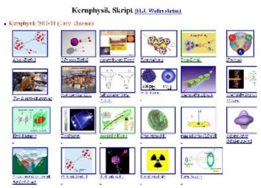
Outline: Hexadecapole collectivity

Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

web-page: https://web-docs.gsi.de/~wolle/ and click on

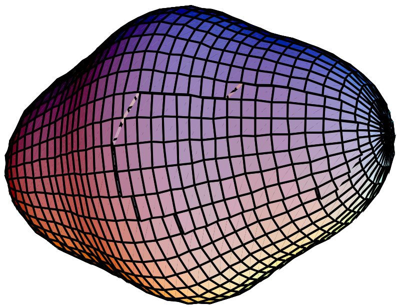


1. Coulomb excitation
2. α-particle spectroscopy
3. E2 and E4 excitation of the 4^+ state
4. hydrodynamical model

Shape parameterization

$$R(\theta,\phi) = R_0 \cdot \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} \cdot Y_{\lambda\mu}(\theta,\phi)\right]$$

Axially symmetric hexadecapole

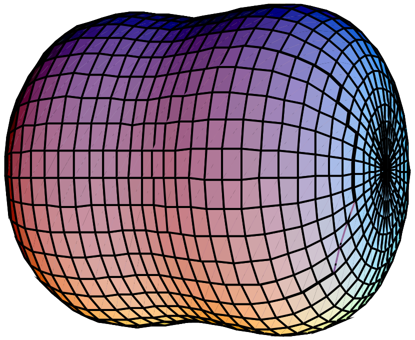


$\lambda = 4$

$\alpha_{40} > 0$, $\alpha_{4\pm1,2,3,4} = 0$

$\alpha_{20} \neq 0$, $\alpha_{2\pm1,2} = 0$

Axially symmetric hexadecapole



$\lambda = 4$

$\alpha_{40} < 0$, $\alpha_{4\pm1,2,3,4} = 0$

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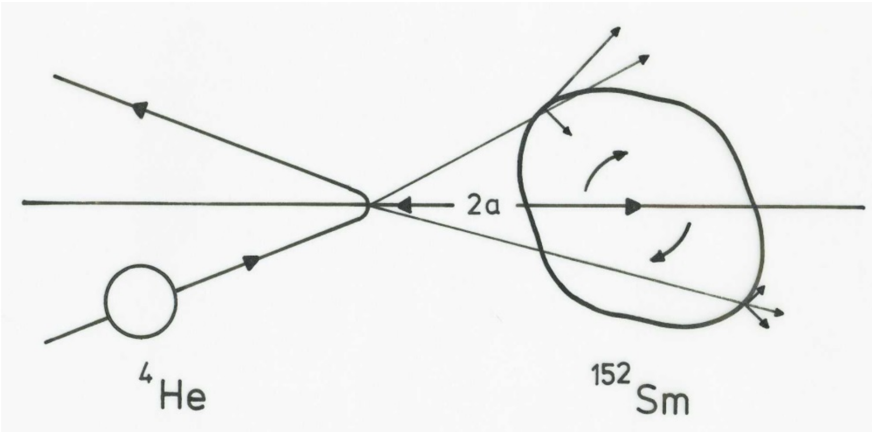
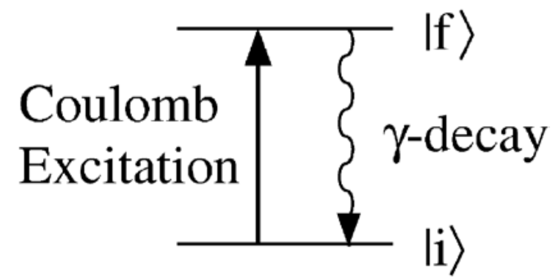
Coulomb excitation

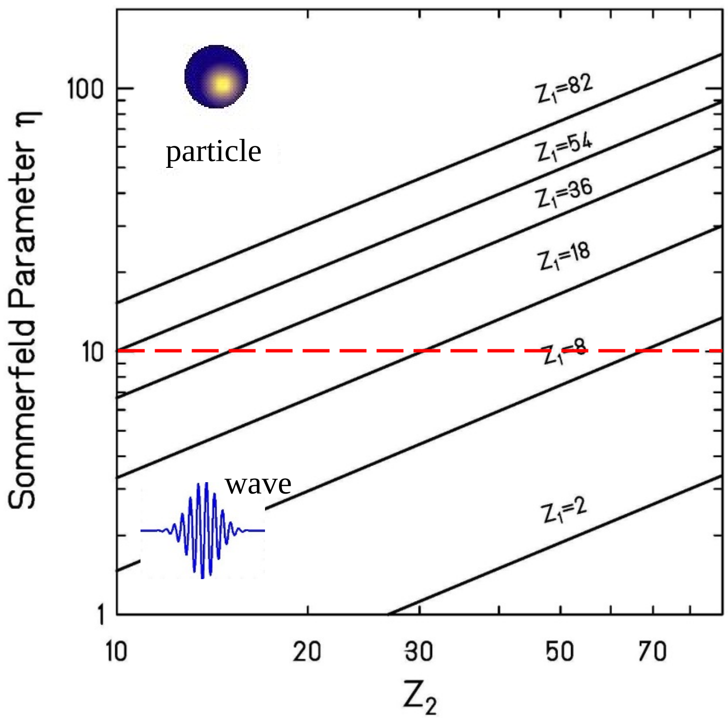




Sommerfeld parameter:

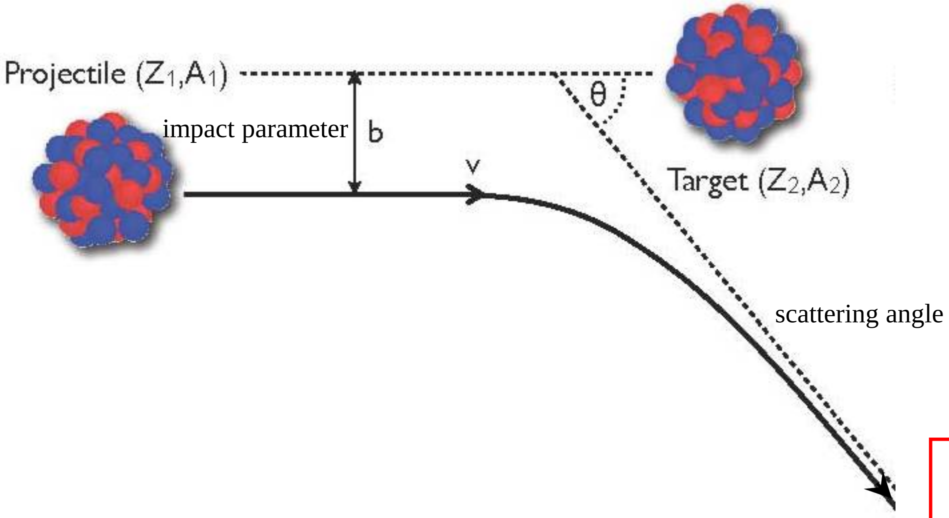
$$\eta = a \cdot k_\infty = \frac{Z_p \cdot Z_t \cdot e^2}{\hbar \cdot v_\infty} \gg 1$$

η >> 1 requirement for a (semi-) classical treatment of equations of motion (hyperbolic trajectories)



^{4}He (**Z_1=2**) projectiles behave like waves
quantummechanical analysis is needed

Classical Coulomb trajectory



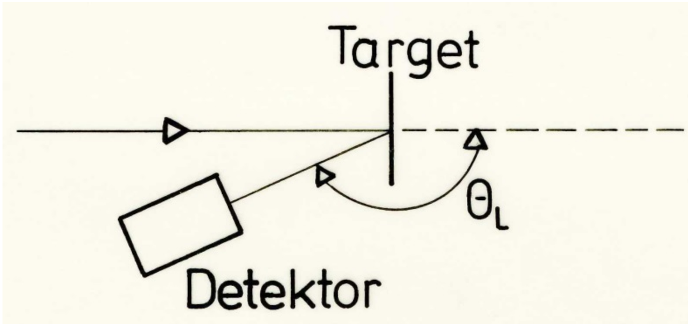
Hyperbolic trajectory:

$$r = a \cdot [\varepsilon \cdot \cosh w + 1] \qquad t = \frac{a}{v_\infty} \cdot [\varepsilon \cdot \sinh w + w]$$

$\varepsilon = \sin^{-1}(\theta_{cm}/2)$ eccentricity of orbit

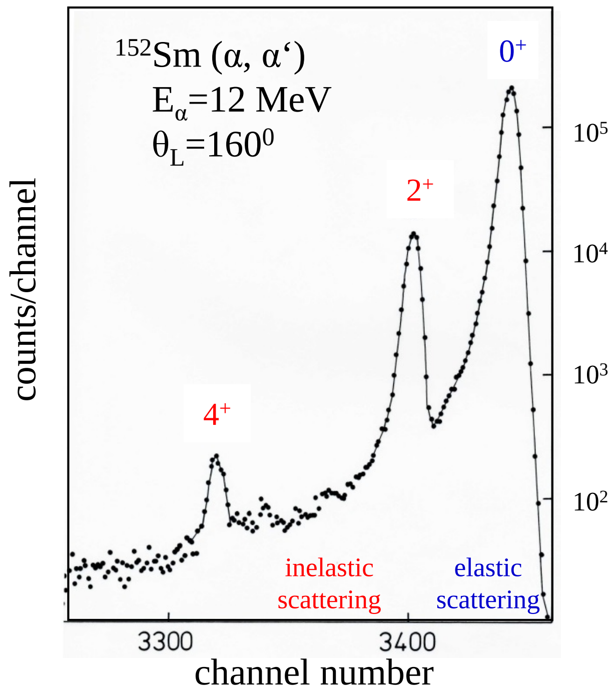
- distance of closest approach: $D(\theta_{cm}) = \frac{a}{\gamma} \cdot \left[1 + \sin^{-1}\left(\frac{\theta_{cm}}{2}\right)\right]$
- impact parameter: $b = \frac{a}{\gamma} \cdot \cot\left(\frac{\theta_{cm}}{2}\right)$
- angular momentum : $\ell = \eta \cdot \cot\left(\frac{\theta_{cm}}{2}\right)$

Coulomb excitation

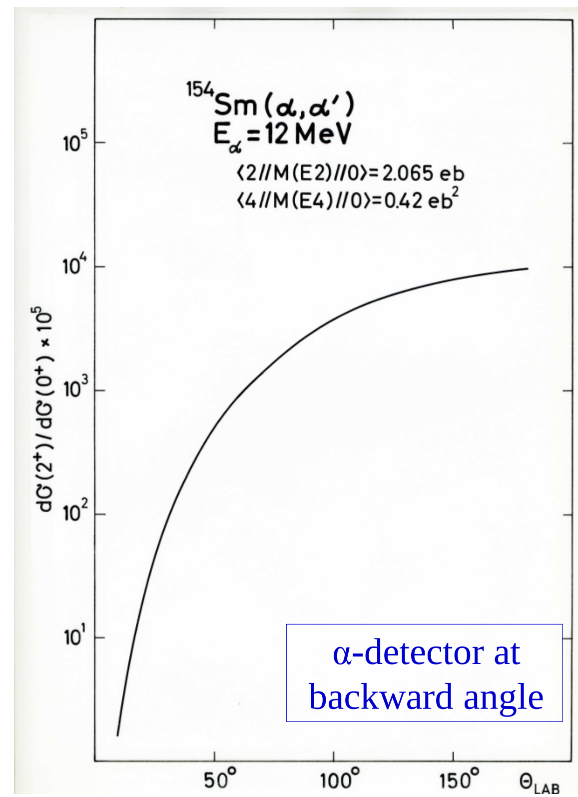


$$\frac{d\sigma_{i\to f}}{d\Omega_{cm}} = P_{i\to f} \cdot \frac{d\sigma_{Ruth}}{d\Omega_{cm}}$$

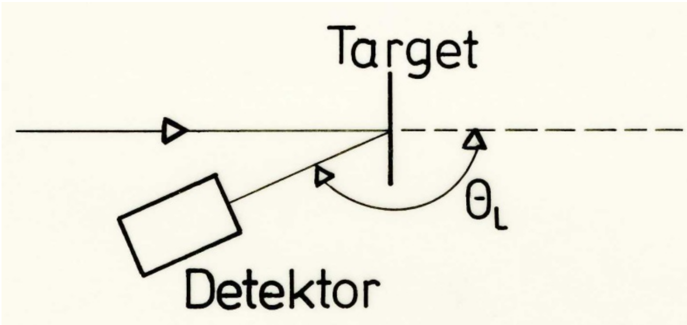
α-particle spectroscopy

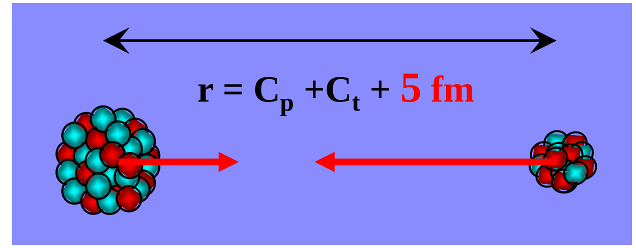


excitation probability $P_{i\to f}$

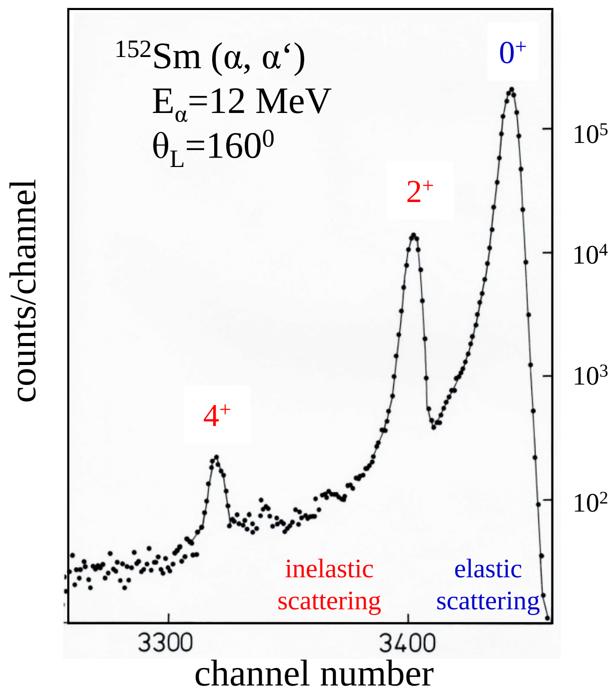


Coulomb excitation

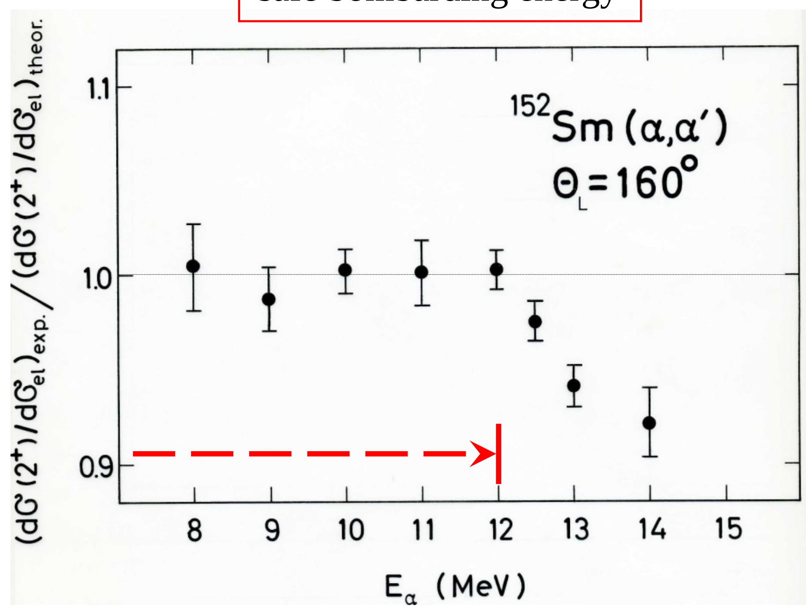




α-particle spectroscopy

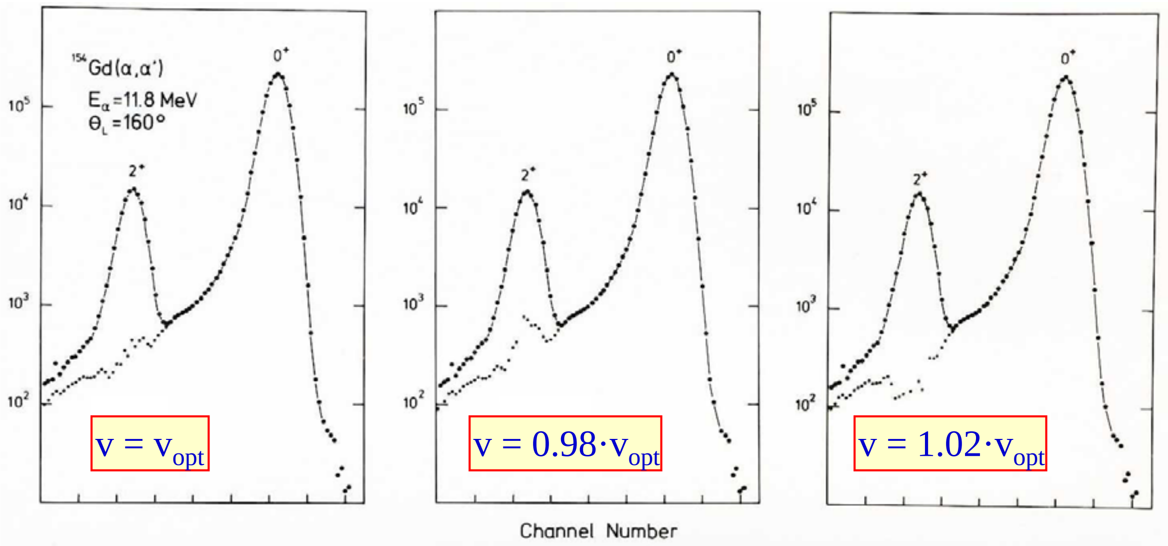


safe bombarding energy



$$E_{lab} = \frac{0.72 \cdot Z_p \cdot Z_t}{C_p + C_t + \Delta} \cdot \frac{A_p + A_t}{A_t} \cdot \left[\sin^{-1}\left(\frac{\theta_{cm}}{2}\right) + 1 \right]$$

Experimental excitation energy



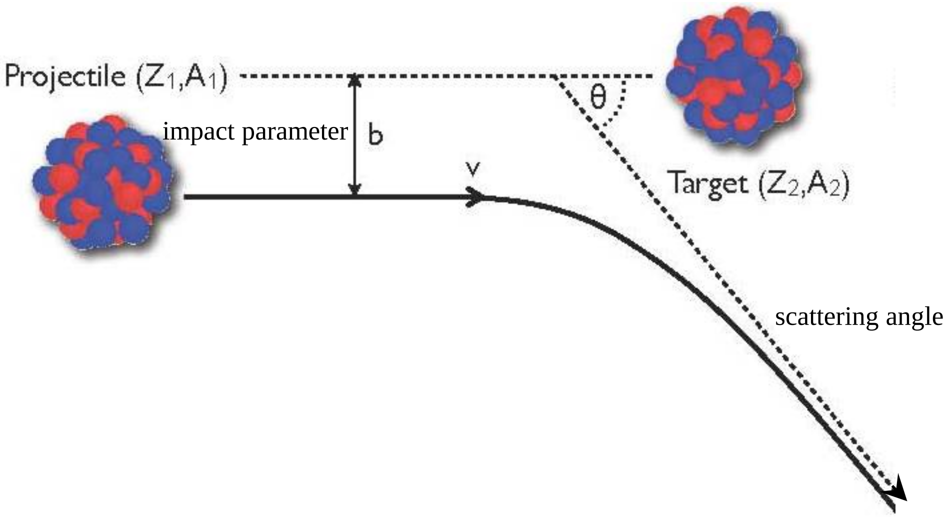
$$r(x) = f(x) + v \cdot f(x+d)$$ r(x) ≡ measured spectrum

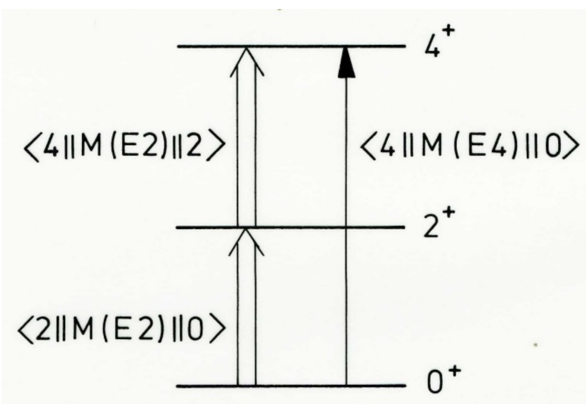
$$f(x) = r(x) - v \cdot f(x+d)$$ f(x) ≡ lineshape of elastic scattering

$$v = \frac{d\sigma_{0\to 2}}{d\sigma_{el}} \cong \frac{d\sigma_{0\to 2}}{d\sigma_{Ruth}} = P_{2^+}$$

$$d\sigma_{E2} \cong 4.819 \cdot \left(1 + \frac{A_1}{A_2}\right)^{-2} \cdot \frac{A_1}{Z_2^2} \cdot E_{MeV} \cdot B(E2; I_i \to I_f) \cdot df_{E2}(\eta, \xi)\ [b]$$

Double-step E2 vs E4 excitation of 4^+ state





reduced transition matrix elements:

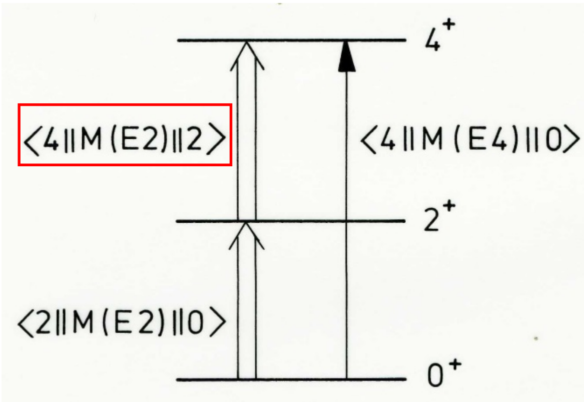
$$\langle I \| M(E\lambda) \| 0 \rangle = \sqrt{\frac{16\pi}{2\lambda+1}} \cdot Q_\lambda \cdot e$$

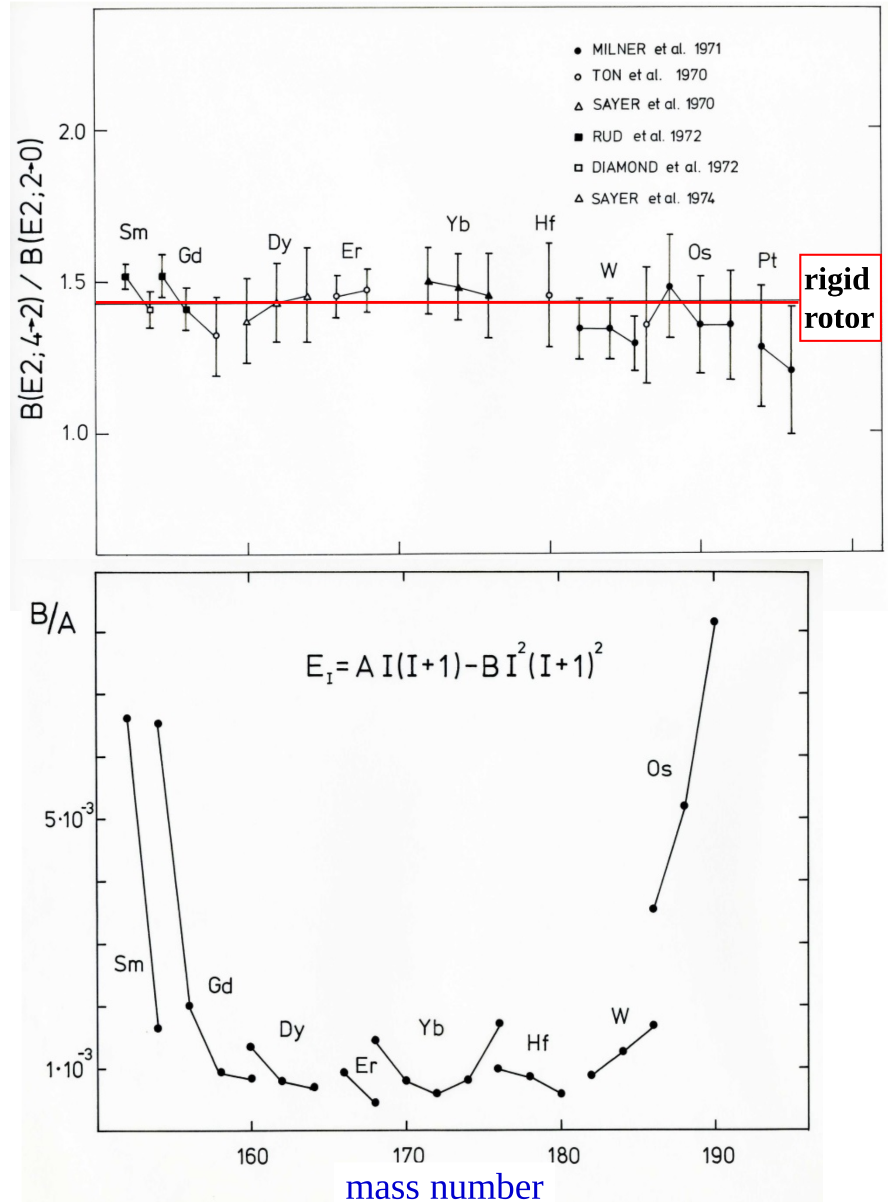
multipole moments Q_λ **(liquid drop):**

$$Q_2 = \frac{3 \cdot Z \cdot R_0^2}{\sqrt{5 \cdot \pi}} \cdot (\beta_2 + 0.360\beta_2^2 + 0.336\beta_3^2 + 0.328\beta_4^2 + 0.967\beta_2\beta_4) \; [fm^2]$$

$$Q_4 = \frac{Z \cdot R_0^4}{\sqrt{\pi}} \cdot (\beta_4 + 0.725\beta_2^2 + 0.462\beta_3^2 + 0.411\beta_4^2 + 0.983\beta_2\beta_4) \; [fm^4]$$

Double-step E2 excitation and rigid rotor model

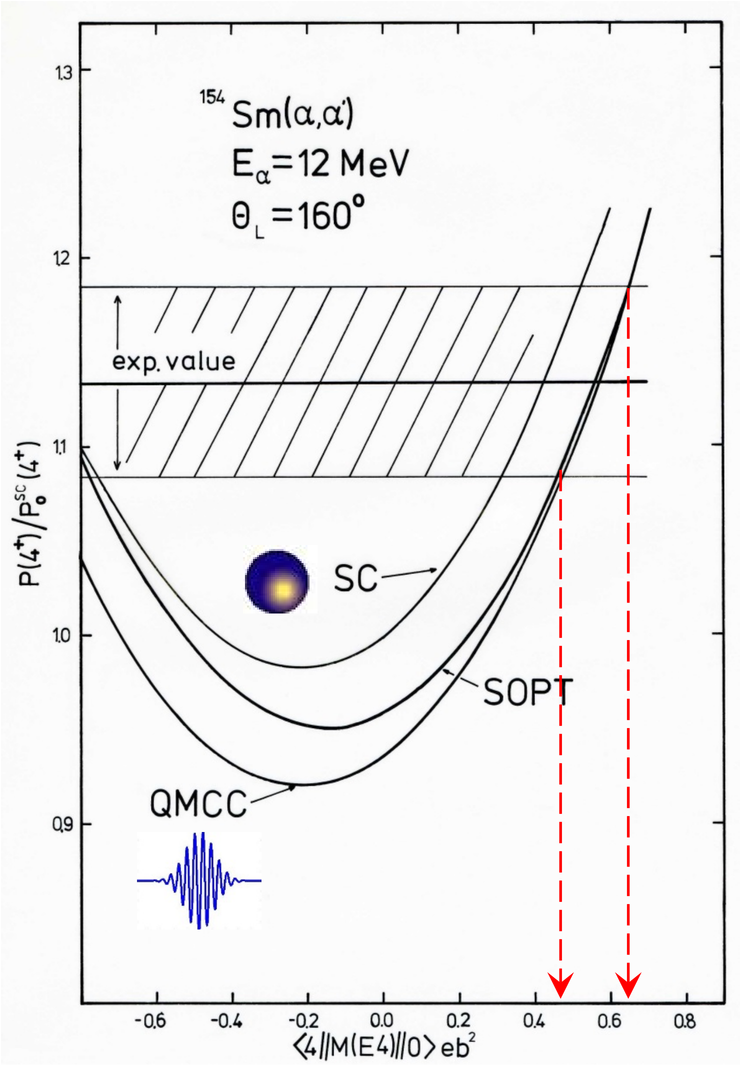




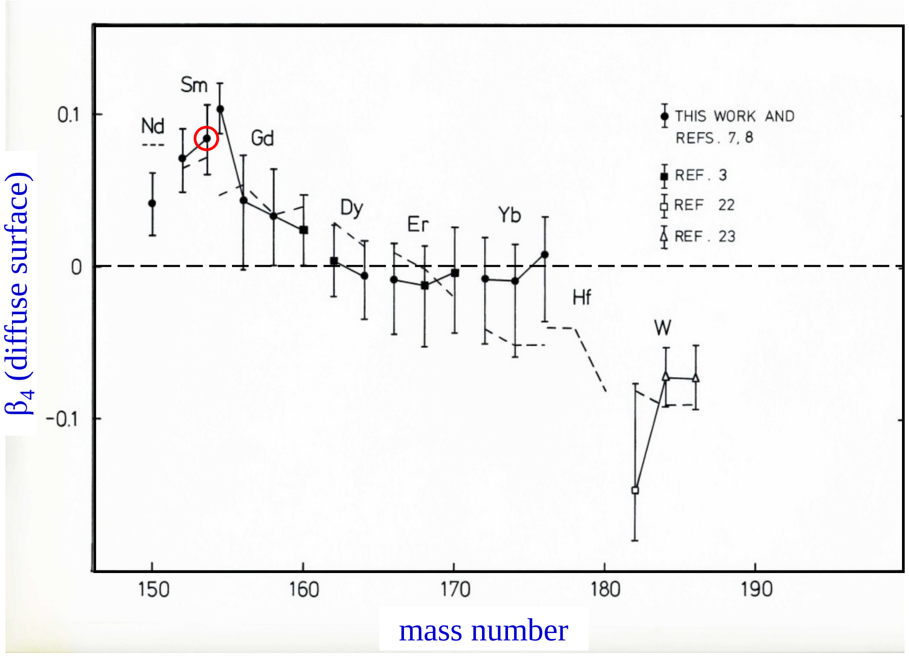
rigid rotor model:

$$\langle I-2\| M(E2)\| I\rangle = \sqrt{\frac{3\cdot I\cdot(I-1)}{2\cdot(2I-1)}}\cdot\frac{3ZeR_0^2\beta_2}{4\pi}$$

Coulomb excitation analysis



$$\langle 4 \| M(E4) \| 0 \rangle = \frac{3 \cdot Z \cdot e \cdot R_0^4}{4 \cdot \pi} \cdot \left(\beta_4 + 0.725\beta_2^2 + 0.411\beta_4^2 + 0.983\beta_2\beta_4 \right)$$



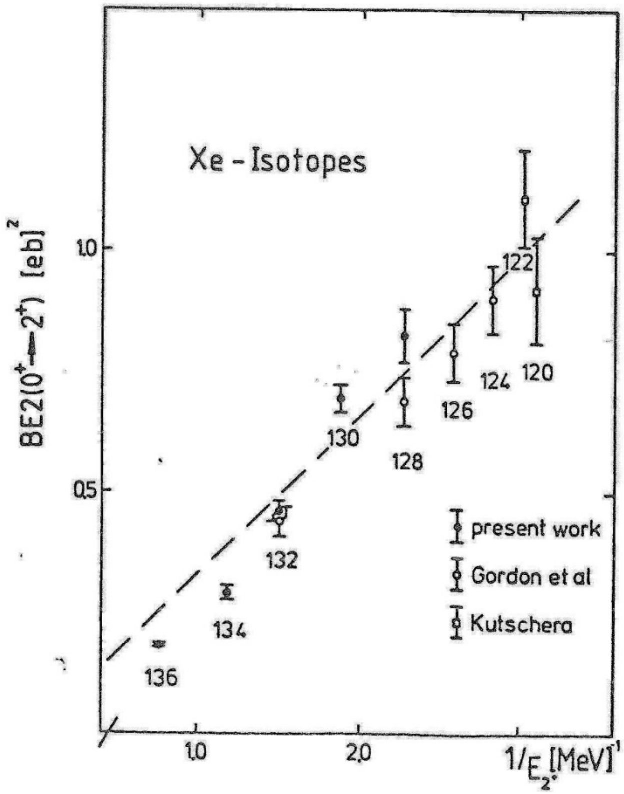
SC ≡ semiclassical SOPT ≡ second order perturbation theory QMCC ≡ quantum mechanical coupled channel

Hydrodynamical model

Reduced transition probability: $B\left(E2;0^{+}\rightarrow 2^{+}\right)=\frac{9Z^{2}e^{2}R_{0}^{4}}{16\pi^{2}}\cdot\beta^{2}$

Excitation energy: $E_{2^{+}}=6\cdot\frac{\hbar^{2}}{2\cdot\Im}$

Moment of inertia: $\Im_{F}=\frac{9}{8\pi}M\,R_{o}^{2}\cdot\beta^{2}$



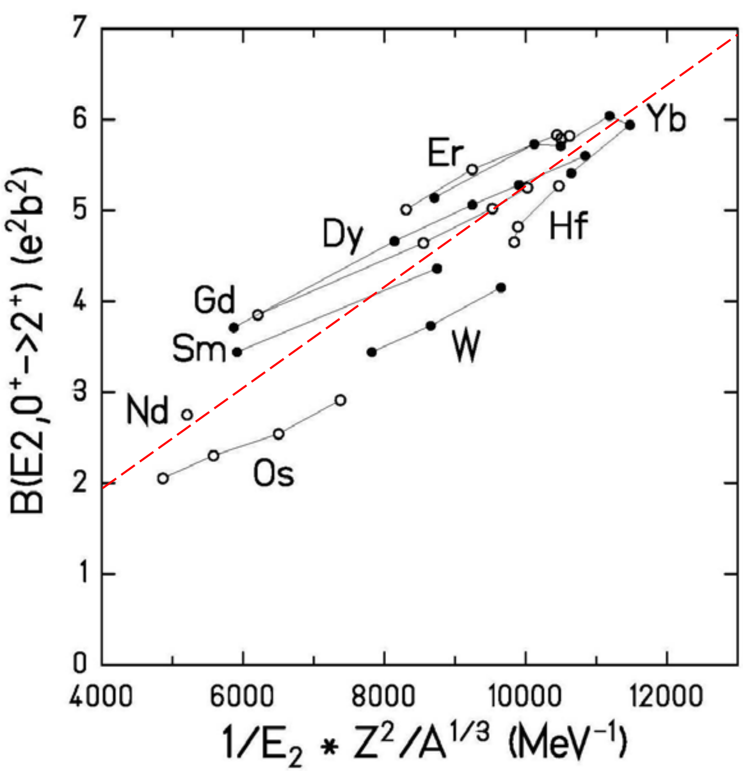
$$B\left(E2;0^{+}\rightarrow 2^{+}\right)=const\cdot\frac{Z^{2}}{A^{1/3}\cdot E_{2^{+}}}$$

REMINDER

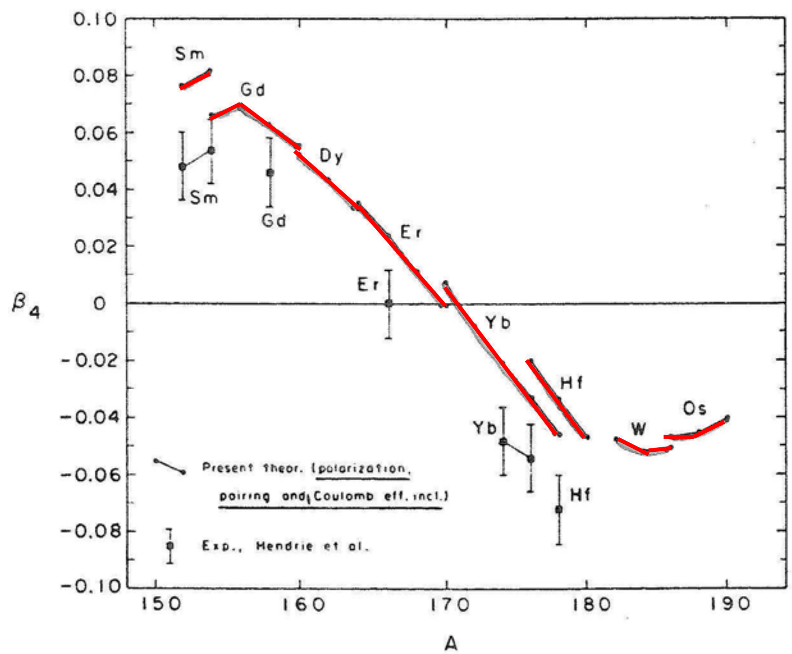
Hydrodynamical model

Reduced transition probability: $B\left(E2;0^{+}\rightarrow 2^{+}\right)=\left(\frac{3ZeR_0^2}{4\pi}\right)^2\cdot\beta_2^2\left\{1+0.36\cdot\beta_2+0.97\cdot\beta_4+0.33\cdot\frac{\beta_4^2}{\beta_2}\right\}^2$

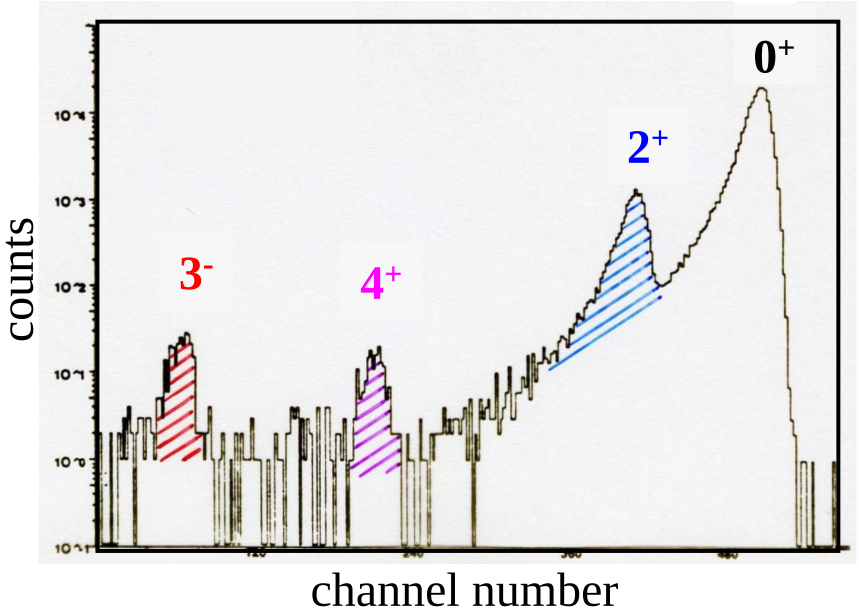
Excitation energy: $E_{2^+}=\frac{8\cdot\pi\cdot\hbar^2}{3\cdot A\cdot M\cdot R_0^2\cdot\left(\beta_2^2+5/3\cdot\beta_4^2\right)}$



first indication of a hexadecapole deformation



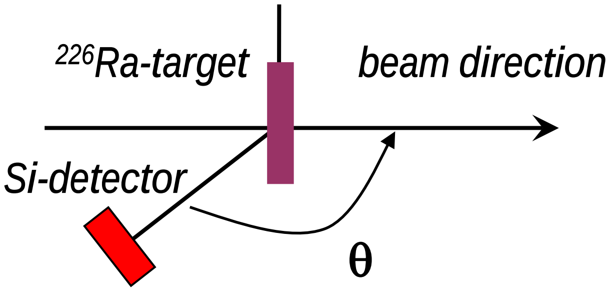
^{226}Ra: quadrupole, octupole and hexadecapole deformation



^{4}He → ^{226}Ra

$E_\alpha = 16$ MeV

$\theta_{lab} = 145^0$



$$P_{i\to f} = \frac{d\sigma_{i\to f}}{d\sigma_{el}} \cong \frac{d\sigma_{i\to f}}{d\sigma_{Ruth}}$$

λ	$\langle \lambda \| M(E\lambda) \| 0 \rangle \; [eb^{\lambda/2}]$	β_λ (exp)	β_λ (theo)
2	2.27 (3)	0.165 (2)	0.164
3	1.05 (5)	0.104 (5)	0.112
4	1.04 (7)	0.123 (8)	0.096