{LD} = -\sqrt{\frac{4\pi}{3}} \cdot e^3 \cdot \frac{3\cdot A\cdot Z}{32\cdot\pi\cdot C_{LD}} \cdot \left\{ \begin{array}{c} \frac{3}{2}\beta_1 + \frac{9}{2}\sqrt{\frac{3}{4\pi}}\frac{3}{\sqrt{35}}\beta_2\beta_3 \\ -\frac{1}{2}\beta_1 + \frac{5}{2}\sqrt{\frac{3}{4\pi}}\frac{3}{\sqrt{35}}\beta_2\beta_3 \\ +\frac{1}{5}\beta_1 \\ +\frac{3}{5}\sqrt{\frac{3}{4\pi}}\frac{3}{\sqrt{35}}\beta_2\beta_3 \\ +\frac{3}{7}\sqrt{\frac{3}{4\pi}}\frac{3}{\sqrt{35}}\beta_2\beta_3 \end{array} \right\}$$

$$Q_1^{LD} = e^3 \cdot \frac{3\cdot A\cdot Z}{32\cdot\pi\cdot C_{LD}} \cdot \frac{60}{35\cdot\sqrt{35}} \cdot \beta_2 \cdot \beta_3$$

$$Q_1^{LD} = 0.01245 \cdot \frac{e\cdot A\cdot Z}{C_{LD}} \cdot \beta_2 \cdot \beta_3 \quad [fm]$$

$C_{LD} \approx 20$ MeV